Strangeness, charm and beauty at finite temperature

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for the Wuppertal-Budapest collaboration

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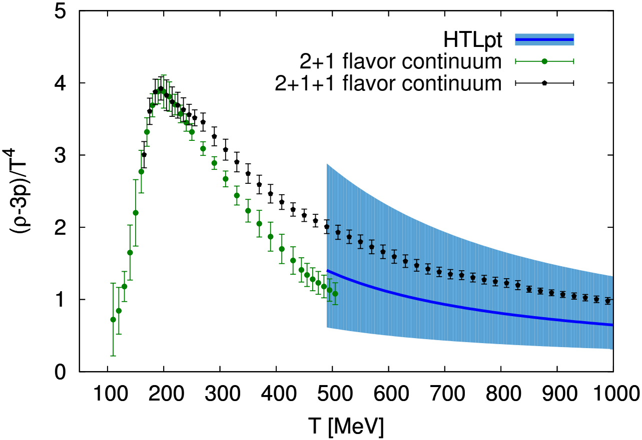
EoS: 2+1 vs 2+1+1 flavour

Continuum extrapolated equation of state with dynamical charm, physical quark masses

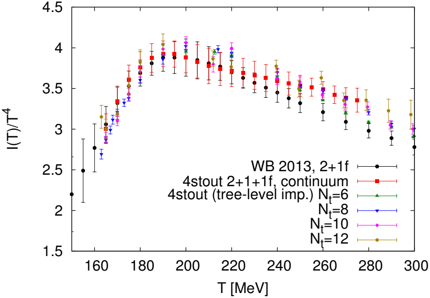


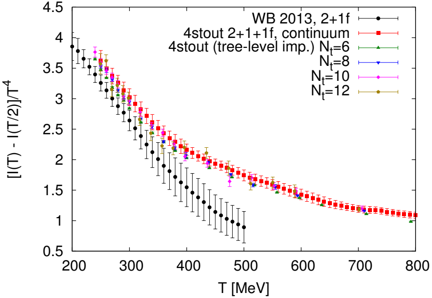
[Wuppertal-Budapest: 1606.07494]

$2+1+1$ flavor equation of state – lattice data

For low temperatures $I(T)/T^4$ is calculated using vacuum subtraction.

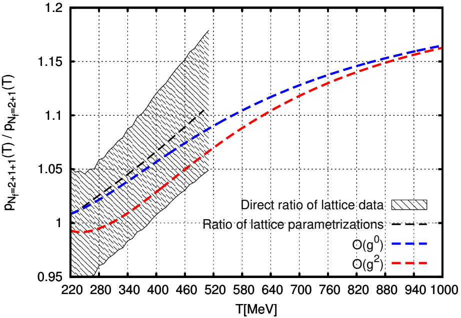
For higher temperatures $[I(T)-I(T/2)]/T^4$ is calculated and continuum extrapolated.





The final trace anomaly is calculated from the continuum extrapolated terms using the formula

$$\frac{I(T)}{T^4} = \sum_{k=0}^{n-1} 2^{-4k}\,\frac{I(T/2^k) - I(T/2^{k+1})}{(T/2^k)^4} + 2^{-4n}\,\frac{I(T/2^n)}{(T/2^n)^4}$$



The tree level formula gives a very good approximation:

$$\frac{p^{(3+1)}(T)}{p^{(3)}(T)} = \frac{SB(3) + F_Q(m_c, T)}{SB(3)}$$

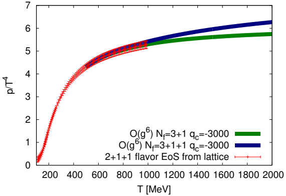
where $F_Q(m, T)$ is dimensionless free energy density of a free massive quark. [Laine&Schröder hep-ph/0603048]

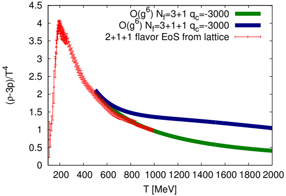
Perturbative parametrization at high temperatures

- Kajantie et al: [hep-ph/0211321]

