Light and strange axial form factors of the nucleon at pion mass 317 MeV

Jeremy Green

Institut für Kernphysik, Johannes Gutenberg-Universität Mainz

in collaboration with Nesreen Hasan, Stefan Meinel, Michael Engelhardt, Stefan Krieg, Jesse Laeuchli, John Negele, Kostas Orginos, Andrew Pochinsky, Sergey Syritsyn

The 34th International Symposium on Lattice Field Theory
July 24–30, 2016

Outline

1. Introduction
2. Renormalization
3. Light and strange G_A
4. Strange quark spin
5. Light and strange G_P
6. Summary and outlook

Nucleon axial form factors

Describe the strength of the coupling of a proton to an axial current:

$$\langle p'|A_\mu^q|p\rangle = \bar{u}(p')\left[\gamma_\mu G_A^q(Q^2) + \frac{(p'-p)_\mu}{2m_N} G_P^q(Q^2)\right]\gamma_5 u(p),$$

where $A_\mu^q = \bar{q}\gamma_\mu\gamma_5 q$.

- Interaction with W boson contains (assuming isospin) isovector A_μ^{u-d}.
  Measured in quasielastic neutrino scattering, e.g. $\bar{\nu}_e p \to e^+ n$,
  and in muon capture, $\mu^- p \to \nu_\mu n$.
- Interaction with Z boson contains A_μ^{u-d-s}.
  Relevant for elastic νp and parity-violating ep scattering.

Quark spin in the proton

$g_A^q \equiv G_A^q(0)$ gives the contribution from the spin of q to the proton's spin. This equals the moment of a polarized parton distribution function:

$$g_A^q = \int_0^1 dx\,(\Delta q(x) + \Delta\bar{q}(x))\,.$$

For the typical phenomenological values:

- g_A^{u-d} is obtained from neutron beta decay.
- g_A^{u+d-2s} is obtained from semileptonic beta decay of octet baryons, assuming $SU(3)$ symmetry.
- A third linear combination is obtained from the integral of polarized PDFs measured in polarized deep inelastic scattering.

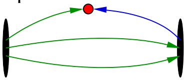
Connected and disconnected diagrams

We need to compute, using an interpolating operator χ,

$$C_{\text{2pt}}(t) = \langle \chi(t)\bar{\chi}(0)\rangle$$
$$C_{\text{3pt}}(\tau, T) = \langle \chi(T) A_\mu^q(\tau) \bar{\chi}(0)\rangle.$$

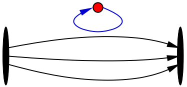
There are two kinds of quark contractions required for C_{3pt}:

- Connected, which we evaluate in the usual way with sequential propagators through the sink.



- Disconnected, which requires stochastic estimation to evaluate the *disconnected loop*,

$$T(\vec{q}, t, \Gamma) = -\sum_{\vec{x}} e^{i\vec{q}\cdot\vec{x}}\,\mathrm{Tr}[\Gamma D^{-1}(x,x)].$$



  We then need to compute the *correlation* between this loop and a two-point correlator.

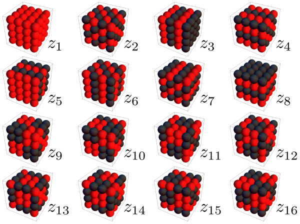
Stochastic estimation for disconnected loop

Estimate the all-to-all propagator stochastically by introducing noise sources η that satisfy $E(\eta\eta^\dagger) = I$. By solving $\psi = D^{-1}\eta$, we get

$$D^{-1}(x, y) = E(\psi(x)\eta^\dagger(y)).$$

We use hierarchical probing to reduce the noise: take the component-wise product $\eta^{[b]} \equiv z_b \odot \eta$ with a specially-constructed spatial Hadamard vector z_b and then replace

$$\eta\eta^\dagger \to \frac{1}{N_b}\sum_b \eta^{[b]}\eta^{[b]\dagger}.$$



red: +1, black: −1

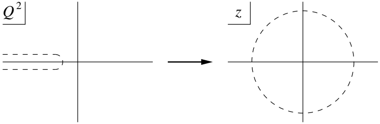
This allows for a progressively increasing amount of spatial dilution.

