Simulation of SU(2) gauge theory with improved domain-wall fermions

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Introduction

- SU(2) gauge theory
  - Many works on confinement mechanism, finite temperature/density
  - Beyond standard model: technicolor, conformal window, dark matter
    - F. Bursa et al. (2011), H.Ohki et al. (2010), Lewis, Pica, Sannino (2012) T. Karavirta et al. (2012), M. Hayakawa et al. (2013)
- Chiral dynamics depending on gauge group and fermion repr.
  - Different symmetry breaking pattern

Fundamental	Adjoint
SU(2): SU(2Nf) → Sp(2Nf) SU(N) N>2: SU(Nf)xSU(Nf) → SU(Nf)	SU(N): SU(2Nf) → SO(2Nf)

  - Dependence on number of flavors
  - Finite temperature/density
  - Eigenvalue distribution – comparison to random matrix theory
- Chiral symmetric fermion is better device
  - Overalp: best symmetry, high numerical cost, involved setup (Aoki phase, etc.) H.M., Kikukawa, Yamada, Nagai, Lattice 2010, 2009
  - Domain-wall: good properties, numerically feasible
    - Approaches to overlap with large Ns
    - Residual mass probes explicit chiral symmetry violation

Domain-wall
- Topology changes
- Large lattice is possible

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Improved domain-wall

Mesurement:
- Spectroscopy
- Eigenvalue/vectors, etc.

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Ovelap
- Fixed topology
- p-regime/ε-regime