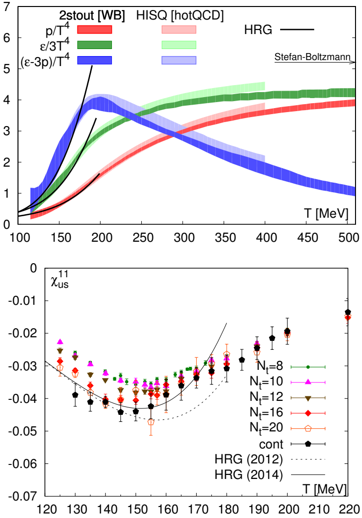
$$\frac{p}{T^4} = \# + \#g^2 + \#g^3 + \#g^4 + \#g^4 \log(g) + \#g^5 + \#g^6 \log(g) + ?g^6$$

- To account for the mass of the charm we use a tree level threshold formula tested in the previous slide.
- The g^6 term has a non-perturbative coefficient. Taking $N_f = 4$ we can fit this to our lattice data, we get $-3200 < q_c < -2700$.
- The fit describes the pressure **and** trace anomaly from 500 MeV.
- Next we can introduce the bottom quark treshold keeping q_c fixed.





Hadron Resonance Gas vs. Lattice



These result: [Wuppertal-Budapest 1112.4416, 1309.5258, 1507.04627] see also [HotQCD 1203.0784,1407.6387]

The Hadron Resonance Gas model

Hadrons are free particles in a heat bath, their interactions are introduced by adding all their resonances to the heat bath, as free particles. [Dashen, Ma, Bernstein 1969]

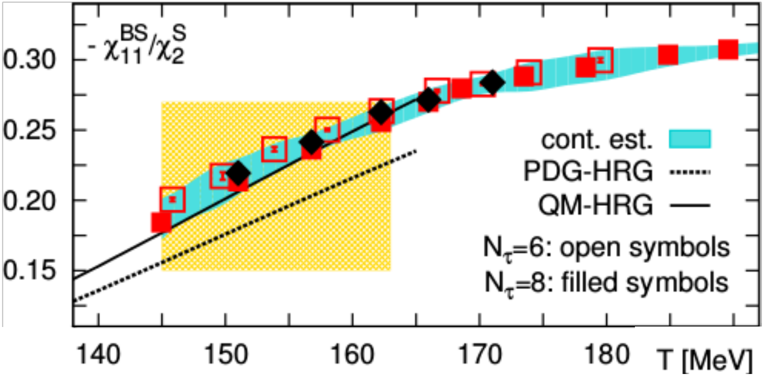
$$\frac{p^{\mathrm{HRG}}}{T^4} = \frac{1}{VT^3} + \sum_{i\in\mathrm{mesons}} \log \mathcal{Z}^M(T, V, m_i, \{\mu\}) + \sum_{i\in\mathrm{baryons}} \log \mathcal{Z}^B(T, V, m_i, \{\mu\})$$

$$\begin{aligned} \log \mathcal{Z}^{M/B}_{m_i} &= \mp \frac{Vd_i}{2\pi^2} \int_0^\infty dk k^2 \log\left(1 \mp z_i e^{-\sqrt{m_i^2+k^2}/T}\right) \\ &= \frac{VT^3}{2\pi^2} d_i \frac{m_i^2}{T^2} \sum_{k=1}^{\infty} (\pm)^{k+1} \frac{z_i^k}{k^2} K_2(km_i/T) \end{aligned}$$

with the fugaciy factor $z_i = \exp(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S)$ and $\hat{\mu} = \mu/T$. For $m_i \gg T$ (e.g. baryons) only the $k = 1$ term is relevant (Boltzmann approximation). The contribution of a particle and antiparticle combined:

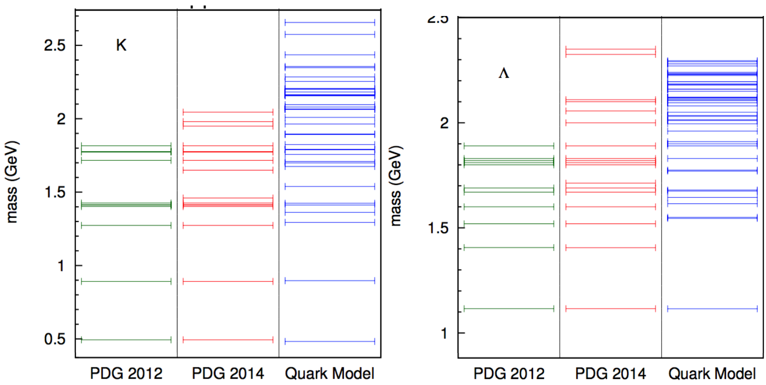
$$\log \mathcal{Z}^{M/B}_{m_i} \approx 2\frac{VT^3}{2\pi^2} d_i \frac{m_i^2}{T^2} K_2(m_i/T) \cosh(B_i\hat{\mu}_B + Q_i\hat{\mu}_Q + S_i\hat{\mu}_S)$$

For some combinations, though, the HRG result did not completely describe the data. [BNL-Bielefeld 1404.6511]

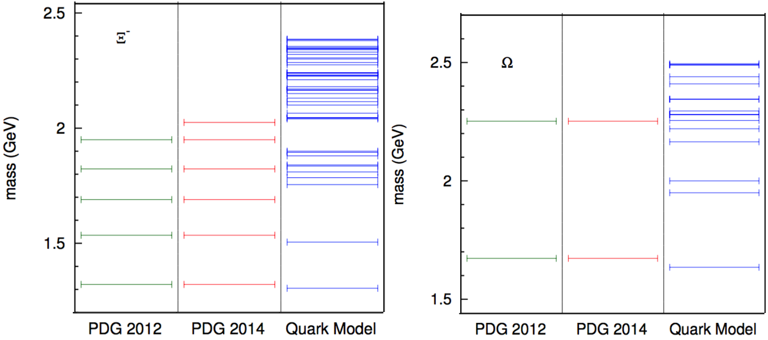


For strange baryons the HRG description does not seem to work.
Idea: it is not the HRG approach that fails, but our knowledge of the resonances is incomplete.

New states appeared in the 2014 edition of the PDG.



But not as many in the $|S| = 2$ and $|S| = 3$ sectors.



Can finite temperature lattice field theory be used to predict extra resonances?

In the Boltzmann approximation and $\mu_Q = 0$ the pressure can be simply written as as sum:

$$\begin{aligned} p^{\mathrm{HRG}}(\hat{\mu}_B, \hat{\mu}_S) \;&=\; p^{\mathrm{mesons}}_{S=0} + p^{\mathrm{baryons}}_{S=0} \cosh(\hat{\mu}_B) \\ &+ p^{\mathrm{mesons}}_{|S|=1} \cosh(\hat{\mu}_S) + p^{\mathrm{baryons}}_{|S|=1} \cosh(\hat{\mu}_B - \hat{\mu}_S) \\ &+ p^{\mathrm{baryons}}_{|S|=2} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) + p^{\mathrm{baryons}}_{|S|=3} \cosh(\hat{\mu}_B - 3\hat{\mu}_S) \end{aligned}$$

As long as this form of the pressure is valid the coefficients can be calculated from the fluctuations at $\mu = 0$. [BNL-Bielefleld 1304.7220]

$$\begin{aligned} p^{\mathrm{mesons}}_{|S|=1}/T^4 &= \chi^S_2 - \chi^{BS}_{22} \\ p^{\mathrm{baryons}}_{|S|=1}/T^4 &= \frac{1}{2}\left(\chi^S_4 - \chi^S_2 + 5\chi^{BS}_{13} + 7\chi^{BS}_{22}\right) \\ p^{\mathrm{baryons}}_{|S|=2}/T^4 &= -\frac{1}{4}\left(\chi^S_4 - \chi^S_2 + 4\chi^{BS}_{13} + 4\chi^{BS}_{22}\right) \\ p^{\mathrm{baryons}}_{|S|=3}/T^4 &= \frac{1}{8}\left(\chi^S_4 - \chi^S_2 + 3\chi^{BS}_{13} + 3\chi^{BS}_{22}\right) \end{aligned}$$