A. Stathopoulos, J. Laeuchli, K. Orginos, SIAM J. Sci. Comput. **35(5)** (2013) S299–S322 [1302.4018]

Fitting Q^2-dependence

We want to fit $G_{A,P}(Q^2)$ with curves to characterize the overall shape of the form factor and determine the axial radius.

- Common approach: use simple fit forms such as a dipole.
- Better: use z-expansion. Conformally map domain where $G(Q^2)$ is analytic in complex Q^2 to $|z| < 1$, then use a Taylor series:



R. J. Hill and G. Paz, Phys. Rev. D **84** (2011) 073006

$$z(Q^2) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}}}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}}}},$$

$$G(Q^2) = \sum_k a_k z(Q^2)^k,$$

with Gaussian priors imposed on the coefficients a_k.

- Leave a_0 and a_1 unconstrained, so that the intercept and slope are not directly constrained.
- For higher coefficients, impose $|a_{k>1}| < 5\max\{|a_0|, |a_1|\}$, and vary the bound to estimate systematic uncertainty.

For G_P, perform the fit to $(Q^2 + m^2)G_P(Q^2)$ to remove the pseudoscalar pole.

Lattice calculation

Previously used for disconnected $G_E(Q^2)$, $G_M(Q^2)$.

JG, S. Meinel, M. Engelhardt, S. Krieg, J. Laeuchli, J. Negele, K. Orginos, A. Pochinsky, S. Syritsyn, Phys. Rev. D **92**, (2015) 031501(R) [1505.01803]

- Ensemble generated by JLab / William & Mary.
- $N_f = 2 + 1$ Wilson-clover fermions
- $a = 0.114$ fm, $32^3 \times 96$
- $m_\pi = 317$ MeV, $m_\pi L = 5.9$
- $m_s \approx m_s^{\text{phys}}$
- 1028 gauge configurations
- Disconnected loops for six source timeslices (16 or 128 Hadamard vectors, plus color+spin dilution).
- Two-point correlators from 96 source positions.
- Connected three-point correlators from 12 source positions.

Control over excited states

$$\text{Recall} \quad C_{\text{3pt}}(\tau, T) = \langle \chi(T) A_\mu^q(\tau) \bar{\chi}(0) \rangle, \quad C_{\text{2pt}}(t) = \langle \chi(t) \bar{\chi}(0) \rangle$$

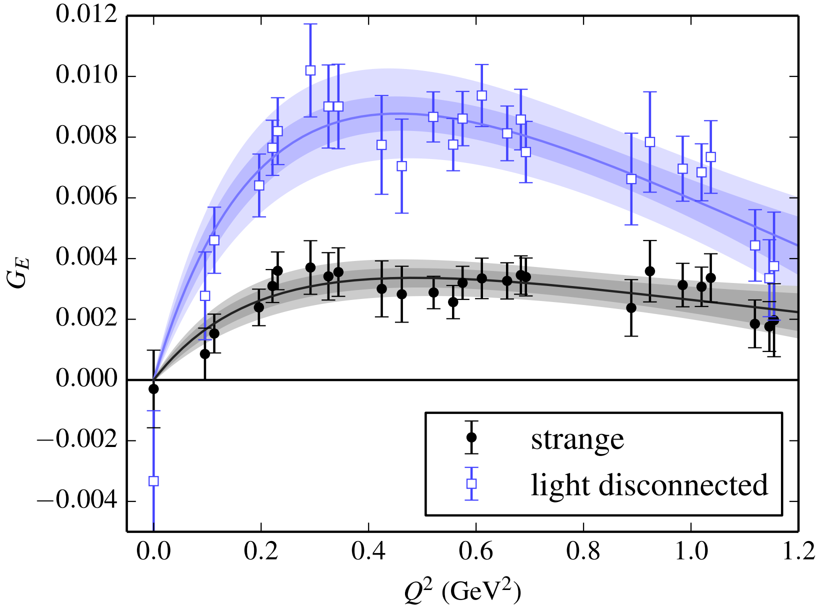
Connected diagrams are evaluated at fixed $T/a \in \{6, 8, 10, 12, 14\}$, which corresponds to T between 0.7 and 1.6 fm. These are obtained for all τ.

