Absence of bilinear condensate in QED_3

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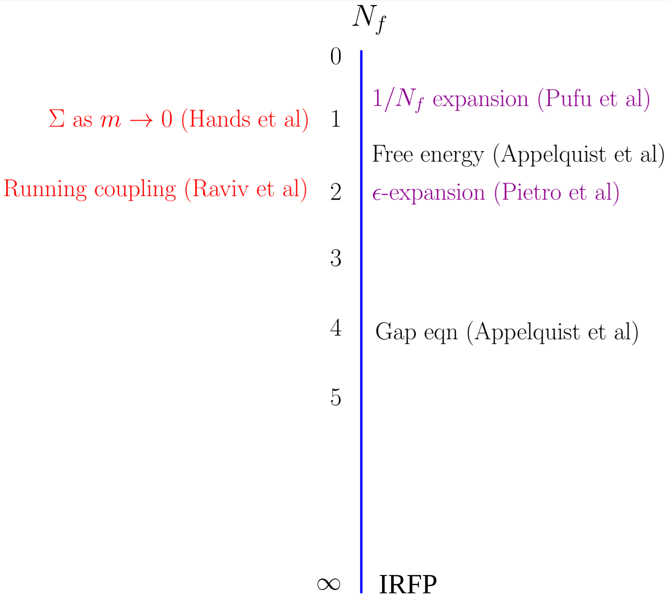
Non-compact parity-invariant QED$_3$ on ℓ^3 Euclidean torus

$$L = \sum_{i=1}^{N_f} \left\{ \overline{u}_i C_{\rm reg} u_i - \overline{d}_i C_{\rm reg}^{\dagger} d_i \right\} + \frac{1}{4g^2} \left(\partial_\mu A_\nu - \partial_\nu A_\mu \right)^2$$

- $u, d \quad \rightarrow \quad$ 2-component fermion field.
- Massless regulated Dirac operator: $\quad C_{\rm reg}$
- $g^2 \quad \rightarrow \quad$ Coupling constant of dimension $[\text{mass}]^1$
  Scale setting: $\quad g^2 = 1 \leftrightarrow$ specify ℓ.
- $U(2N_f)$ flavor symmetry in the continuum limit since $C = -C^\dagger$.
- $N_f \rightarrow \infty$ has an IR fixed point. What is the effect of finite N_f corrections? Spontaneously break $U(2N_f) \longrightarrow U(N_f) \times U(N_f)$?

Transition from massive to conformal phase *plausible*



Tell-tale signs for bilinear condensate Σ

(Shuryak and Verbaarschot '93) Spontaneous flavor symmetry breaking $\Rightarrow$ Chiral lagrangian at finite ℓ $\Rightarrow$ Random matrix theory for low eigenvalues ($z = \Sigma\lambda\ell^3$)

- Finite-size scaling of low-lying eigenvalues of the Dirac operator:

$$\lambda\ell \sim \frac{1}{\ell^2}.$$

- IPR: eigenvectors Ψ_λ of the Dirac operator are completely delocalized $\Rightarrow$

$$I_2 \equiv \int \langle|\psi(x)|^4\rangle d^3x \sim \frac{1}{\ell^3}$$

- Ergodic behaviour of number-variance $\Sigma_2(n)$, the variance in the number of eigenvalues n below a value λ.

$$\Sigma_2(n) \sim \log(n)$$

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Parity-invariant Wilson-Dirac fermions

Regularize at the level of two-component fermions (as opposed to the equivalent four component fermions):

$$L = \overline{u} C_w u - \overline{v} C_w^\dagger v; \qquad C_w = C_n + B - m$$

Corresponding 4-component Hermitian Wilson-Dirac operator:

$$H_w = \begin{bmatrix} 0 & C_w(m) \\ C_w^\dagger(m) & 0 \end{bmatrix} \longrightarrow \text{eigenvalues } \lambda.$$

m $\rightarrow$ tune mass to zero as Wilson fermion has additive renormalization.

Advantage: All even flavors $2N_f$ can be simulated without involving square-rooting.

Parity-invariant overlap fermions

Start from multi-particle Hamiltonians $\mathcal{H}_\pm = -a^\dagger H_\pm a$ where
$H_+ = H_w; \qquad H_- = \gamma_5.$

With one choice of phase, the gauge-invariant overlap has an explicit formula in 3d:

$$\langle + | - \rangle = \det\left(\frac{1+V}{2}\right); \quad V = \frac{1}{\sqrt{C_w C_w^\dagger}} C_w.$$

Parity-invariant fermion determinant: $\{\langle + | - \rangle\}_u \{\langle - | + \rangle\}_v$.

