Sea quark QED effects and twisted mass fermions

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Outline of the talk

1. Motivation & Introduction
   - About twisted mass sea fermions coupled to photons
2. EM & strong $U_A(1)$ anomalies
   - QCD+QED with no θ term at maximal twist
3. Maximal twist by symmetry recovery
   - How to fix the critical mass beyond electro-quenching
4. Strategy for leading isospin breaking (LIB) effects
   - Mixed action and RM123 insertion method

Motivation & Introduction

Q(C+E)D with maximal twisted mass (MTM) fermions: $\psi^t = (u, d)$

$$S_F^{\mathrm{Q(C+E)D}}(\psi, \bar{\psi}, U, E) = a^4 \sum_x \bar{\psi}(x)\left[\gamma \cdot \widetilde{\nabla} - i\gamma_5 \tau_3 W_{\mathrm{cr}} + \mu + \tau_3 \epsilon\right]\psi(x)$$

- $\gamma \cdot \widetilde{\nabla} \equiv \frac{1}{2} \sum_\mu \gamma_\mu (\nabla^\star_\mu + \nabla_\mu)\,, \quad W_{\mathrm{cr}} \equiv -\frac{a}{2} \sum_\mu \nabla^\star_\mu \nabla_\mu + M_{\mathrm{cr}}$
- $\nabla_\mu \psi(x) \equiv \frac{1}{a}\left[\mathcal{U}_\mu(x)\psi(x + a\hat{\mu}) - \psi(x)\right]$

  $\nabla^*_\mu \psi(x) \equiv \frac{1}{a}\left[\psi(x) - \mathcal{U}^\dagger_\mu(x - a\hat{\mu})\psi(x - a\hat{\mu})\right]$

$$\mathcal{U}_\mu(x) = E_\mu(x) U_\mu(x)\,, \quad E_\mu(x) = \frac{1\!\!1 + \tau_3}{2} e^{-ieq_{up}\mathcal{A}_\mu(x)} + \frac{1\!\!1 - \tau_3}{2} e^{-ieq_{dn}\mathcal{A}_\mu(x)}$$

$q_{up} \neq q_{dn} \;\Rightarrow$ flavour structure $(1\!\!1 \pm \tau_3)/2$ in both $\bar{\psi}\gamma \cdot \widetilde{\nabla}\psi$ and $\bar{\psi} W_{\mathrm{cr}} \psi$

Note 1- Mass splitting with τ_3 and not τ_1 forced by gauge invariance

Note 2 - Q of an up-down doublet has the form $e(1\!\!1/6 + \tau_3/2)$

EM and strong $U_A(1)$ anomalies

- We want to analyze how EM and strong $U_A(1)$ anomalies affect
  - the form of the (continuum fermionic) effective action, $\Gamma_{\rm F}^{\rm Q(C+E)D}$
  - its dependence on the vacuum θ-angle
- We start from the general action (above we had $\theta_u = -\theta_d = \frac{\pi}{2}$)

$$S^{\rm Q(C+E)D}(\psi, \bar{\psi}, U, E) = a^4 \sum_x \Big[FF|_E(x) + {\rm tr}(GG)|_U(x) \Big] +$$
$$+ a^4 \sum_x \bar{\psi}_u(x) \Big[\gamma \cdot \widetilde{\nabla}^u + e^{-i\theta_u\gamma_5} W_{\rm cr}^u + M_u \Big] \psi_u(x) +$$
$$+ a^4 \sum_x \bar{\psi}_d(x) \Big[\gamma \cdot \widetilde{\nabla}^d + e^{-i\theta_d\gamma_5} W_{\rm cr}^d + M_d \Big] \psi_d(x)$$

- Phases can be moved to masses through singlet axial lattice rotations

$$\psi_u = e^{i\gamma_5\theta_u/2}\chi_u \qquad \bar{\psi}_u = \bar{\chi}_u e^{i\gamma_5\theta_u/2}$$
$$\psi_d = e^{i\gamma_5\theta_d/2}\chi_d \qquad \bar{\psi}_d = \bar{\chi}_d e^{i\gamma_5\theta_d/2}$$