- For the ratio method, we compute $R(\tau, T) \sim C_{\text{3pt}}(\tau, T)/C_{\text{2pt}}(T)$.
  For each T average over the three points near $\tau = T/2$.
  Excited-state contamination will decay asymptotically as $e^{-\Delta E\, T/2}$.
- For the summation method, compute $S(T) = \sum_{\tau=a}^{T-a} R(\tau, T)$.
  Fit a line to $S(T)$ at three adjacent values of T and take its slope.
  Excited-state contamination will decay asymptotically as $Te^{-\Delta E\, T}$.

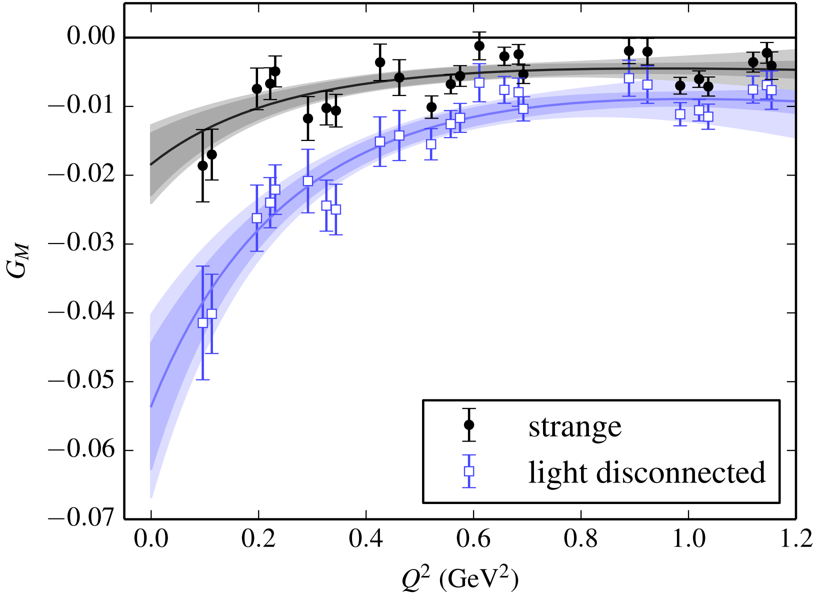
Disconnected diagrams are evaluated at fixed $\tau/a \in \{3, 4, 5, 6, 7\}$ (light) or $\{4, 5, 6\}$ (strange) and obtained for all T.

- Use the ratio method.
  For each T average over the two or three points near $\tau = T/2$.

Previous result: G_E



Previous result: G_M



Renormalization of the axial current: massless case

Flavour-singlet and nonsinglet axial currents renormalize differently.

- Nonsinglet has zero anomalous dimension and matching between "good" schemes that satisfy the axial Ward identity is trivial, e.g.:

$$\frac{Z_A^{\overline{\mathrm{MS}}}}{Z_A^{\mathrm{RI\text{-}SMOM}}} = 1 = \frac{Z_A^{\overline{\mathrm{MS}}}}{Z_A^{\mathrm{RI'\text{-}MOM}}},$$

  to all orders in perturbation theory.
- Singlet has an anomalous dimension starting at $O(\alpha^2)$. "Good" schemes should satisfy the anomalous Ward identity. We know that

$$\frac{Z_A^{\overline{\mathrm{MS}}}}{Z_A^{\mathrm{RI\text{-}SMOM}}} \overset{*}{=} 1 + O(\alpha^2) = \frac{Z_A^{\overline{\mathrm{MS}}}}{Z_A^{\mathrm{RI'\text{-}MOM}}}.$$

$*$ T. Bhattacharya, V. Cirigliano, R. Gupta, E. Mereghetti and B. Yoon, Phys. Rev. D **92**, 114026 (2015)

Renormalization of the axial current: $N_f = 2 + 1$

For a single ensemble with $m_u = m_d \neq m_s$, define $A_\mu^a = \bar{\psi}\gamma_\mu\gamma_5\lambda^a\psi$, where

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}, \quad \lambda^3 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \quad \lambda^0 = \frac{1}{\sqrt{3}} I.$$

