Meson masses and decay constants at large N

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Meson masses and decay constants in the large N limit of SU(N) gauge theory is estimated from the twisted space-time reduced model. To this end, we introduce a new smearing method which enables us to obtain reliable values for these quantities.

Twisted Eguchi-Kawai model (TEK-model) is the lattice theory having only one site with twisted boundary condition, which has been shown to be equivalent to the usual gauge theory in the large N limit.

Last year, two of us proposed a new method to calculate meson correlators in this model.

This was actually a quite challenging problem, because

Twist is the property of SU(N)/Z(N).   It is not clear how to introduce quarks in the fundamental representation in our framework.

Meson propagators are space-time extended object. It is not clear how to introduce space-time extended object in the one site model.

For simplicity, we used point-point meson correlators.   We could calculate meson masses, however, it is not possible to reliably estimate decay constant due to contaminations of heavier states.

In this talk, we apply a new smearing method, which makes it possible to reliably calculate meson masses and decay constant.

● Twisted Eguchi-Kawai model
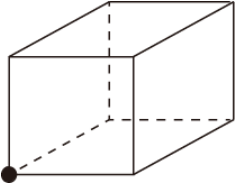
Eguchi-Kawai model can be viewed as the usual Wilson gauge theory having only one site with the periodic boundary conditions.

We introduce twisted boundary conditions in $SU(N), N = L^2$ theory

$$S_{TEK} = bN \sum_{\mu \neq \nu = 1}^{d} Tr\left(z_{\nu\mu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger \right)$$

$$z_{\mu\nu} = \exp\left(k \frac{2\pi i}{L} \right) \in Z(L), \quad z_{\nu\mu}^* = -z_{\mu\nu}, \qquad \mu < \nu$$

k , L : co-prime, k/L fixed as we go $N = L^2 \rightarrow \infty$

This model is related to ordinary SU(N) lattice theory on a lattice with space-time volume $V = L^4$ up to $O(1/N^2)$ corrections.

The number of degree of freedom of SU(N) matrix is $N^2 = L^4$

The vacuum configuration $U_\mu^{(0)} \equiv \Gamma_\mu$ of this model satisfy

$$z_{\nu\mu}\Gamma_\mu\Gamma_\nu\Gamma_\mu^\dagger\Gamma_\nu^\dagger = 1 \qquad \left[S_{TEK} = bN\sum \mathrm{Tr}\left(z_{\nu\mu}U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right) \right]$$

Γ_μ are four $L^2 \times L^2 (= N \times N)$ matrices with

$$\Gamma_\mu\Gamma_\nu = z_{\mu\nu}\Gamma_\nu\Gamma_\mu$$

Γ_μ play a central role in the construction of meson propagators.

We suppose that fundamental fermion lives in a lattice with volume $V = L^3 \times \ell_0 L, \quad \ell_0 :$ positive integer .

Then, point-point meson correlator in γ_A and γ_B channel is given by

$$C_{AB}(t) = \frac{1}{\ell_0 N^{3/2}} \sum_{\rho_0} \exp(i\rho_0 t) \, \mathrm{Tr}\left[\gamma_A D^{-1}(\rho_0) \gamma_B D^{-1}(0) \right]$$
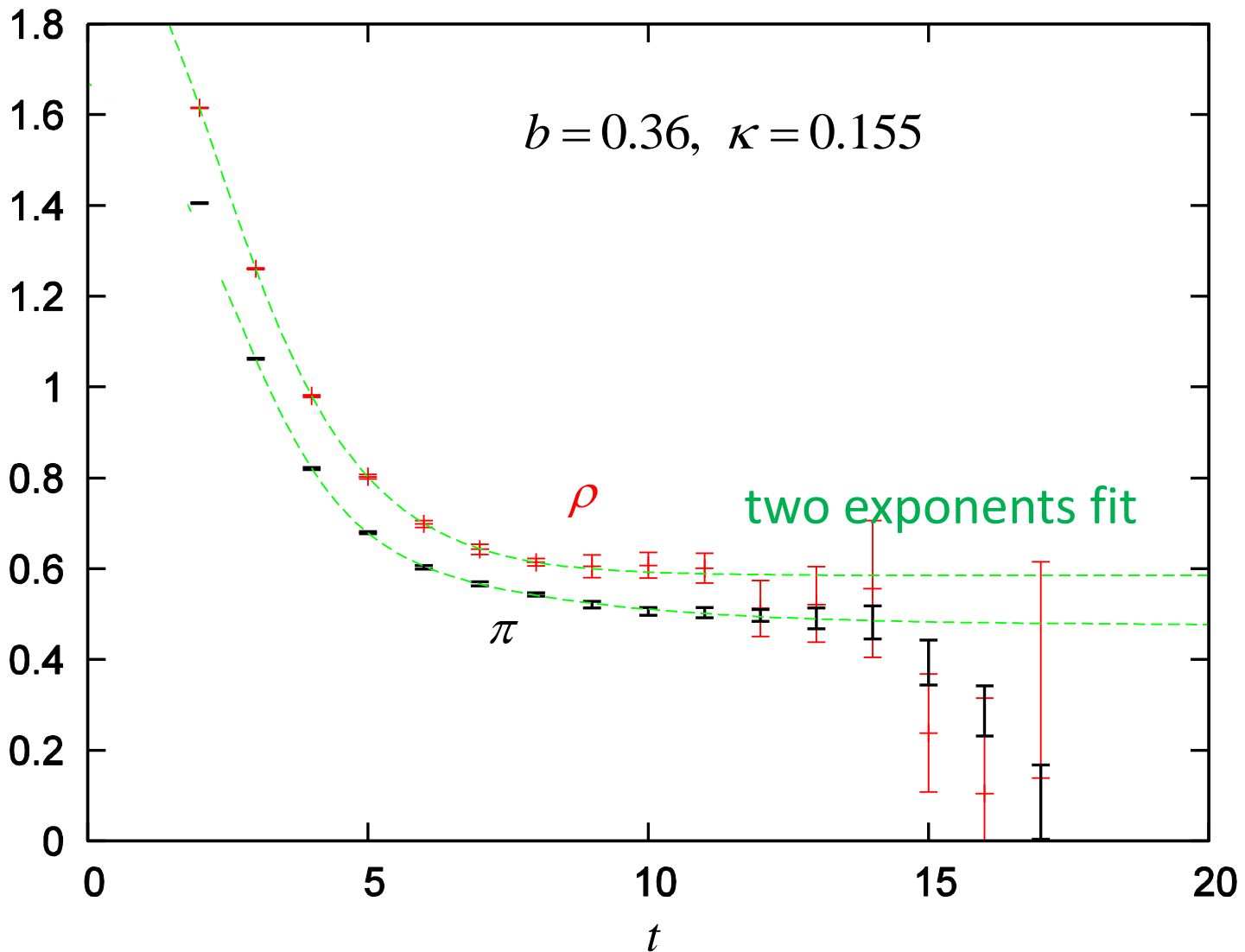
with