This method has a limited accuracy:

$$p^{\mathrm{HRG}} \gg p^{\mathrm{mesons}}_{|S|=1} \gg p^{\mathrm{baryons}}_{|S|=1} \gg p^{\mathrm{baryons}}_{|S|=2} \gg p^{\mathrm{baryons}}_{|S|=3}$$

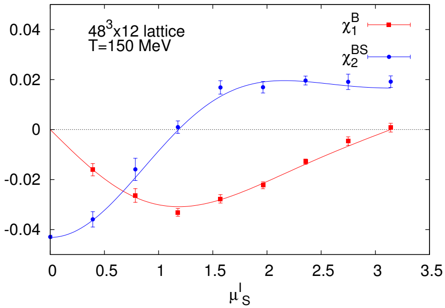
For $|S| > 1$ there is a large cancellation between the terms in the linear combination.

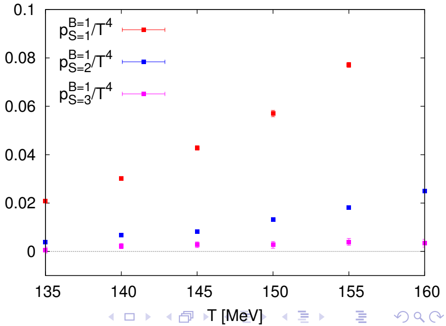
Extraction of the partial pressure sector by sector

Intorducing an **imaginary strangeness** chemical potential at vanishing baryo-chemical potential and fixed temperature:

$$
\begin{aligned}
\mathrm{Im}\ \chi_1^B &= +p^{\mathrm{B=1}}_{|S|=1}\sin(\hat\mu_B^I-\hat\mu_S^I) \\
&\quad +p^{\mathrm{B=1}}_{|S|=2}\sin(\hat\mu_B^I-2\hat\mu_S^I)+p^{\mathrm{B=1}}_{|S|=3}\sin(\hat\mu_B^I-3\hat\mu_S^I) \\
-\mathrm{Re}\ \chi_{11}^{BS} &= +p^{\mathrm{B=1}}_{|S|=1}\cos(\hat\mu_B^I-\hat\mu_S^I) \\
&\quad +2p^{\mathrm{B=1}}_{|S|=2}\cos(\hat\mu_B^I-2\hat\mu_S^I)+3p^{\mathrm{B=1}}_{|S|=3}\cos(\hat\mu_B^I-3\hat\mu_S^I)
\end{aligned}
$$

Corresponding data on a $48^3 \times 12$ lattice:

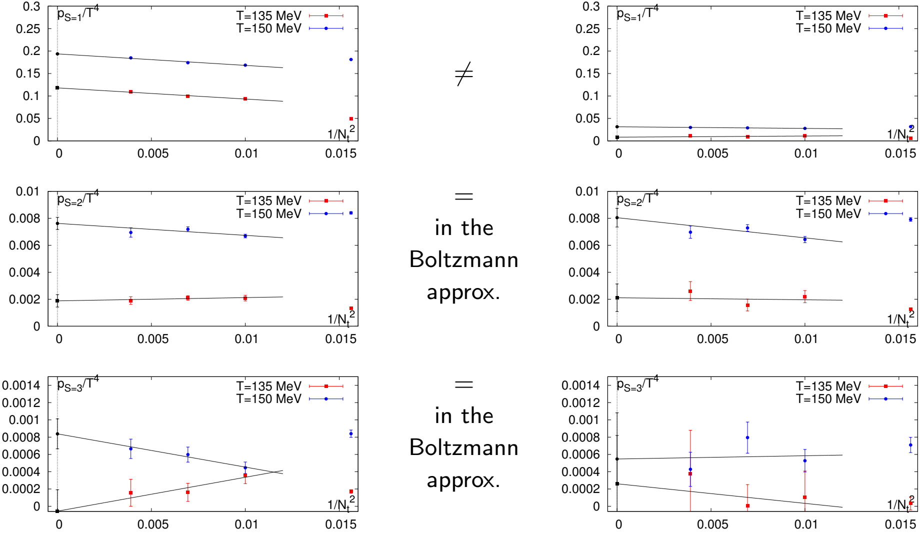


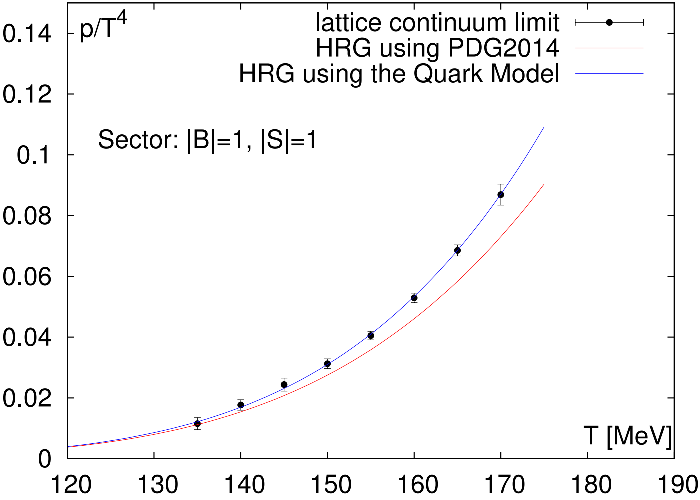


Continuum limit: temperature by temperature

Fourier analysis of χ_2^S:
$|S| = 1, 2, 3, \ldots$ baryons and mesons.

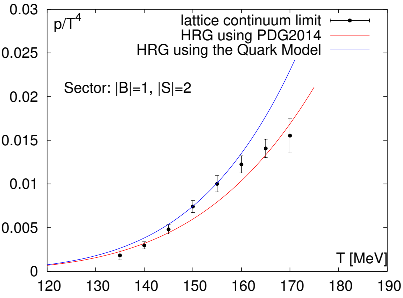
Fourier analysis of χ_{11}^{BS}:
$|S| = 1, 2, 3, \ldots$ baryons.

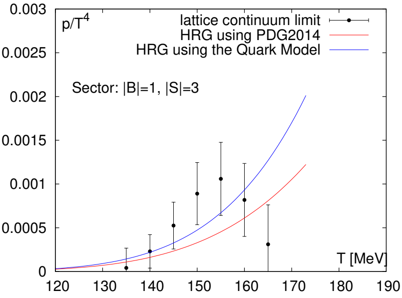


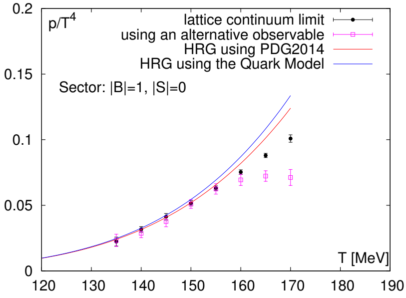


The $|S| = 1$ baryons are significantly better described by the Quark Model

Results: strange baryons



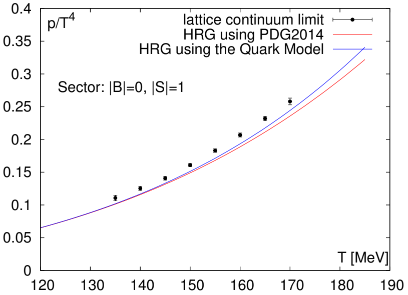


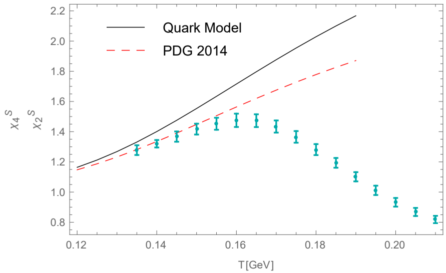


The $|S| = 2, 3$ baryon data also require extra states

For the $|S| = 0$ baryons PDG as well as Quark Model describe the data

While the Quark Model can describe all baryon data, both the PDG and the Quark Model under-estimate the strange meson contribution.