This gives the renormalization pattern

$$\begin{pmatrix} A_\mu^{R,3} \\ A_\mu^{R,8} \\ A_\mu^{R,0} \end{pmatrix} = \begin{pmatrix} Z_A^{3,3} & 0 & 0 \\ 0 & Z_A^{8,8} & Z_A^{8,0} \\ 0 & Z_A^{0,8} & Z_A^{0,0} \end{pmatrix} \begin{pmatrix} A_\mu^3 \\ A_\mu^8 \\ A_\mu^0 \end{pmatrix}.$$

In the $SU(3)_f$ limit, $Z_A^{3,3} = Z_A^{8,8}$ and $Z_A^{8,0} = Z_A^{0,8} = 0$.

Rome-Southampton method

On Landau-gauge-fixed configurations, compute

$$S_i(p) = \sum_x e^{-ip\cdot x}\langle \psi_i(x)\bar\psi_i(0)\rangle, \quad G^{O}_{ij}(p',p) = \sum_{x,y} e^{-ip'\cdot x + ip\cdot y}\langle \psi_i(x) O(0) \bar\psi_j(y)\rangle$$

$$\text{and} \qquad \Lambda^{O}_{ij}(p',p) = S_i^{-1}(p') G^{O}_{ij}(p',p) S_j^{-1}(p)$$

These renormalize as

$$A_\mu^{R,a} = Z_A^{ab} A_\mu^b, \quad S_i^R(p) = Z_q^i S_i(p) \implies \Lambda^{A_\mu^a}_{R,ij}(p',p) = \frac{Z_A^{ab}}{\sqrt{Z_q^i Z_q^j}} \Lambda^{A_\mu^b}_{ij}(p',p).$$

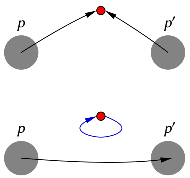
RI′-MOM or RI-SMOM schemes define a projector P_ν for specific kinematics K at scale μ. The condition for $Z_A^{ab}(\mu)$ becomes

$$\sum_\nu \mathrm{Tr}_{\text{col,spin,flav}} \left[\lambda^a \Lambda_R^{A_\nu^b} P_\nu \right]_K = \delta^{ab}.$$

Rome-Southampton method, cont.

For the axial current:

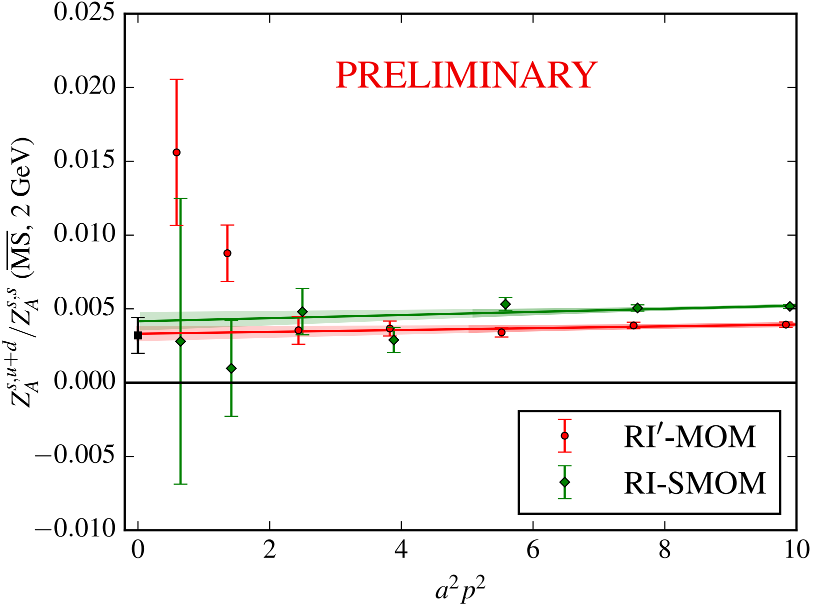
- Evaluate $S_i(p)$ and the connected contributions to $G_{ij}^{A_\mu^a}(p',p)$ using 4d plane-wave sources.
- Correlate the plane-wave-source propagators with previously-computed disconnected loops to get the disconnected contributions to $G_{ij}^{A_\mu^a}(p',p)$. Results obtained from about 200 configurations.



