Lambda-Nucleon and Sigma-Nucleon interactions from lattice QCD with physical masses

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Outline

- Introduction
    - Brief introduction of HAL QCD method
- Effective block algorithm for various baryon-baryon channels [arXiv:1510.00903(hep-lat)]
- Preliminary results of LN-SN potentials at nearly physical point
    - LN-SN(I=1/2), central and tensor potentials
    - SN(I=3/2), central and tensor potentials
- Summary

Plan of research

QCD

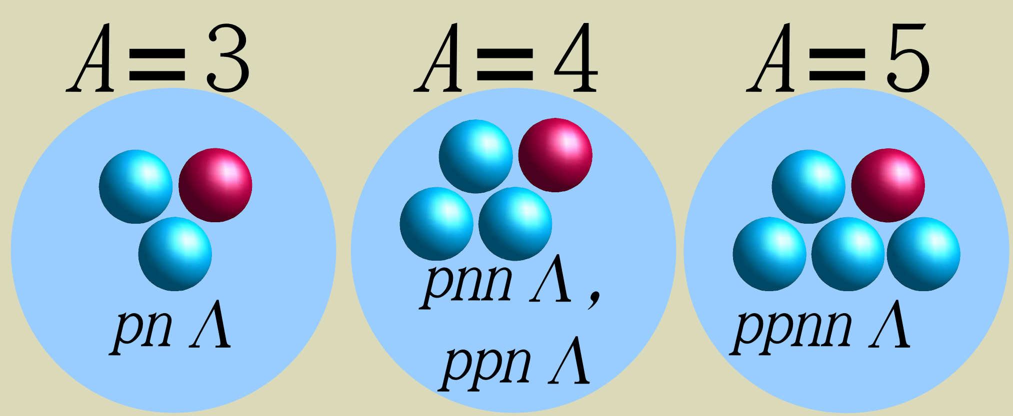
Baryon interaction

J-PARC,
JLab, GSI, MAMI, ...
YN scattering,
hypernuclei

Structure and reaction of (hyper)nuclei

Equation of State (EoS) of nuclear matter

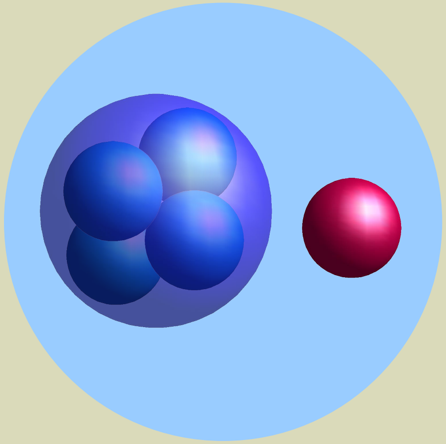
Neutron star and supernova



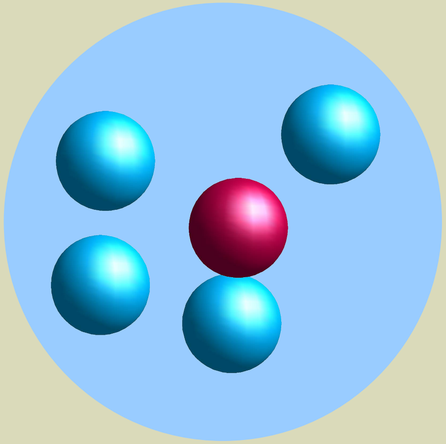
What is realistic picture of hypernuclei?

- $B(\text{total}) = B(^4\text{He}) + B_\Lambda(^5_\Lambda\text{He})$

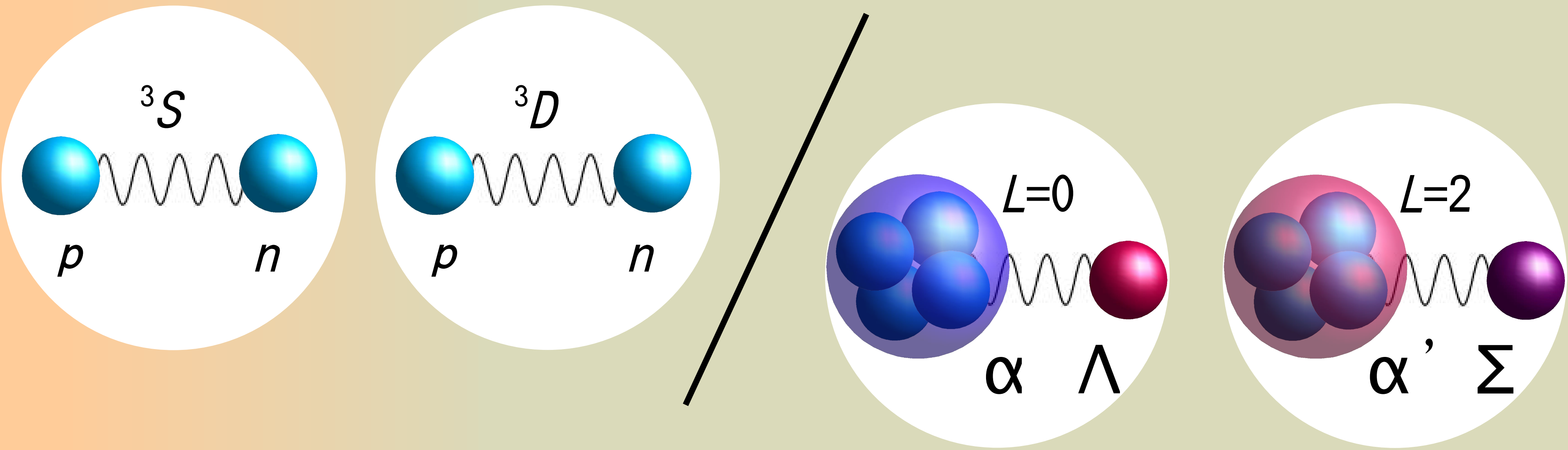
- A conventional picture:
  $B(\text{total})$
  $= B(^4\text{He}) + B_\Lambda(^5_\Lambda\text{He})$
  = 28+3 MeV.



- A (probably realistic) picture:
  $B(\text{total})$
  $= (B(^4\text{He}) - \Delta E_c) + (B_\Lambda(^5_\Lambda\text{He}) + \Delta E_c)$
  = ??+?? MeV.



Comparison between d=p+n and core+Y



	$\langle T_S \rangle$ (MeV)	$\langle T_D \rangle$ (MeV)	$\langle V_{NN}(\mathrm{central}) \rangle$ (MeV)	$\langle V_{NN}(\mathrm{tensor}) \rangle$ (MeV)	$\langle V_{NN}(\mathrm{LS}) \rangle$ (MeV)
AV8	8.57	11.31	−4.46	−16.64	−1.02
G3RS	10.84	5.64	−7.29	−11.46	0.00

	$\langle T_{Y\text{-}c} \rangle_\Lambda$	$\langle T_{Y\text{-}c} \rangle_\Sigma + \Delta \langle H_C \rangle$	$\langle V_{YN}$(のこり)$\rangle$	$2\langle V_{\Lambda N-\Sigma N}(\mathrm{tensor}) \rangle$
$^5_\Lambda$He	9.11	3.88+4.68	−0.86	−19.51
$^4_\Lambda$H*	5.30	2.43+2.02	0.01	−10.67
$^4_\Lambda$H	7.12	2.94+2.16	−5.05	−9.22

HN, Akaishi, Suzuki, PRL89, 142504 (2002).

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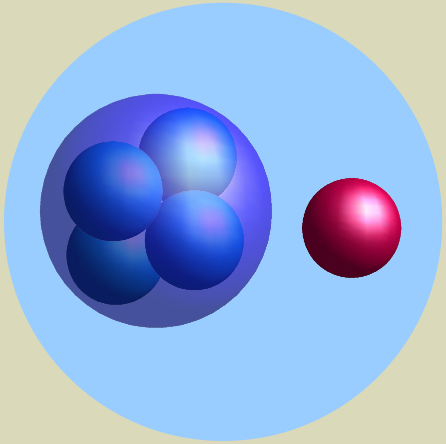
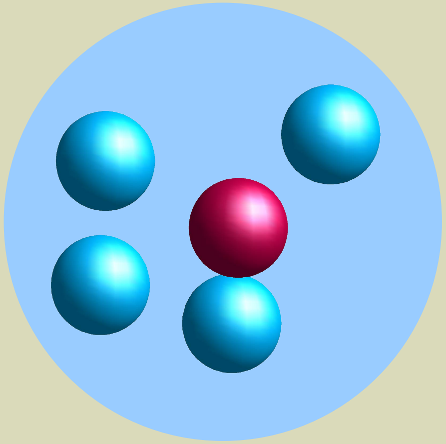
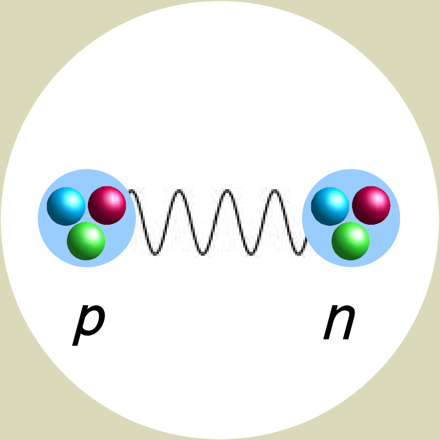


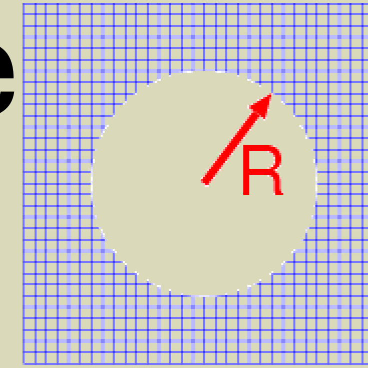
- A (probably realistic) picture:
  $B(\text{total})$
  $= (B(^4\text{He}) - \Delta E_c) + (B_\Lambda(^5_\Lambda\text{He}) + \Delta E_c)$
  $=$ 24+7 MeV.