This explains, why the kurtosis (χ_4^S/χ_2^S) is over-estimated by the Quark Model: it contains the right abundance of multi-strange baryons, but lacks single-strange mesons that would bring the curve down.

Conclusions

- The 2+1+1 flavor equation of state has been calculated in the continuum limit.
- The charm threshold is very close to the tree-level expectation
- With a single fit parameter the perturbative formula describes the data in a broad range: 500-1000 MeV
- This allows to continue the equation of state up to the electroweak by adding the tree-level bottom threshold

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- We developed a method to extract the partial pressure in various strangeness sectors.
- We compared the results to two versions of the Hadron Resonance Gas model
- The comparison suggests that the so far undiscovered strange baryons are necessary de describe lattice data
- There are potentially undiscovered strange mesons than what the Quark Model predicts

backup

4stout program for the equation of state

We calculate the equation of state with physical quark masses in the continuum.

2nd generation staggered program:

- `4stout` staggered action (taste breaking similar to HISQ), tree-level Symanzik gauge action, smeared one-link fermions
- 2+1+1 dynamical flavors
- Bare masses tuned to M_π/f_π, M_K/f_π, scale setting: f_π and w_0
- Charm mass set to $m_c/m_s = 11.85$ [HPQCD: 0910.3102]
- Details of the tuning and LCP: [1507.04627].

In this talk I will use $N_t = 6, 8, 10$ and 12 in the continuum extrapolation.

This action has already been used
in the context of fluctuations [1507.04627]
and the for μ_B dependence of the thermodynamic observables
[1507.07510,1607.02493].

The trace anomaly at high temperatures

- To calculate the trace anomaly we need to know: the gauge action, the chiral condensate, and the running couplings and masses. The formula is then:

$$N_t^4 \left(-\frac{d\beta}{d\log a}\right)\left[\frac{\partial}{\partial\beta} + \sum_f \left(\frac{d\log m_f}{d\beta}\right) m_f \frac{\partial}{\partial m_f}\right] \frac{\log Z}{N_t N_s^3}$$

The systematics of the LCP choice are particularly important here.

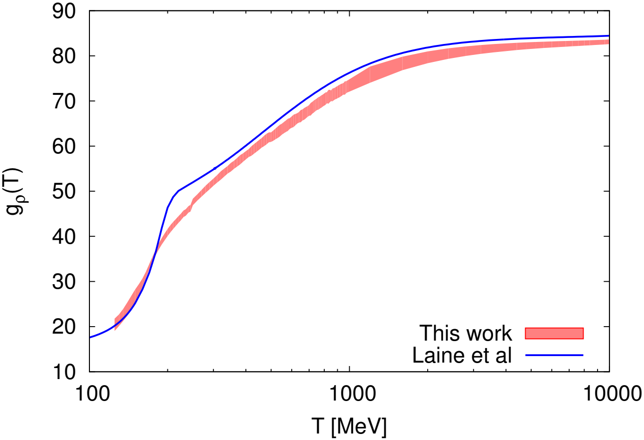
- There is an additional additive UV divergence, that is commonly renormalized by subtracting the zero temperature result. In this scheme, $p(T=0)=0$. Technical problem: renormalization with $T=0$ lattices becomes prohibitively expensive for high temperatures
- So we calculate the continuum limit of $\left[I\left(T\right) - I\left(T/2\right)\right]/T^4$ instead. The trace anomaly is given by:

$$\frac{I(T)}{T^4} = \sum_{k=0}^{n-1} 2^{-4k} \frac{I(T/2^k) - I(T/2^{k+1})}{(T/2^k)^4} + 2^{-4n}\frac{I(T/2^n)}{(T/2^n)^4}$$

- The pressure is: $p(T_1)/T_1^4 - p(T_2)/T_2^4 = \int_{T_2}^{T_1} \frac{\epsilon - 3p}{T^5}$