$$D(\rho_0) = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu)\tilde{U}_\mu \Gamma_\mu^* + (1+\gamma_\mu)\tilde{U}_\mu^\dagger \Gamma_\mu^t \right]$$

$$\tilde{U}_{\mu=0} = \exp(i\rho_0)U_{\mu=0}, \quad \tilde{U}_{\mu=1,2,3} = U_{\mu=1,2,3}$$

$$\rho_0 = \frac{2\pi m}{\ell_0 L}; \quad 0 \le m \le \ell_0 L - 1,$$

$D(\rho_0)$ are $4L^4 \times 4L^4$ matrix. We have to invert $\ell_0 L$ such matrices.

$$m_{eff} = \log\left[C(t-1)/C(t)\right]$$

$b = 0.36, \quad \kappa = 0.155$

ρ

π

two exponents fit

t

We propose a new smearing method by replacing γ_A and γ_B in

$$C_{AB}(t) = \frac{1}{\ell_0 N^{3/2}} \sum_{\rho_0} \exp(i\rho_0 t) \, \mathrm{Tr}\left[\gamma_A D^{-1}(\rho_0)\gamma_B D^{-1}(0)\right]$$

by the operator having the same quantum number

$$\gamma_A \to D_s^\ell \gamma_A, \quad D_s = \frac{1}{1+6c}\left[1 + c\sum_{i=1}^{3}\left(\bar{U}_i'\Gamma_i^* + \bar{U}_i'^\dagger\Gamma_i^t\right)\right]$$

ℓ is the smearing level and U_i' is the ape-smeared spatial link after making the following transformation several times

$$U_i' = \mathrm{Proj}_{\mathrm{SU(N)}}\left[(1-f)U_i + \frac{f}{4}\sum_{j\neq i} j\left(U_j^\dagger U_i U_j + U_j^\dagger U_i U_j\right)\right]$$

Finally, the smeared correlators are given by

$$C_{AB}^{\ell\ell'}(t) = \frac{1}{\ell_0 N^{3/2}} \sum_{\rho_0} \exp(i\rho_0 t) \, \mathrm{Tr}\left[D_s^\ell \gamma_A D^{-1}(\rho_0) D_s^{\ell'} \gamma_B D^{-1}(0)\right]$$

To obtain good signals for ground state, we first solve the generalized eigenvalue problem

$$C(t_1)\,\mathrm{v}^\alpha = \lambda^\alpha C(t_0)\,\mathrm{v}^\alpha$$

with $t_0 = 1,\ t_1 = 2.$ Them, from the eigenvector v^1 corresponding to the largest eigenvalue λ^1, we construct the smeared-smeared and smeared-local correlators as

$$C_{AB}^{SS}(t) = (\mathrm{v}^1)_\ell^* \, C_{AB}^{\ell\ell'}(t)(\mathrm{v}^1)_{\ell'}, \quad C_{AB}^{LS}(t) = C_{AB}^{0\ell'}(t)(\mathrm{v}^1)_{\ell'}$$

Actually, eigenvalue problem is solved by selecting 10 smearing levels

$$\ell = 0, 1, 2, 3, 4, 5, 10, 20, 50, 100.$$

To test our new formula, we made simulations at $N = 289 = L^2 = 17^2$.