Lattice QCD calculation



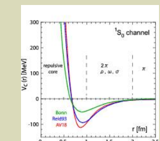
Multi-hadron on lattice



i) basic procedure:

asymptotic region

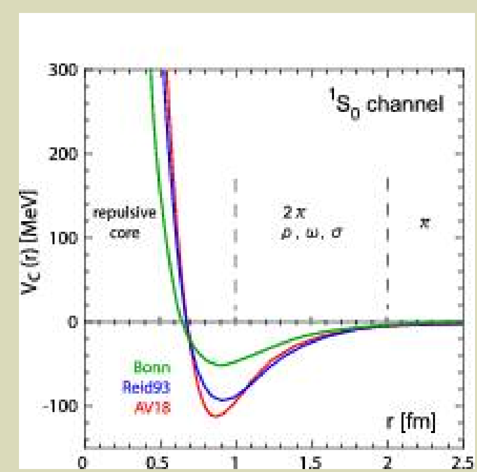
--> phase shift



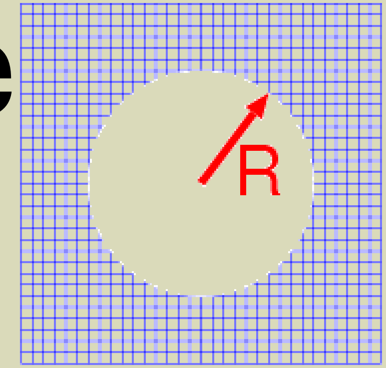
ii) HAL's procedure:

interacting region

--> potential



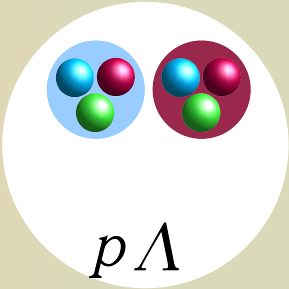
Multi-hadron on lattice

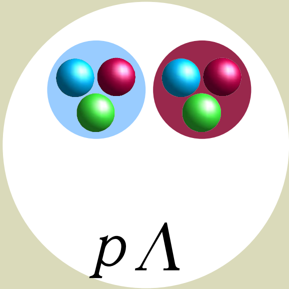


Lattice QCD simulation

$$L = -\frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^{\mu}(i\partial_{\mu} - g t^{a} A^{a}_{\mu}) q - m\bar{q}q$$

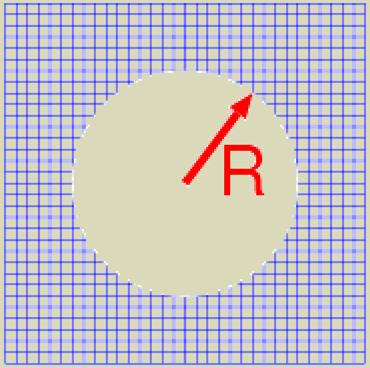
$$\begin{aligned}\langle O(\bar{q}, q, U)\rangle &= \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \\ &= \int dU \det D(U) e^{-S_U(U)} O(D^{-1}(U)) \\ &= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))\end{aligned}$$

$\rightarrow \langle$ 

(t) 

$(t_0)\rangle$

Multi-hadron on lattice



i) basic procedure:

asymptotic region

(or temporal correlation)

--> scattering energy

--> phase shift

$$E = \frac{k^2}{2\mu}$$

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in Z^3} \frac{1}{(n^2 - q^2)^s} \qquad \Re\, s > \frac{3}{2}$$

Luscher, NPB354, 531 (1991).
Aoki, et al., PRD71, 094504 (2005).

Multi-hadron on lattice

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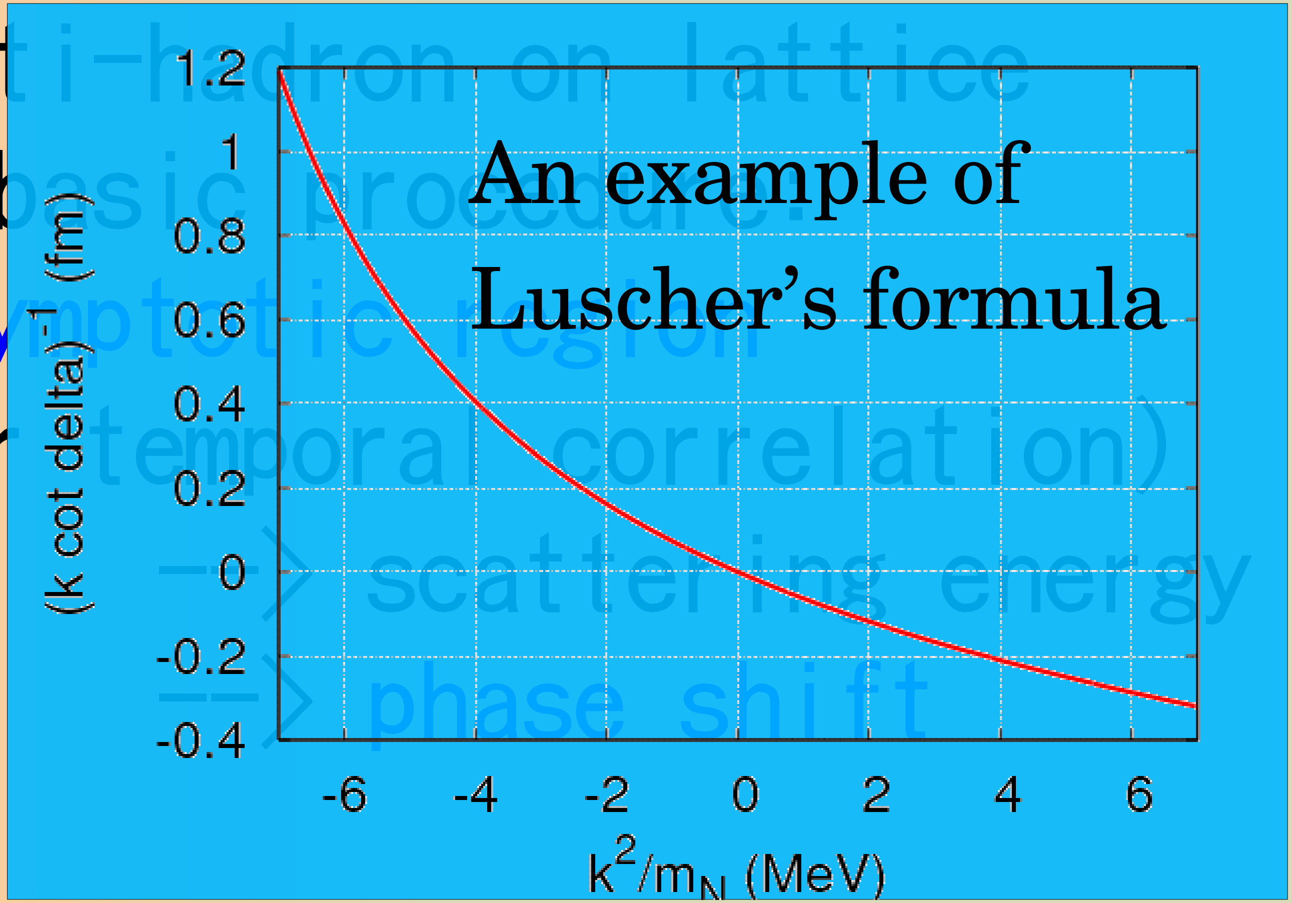

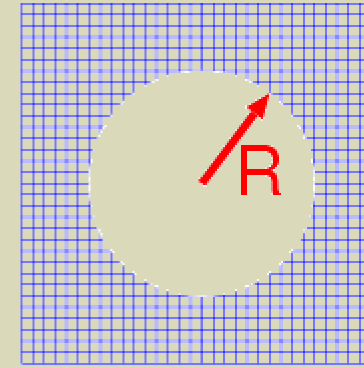

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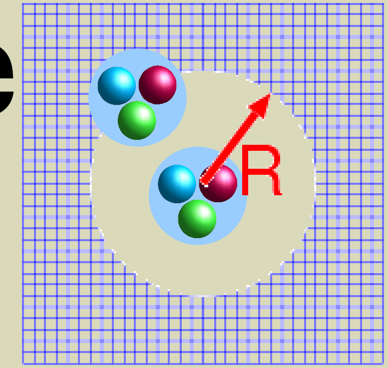
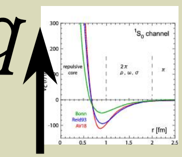


Lattice QCD simulation

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$



$$\langle O(\bar{q}, q, U)\rangle = \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U)$$
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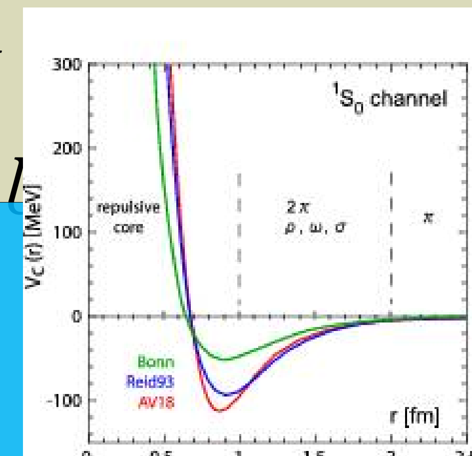
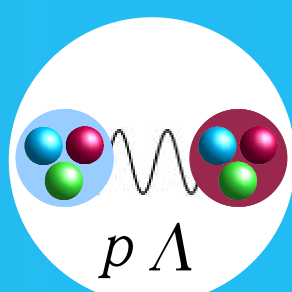
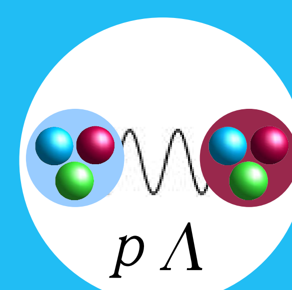


$$F^{(JM)}_{\alpha\beta}(\vec{r}, t - t_0)$$

$$\rightarrow \langle$$ 

$$(\vec{r}, t)\ \overline{}$$ 

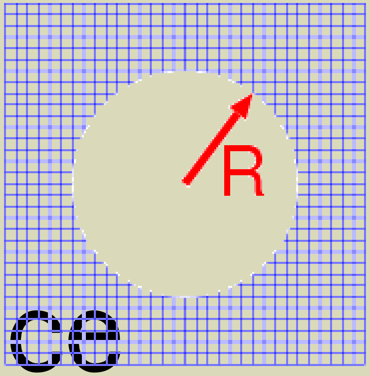
$$(t_0)\rangle$$

Calculate the scattering state

Multi-hadron on lattice

ii) HAL's procedure:

make better use of the lattice output ! (wave function)



interacting region

--> potential

Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., PTP123, 89 (2010).

NOTE:

> Potential is not a direct experimental observable.

> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

....

Multi-hadron on lattice

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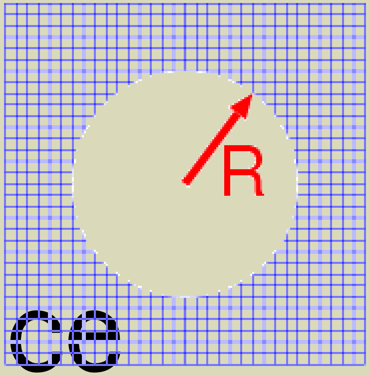

interacting region

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Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., PTP123, 89 (2010).