Once we've obtained $A_\nu^{R,a}$ in some scheme at scale p:

1. Perform the (trivial) one-loop matching to $\overline{\mathrm{MS}}$ at scale p.
2. Apply two-loop running of A_μ^0 to the target scale $\mu = 2$ GeV.
3. Rotate from the $\{3,8,0\}$ basis to $\{u-d, u+d, s\}$.
4. Extrapolate the matching point p to zero to remove $O(a^2p^2)$ artifacts.

Mixing of light with strange



Renormalization matrix

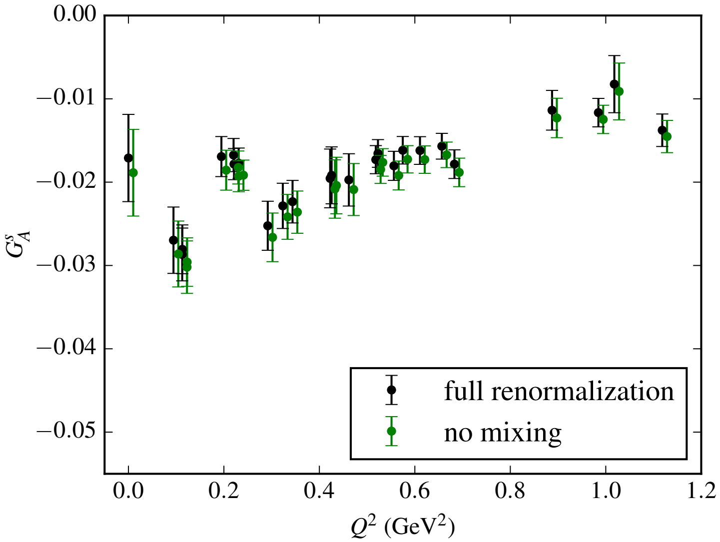
In $\overline{\text{MS}}$ at 2 GeV:

$$\begin{pmatrix} A_\mu^{R,u-d} \\ A_\mu^{R,u+d} \\ A_\mu^{R,s} \end{pmatrix} = \begin{pmatrix} Z_A^{3,3} & 0 & 0 \\ 0 & Z_A^{u+d,u+d} & Z_A^{u+d,s} \\ 0 & Z_A^{s,u+d} & Z_A^{s,s} \end{pmatrix} \begin{pmatrix} A_\mu^{u-d} \\ A_\mu^{u+d} \\ A_\mu^{s} \end{pmatrix}$$
$$= \begin{pmatrix} 0.8623(1)(71) & 0 & 0 \\ 0 & 0.8662(26)(45) & 0.0067(8)(5) \\ 0 & 0.0029(10)(5) & 0.9126(11)(98) \end{pmatrix} \begin{pmatrix} A_\mu^{u-d} \\ A_\mu^{u+d} \\ A_\mu^{s} \end{pmatrix}.$$

Systematic error estimated from different fits and from different intermediate lattice schemes.

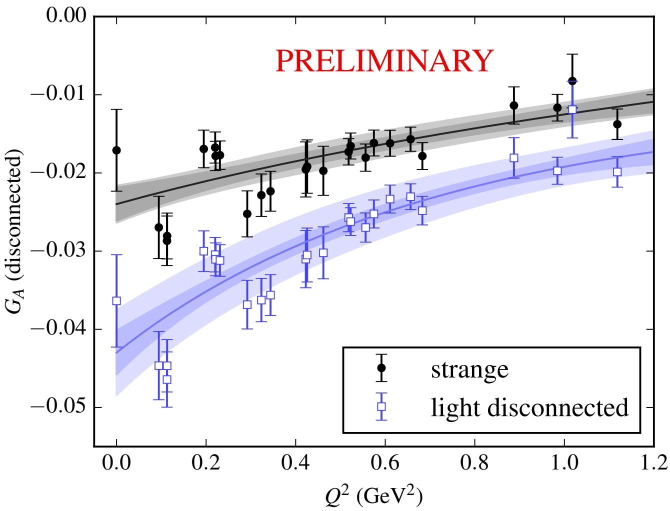
To study the disconnected contribution to the light-quark currents, consider a third partially-quenched light quark, r, with $m_r = m_u = m_d$. Then $A_\mu^{u+d,\text{conn}} = A_\mu^{u+d-2r}$, which renormalizes diagonally with $Z_A^{3,3}$. Writing $A_\mu^{R,u+d,\text{disc}} = A_\mu^{R,u+d} - A_\mu^{R,u+d,\text{conn}}$, shows that the mixing of connected into disconnected is controlled by $Z_A^{u+d,u+d} - Z_A^{3,3} = 0.0061(18)(10)$.