We take two values of inverse 'tHooft coupling $(b = 1 / g_0^2 N)$
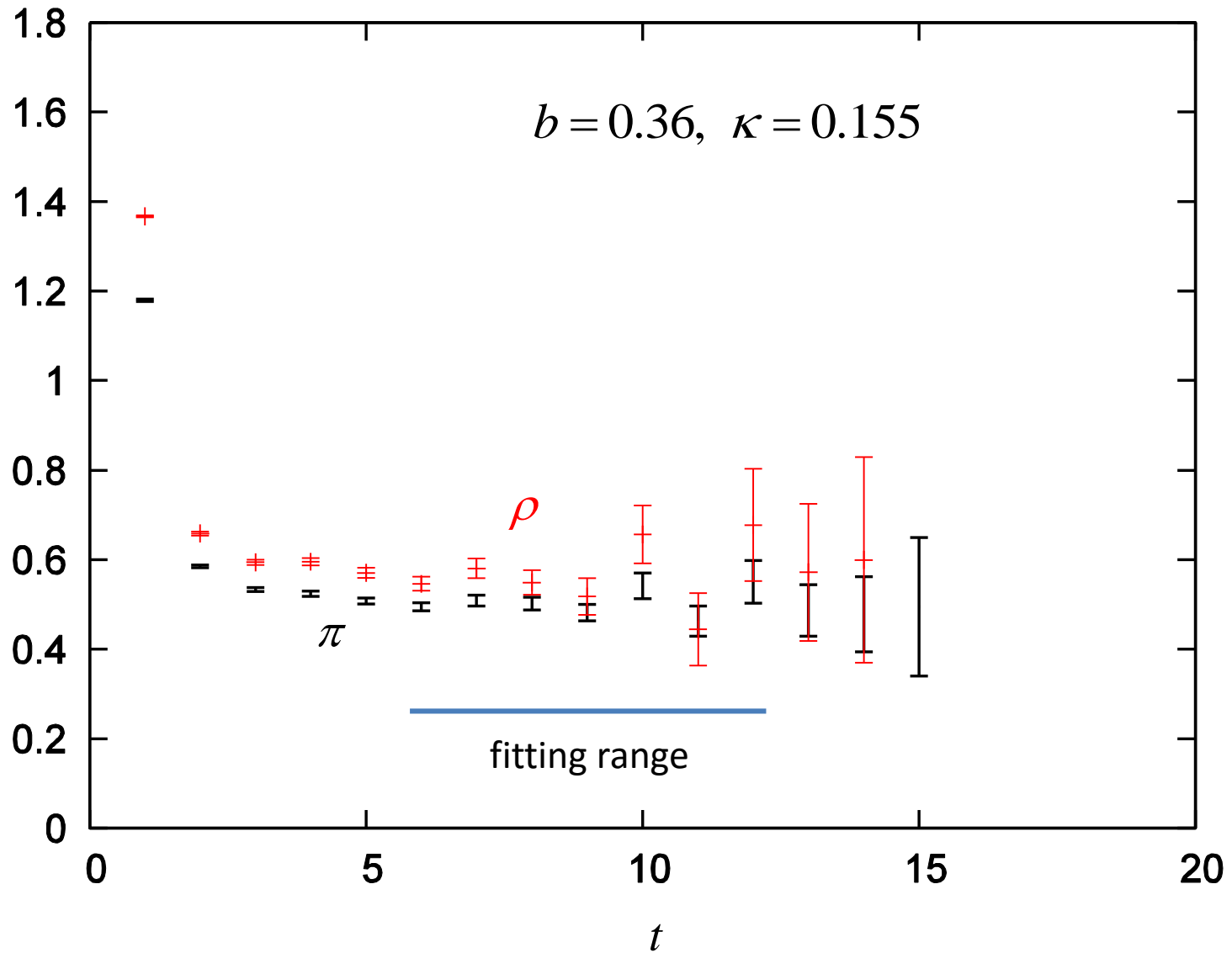and two or three values of κ , with $\ell_0 = 1$ and 2 .