QCD contribution to effective degrees of freedom

Effective degrees of freedom: energy normalized to the SB limit of one bosonic degree of freedom: $g = \epsilon \frac{30}{\pi^2 T^4}$



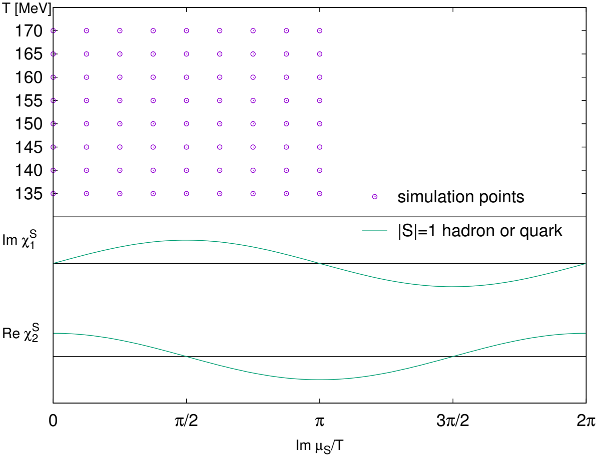
In the space of imaginary chemical potential the decomposition into strangeness and baryon number sector follows the pattern of a Fourier representation:

$$\begin{aligned} p^{\text{full}}(\hat{\mu}_B^I, \hat{\mu}_S^I) \;=\; & p_{S=0}^{\mathrm{B}=0} + p_{S=0}^{\mathrm{B}=1} \cos(\hat{\mu}_B^I) \\ & + p_{|S|=1}^{\mathrm{B}=0} \cos(\hat{\mu}_S^I) + p_{|S|=1}^{\mathrm{B}=1} \cos(\hat{\mu}_B^I - \hat{\mu}_S^I) \\ & + p_{|S|=2}^{\mathrm{B}=1} \cos(\hat{\mu}_B^I - 2\hat{\mu}_S^I) + p_{|S|=3}^{\mathrm{B}=1} \cos(\hat{\mu}_B^I - 3\hat{\mu}_S^I) \end{aligned}$$

Used observables: baryon and strangeness fluctuations

$$\begin{aligned} \mathrm{Im}\ \chi_1^S \;=\; & p_{|S|=1}^{\mathrm{B}=0} \sin(\hat{\mu}_S^I) + p_{|S|=1}^{\mathrm{B}=1} \sin(\hat{\mu}_B^I - \hat{\mu}_S^I) \\ & + 2p_{|S|=2}^{\mathrm{B}=1} \sin(\hat{\mu}_B^I - 2\hat{\mu}_S^I) + 3p_{|S|=3}^{\mathrm{B}=1} \sin(\hat{\mu}_B^I - 3\hat{\mu}_S^I) \\ \mathrm{Re}\ \chi_2^S \;=\; & p_{|S|=1}^{\mathrm{B}=0} \cos(\hat{\mu}_S^I) + p_{|S|=1}^{\mathrm{B}=1} \cos(\hat{\mu}_B^I - \hat{\mu}_S^I) \\ & + 4p_{|S|=2}^{\mathrm{B}=1} \cos(\hat{\mu}_B^I - 2\hat{\mu}_S^I) + 9p_{|S|=3}^{\mathrm{B}=1} \cos(\hat{\mu}_B^I - 3\hat{\mu}_S^I) \\ -\mathrm{Re}\ \chi_{11}^{BS} \;=\; & + p_{|S|=1}^{\mathrm{B}=1} \cos(\hat{\mu}_B^I - \hat{\mu}_S^I) \\ & + 2p_{|S|=2}^{\mathrm{B}=1} \cos(\hat{\mu}_B^I - 2\hat{\mu}_S^I) + 3p_{|S|=3}^{\mathrm{B}=1} \cos(\hat{\mu}_B^I - 3\hat{\mu}_S^I) \end{aligned}$$

We simulate at 8 different imaginary **strangeness** chemical potentials.



We use the 4stout staggered action with physical quark masses.
Continuum extrapolation from $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$.