- Lattice rotations are not anomalous
- $S_{\rm F}^{\rm Q(C+E)D}$ is periodic in θ_u and θ_d

Action and effective action

The action becomes (recall $M_{u,d} > 0$, Wilson terms $W_{cr}^{u,d}$ are critical)

$$S^{\mathrm{Q(C+E)D}}(\chi,\bar\chi,U,E) = a^4 \sum_x \Big[FF|_E(x) + \mathrm{tr}(GG)|_U(x) \Big] +$$

$$+a^4 \sum_x \bar\chi_u(x) \Big[\gamma\cdot\widetilde\nabla^u + W^u_{\mathrm{cr}} + e^{i\theta_u\gamma_5} M_u\Big]\chi_u(x) +$$

$$+\,a^4 \sum_x \bar\chi_d(x) \Big[\gamma\cdot\widetilde\nabla^d + W^d_{\mathrm{cr}} + e^{i\theta_d\gamma_5} M_d\Big]\chi_d(x)$$

Note: $\quad M_{u,d}\cos\theta_{u,d} \equiv m_{u,d} \overset{\star}{\sim} \dfrac{m^{\mathrm{ren}}_{u,d}}{Z_{m_{u,d}}}, \quad M_{u,d}\sin\theta_{u,d} \equiv \mu_{u,d} = \dfrac{\mu^{\mathrm{ren}}_{u,d}}{Z_{\mu_{u,d}}}$

$\star$ Actually $m^{\mathrm{ren}}_{u,d} = Z_{m_{u,d}}[m_{u,d}(1+\rho_m^{u,d}) + m_{d,u}\tilde\rho_m^{d,u}]$ – see e.g. Horkel & Sharpe, Phys.Rev. D92 (2015) 7, 074501

Symmetries (see below) $\Rightarrow$ *continuum* local effective action reads

[#] $$S^{\mathrm{Q(C+E)D}}_{cont} = \int d^4x \left\{ FF|_{\mathcal{A}}(x) + \mathrm{tr}(GG)|_{\mathcal{G}}(x) \right\} +$$

$$+\int d^4x \left\{\bar\chi_u(x)\Big[\gamma\cdot D^u + e^{i\hat\theta_u\gamma_5}\hat M_u\Big]\chi_u(x) + \bar\chi_d(x)\Big[\gamma\cdot D^d + e^{i\hat\theta_d\gamma_5}\hat M_d\Big]\chi_d(x)\right\}$$

$$\hat M_{u,d} = Z^{cont}_{S\;u,d}\sqrt{(m^{\mathrm{ren}}_{u,d})^2 + (\mu^{\mathrm{ren}}_{u,d})^2}\,, \quad \hat\theta_{u,d} = \arctan\left(\frac{Z_{\mu_{u,d}}}{Z_{m_{u,d}}}\tan\theta_{u,d}\right)$$

$$\text{Recall} \quad S_{\rm F,\,cont}^{\rm Q(C+E)D} = \int d^4x\, \bar\chi(x)\Big[\gamma\cdot D + e^{i\alpha\gamma_5}\mu\Big]\chi(x) =$$

$$= \int d^4x\, \bar\chi(x) e^{i\gamma_5\frac{\alpha}{2}}\Big[\gamma\cdot D + \mu\Big] e^{i\gamma_5\frac{\alpha}{2}}\chi(x) \quad \Longrightarrow$$

$$\Longrightarrow \quad S_{\rm F,\,cont}^{\rm Q(C+E)D} = \int d^4x\, \bar\psi(x)\Big[\gamma\cdot D + \mu\Big]\psi(x) +$$

$$+ i\frac{\alpha}{32\pi^2}\int d^4x \Big[e^2\tilde F F|_{\mathcal A}(x) + g^2 {\rm tr}[\tilde G G|_{\mathcal G}(x)]\Big]$$