=>

> Phase shift
> Nuclear many-body problems

In lattice QCD calculations, we compute the normalized four-point correlation function

$$R^{(J,M)}_{\alpha\beta}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t) B_{2,\beta}(\vec{X},t) \overline{\mathcal{J}^{(J,M)}_{B_3B_4}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$p = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) u_c, \qquad n = -\varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) d_c, \tag{2}$$

$$\Sigma^+ = -\varepsilon_{abc}\left(u_a C\gamma_5 s_b\right) u_c, \qquad \Sigma^- = -\varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) d_c, \tag{3}$$

$$\Sigma^0 = \frac{1}{\sqrt{2}}\left(X_u - X_d\right), \qquad \Lambda = \frac{1}{\sqrt{6}}\left(X_u + X_d - 2X_s\right), \tag{4}$$

$$\Xi^0 = \varepsilon_{abc}\left(u_a C\gamma_5 s_b\right) s_c, \qquad \Xi^- = -\varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) s_c, \tag{5}$$

where

$$X_u = \varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) u_c, \quad X_d = \varepsilon_{abc}\left(s_a C\gamma_5 u_b\right) d_c, \quad X_s = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) s_c, \tag{6}$$

An improved recipe for NY potential:

- cf. Ishii (HAL QCD), PLB712 (2012) 437.

- Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu}\nabla^2 R(t,\vec{r}) + \int d^3 r' \, U(\vec{r},\vec{r}\,') R(t,\vec{r}\,') = -\frac{\partial}{\partial t} R(t,\vec{r})$$

$$\rightarrow \frac{k^2}{2\mu} R(t,\vec{r})$$

$$U(\vec{r},\vec{r}\,') = V_{NY}(\vec{r},\nabla)\,\delta(\vec{r}-\vec{r}\,')$$

- A general expression of the potential:

$$V_{NY} = V_0(r) + V_\sigma(r)(\vec{\sigma}_N \cdot \vec{\sigma}_Y) + V_T(r) S_{12} + V_{LS}(r)(\vec{L}\cdot\vec{S}_+) + V_{ALS}(r)(\vec{L}\cdot\vec{S}_-) + O(\nabla^2)$$

Determination of baryon-baryon potentials at nearly physical point

Effective block algorithm for various baryon-baryon calculations

arXiv:1510.00903(hep-lat)

Numerical cost (# of iterative operations) in this algorithm

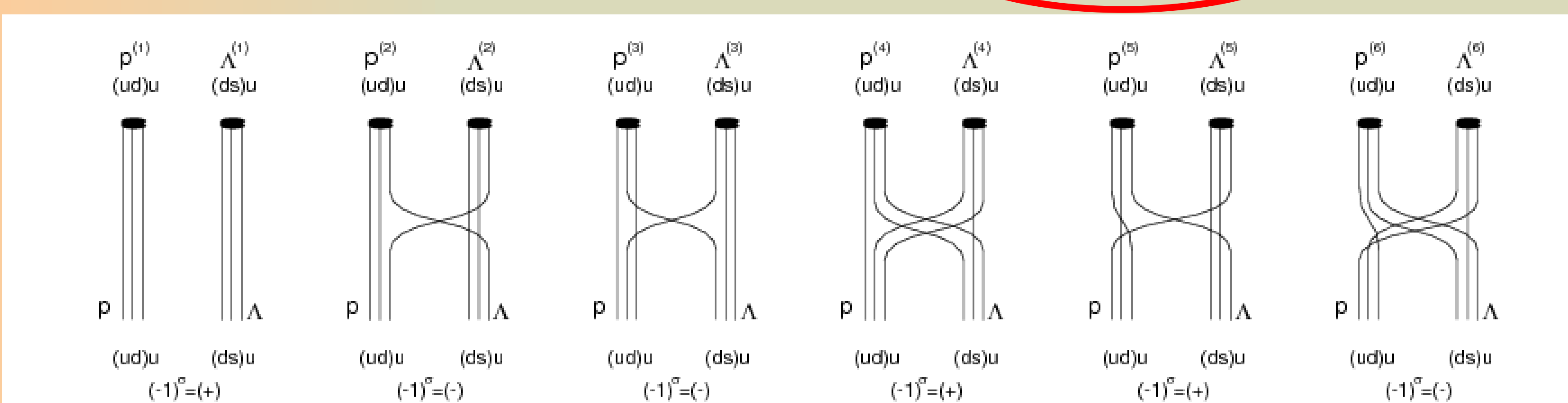
$$1+N_c^2+N_c^2 N_\alpha^2+N_c^2 N_\alpha^2+N_c^2 N_\alpha+N_c^2 N_\alpha=370$$

In an intermediate step:

$$(N_c!\, N_\alpha)^B \times N_u!\, N_d!\, N_s! \times 2^{N_\Lambda+N_{\Sigma^0}-B}=3456$$

In a naïve approach:

$$(N_c!\, N_\alpha)^{2B} \times N_u!\, N_d!\, N_s! = 3,981,312$$



Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{array}{lll}
\langle p\Lambda\overline{p\Lambda}\rangle, & \langle p\Lambda\overline{\Sigma^+ n}\rangle, & \langle p\Lambda\overline{\Sigma^0 p}\rangle, \\
\langle \Sigma^+ n\overline{p\Lambda}\rangle, & \langle \Sigma^+ n\overline{\Sigma^+ n}\rangle, & \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle, \\
\langle \Sigma^0 p\overline{p\Lambda}\rangle, & \langle \Sigma^0 p\overline{\Sigma^+ n}\rangle, & \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle,
\end{array} \tag{4.2}$$

$$\begin{array}{llllll}
\langle \Lambda\Lambda\overline{\Lambda\Lambda}\rangle, & \langle \Lambda\Lambda\overline{p\Xi^-}\rangle, & \langle \Lambda\Lambda\overline{n\Xi^0}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle, & \\
\langle p\Xi^-\overline{\Lambda\Lambda}\rangle, & \langle p\Xi^-\overline{p\Xi^-}\rangle, & \langle p\Xi^-\overline{n\Xi^0}\rangle, & \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle, \\
\langle n\Xi^0\overline{\Lambda\Lambda}\rangle, & \langle n\Xi^0\overline{p\Xi^-}\rangle, & \langle n\Xi^0\overline{n\Xi^0}\rangle, & \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle, & \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle, \\
\langle \Sigma^+\Sigma^-\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle, & \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Lambda}\rangle, \\
\langle \Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle, & \langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle, & \langle \Sigma^0\Sigma^0\overline{\Sigma^0\Sigma^0}\rangle, & \\
 & \langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle, & \langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle, & \langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle, & & \langle \Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle,
\end{array} \tag{4.3}$$

$$\begin{array}{lll}
\langle \Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle, & \langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle, & \langle \Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle, \\
\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda}\rangle, & \langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle, & \langle \Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle, \\
\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle, & \langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle, & \langle \Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle,
\end{array} \tag{4.4}$$

$$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

Make better use of the computing resources!

arXiv:1510.00903(hep-lat), (see also 1604.08346)

Almost physical point lattice QCD calculation using N_F=2+1 clover fermion + Iwasaki gauge action

- APE-Stout smearing (ρ=0.1, n_{stout}=6)
- Non-perturbatively O(a) improved Wilson Clover action at β=1.82 on $96^3 \times 96$ lattice
- $1/a$ = 2.3 GeV (a = 0.085 fm)
- Volume: $96^4 \rightarrow (8\text{fm})^4$
- m_π=145MeV, m_K=525MeV
- DDHMC(ud) and UVPHMC(s) with preconditioning
- K.-I.Ishikawa, et al., PoS LAT2015, 075; arXiv:1511.09222 [hep-lat].

- NBS wf is measured using wall quark source with Coulomb gauge fixing, spatial PBD and temporal DBC; #stat=207configs x 4rotation x Nsrc (Nsrc=4 → 20 → 96 (2015FY))

LN-SN potentials at nearly physical point

The methodology for coupled-channel V is based on:
Aoki, et al., Proc.Japan Acad. B87 (2011) 509.
Sasaki, et al., PTEP 2015 (2015) no.11, 113B01.
Ishii, et al., JPS meeting, March (2016).

#stat: (this/scheduled in FY2015) < 0.05 → 0.2 for

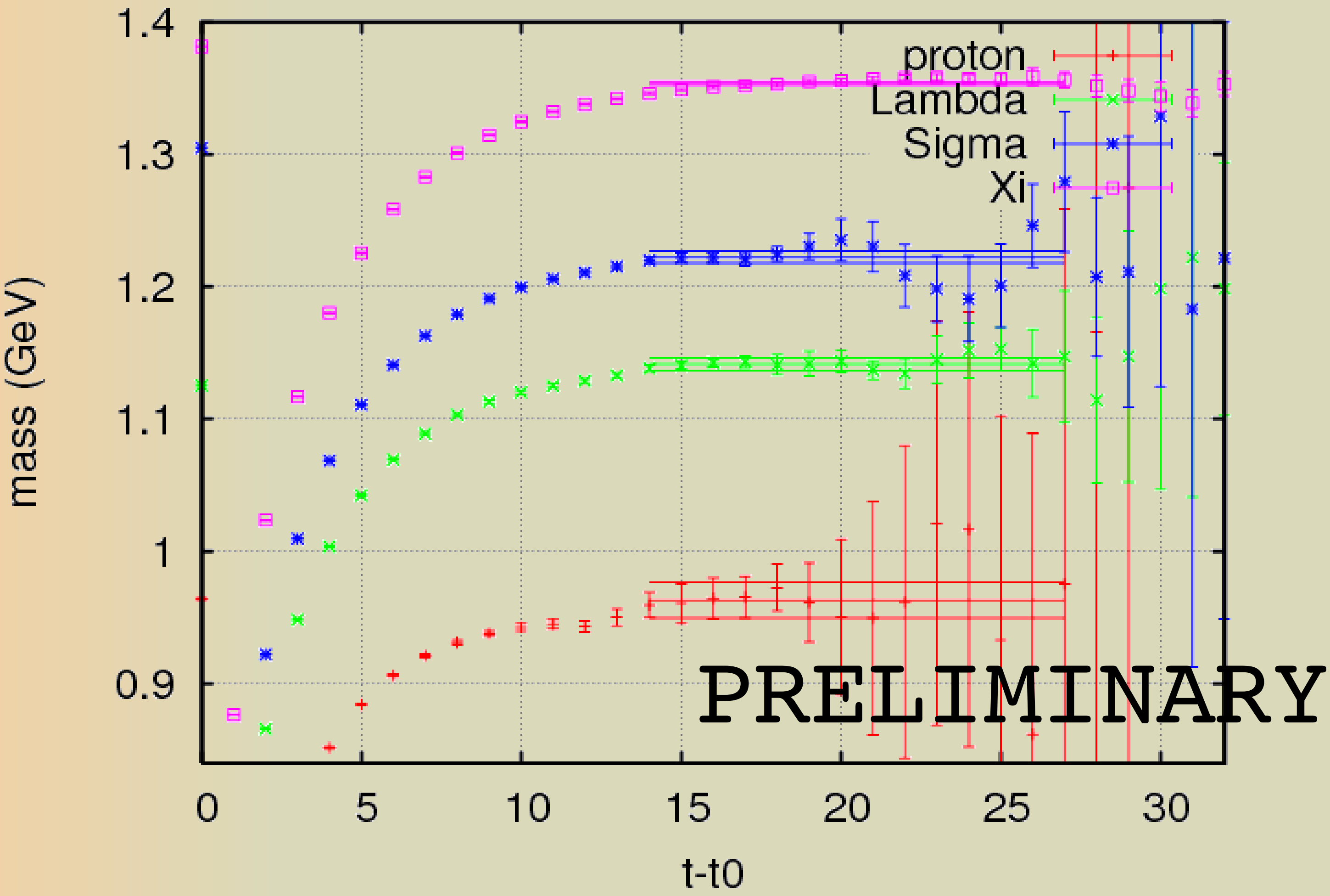
$\Lambda N - \Sigma N \ (I = 1/2)$

$V_C(^1S_0)$ $V_C(^3S_1 - {}^3D_1)$ $V_T(^3S_1 - {}^3D_1)$

$\Sigma N \ (I = 3/2)$

$V_C(^1S_0)$ $V_C(^3S_1 - {}^3D_1)$ $V_T(^3S_1 - {}^3D_1)$

Effective mass plot of the single baryon's correlation function



Potentials obtained at t-t0 = 5 to 12 will be shown.