$$b = 0.36 \; , \; \kappa = 0.155, 0.157, 0.1585$$
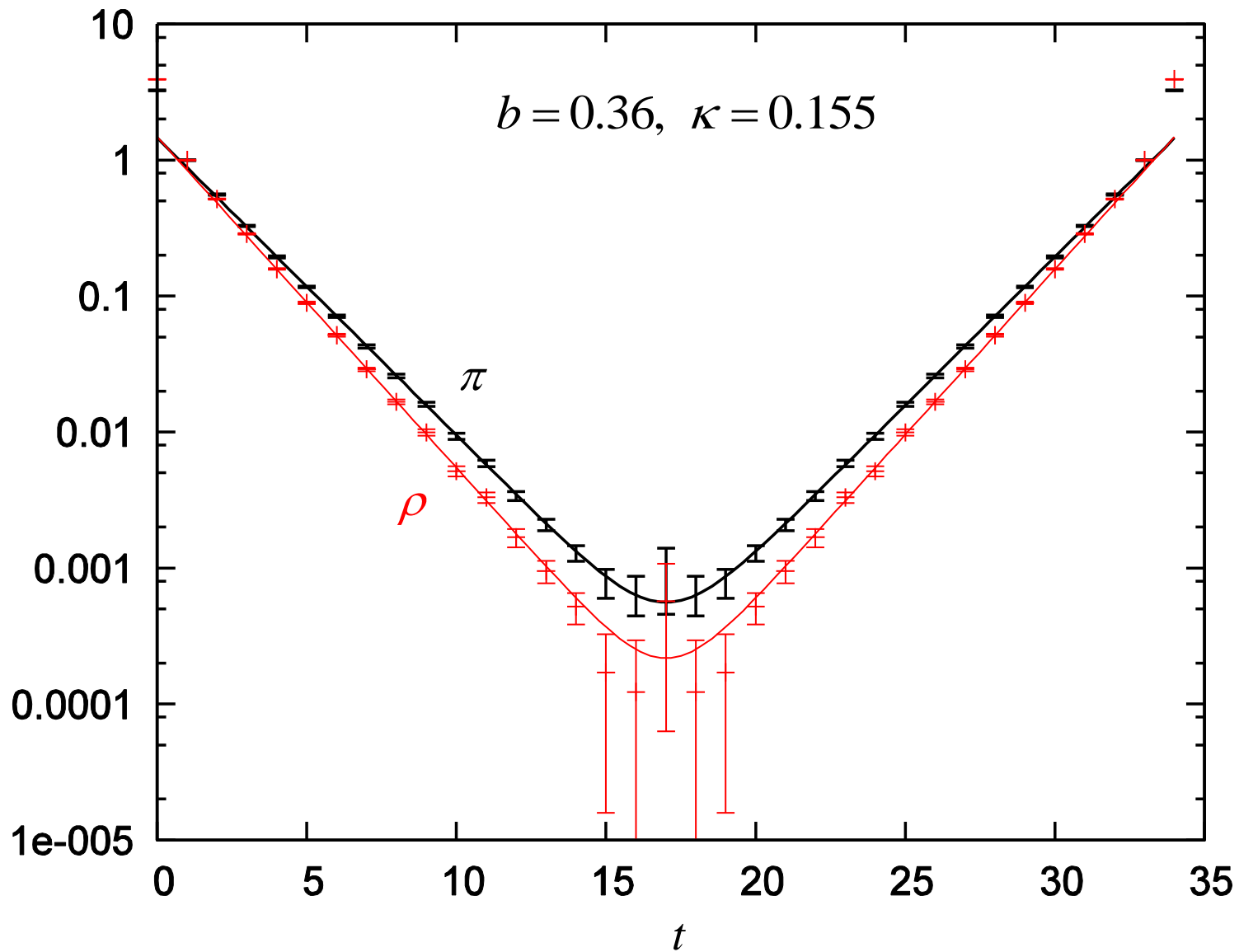
$$b = 0.365, \; \kappa = 0.1535, 0.1555$$

For each parameter set, we calculate meson propagators with 800
configurations. Each configuration is separated by 1000 MC sweeps.