- Continuum theory property above implies in our case

$$S_{cont}^{\rm Q(C+E)D} = \int d^4x \left\{ FF|_{\mathcal A}(x) + {\rm tr}(GG)|_{\mathcal G}(x) \right\} +$$

$$= \int d^4x\, \bar\psi_u(x)\Big[\gamma\cdot D^u + \hat M_u\Big]\psi_u(x) + \int d^4x\, \bar\psi_d(x)\Big[\gamma\cdot D^d + \hat M_d\Big]\psi_d(x) +$$

$$+ i\frac{e^2}{32\pi^2}(\hat\theta_u + \hat\theta_d)\int d^4x \tilde F F(x) + i\frac{g^2}{32\pi^2}(\hat\theta_u + \hat\theta_d)\int d^4x {\rm tr}[\tilde G G(x)]$$

- Note: $\theta_u = -\theta_d = \frac{\pi}{2}$ (MTM) $\leftrightarrow$ $\hat\theta_u = -\hat\theta_d = \frac{\pi}{2}$ (no P–breaking)

A few observations

1. $\hat{M}_u = \hat{M}_d = 0 \rightarrow$ only the trivial topological sector contributes
   $\hat{\theta}_u + \hat{\theta}_d = 0 \rightarrow$ all sectors contribute with equal weight
2. No CP-breaking term if $\hat{\theta}_u = -\hat{\theta}_d$. Indeed, one could set to zero both mass phases with a non-anomalous axial-τ_3 rotation
3. MTM case ($\theta_u = -\theta_d = \pi/2$ or $\mu_u > 0$, $\mu_d < 0$, $m_{u,d} \rightarrow 0^+$) amounts to $\hat{\theta}_u = -\hat{\theta}_d = \pi/2$ (in all renormalization schemes)
   $\Rightarrow$ MTM quarks well suited for QCD+QED via RM123 approach

Agreement with findings by Horkel & Sharpe [Phys.Rev. D92 (2015) 7, 074501 and 9, 094514]

Incidentally: within lattice QCD the action

$$S^{\rm QCD}(\psi, \bar{\psi}, U) \;=\; a^4 \sum_x \mathrm{tr}(GG)|_U(x) \;+$$

$$+a^4 \sum_x \bar{\psi}(x)\Big[\gamma \cdot \widetilde{\nabla} + e^{-i\theta\gamma_5} W_{\rm cr} + M\Big]\psi(x)$$

was shown to lead to S^{QCD}_{cont} with θ-vacuum (Seiler&Stamatescu '81)

Form [#] of $\Gamma^{\mathrm{Q(C+E)D}}$: sketch of the proof

Setting $\hat{M}_f \cos\hat{\theta}_f \equiv \hat{m}_f\,,\ \hat{M}_f \sin\hat{\theta}_f \equiv \hat{\mu}_f$ the thesis reads

[#]
$$S^{\mathrm{Q(C+E)D}}_{cont} = \int d^4x \left\{ FF|_{\mathcal{A}}(x) + \mathrm{tr}(GG)|_{\mathcal{G}}(x) \right\} +$$
$$+ \int d^4x \left\{ \sum_{f=u,d} \bar{\chi}_f(x)\left[\gamma\cdot D^f + \hat{m}_f + i\gamma_5\hat{\mu}_f\right]\chi_f(x)\right\}$$

with $\hat{m}_f \overset{\star}{\sim} Z^{cont}_{S\,f} Z_{m_f} m_f\,,\ \hat{\mu}_f = Z^{cont}_{S\,f} Z_{\mu_f}\mu_f$. Other $d=4$ terms:

- $\bar{\chi}_f\gamma_5\gamma\cdot D^f\chi_f$ is ruled out by charge conjugation
- $\bar{\chi}_f\gamma_5\chi_f\,,\ \mathrm{tr}(\widetilde{G}G)\,,\ \widetilde{F}F$ are ruled out by $\widetilde{P}\times(\mu_f\to-\mu_f)$
- $\sum_{\mu\nu}\bar{\chi}_f\gamma_\nu\gamma_\mu D^f_\mu\chi_f$ are excluded by $\widetilde{P}\times(\mu_f\to-\mu_f)$ and $H(4)$

while $a^{-1}\bar{\chi}_f\chi_f$ is allowed & canceled by $M^f_{cr}\bar{\chi}_f\chi_f$. Here we defined

$$\widetilde{P} \ :\ \chi_f(x)\to\gamma_0\chi_f(x_P)\,,\ \bar{\chi}_f(x)\to\bar{\chi}_f(x_P)\gamma_0\,,\ x_P=(x_0,-\vec{x})$$
$$U_0(x)\to U_0(x_P)\,,\ U_k(x)\to U^\dagger_k(x-a\hat{k})$$
$$E_0(x)\to E_0(x_P)\,,\ E_k(x)\to E^\dagger_k(x-a\hat{k})$$