TABLE 4

The eigenvalues of the normalization kernel in eq. (3.3) for $S=-1$ two-baryon (BB) system

$S=-1$

I	J	BB	Eigenvalues (uncoupled)	Eigenvalues (coupled)
$\frac{1}{2}$	0	$N\Lambda$	1	0 $\frac{10}{9}$
		$N\Sigma$	$\frac{1}{9}$	
$\frac{1}{2}$	1	$N\Lambda$	1	$\frac{8}{9}$ $\frac{10}{9}$
		$N\Sigma$	1	
$\frac{3}{2}$	0	$N\Sigma$	$\frac{10}{9}$	
$\frac{3}{2}$	1	$N\Sigma$	$\frac{2}{9}$	

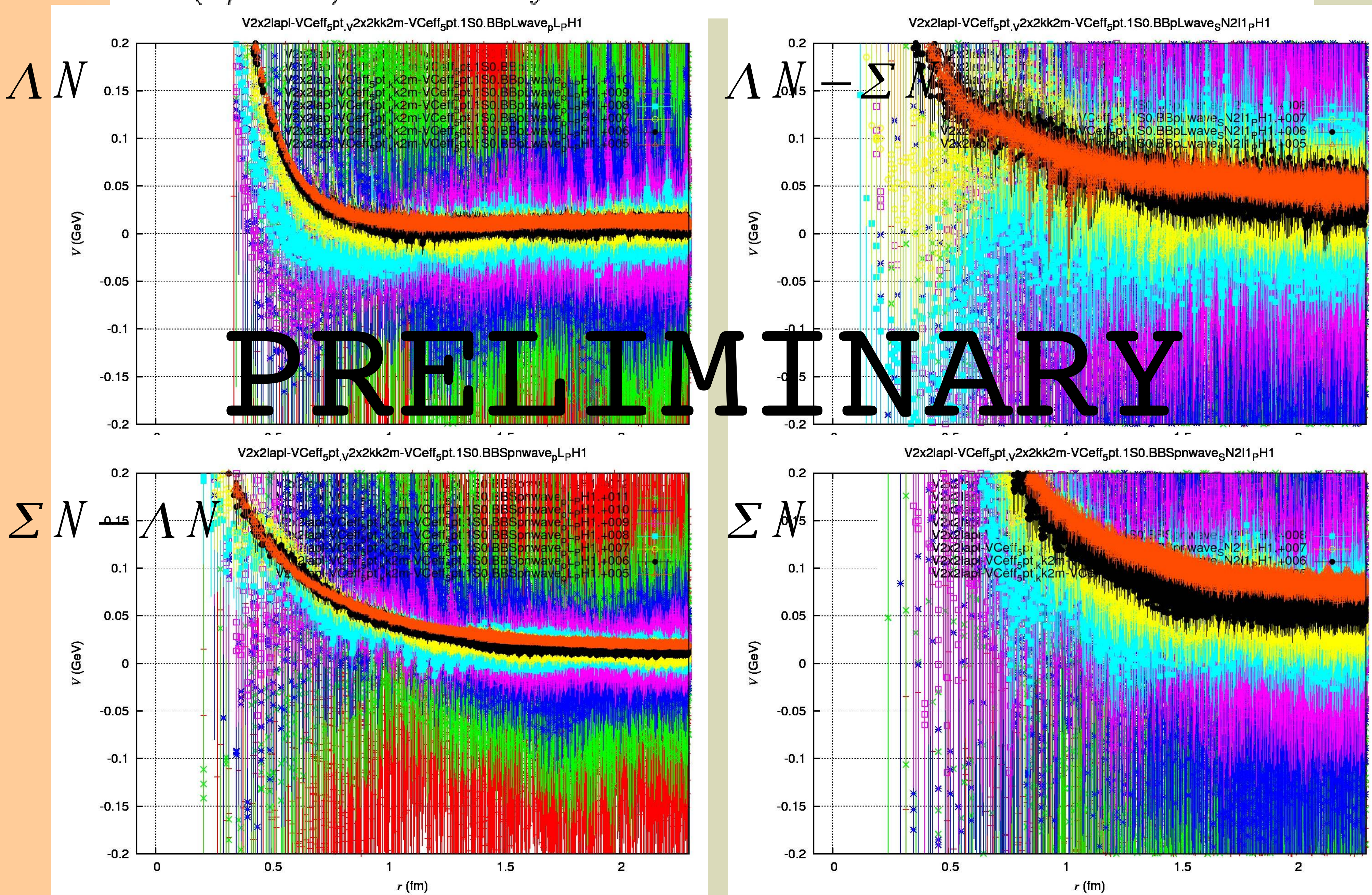
Eigenvalues of single and coupled channels are given.

Oka, Shimizu and Yazaki (1987)

Very preliminary result of LN potential at the physical point

$V_C(\,^1S_0)$

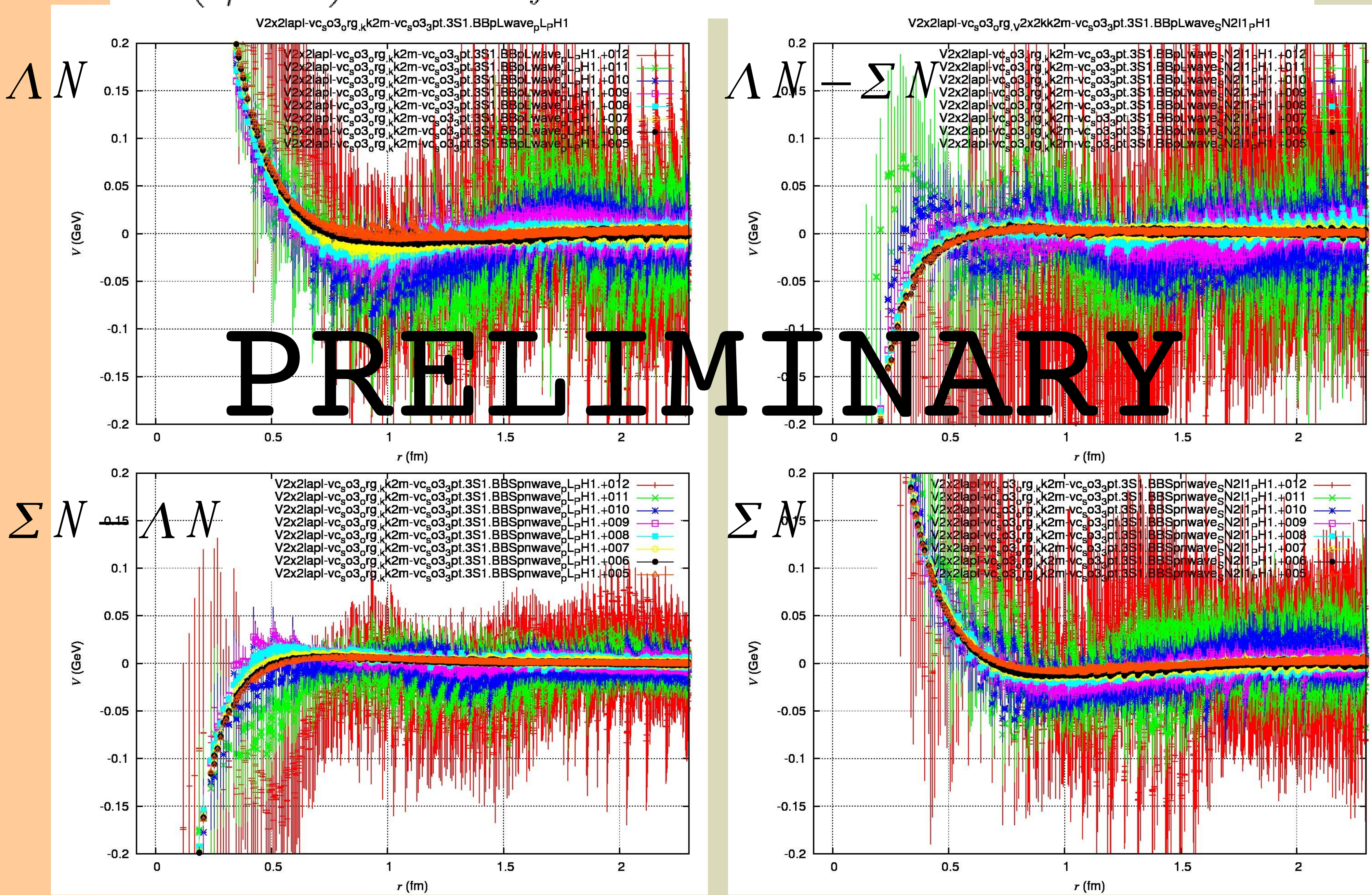
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdot (8)$$