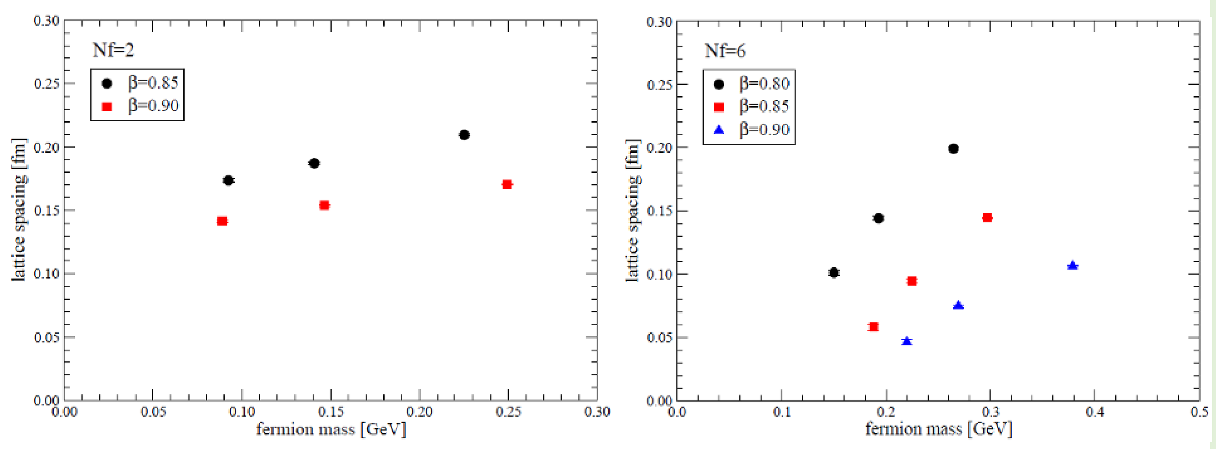
Previous result

H.Matsufuru, K-i Nagai, N.Yamada, Lattice 2013, 2014, 2015

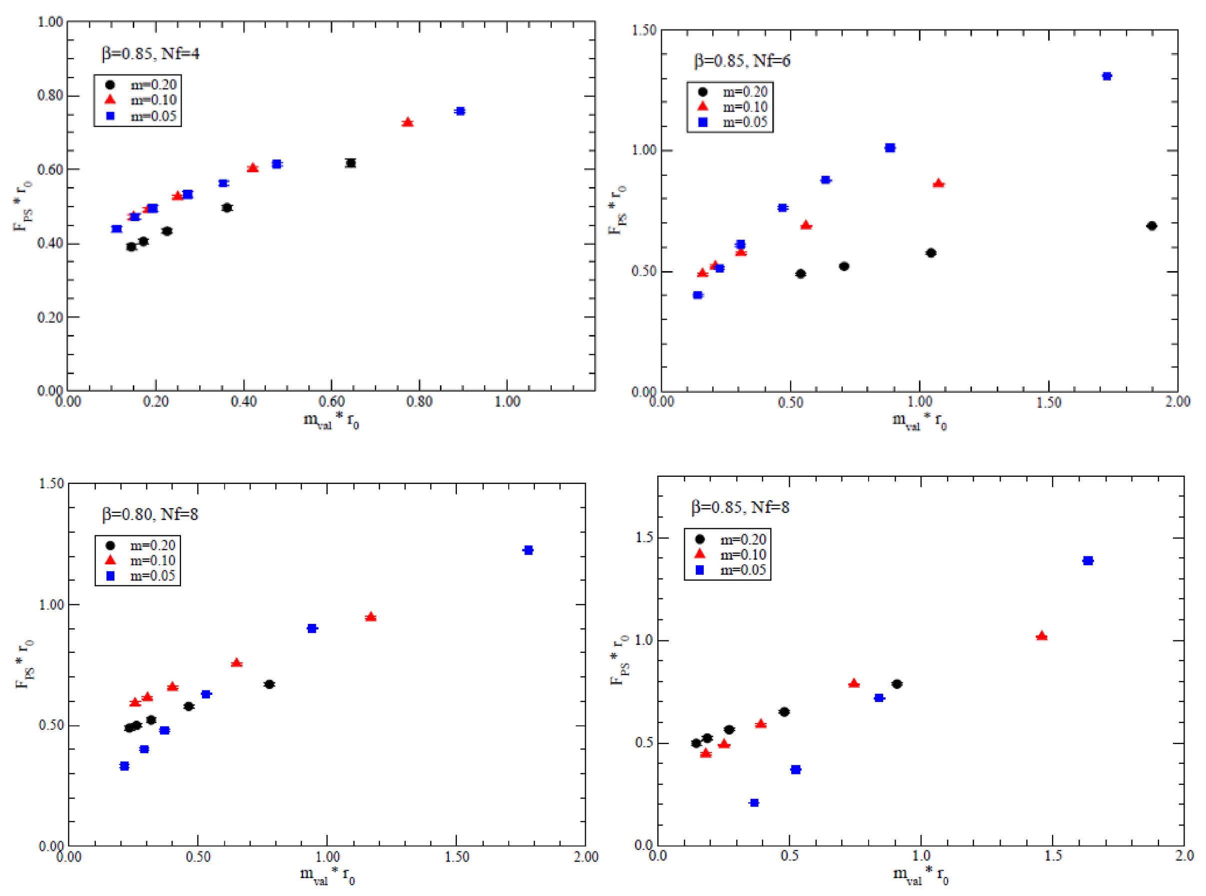
- Lattice actions:
  - Iwasaki gauge action
  - Standard domain-wall fermions: Nf=2, 4, 6, 8
- Lattice size: 16^3x32, Ns=16
- Survey of Nf-dependence with fixed setup

Nf	beta	m
2	0.85, 0.90	0.20, 0.10, 0.05
4	0.85, 0.90	0.20, 0.10, 0.05, (0.03)
6	0.80, 0.85, 0.90	0.20, 0.10, 0.05
8	0.80, 0.85	0.20, 0.10, 0.05

- Static potentail
  - Extrapolation to massless limit seems successful for Nf=2, while not for Nf=6 and 8.
  - Confining feature disappears as massless limit



- Decay constant (scaled by Sommer's scale r_0)
  - Nf=4: approaches to finite value as valence mass goes to zero
  - Nf=6: sea fermion mass dependence becomes larger
  - Nf=8: no finite massless limit