Critical mass $M_{cr}^{u,d}$ in Q(C+E)D with MTM quarks

Recall QCD+QED action for two distinct flavours: $\psi^t = (u, d)$

$$S_{\rm F}^{\rm Q(C+E)D}(\psi, \bar\psi, U, E) = a^4 \sum_x \bar\psi(x)\left[\gamma\cdot\widetilde\nabla - i\gamma_5\tau_3 W_{\rm cr} + \mu + \tau_3\epsilon\right]\psi(x)$$

$$\gamma\cdot\widetilde\nabla \equiv \frac{1}{2}\sum_\mu \gamma_\mu(\nabla_\mu^\star + \nabla_\mu)\,, \quad W_{\rm cr} \equiv -\frac{a}{2}\sum_\mu \nabla_\mu^\star\nabla_\mu + M_{\rm cr}$$

Complex quark determinant $\Rightarrow$ LIB effects via RM123 method

Here we discuss convenient conditions to fully fix M_{cr}^u and M_{cr}^d

$$M_{cr} = M_{cr}^u \frac{1\!\!1 + \tau_3}{2} + M_{cr}^d \frac{1\!\!1 - \tau_3}{2} \equiv m_{cr} 1\!\!1 + \tilde m_{cr}\tau_3$$

$$m_{cr} = m_{cr}^{LQCD} + \alpha_{em}\frac{\delta_{em}(g^2)}{a} + O(\alpha_{em}^2)$$

$$\tilde m_{cr} = \alpha_{em}\frac{\tilde\delta_{em}(g^2)}{a} + O(\alpha_{em}^2)$$

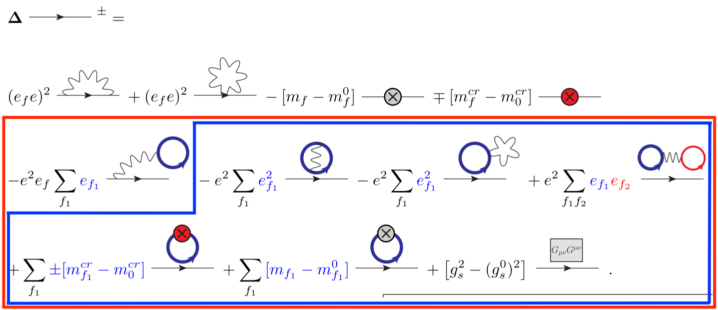
where $m_{cr}^{LQCD} = \frac{w(g^2)}{a} + w_1(g^2)\Lambda_{QCD} + O(a)$

LIB effects à la RM123 [JHEP 1204(2012), Phys.Rev. D87(2013)]

- Leading Isospin Breaking (LIB) effects can be calculated directly by expanding the lattice path-integral in powers of α_{em} and $(m_d - m_u)$

$$\mathcal{O}(\vec{g}) = \frac{\langle\ R[U,A;\vec{g}]\ O[U,A;\vec{g}]\ \rangle^{A,\vec{g}^0}}{\langle\ R[U,A;\vec{g}]\ \rangle^{A,\vec{g}^0}} = \frac{\left\langle \left(1+\dot{R}+\ldots\right)\left(O+\dot{O}+\ldots\right)\right\rangle}{\langle 1+\dot{R}+\ldots\rangle} = \mathcal{O}(\vec{g}^0) + \Delta\mathcal{O}$$

