Non-Local effective SU(2) Polyakov loop model from inverse Monte-Carlo methods

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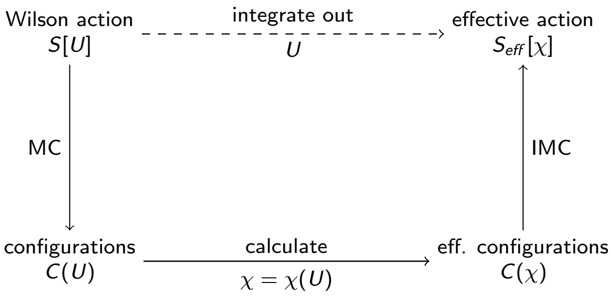
Content

Motivation

- QCD difficult to solve. Sign problem for finite μ
- There are many ideas to deal with this, e.g. effective models
- Sign problem in Polyakov loop models are expected to be less severe
- Problem: How do we get the effective action from a known action?

Theory

Inverse Monte-Carlo Method I



Inverse Monte-Carlo Method II

- Take an effective action $S_{eff}(\lambda)$ with yet to find coupling constants λ
- Remember how DSEs are derived

$$\left\langle \frac{\delta S}{\delta \varphi}(\lambda) \right\rangle_{eff} = 0$$

- Demand that the effective theory approximates the full theory well, i.e.

$$\left\langle \frac{\delta S}{\delta \varphi}(\lambda) \right\rangle_{full} = 0$$

- Solve this equation numerically for λ

Geometric Ward-Identities I

- Left invariance of the Haar measure yields the (mathematical) identity

$$\int d\mu(g)(L_a f)(g) = 0, f \in L_2(G).$$

- For class functions F, $\tilde{F}$ we obtain

$$\int d\mu_{\text{red}} \underbrace{\vec{L}\cdot(F\vec{L}\tilde{F})}_{\text{class function}} = \int d\mu_{\text{red}}(F\vec{L}^2\tilde{F} + \vec{L}F\cdot\vec{L}\tilde{F}) = 0$$

Geometric Ward-Identities II

- Use character expansion for class functions

$$F(g) = F(\chi_1(g), ..., \chi_r(g)), \qquad r = rank(G)$$
$$L_a F(\chi) = \sum_q \frac{\partial F(\chi)}{\partial \chi_q(g)} L_a \chi_q(g)$$

- Set $\tilde{F} = \chi_p$, with $p \in \{1, ..., r\}$
- Use

$$\chi_\mu \chi_\nu = \sum_\lambda C^\lambda_{\mu\nu} \chi_\lambda, \qquad \sum_a L_a^2 \chi_p(g) = -c_p \chi_p(g)$$

Geometric Ward-Identity

$$0 = \int_G d\mu_{red} \left\{ \frac{1}{2} \sum_q \underbrace{\left[(c_p + c_q)\chi_p \chi_q - \sum_\lambda C^\lambda_{\mu\nu} c_\lambda \chi_\lambda \right]}_{=:K_q} \frac{\partial F(\chi)}{\partial \chi_q(g)} - c_p \chi_p(g) F \right\}$$

Geometric DSEs

- Insert $\exp(-S_{eff})$ and take sum over all lattice points

$$V^{-1}\sum_{i\in L}\left\langle \frac{1}{2}\sum_{q} K_{q,i}\frac{\partial F_i}{\partial \chi_{q,i}}\exp(+S_{eff}) - c_p\chi_{p,i}F_i\exp(+S_{eff})\right\rangle_{eff} = 0$$

- Take this DSE for IMC-method to calculate $\vec{\lambda}$ via

Geometric DSEs

$$V^{-1}\sum_{i\in L}\left\langle \frac{1}{2}\sum_{q} K_{q,i}\frac{\partial \vec{F}_i}{\partial \chi_{q,i}}\exp(+S_{eff}(\vec{\lambda})) - c_p\chi_{p,i}\vec{F}_i\exp(+S_{eff}(\vec{\lambda}))\right\rangle_{full} = 0$$

(Need $\dim(\vec{F}) = \dim(\vec{\lambda})$ different class-functions F_i to solve the equations for $\vec{\lambda}$)

Logarithmic SU(2) Polyakov Models

- Integrating out all spatial links and applying the strong coupling expansion yields

The linear Polyakov model

$$S = \sum_{p} \sum_{r} \sum_{<i,j>=r} \lambda_{p,r} \chi_{p,i} \chi_{p,j},$$

- Expanding the action term and applying a resummation of higher order terms yields

The logarithmic Polyakov model

$$S = -\sum_{p} \sum_{r} \sum_{<i,j>=r} \log \left(1 + g_{p,r} \chi_{p,i} \chi_{p,j}\right)$$