Simulations have been made by Hitachi SR16000 at KEK
and NEC SX-ACE at RCNP, Osaka university.
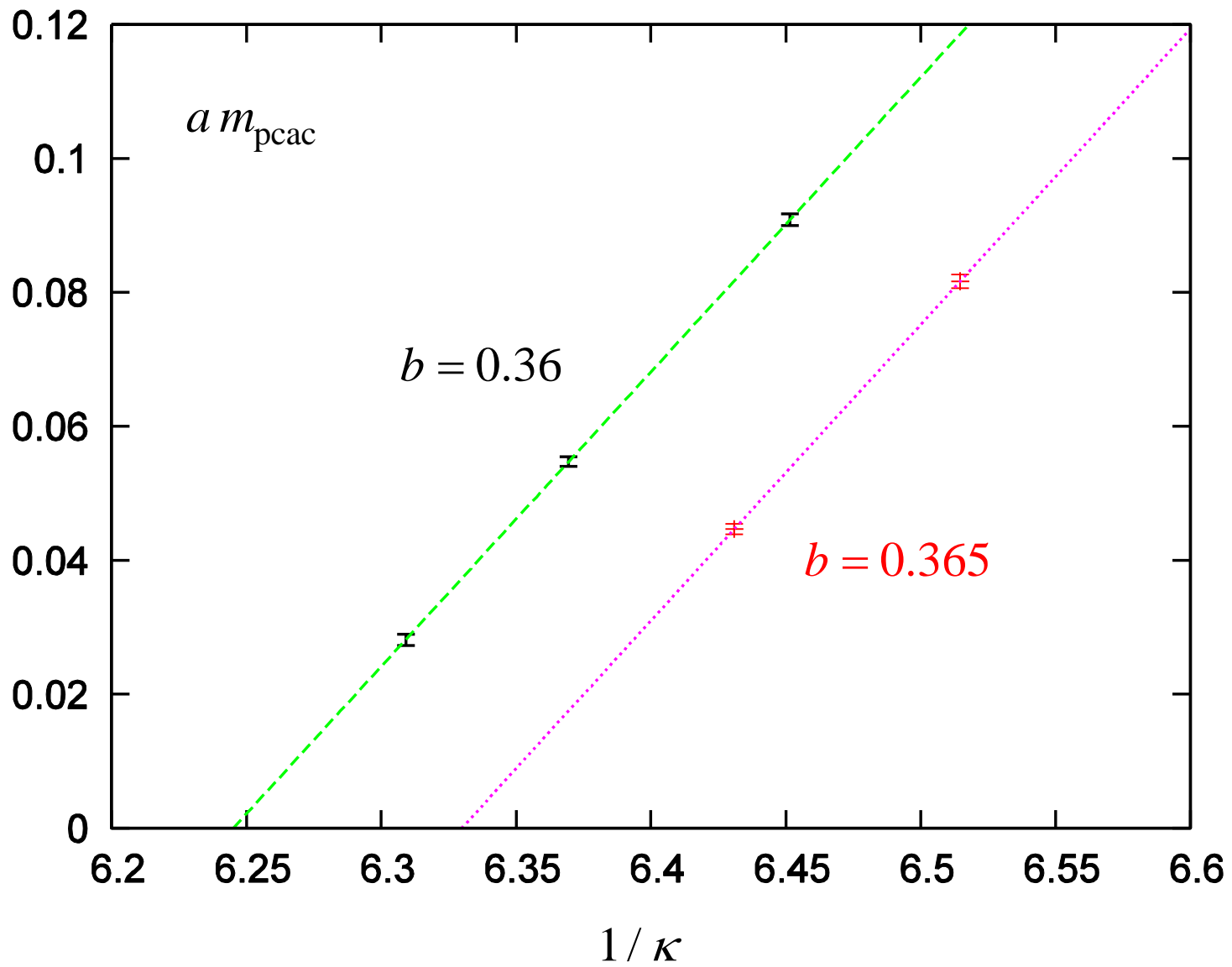
$$m_{eff} = \log\left[C^{SS}(t-1)/C^{SS}(t)\right]$$

$b = 0.36, \ \kappa = 0.155$

ρ

π

fitting range

t

$C^{SS}(t)$

$b = 0.36, \; \kappa = 0.155$

π

ρ

The first quantity we study is the pcac-quark mass.
We compute it from

$$a\,m_{\mathrm{pcac}}(t) = \frac{C^{LS}_{\gamma_0\gamma_5,\gamma_5}(t+a) - C^{LS}_{\gamma_0\gamma_5,\gamma_5}(t-a)}{4C^{LS}_{\gamma_5,\gamma_5}(t)}$$