- sea quark e.m. effects via (noisy) fermion disconnected diagrams



M_{cr} determination – Non-singlet chiral WTIs

- QCD+QED continuum chiral WTIs $\qquad (\delta q = q_{up} - q_{dn})$

$$\partial_\mu V^3_\mu = 0 \qquad \partial_\mu A^3_\mu - 2\mu P^3 - 2\epsilon P^0 = 0$$

$$\partial_\mu V^1_\mu + e\,\delta q\,\mathcal{A}_\mu i V^2_\mu + 2\epsilon i S^2 = 0 \qquad \partial_\mu A^1_\mu + e\,\delta q\,\mathcal{A}_\mu i A^2_\mu - 2\mu P^1 = 0$$

$$\partial_\mu V^2_\mu - e\,\delta q\,\mathcal{A}_\mu i V^1_\mu - 2\epsilon i S^1 = 0 \qquad \partial_\mu A^2_\mu - e\,\delta q\,\mathcal{A}_\mu i A^1_\mu - 2\mu P^2 = 0$$

with e.g. $P^3 = \bar\psi\gamma_5\tau_3\psi$ and $\mu + \tau_3\epsilon = \mathrm{diag}(M_u, M_d)$

- A way to fix $M_{cr}^{u,d}$ could be to impose two continuum WTIs, e.g.

$$\langle\partial_\mu V^1_\mu(x)P^2(0)\rangle + ie\,\delta q\,\langle\mathcal{A}_\mu V^2_\mu(x)P^2(0)\rangle + 2i\epsilon\langle S^2(x)P^2(0)\rangle = 0$$

$$\langle\partial_\mu A^1_\mu(x)S^1(0)\rangle - ie\,\delta q\,\langle\mathcal{A}_\mu A^2_\mu(x)S^1(0)\rangle - 2\mu\langle P^1(x)S^1(0)\rangle = 0$$

with $\mathcal{A}_\mu$ = photon. Need chiral-covariant renormalized operators ...
In the conditions above all relevant correlators are parity-violating

- Another way to fix $M_{\rm cr}^{u,d}$:
imposing one chiral WTI [RM123, 2013: electroquenched approximation]
& minimizing $m_\pi^\pm$ wrt bare $m_{u,d}$ [Horkel & Sharpe, 2015]

Critical mass from parity/flavour restoring in tmLQCD

Inspiration from Wilson twisted mass lattice QCD: its effective action, $\Gamma_{\rm latt}$, contains a local term $\sum_x [\frac{w_0(g_0^2)}{a} - M_0][\bar\psi i\gamma_5\tau_3\psi](x)$ plus further parity(P)-breaking local terms with $d = 5, 7, \ldots \quad \Longrightarrow$

Enforcing P-restoration in correlators fixes $M_0 = m_{\rm cr}^{LQCD}$ up to O(a)

E.g. impose $\sum_{\vec x}\langle V_0^1(x)P^2(0)\rangle_{M_0}^{\rm latt} = 0$ (for all $x_0 \gg a$: optimal $m_{\rm cr}$)

Lattice theory @ $M_0 \sim m_{\rm cr}$ described by a continuum effective action

$$S_{LEL}^{\rm eff} = \int d^4y\{L_4^{QCD}(y) + [a^{-1}w(g^2) - M_0]\,[\bar\psi i\gamma_5\tau_3\psi](y) + aL_5(y) + \ldots\}$$

s.t. lattice correlators admit formal expansion in a and $M_0 - m_{\rm cr}$, e.g.

$$\langle V_0^1(x)P^2(0)\rangle_{M_0}^{\rm latt} = \langle V_0^1(x)P^2(0)\rangle|^{L_4} + \mathrm{O}(a) +$$

$$+\ (m_{\rm cr} - M_0)\int d^4y\langle V_0^1(x)P^2(0)[\bar\psi i\gamma_5\tau_3\psi](y)\rangle|^{L_4}$$

Note: L_4 is P-invariant. Here $M_0 \to m_{\rm cr}$ limit to be taken before $a \to 0$ limit.