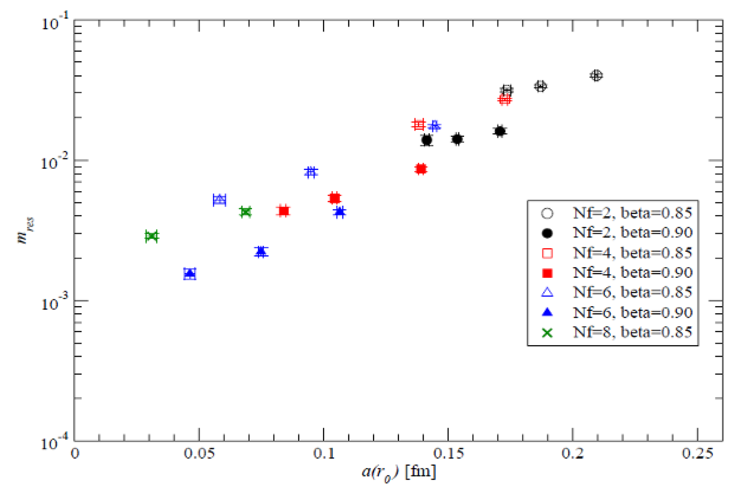
Improved fermion

- Standard domain-wall fermion action

$$S_{DW} = \sum_{x,s}\bar\psi(x,s)D_W(x,y;-M_0)\psi(y,s) -\frac{1}{2}\sum_{x,s}\bar\psi(x,s)\left[(1-\gamma_5)\psi(x,s+1)+(1+\gamma_5)\psi(x,s-1)-2\psi(x,s)\right] +m\left[\bar\psi(x,1)P_R\psi(x,L_s)+\bar\psi(x,L_s)P_L\psi(x,1)\right]$$

$$D_W(x,y;M) = M\delta_{x,y} - \frac{1}{2}\sum_{\mu=1}^{4}\left\{(1-\gamma_\mu)U_\mu(x)\delta_{x+\hat\mu,y}+(1+\gamma_\mu)U_\mu^\dagger(x-\hat\mu)\delta_{x-\hat\mu,y}-4\delta_{x,y}\right\}$$

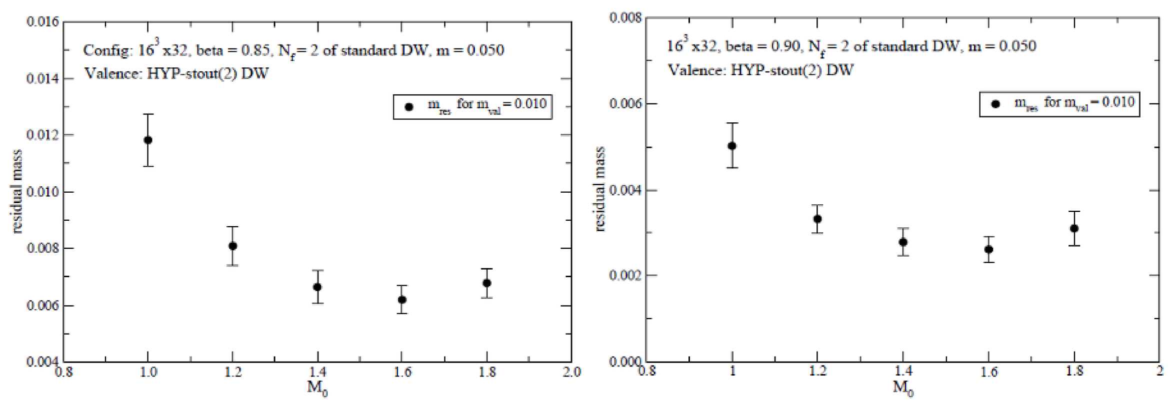
  - M_0: domain-wall height, m: fermion mass
  - Ls: extent of 5-th direction
  - Boundary conditions: $P_R\psi(s=0)=P_L\psi(s=L_s+1)=0$
  - 4D fermion field: $q(x) = P_L\psi(x,s=1)+P_R\psi(x,s=L_s)$, $\bar q(x) = \bar\psi(x,s=1)P_R+\bar\psi(x,s=L_s)P_L$
- For more detailed analyses, small mass region must be explored
  - Rahter large residual mass → improved fermion action is necessary