Effect of mixing: G_A^s



Largest effect: mixing of (large) G_A^{u+d} into (small) G_A^s.

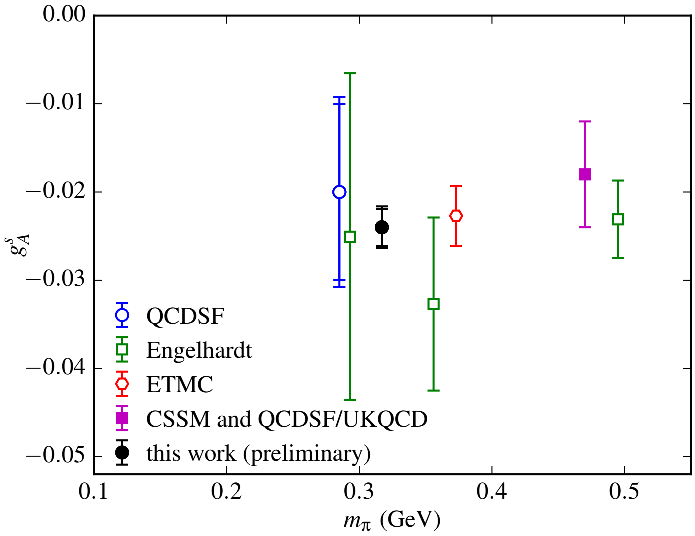
G_A, disconnected



Fit (using z-expansion) produces more precise result at $Q^2 = 0$.

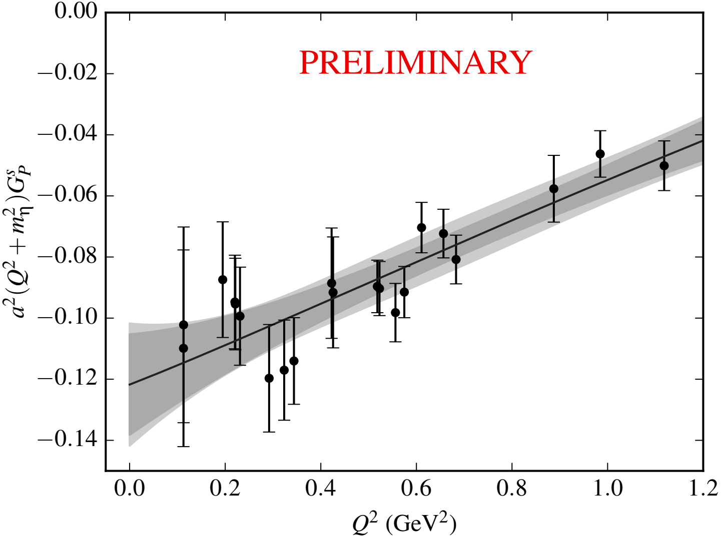
Systematic uncertainty for $G_A^{u,\mathrm{disc}} = G_A^{d,\mathrm{disc}}$ dominated by excited states.