Very preliminary result of LN potential at the physical point

$V_C({}^3S_1 - {}^3D_1)$

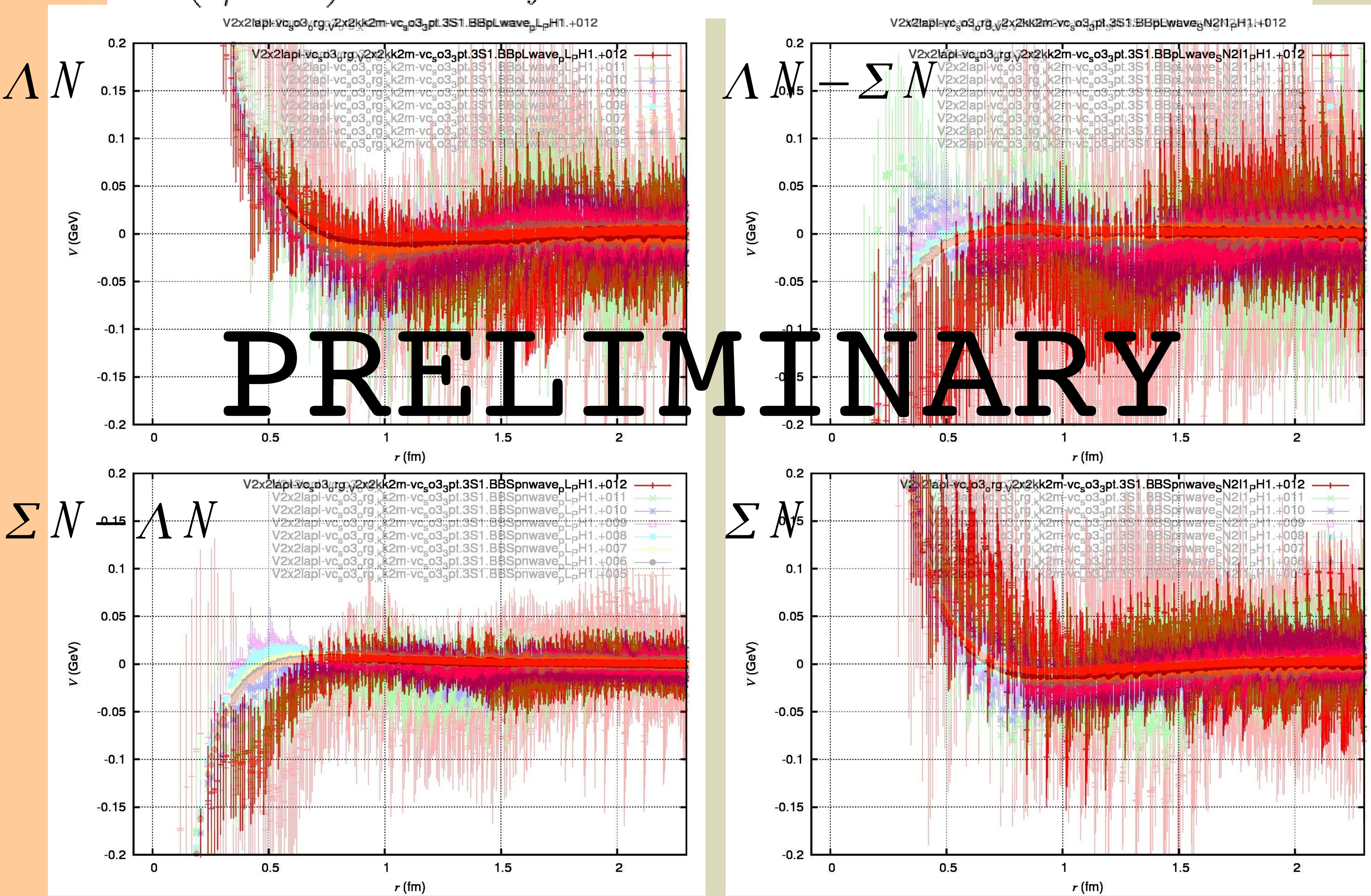
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_C(^3S_1 - ^3D_1)$

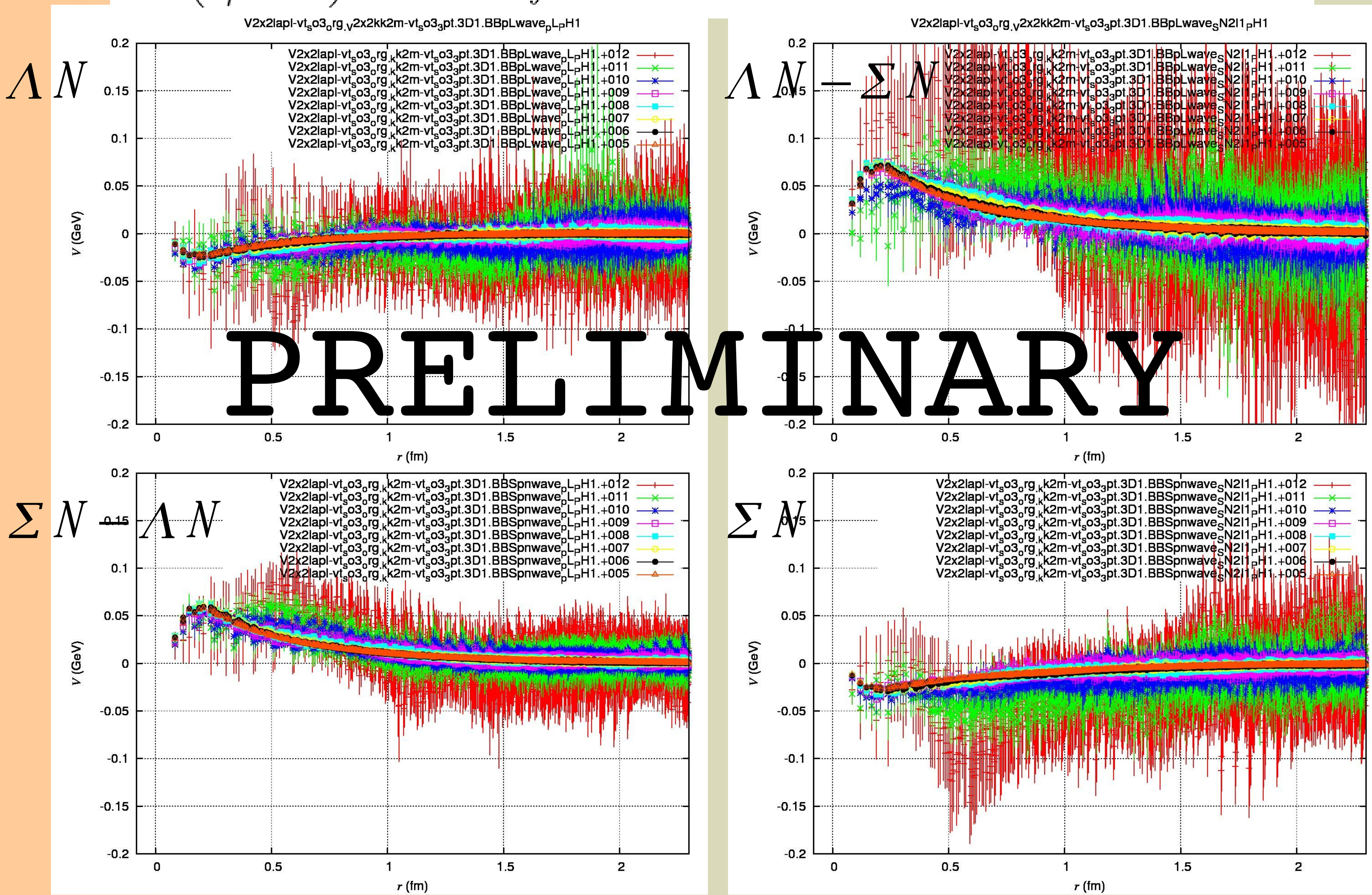
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r' \, U(\vec{r},\vec{r}') R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r}) R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_T({}^3S_1 - {}^3D_1)$

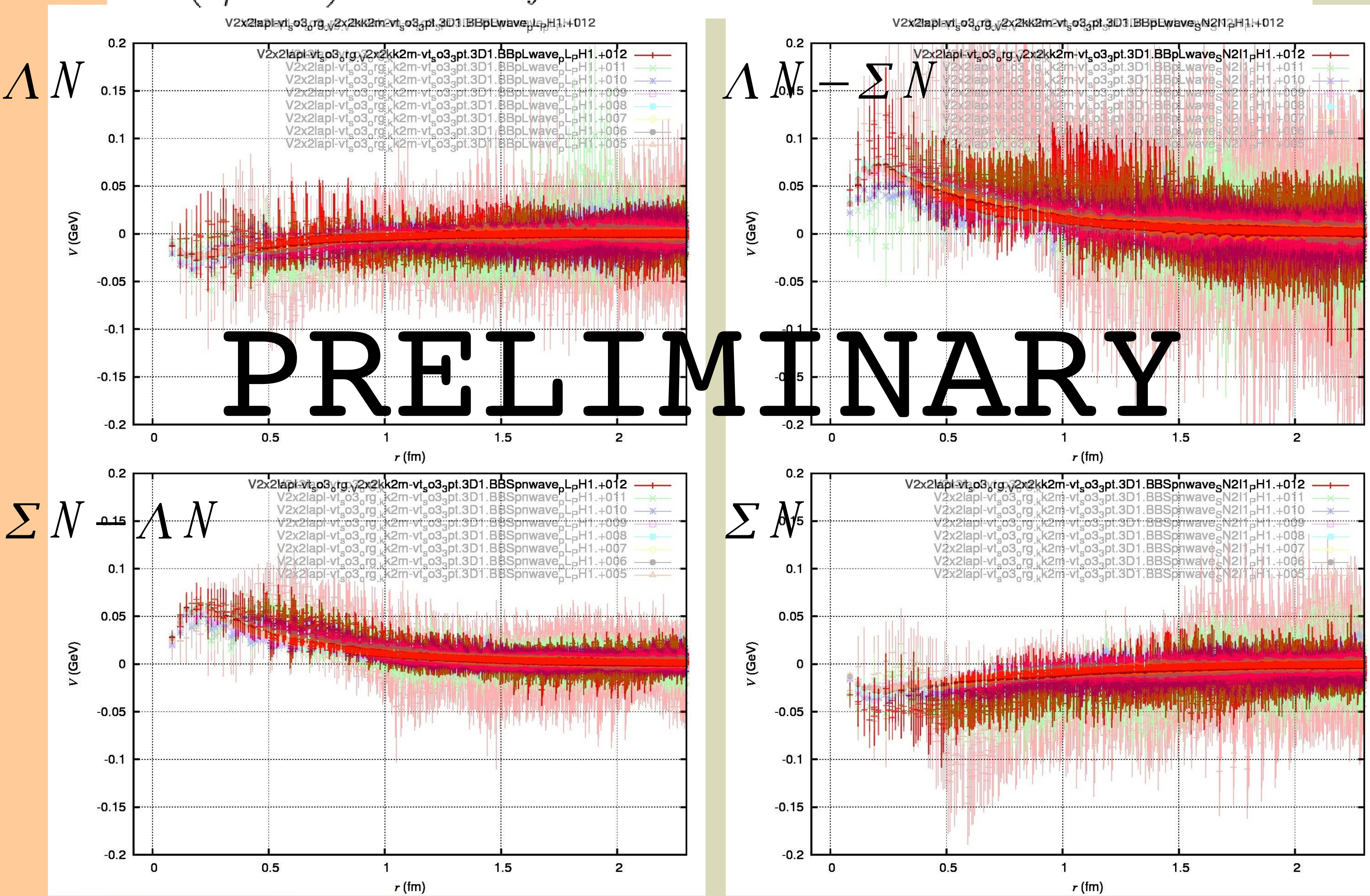
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r' \, U(\vec{r},\vec{r}') R(\vec{r}',t) + O(k^4) = V_{\mathrm{LO}}(\vec{r}) R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_T({}^3S_1 - {}^3D_1)$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$