[J. Langelage, S. Lottini, O. Philipsen 2010]

Geometric DSEs for the Logarithmic Polyakov Model

- Neglecting terms with $r \geq r_{max}$, and represenations with $p \geq p_{max}$ yields

$$e^{-S} = \prod_{p=1}^{p_{max}} \prod_{r=1}^{r_{max}} \prod_{<i,j>=r} \exp\left(-\lambda_{p,r}\chi_{p,i}\chi_{p,j}\right),$$

$$e^{-S} = \prod_{p=1}^{p_{max}} \prod_{r=1}^{r_{max}} \prod_{<i,j>=r} \left(1 + g_{p,r}\chi_{p,i}\chi_{p,j}\right),$$

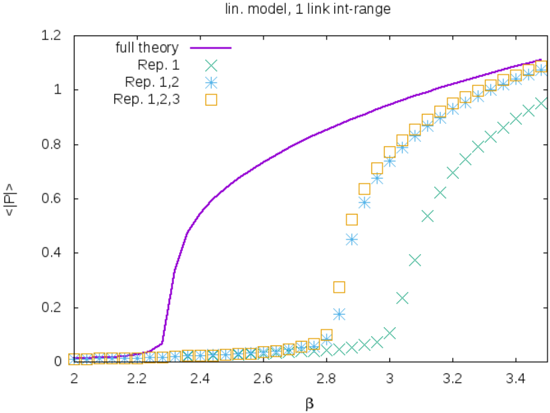
- Insert into geometric DSE and set $\vec{F}_i = \vec{f}_i \exp(-S_{eff})$, with $\vec{f}_i = \{f_{p,r,i}\}$

$$f_{p,r,i} = \frac{1}{g_{p,r}} \frac{\partial(e^{-S})_{p,r,i}}{\partial\chi_{1,i}}$$

- Now match the effective model to the full theory

NUMERIC RESULTS

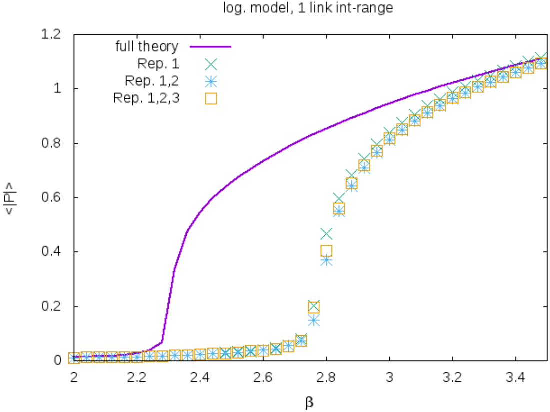
Local Polyakov Models: Linear vs. Logarithmic



Lattice: $20^3 \times 4$, Configs: 10,000

- Lin. model improves if we add represenations

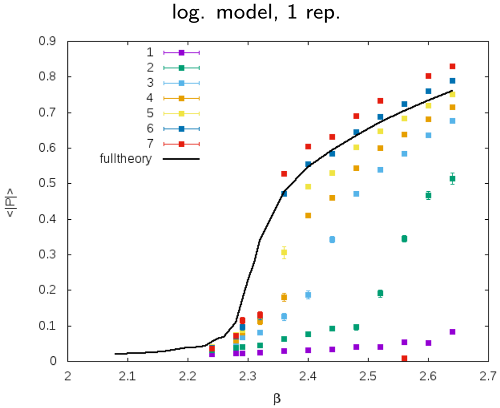
Local Polyakov Models: Linear vs. Logarithmic



Lattice: $20^3 \times 4$, Configs: 10,000

- Log. resummation seems to work quite well. No higher represenations needed.
- For small β the log. resummation is expected to improve results.
- Still far from the full theory for large β. → Try non-local models

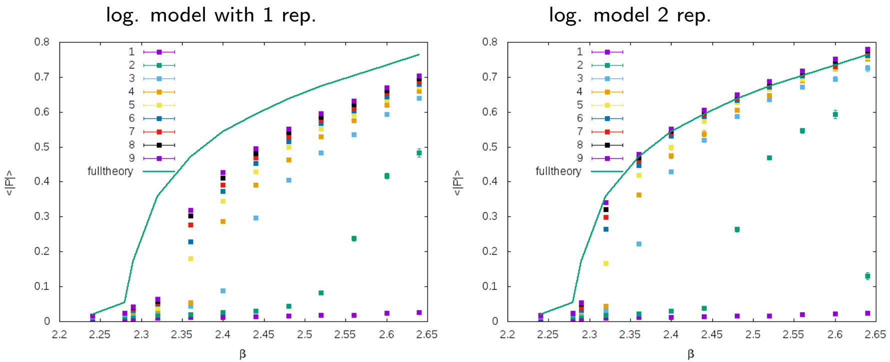
Non-local Logarithmic Polyakov Model



$16^3 \times 4$ lattice, 1000 configs

- Adding larger distances for the interaction improves the result.
- We “overshoot” when we include too large distances.