Strange quark spin



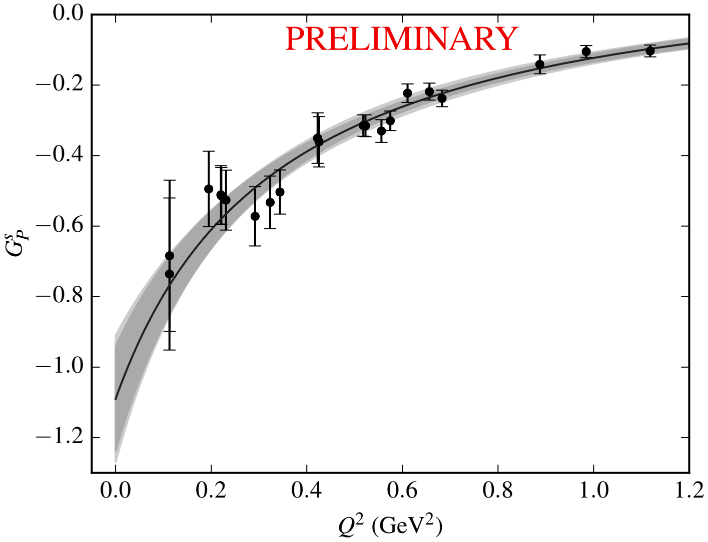
Comparison with published results.

Fitting to $G_P^s(Q^2)$



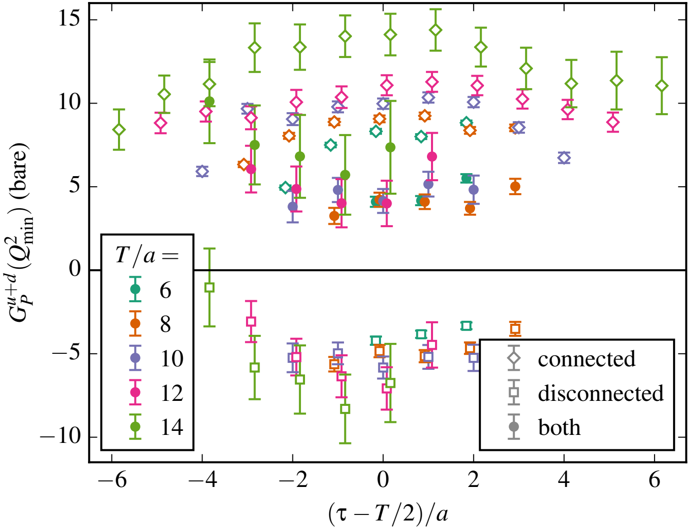
First remove the pole at $Q^2 = -m_\eta^2$, then fit using the z-expansion …

Fitting to $G_P^s(Q^2)$



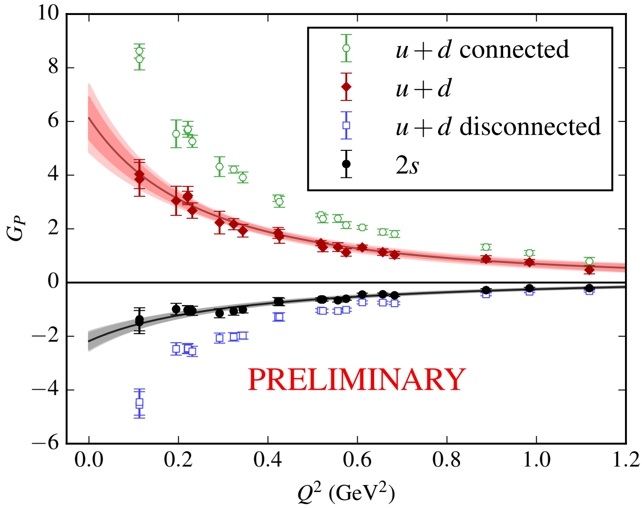
…finally, restore the pole.

$G_P^{u+d}(Q^2_{\mathrm{min}})$: excited states



Cancellation between connected and disconnected.

G_P, isoscalar



Connected $u + d$ seems to have a pion pole, cancelled by disconnected part.

Disconnected contributions are too large to neglect.

Summary

- Disconnected contributions to Rome-Southampton style renormalization can be evaluated with reasonable errors using volume sources.
- Effect of mixing between light and strange axial currents is small but may be important for precise results.
- Good signal obtained for disconnected G_A and G_P at pion mass 317 MeV.
- Preliminary value: $g_A^s = -0.0240(21)(11)$.
- Significant cancellation occurs between connected and disconnected G_P, particularly at low Q^2. This may be caused by cancellation of a pion-pole contribution.
- Calculations at lower pion masses are essential for connecting with experiment.