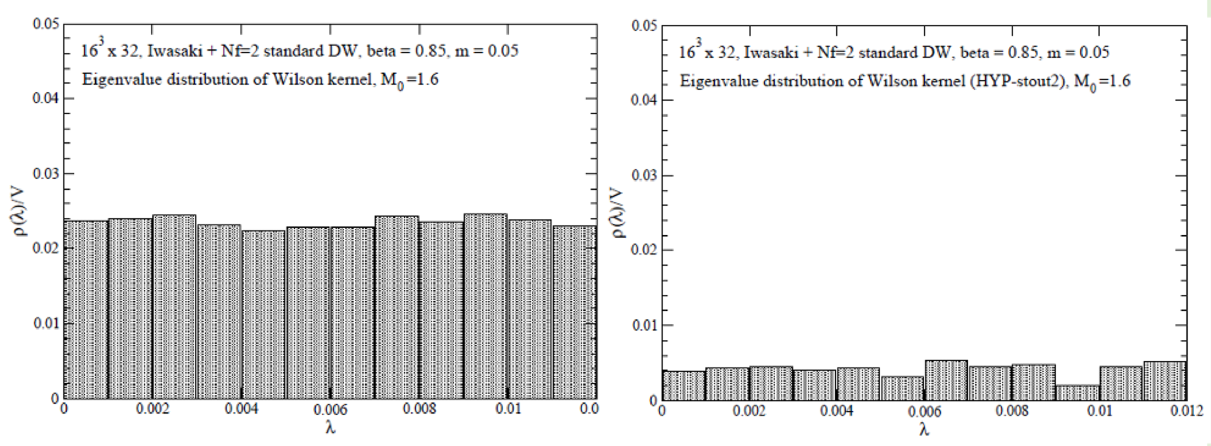
- HYP-stout (HEX)
  - HYP-stout link smearing is twice applied
    Hasenfratz and Knechtli (2001)
    Morningstar and Peardon (2004)
- Optimal domain-wall
  - Equivalent to overlap fermion (except for low-lying mode og H_W)
    T-W Chiu (2003)

On previous configuration

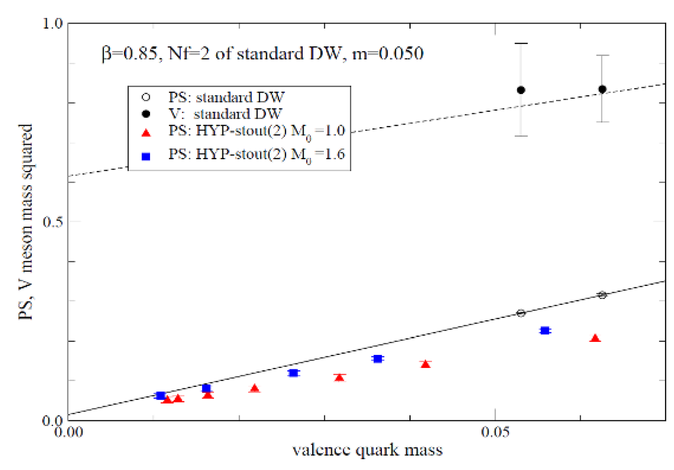
- Previously generated configs.: 16^3 x 32 lattice, Nf=2 DW
  - Valence HYP-stout(2) DW
- Residual mass
  - Reduced about 5 times
  - Optimal around M_0=1.6



- Eigenvalue distribution of H_W
  - Link smearing reduces low-lying modes several times
  - Well explains reduction of residual mass