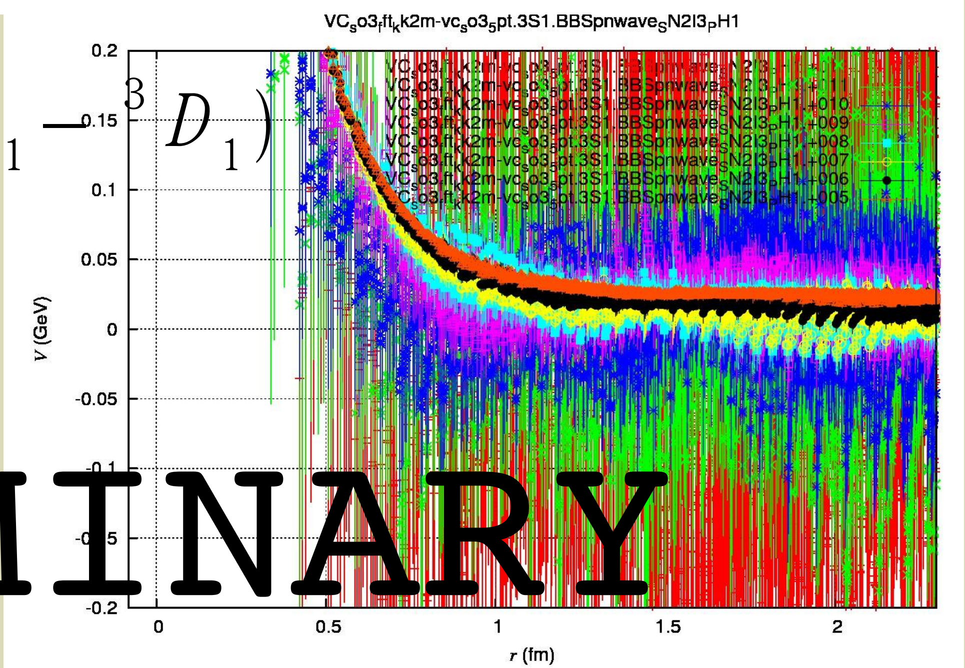


Very preliminary result of LN potential at the physical point

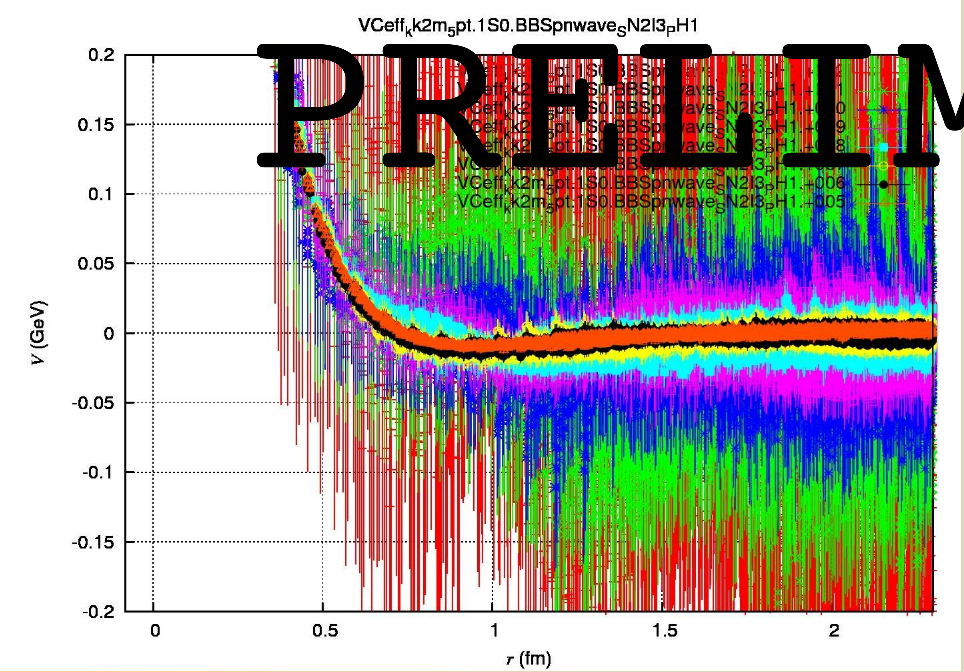
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}\,')R(\vec{r}\,',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots \quad (8)$$

$\Sigma N\,(I=3/2)$

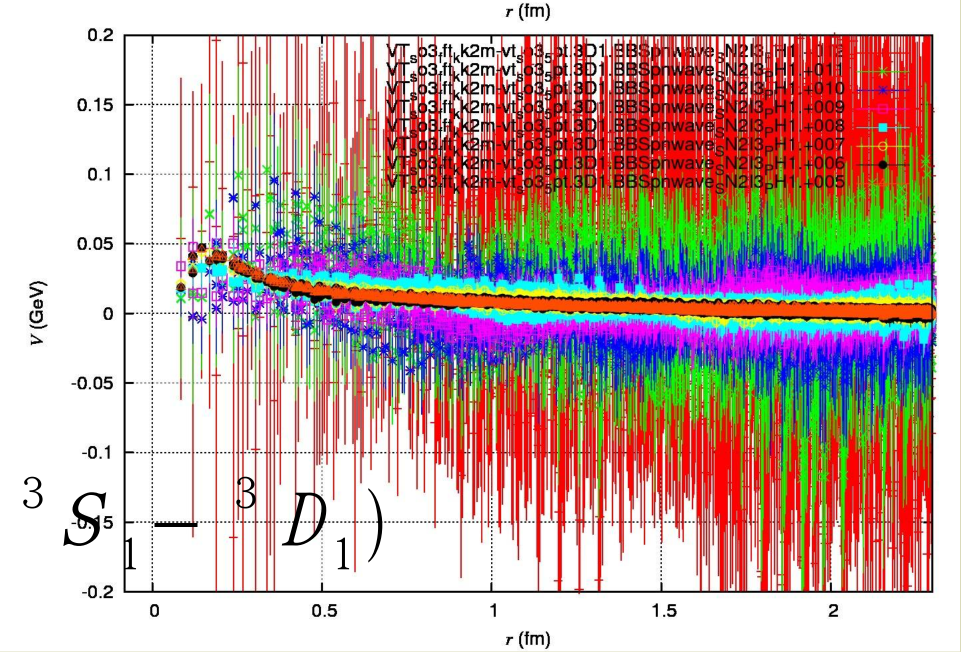
$V_C(\,^3S_1 - \,^3D_1)$



PRELIMINARY



$V_C(\,^1S_0)$



$V_T(\,^3S_1 - \,^3D_1)$

Summary

(I-1) Preliminary results of LN-SN potentials at nearly physical point. (Lambda-N, Sigma-N: central, tensor)
Statistics approaching to 0.2 (=present/scheduled)

Several interesting features seem to be obtained with more high statistics.

(I-2) Effective hadron block algorithm for the various baron-baryon interaction

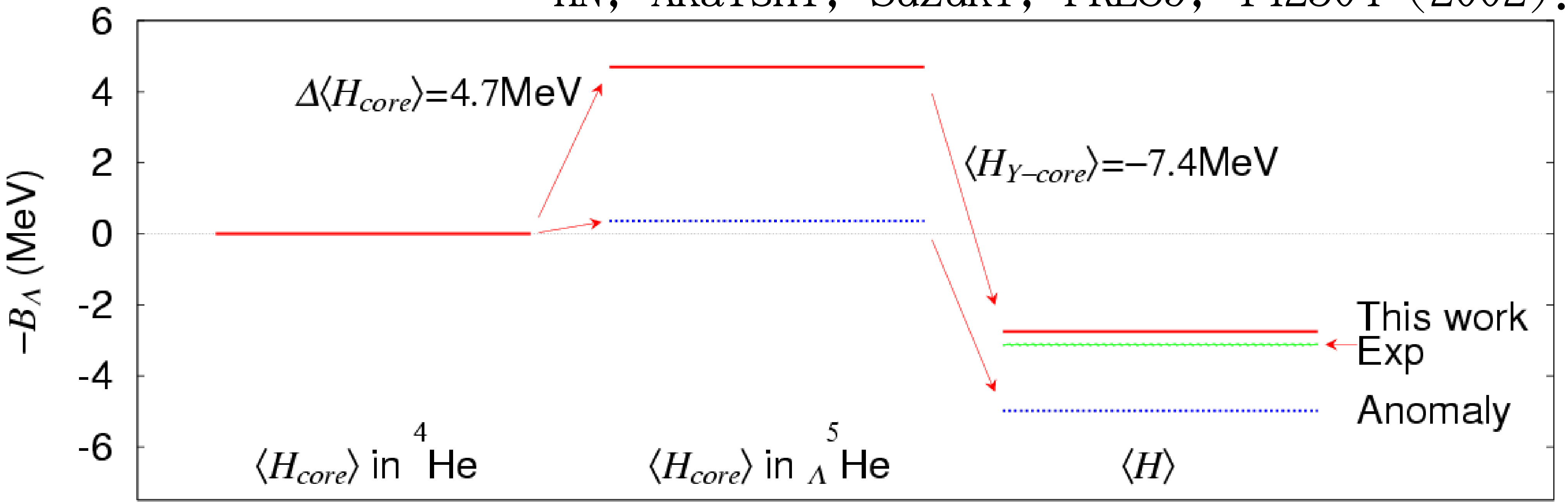
Paper available from [arXiv:1510.00903(hep-lat)]

Future work:
(II-1) Physical quantities including the binding energies of few-body problem of light hypernuclei with the lattice YN potentials

Backup slides

Rearrangement effect of ${}^{5}_{\Lambda}$He

HN, Akaishi, Suzuki, PRL89, 142504 (2002).



$$H = \sum_{i=1}^{A}\left(m_i c^2 + \frac{\boldsymbol{p}_i^2}{2\,m_i}\right) - T_{CM} + \sum_{i<j}^{A-1} v_{ij}^{(NN)} + \sum_{i=1}^{A-1} v_{iY}^{(NY)} = H_{core} + H_{Y-core} \quad ,$$

$$H_{core} = \sum_{i=1}^{A-1}\frac{\boldsymbol{p}_i^2}{2\,m_N} - \frac{\left(\sum_{i=1}^{A-1}\boldsymbol{p}_i\right)^2}{2(A-1)\,m_N} + \sum_{i<j}^{A-1} v_{ij}^{(NN)} = T_{core} + V_{NN} \,.$$