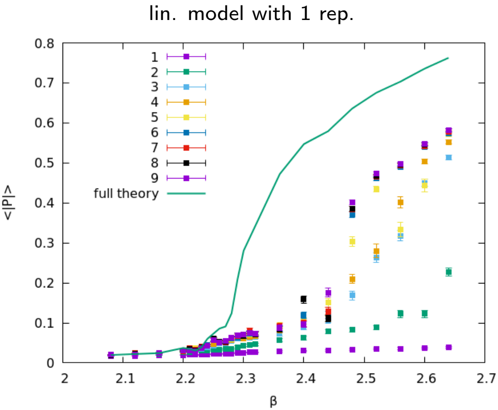
Non-local Logarithmic Polyakov Model



$32^3 \times 4$ lattice, 1000 configs

- Larger Lattice seems to fix overshooting.
- But: Higher represenations change the result. We overshoot again.
- Logarithmic resummation not doing well for large β (many non-local terms)

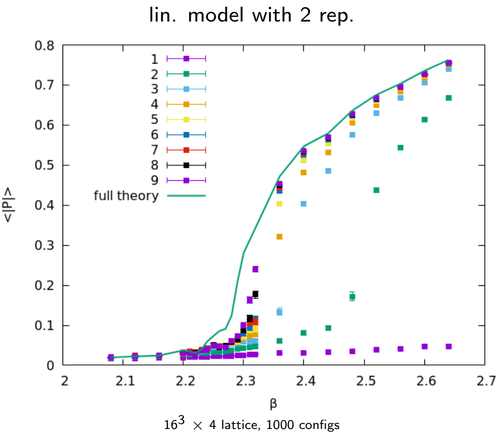
Non-local Linear Polyakov Models



$16^3 \times 4$ lattice, 1000 configs

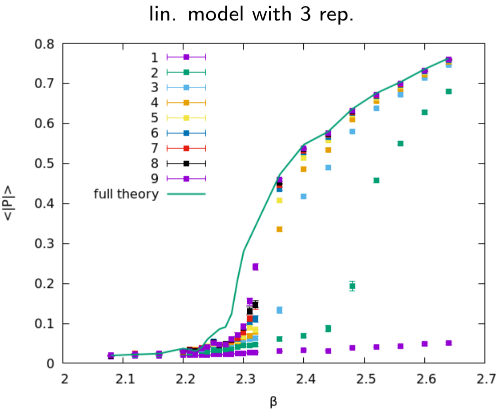
- Linear model does not overshoot, even on the smaller lattice.

Non-local Linear Polyakov Models



- We get close to the full theory with 2 represenations.

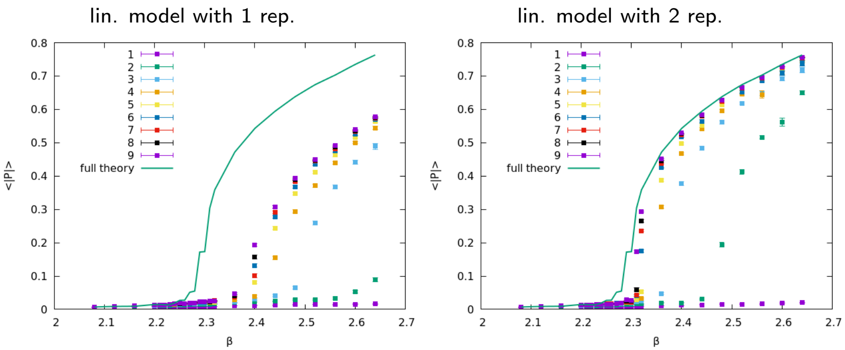
Non-local Linear Polyakov Models



$16^3 \times 4$ lattice, 1000 configs

- Adding more rep. seems not to spoil the result. Still close to full theory.
- Approaches the full theory very slowly near β_c (large correlation length)

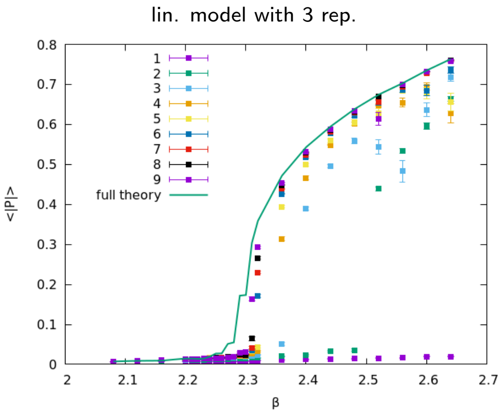
Non-local Linear Polyakov Models



$32^3 \times 4$ lattice, 1000 configs

- Same result for larger lattice. No overshooting. Close to full theory with 2 rep.