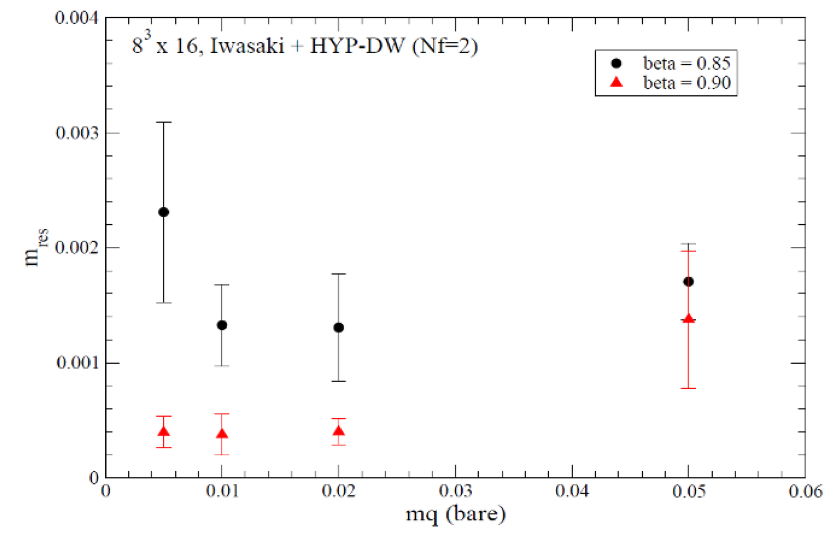


- Spectroscopy is underway

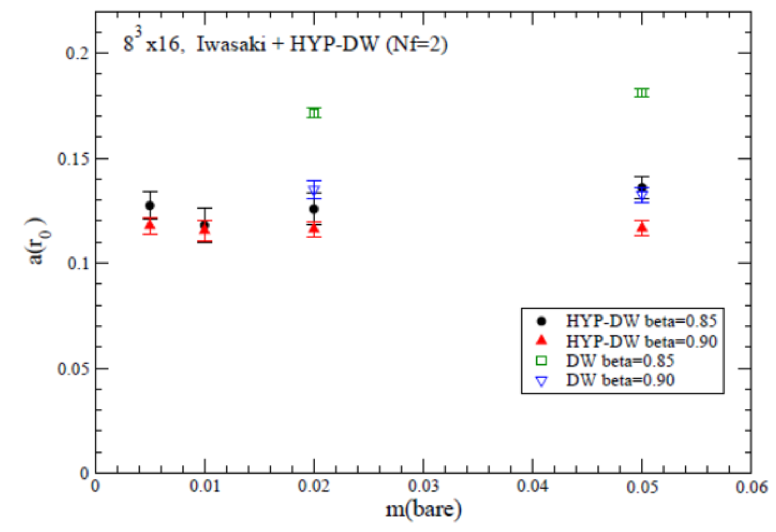


Dynamical simulations

- Dynamical simulations with HYP-stout(2) underway
  - Parameter search at Nf=2 on 8^3 x 16 lattice
  - Nf=2 on 16^3 x 32 lattice
  - Extension to Nf > 2 is planned
- Residual mass
  - Small residual mass is verified



- Static potential
  - HYP-stout (2) gives slightly smaller lattice spacing than standard DW.
  - At Nf=2, mass dependence is small



Outlook

- Improved DW fermions
  - HYP-stout DW significantly reduces residual mass
  - Application of optimal domain-wall is underway
  - Smaller mass region than standard DW is able to explore
  - Application to Nf > 2 and adjoint fermion are underway
- Toward ε-regime simulation
  - $1/m_\pi$ > L > $1/F_\pi$ required
  - HYP-stout(2) DW may give sufficiently small mass, while to be confirmed in practice
  - Optimal DW would be employed in the vicinity of m=0

Resources/environment

- Machines
  - Hitachi SR16000, IBM Blue Gene/Q at KEK
  - φ at KMI, Nagoya Univ.
- Code:
  - Bridge++ (C++)
- JLDG (Japan Lattice Data Grid)
  - For fast data transfer and unifined data management