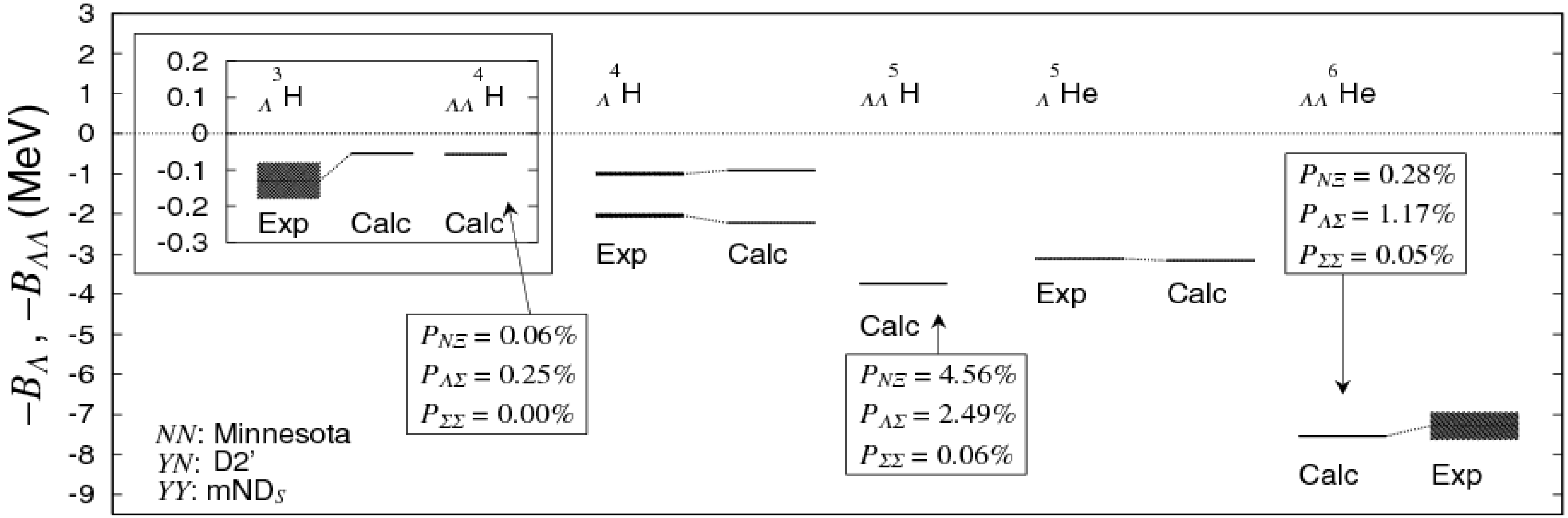


FIG. 1. Λ and $\Lambda\Lambda$ separation energies of $A = 3 - 6$, $S = -1$ and -2 s-shell hypernuclei. The Minnesota NN, D2′ YN, and mND_S YY potentials are used. The width of the line for the experimental B_Λ or $B_{\Lambda\Lambda}$ value indicates the experimental error bar. The probabilities of the $N\Xi$, $\Lambda\Sigma$, and $\Sigma\Sigma$ components are also shown for the $\Lambda\Lambda$ hypernuclei.

Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

[2] In this paper, we take a conventional choice of the baryon's interpolating field given in Eqs. (2), (8)–(9) which is expected to have large overlap with the single baryon's ground state. Utilising more general form of the baryon's interpolating field is straightforward. We may replace, for example, the baryon's interpolating field as

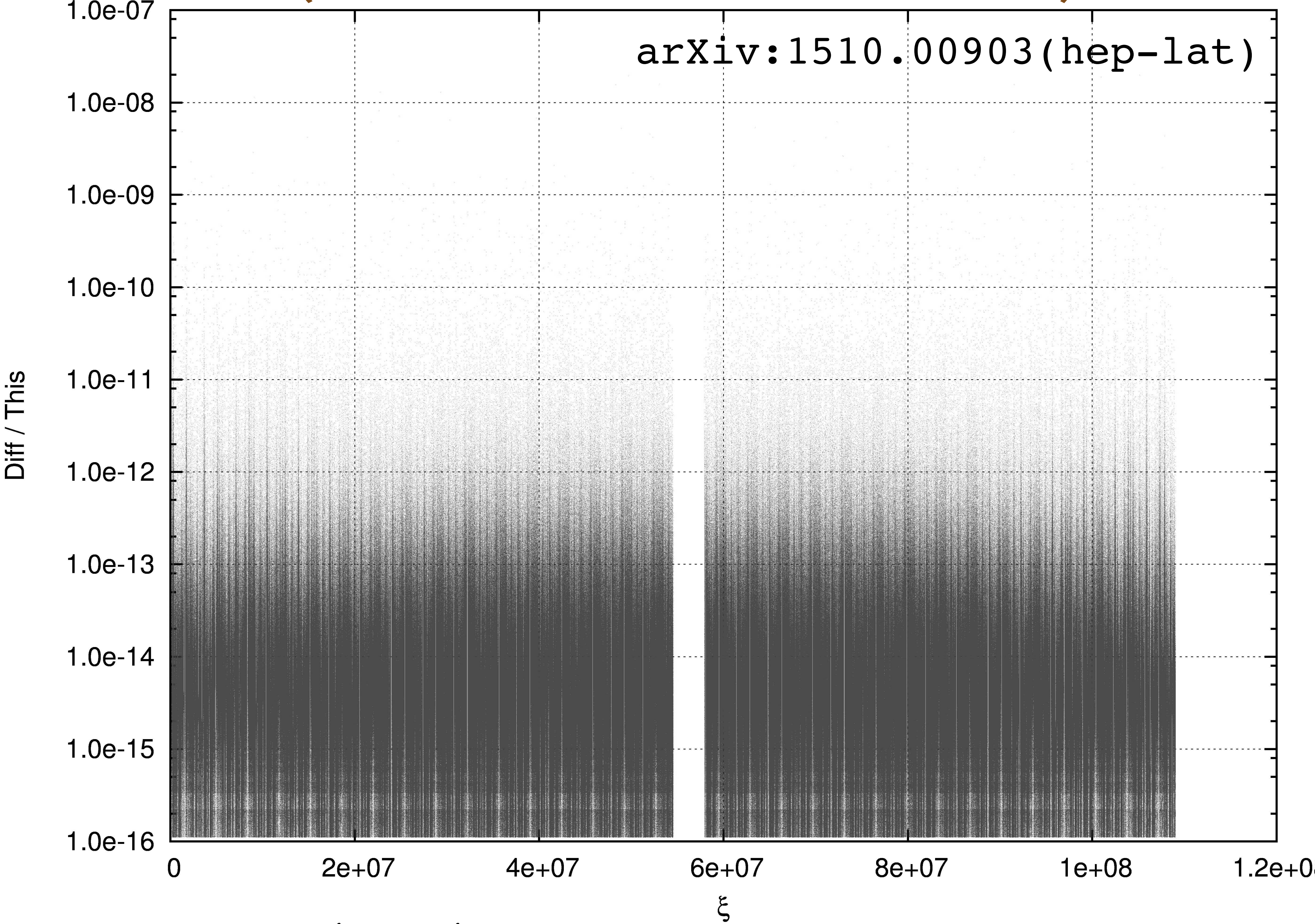
$$B_\gamma = \varepsilon_{abc}\left((q_{1,a}\Gamma_1 q_{2,b})\,\Gamma_2 q_{3,c}\right), \tag{11}$$

where q_1, q_2, and q_3 denote particular quark flavours to form baryon B and the set of gamma matrices $\{\Gamma_1, \Gamma_2\}$ is appropriately taken so as to carry the quantum numbers of baryon B with combined spinor-space-time subscript γ. Even for the general case, we can follow the procedure in this section with taking two replacements everywhere: (i) $(C\gamma_5)(\alpha,\alpha') \to \Gamma_1(\alpha,\alpha')$ and (ii) $\delta(\alpha,\alpha') \to \Gamma_2(\alpha,\alpha')$.

[3] In this paper, we focus on the $2+1$ flavour lattice QCD calculation for the study of the octet-baryon-octet-baryon interactions in the isospin symmetric limit. An extension to the other charge states than the channels given in Eqs. (34)–(38) is straightforward. Moreover, even though the system comprises decuplet baryons such as Ω^-'s, we can take Eq. (11) and the gamma matrices $\Gamma_1 = C\gamma_\ell$ and $\Gamma_2 = 1$ with spatial vector index ℓ.

arXiv:1510.00903(hep-lat)

Benchmark (from NN to XiXi channels)



$$\xi = \alpha + 2(\beta + 2(\alpha' + 2(\beta' + 2(x + 16(y + 16(z + 16(c + 52(t - t_0))))))))$$

TABLES

Fujiwara, et al., (2002)

TABLE I. The relationship between the isospin basis and the flavor-SU_3 basis for the B_8B_8 systems. The flavor-SU_3 symmetry is given by the Elliott notation $(\lambda\mu)$. $\mathcal{P}$ denotes the flavor exchange symmetry, and I the isospin.

S	B_8B_8 (I)	$\mathcal{P}=+1$ (symmetric)	$\mathcal{P}=-1$ (antisymmetric)
		^1E or ^3O	^3E or ^1O
0	NN $(I=0)$	–	(03)
	NN $(I=1)$	(22)	–
-1	ΛN	$\frac{1}{\sqrt{10}}[(11)_s+3(22)]$	$\frac{1}{\sqrt{2}}[-(11)_a+(03)]$
	ΣN $(I=1/2)$	$\frac{1}{\sqrt{10}}[3(11)_s-(22)]$	$\frac{1}{\sqrt{2}}[(11)_a+(03)]$
	ΣN $(I=3/2)$	(22)	(30)
-2	$\Lambda\Lambda$	$\frac{1}{\sqrt{5}}(11)_s+\frac{9}{2\sqrt{30}}(22)+\frac{1}{2\sqrt{2}}(00)$	–
	ΞN $(I=0)$	$\frac{1}{\sqrt{5}}(11)_s-\sqrt{\frac{3}{10}}(22)+\frac{1}{\sqrt{2}}(00)$	$(11)_a$
	ΞN $(I=1)$	$\sqrt{\frac{3}{5}}(11)_s+\sqrt{\frac{2}{5}}(22)$	$\frac{1}{\sqrt{3}}[-(11)_a+(30)+(03)]$
	$\Sigma\Lambda$	$-\sqrt{\frac{2}{5}}(11)_s+\sqrt{\frac{3}{5}}(22)$	$\frac{1}{\sqrt{2}}[(30)-(03)]$
	$\Sigma\Sigma$ $(I=0)$	$\sqrt{\frac{3}{5}}(11)_s-\frac{1}{2\sqrt{10}}(22)-\sqrt{\frac{3}{8}}(00)$	–
	$\Sigma\Sigma$ $(I=1)$	–	$\frac{1}{\sqrt{6}}[2(11)_a+(30)+(03)]$
	$\Sigma\Sigma$ $(I=2)$	(22)	–
-3	$\Xi\Lambda$	$\frac{1}{\sqrt{10}}[(11)_s+3(22)]$	$\frac{1}{\sqrt{2}}[-(11)_a+(30)]$
	$\Xi\Sigma$ $(I=1/2)$	$\frac{1}{\sqrt{10}}[3(11)_s-(22)]$	$\frac{1}{\sqrt{2}}[(11)_a+(30)]$
	$\Xi\Sigma$ $(I=3/2)$	(22)	(03)
-4	$\Xi\Xi$ $(I=0)$	–	(30)
	$\Xi\Xi$ $(I=1)$	(22)	–

Re

TABLE 8

Summary of the eigenvalues of the normalization kernel, the adiabatic potential V at $R=0$ due to the color magnetic interaction and the effective hard core radius r_c.