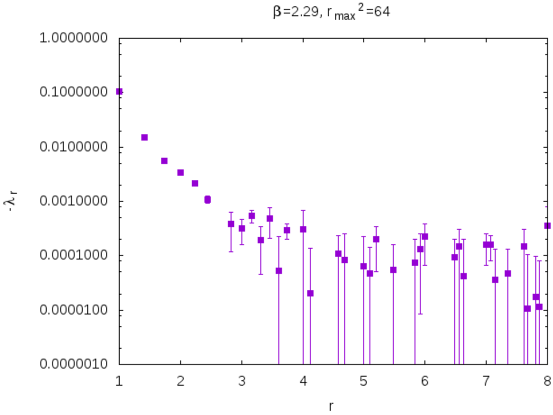
Non-local Linear Polyakov Models



$32^3 \times 4$ lattic, 1000 configs

- Higher represenations seem not to spoil the result.
- Adding non-local terms for large β works much better than for log. model
- Approaches the full thery very slowly around β_c

Long-Distance Behaviour of Non-Local Couplings



$16^3 \times 4$ lattice, 1000 configs

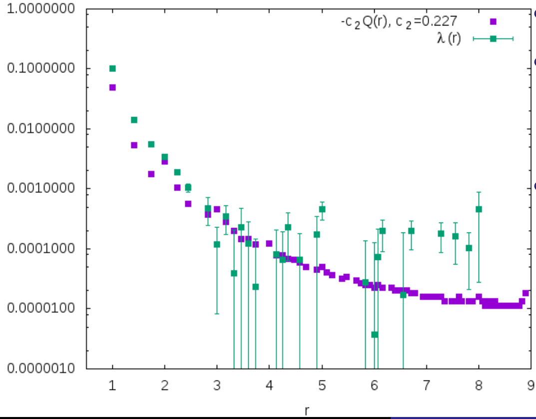
- Look at long-distance behaviours of couplings to make model predictable and compare to analytical models

Compare to Greensite's and Langfeld's analytical model

$$S = c_o \sum_x P_x - \frac{1}{2} c_1 \sum_x P_x^2 - 2c_2 \sum_{x,y} P_x Q(x-y) P_y,$$

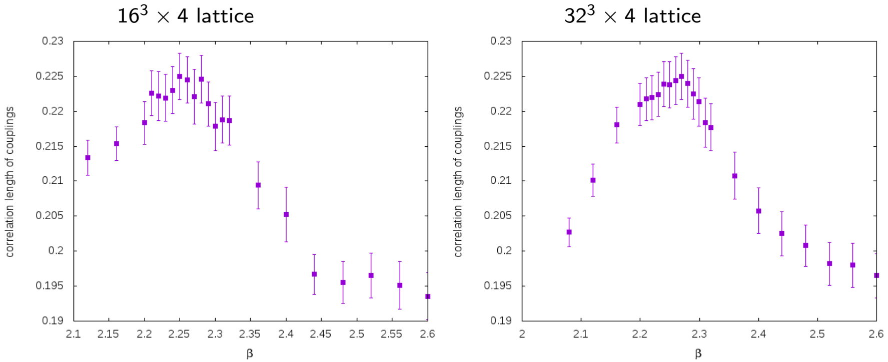
$$Q(x-y) = \begin{cases} (\sqrt{-\nabla^2})_{xy} & |x-y| \le r_{max} \\ 0 & |x-y| > r_{max} \end{cases}$$

$\beta = 2.22$



- Shape agrees quite well.
- Fitted coupling $c_2 \approx 0.227(53)$ does not agree with prediction of 0.491(1)
- Maybe if we insert addtional terms (linear P-term)?

Long-Distance Behaviour of Non-Local Couplings



lin. model with 1 rep.

- Fitting linear part and extracting "correlation length" yields peak around $\beta_c \approx 2.29$
- Dependence seems not to scale with the volume
- $\rightarrow$ Might suggest model becomes local again in the contiuum

Conclusion

- IMC-method works well to fix theories
- Logarithmic resummation does not work well for $\beta \gtrapprox \beta_c$
- Non-local linear Polyakov model seems to work well for $\beta > \beta_c$.
- Difficult around β_c. Need more non-local terms.
- Model might become local in the contiuum limit

Outlook

- Improvements around β_c
- Maybe add addtional terms in our ansatz (linear polyakov term)
- Check larger lattices and contiuum limit
- Add fermions
- Other gauge groups ($SU(3)$, G_2)

END

THANK YOU FOR YOUR ATTENTION