Critical mass from parity restoring in tmLQ(C+E)D

- Need to eliminate $a^{-1}\bar{\psi} i\gamma_5\tau_3\psi$ and $a^{-1}\bar{\psi} i\gamma_5\psi$ from $\Gamma_{\rm latt}^{QCD+QED}$
- P-restoration fixes $m_0 \to m_{cr}$ & $\tilde{m}_0 \to \tilde{m}_{cr}$ in $M_0 = m_0 1\!\!1 + \tilde{m}_0\tau_3$
- Impose $\sum_{\vec{x}}\langle V_0^1(x)P^2(0)\rangle_{M_0}^{\rm latt} = 0$ and $\sum_{\vec{x}}\langle S^1(x)P^1(0)\rangle_{M_0}^{\rm latt} = 0$

Latt. theory @ $\epsilon = 0$, $(m_0, \tilde{m}_0) \sim (m_{cr}, \tilde{m}_{cr})$ described by cont. LEL

$$L_4^{QCD+QED}(y) + [m_{cr} - m_0]\,[\bar{\psi} i\gamma_5\tau_3\psi](y) + [\tilde{m}_{cr} - \tilde{m}_0]\,[\bar{\psi} i\gamma_5\psi](y) + aL_5(y) + \dots$$

s.t. correlators admit a formal expansion in a, $m_0 - m_{cr}$, $\tilde{m}_0 - \tilde{m}_{cr}$, e.g.

$$\langle V_0^1(x)P^2(0)\rangle_{M_0}^{\rm latt} = (m_{cr}^{LQCD} + \alpha_{em}a^{-1}\delta_{em} - m_0)\int d^4z\langle V_0^1(x)P^2(0)\bar{\psi} i\gamma_5\tau_3\psi(z)\rangle|^{L_4} +$$

$$+(\alpha_{em}a^{-1}\tilde{\delta}_{em} - \tilde{m}_0)\int d^4z\langle V_0^1(x)P^2(0)\bar{\psi} i\gamma_5\psi(z)\rangle|^{L_4} + \mathrm{O}(a)$$

$$\langle S^1(x)P^1(0)\rangle_{M_0}^{\rm latt} = (\alpha_{em}a^{-1}\tilde{\delta}_{em} - \tilde{m}_0)\int d^4z\langle S^1(x)P^1(0)\bar{\psi} i\gamma_5\psi(z)\rangle|^{L_4} +$$

$$+(m_{cr}^{LQCD} + \alpha_{em}a^{-1}\delta_{em} - m_0)\int d^4z\langle S^1(x)P^1(0)\bar{\psi} i\gamma_5\tau_3\psi(z)\rangle|^{L_4} + \mathrm{O}(a)$$

P-invariance (isospin symm. as $\alpha_{em} \to 0$) of $L_4^{QCD+QED}$ was (can be) used

RM123 approach for $N_f = 2{+}1{+}1$ MTM LQ(C+E)D

Action with two-flavours Dirac operators $D_{33}^{h,\ell} \supset -i\gamma_5\tau_3 + \mu_{h,\ell} + \epsilon_{h,\ell}\tau_3$

- is necessary to preserve e.m. gauge invariance
- in general has complex fermionic determinant

The RM123 method for LIB effects allows to extract physical info from correlation funcions (with suitable operator insertions) evaluated

⋆ in isosymmetric lattice theory for $N_f = 2$: real fermionic determinant

⋆ in a mixed action lattice theory with $e = \epsilon_\ell = 0$ for $N_f = 2 + 1 + 1$:

$$S_{\rm mix} = S_{33}^{h,\ell}|_{e=\epsilon_\ell=0} + \left(\bar{\psi}_h^{sea}, [\gamma \cdot \widetilde{\nabla} - i\gamma_5\tau_3 W_{\rm cr} + \mu_h + \epsilon_h \tau^1]\psi_h^{sea}\right)|_{e=0} +$$

$+S_{\rm ghost}(\Phi_h; \mu_h + \epsilon_h\tau_3)|_{e=0}$ has real fermionic determinant too

Suitable operator insertions in correlators reproduce all LIB effects due to $\epsilon_\ell \neq 0$ and $\alpha_{em} > 0$, included those from electro-unquenching (which need fermion disconnected diagrams evaluation) with only O(a^2) lattice artifacts

[Proof along the lines of Phys.Rev. D87 (2013) 11, 114505 (RM123) & JHEP 0410 (2004) 070 (Frezzotti-Rossi)]

Thank you