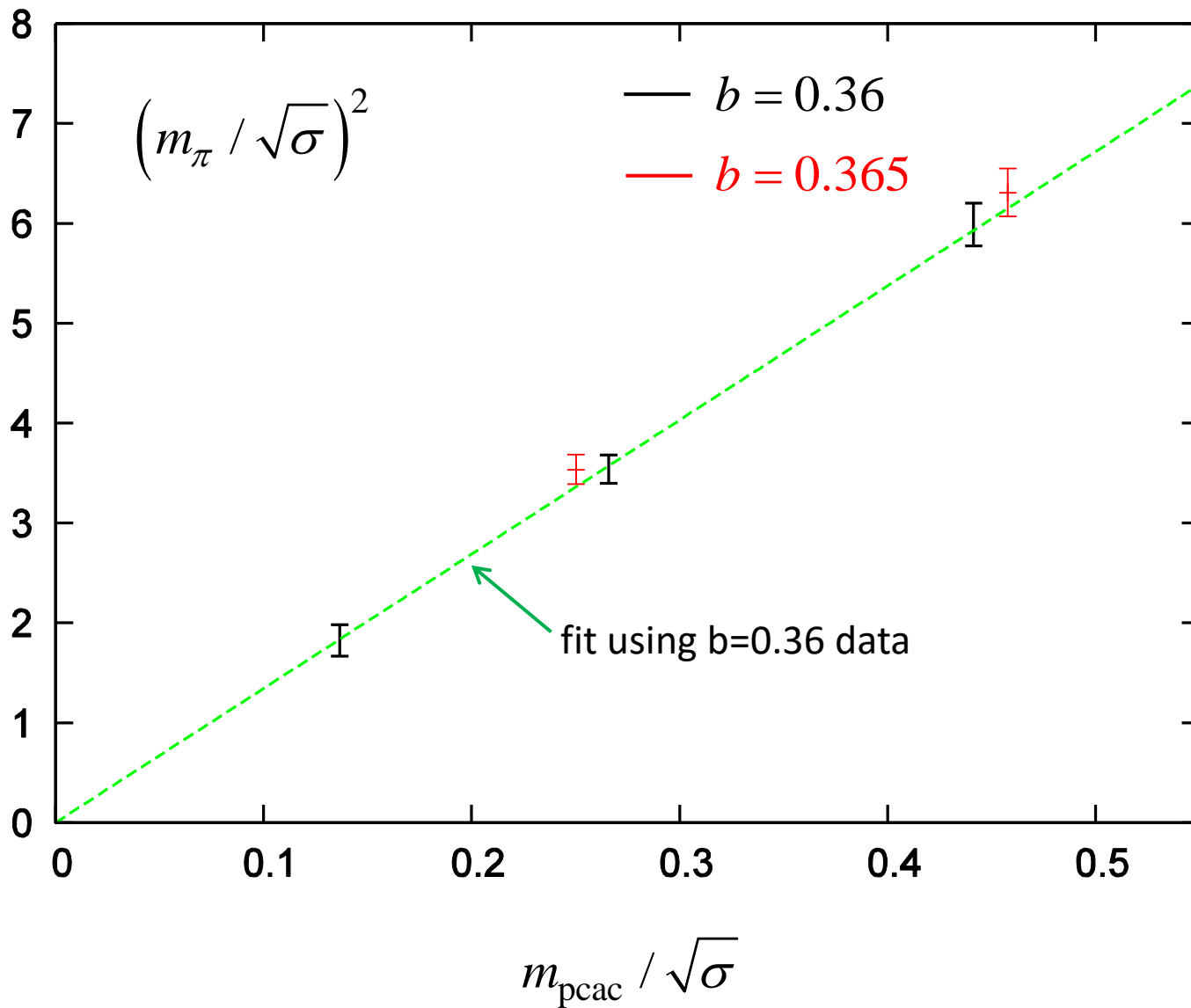by fitting this to a constant in the fitting range $6 \leq t \leq 12$.
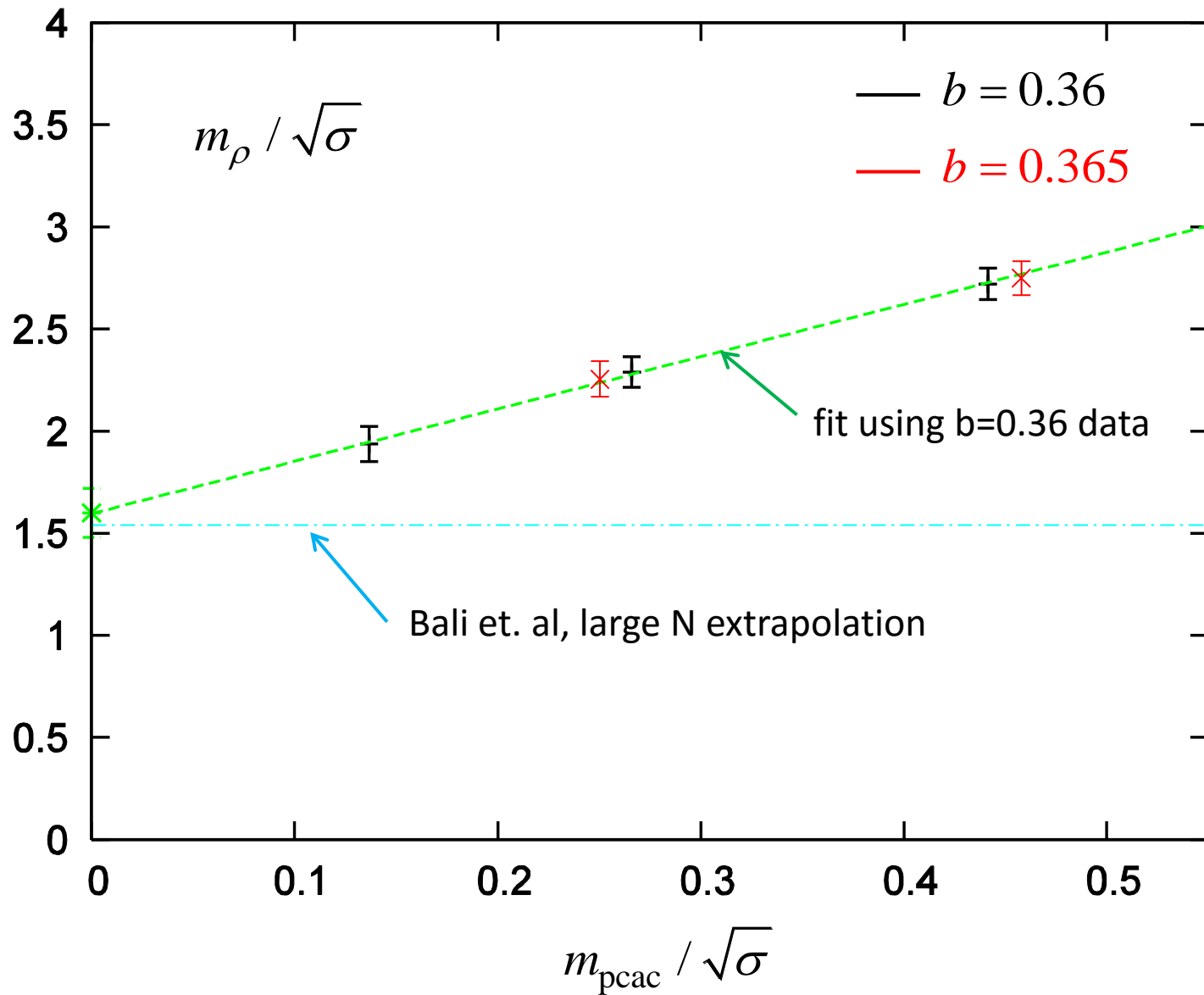
We, then, calculate $\left(m_\pi / \sqrt{\sigma}\right)^2$ and $m_\rho / \sqrt{\sigma}$ as functions of $m_{\text{pcac}} / \sqrt{\sigma}$ with