I	J	BB	Eigenvalue	$V(R=0)$ [MeV]	r_c [fm]
$\frac{1}{2}$	0	NΛ	1	381	0.44
		NΣ	$\frac{1}{9}$	303	0.72
$\frac{1}{2}$	1	NΛ	1	264	0.37
		NΣ	1	215	0.30
$\frac{3}{2}$	0	NΣ	$\frac{10}{9}$	391	0.40
$\frac{3}{2}$	1	NΣ	$\frac{2}{9}$	346	0.77
0	1	NΞ	$\frac{8}{9}$	93	0.29
1	0	NΞ	$\frac{4}{9}$	342	0.68
		$\Lambda\Sigma$	$\frac{6}{9}$	298	0.56
1	1	NΞ	$\frac{20}{27}$	219	0.53
		$\Lambda\Sigma$	$\frac{18}{27}$	245	0.57
		$\Sigma\Sigma$	$\frac{22}{27}$	95	0.33
2	0	$\Sigma\Sigma$	$\frac{10}{9}$	336	0.41

Oka, Shimizu and Yazaki (1987)

TABLE 5

Same as table 4 for $S=-2$ two-baryon system

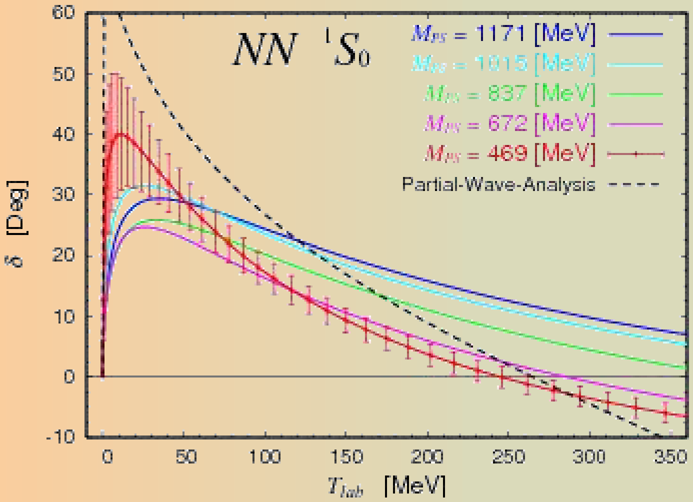
$S=-2$

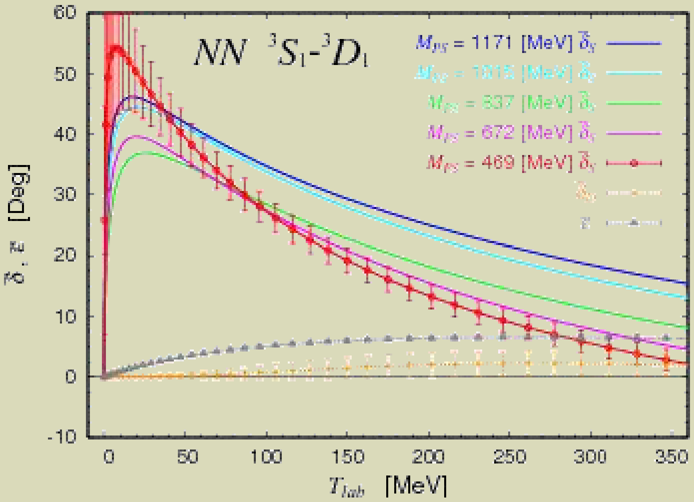
I	J	BB	Eigenvalues (uncoupled)	Eigenvalues (coupled)
0	0	$\Lambda\Lambda$	1	0 $\frac{10}{9}$ 2
		$N\Sigma$	$\frac{4}{3}$	
		$\Sigma\Sigma$	$\frac{7}{9}$	
0	1	$N\Xi$	$\frac{8}{9}$	
1	0	$N\Xi$	$\frac{4}{9}$	0 $\frac{10}{9}$
		$\Lambda\Sigma$	$\frac{2}{3}$	
1	1	$N\Xi$	$\frac{20}{27}$	$\frac{2}{9}$ $\frac{8}{9}$ $\frac{10}{9}$
		$\Lambda\Sigma$	$\frac{2}{3}$	
		$\Sigma\Sigma$	$\frac{22}{27}$	
2	0	$\Sigma\Sigma$	$\frac{10}{9}$	

Oka, Shimizu and Yazaki (1987)

Stochastic variational calculation of 4He with using a lattice potential

- For NN potential, we use the SU(3) potential at the lightest quark mass(m_ps = 469 MeV), which has been reported to have a 4N bound state (about 5.1MeV) within a tensor-included effective central potential; NPA881, 28-43 (2011).





Benchmark test calculation of a four-nucleon bound state,
Phys. Rev. C64, 044001 (2001).

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

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Spin–orbit force from lattice QCD

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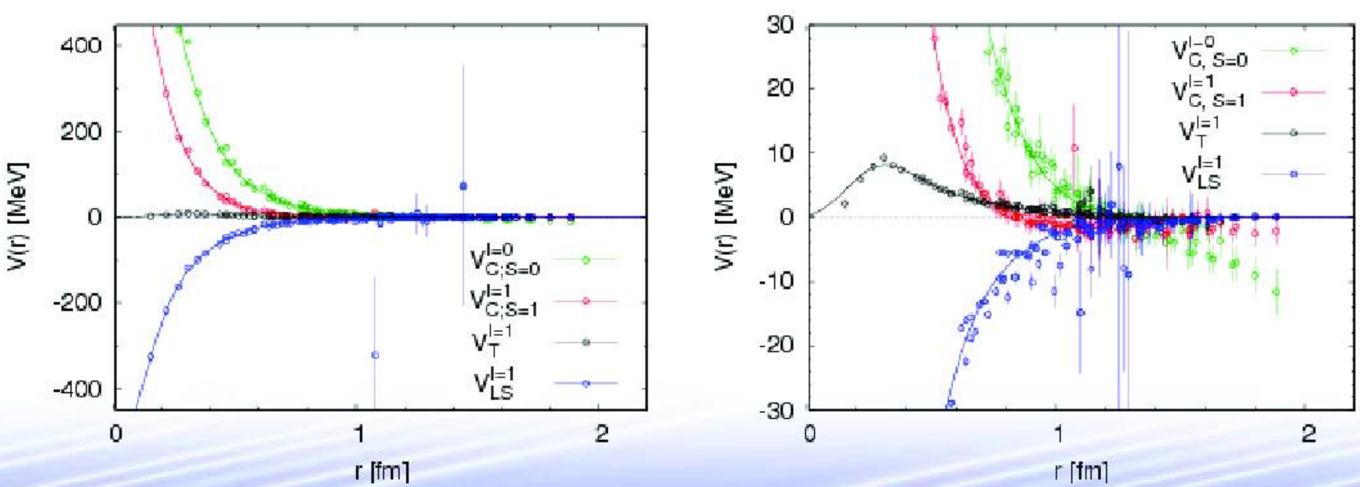
A B S T R A C T

We present a first attempt to determine nucleon–nucleon potentials in the parity-odd sector which appear in the $^1P_1$, $^3P_0$, $^3P_1$, $^3P_2$–$^3F_2$ channels, in $N_f = 2$ lattice QCD simulations. These potentials are constructed from the Nambu–Bethe–Salpeter wave functions for $J^P = 0^-$, 1^- and 2^-, which correspond to the A_1^-, T_1^- and $T_2^- \oplus E^-$ representation of the cubic group, respectively. We have found a large and attractive spin–orbit potential $V_{LS}(r)$ in the isospin-triplet channel, which is qualitatively consistent with the phenomenological determination from the experimental scattering phase shifts. The potentials obtained from lattice QCD are used to calculate the scattering phase shifts in the $^1P_1$, $^3P_0$, $^3P_1$ and $^3P_2$–$^3F_2$ channels. The strong attractive spin–orbit force and a weak repulsive central force in spin-triplet P-wave channels lead to an attraction in the $^3P_2$ channel, which is related to the P-wave neutron paring in neutron stars.



Inoue san's NN potential

- Central and spin-oribt potentials

$$V_{C,LS}(r) = V_1 \exp(-\alpha_1 r^2) + V_2 \exp(-\alpha_2 r^2) - V_3 (1 - \exp(-\alpha_3 r^2))^2 (\exp(-\alpha_4 r)/r)^2$$

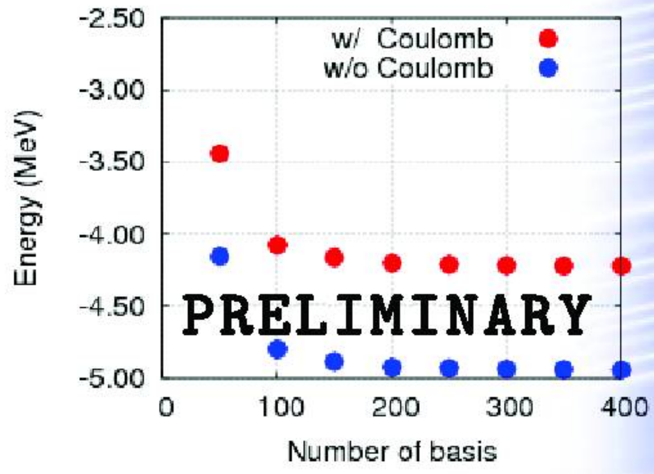
- Tensor potential

$$V_T(r) = V_1 (1 - \exp(-\alpha_1 r^2))^2 \left(1 + \frac{3}{\alpha_2 r} + \frac{3}{(\alpha_2 r)^2}\right) \frac{\exp(-\alpha_2 r)}{r} + V_2 (1 - \exp(-\alpha_3 r^2))^2 \left(1 + \frac{3}{\alpha_4 r} + \frac{3}{(\alpha_4 r)^2}\right) \frac{\exp(-\alpha_4 r)}{r}$$

Results of few-body calculation

☆ Inputs:

- m=1161.0 MeV,
- hbar c = 197.3269602 MeV fm
- hbar c/e^2 = 137.03599976
- V_NN consists of AV8 type operators, determined from {1S0, 3S1, 3SD1, 1P1, 3P0, 3P1, 3PF2}.
  - V_0, V_σ, V_τ, $V_{\sigma\tau}$, V_T, $V_{T\tau}$, V_{LS}^{odd} are determined



☆ Preliminary results:

- B(4He)=4.23 MeV (w/ Coulomb) (old: 4.37MeV)
  - Probabilities of (S, P, D) waves = (98.8%, 0.002%, 1.2%)
  - cf. roughly speaking (S,P,D)~(<90%, <0.1%, >10%) for a realistic NN force
- B(4He)=4.95 MeV (w/o Coulomb) (old: 5.09MeV)
  - Probabilities of (S, P, D) waves = (98.8%, 0.002%, 1.2%)