$$a^2 \sigma(b = 0.36) \ = 0.04234(103)$$
$$a^2 \sigma(b = 0.365) = 0.03181(60)$$

by fitting propagators to $\cosh$ in the fitting range $6 \le t \le 12$.

Finally, we calculate pion decay constant f_π^{lat}

$$C_{\gamma_0\gamma_5,\gamma_5}^{LS}(t) \simeq \frac{1}{2m_\pi}\langle 0|A_0|\pi\rangle\langle \pi|\pi^\dagger|0\rangle\exp(-tm_\pi) = C_{A_0}\exp(-tm_\pi)$$

$$C_{\gamma_5,\gamma_5}^{LS}(t) \simeq \frac{1}{2m_\pi}\langle 0|\pi|\pi\rangle\langle \pi|\pi^\dagger|0\rangle\exp(-tm_\pi) = C_\pi\exp(-tm_\pi)$$
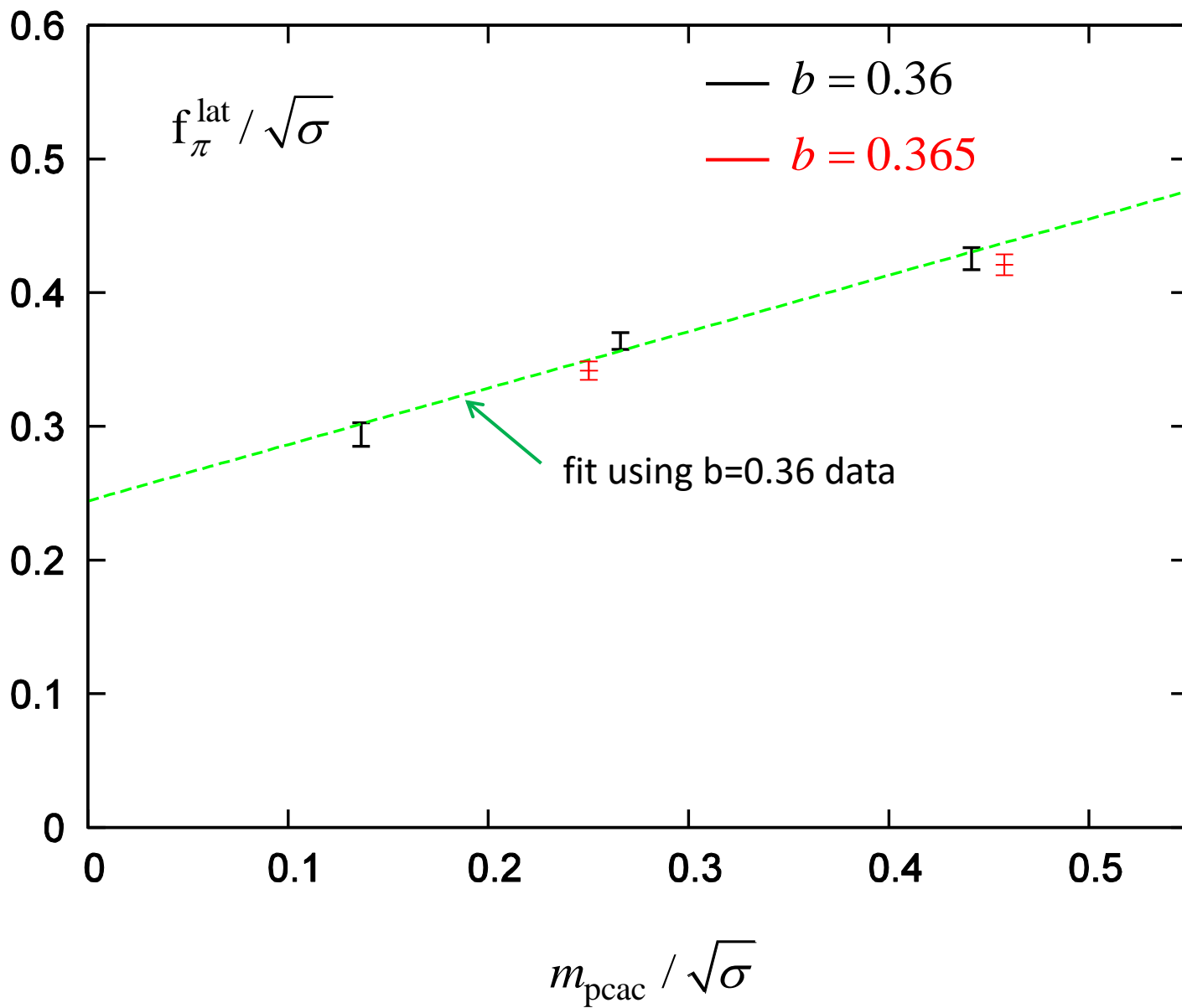
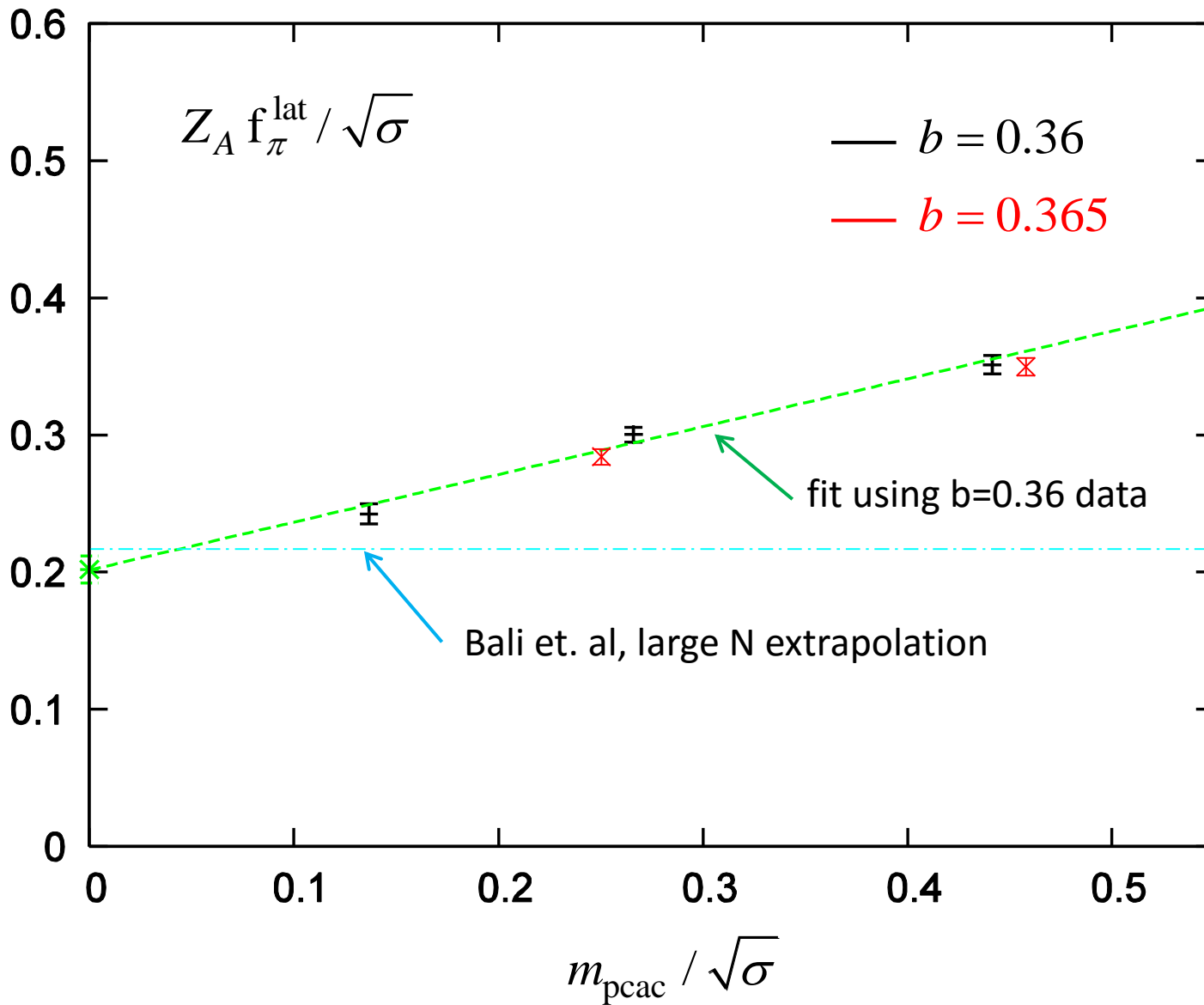$$f_\pi^{\text{lat}} = C_{A_0}\sqrt{\frac{1}{m_\pi C_\pi}}\sqrt{\frac{3}{N}}$$

with one-loop improved *Z*-factor

$$Z_A = 1 - 0.4694\frac{\lambda_E}{4\pi}, \quad \lambda_E = -8\ln U_P$$

$$Z_A(b = 0.36) = 0.8256$$
$$Z_A(b = 0.365) = 0.8314$$

Conclusion

- We have shown that our new smearing method works quite well and succeeded in evaluating

  pion and rho masses, and pion decay constant.

- We are ready to evaluate masses of other heavier channels such as scalar, tensor, axial-vector and also excited states with full use of the generalized eigenvalue problem.

- It is quite interesting to calculate meson spectrum in theories having adjoint fermions with various number of flavors.

  It is a straightforward task for us, since configurations with dynamical quarks are already generated.