Propagator with the full U($2N_f$) symmetry:

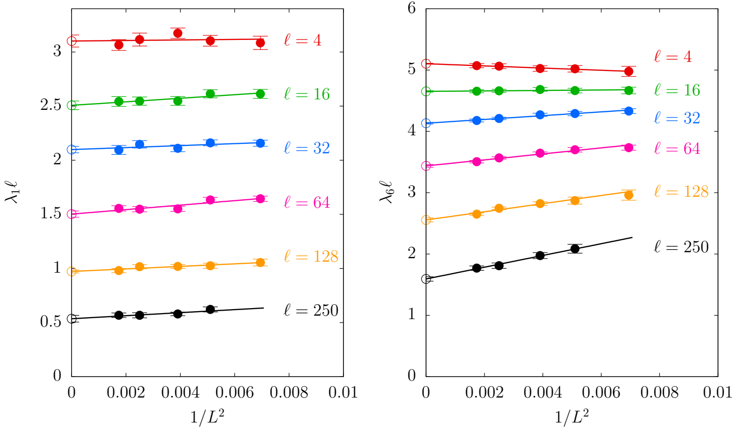
$$\begin{bmatrix} 0 & \frac{1-V}{1+V} \\ \frac{1-V}{1+V} & 0 \end{bmatrix} \rightarrow \text{eigenvalues} \frac{1}{i\lambda}.$$

Continuum limit at fixed ℓ

- L^3 periodic lattice with physical volume ℓ^3.
- Non-compact gauge-action: $S_g = \frac{L}{\ell} \sum_n \sum_{\mu \neq \nu} \left(\Delta_\mu \theta_\nu(n) - \Delta_\nu \theta_\mu(n)\right)^2$
- Continuum limit at fixed ℓ by taking $L \to \infty$.
- L =12,14,16,20,24 at different $4 < \ell < 250$.
- HYP smeared θ in Dirac operator.
- Dynamical fermion simulation using standard HMC with both massless Wilson and overlap fermions.

Continuum limit at fixed ℓ

$\lambda_j \quad \rightarrow$ eigenvalues of Hermitian Dirac operator



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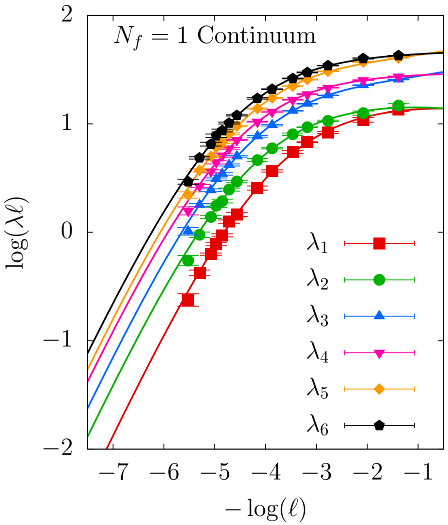
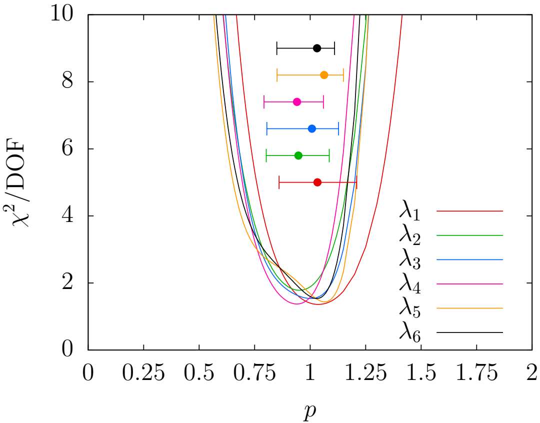
If bilinear condensate: $\lambda\ell \sim \ell^{-2}$

$\lambda\ell \sim \ell^{-p} F(1/\ell) \longrightarrow$ approximate by [1/1] Padé



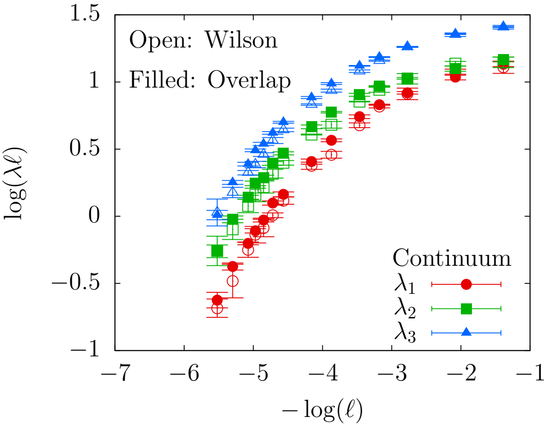
If bilinear condensate: $\lambda\ell \sim \ell^{-2}$

Likelihood of different values of p as $\ell \to \infty$



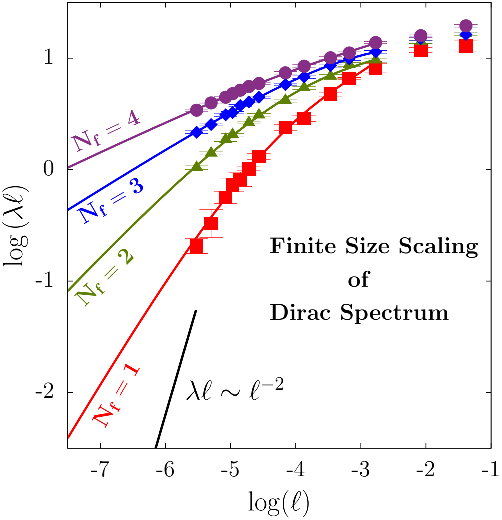
Expectation when condensate: $p = 2 \longrightarrow$ seems to be ruled out.

Agreement between Wilson and overlap fermion formulations



On lattice, Wilson fermions break $U(2N_f) \to U(N_f)\times U(N_f)$. Overlap has exact $U(2N_f)$. The agreement shows continuum limits are under control.

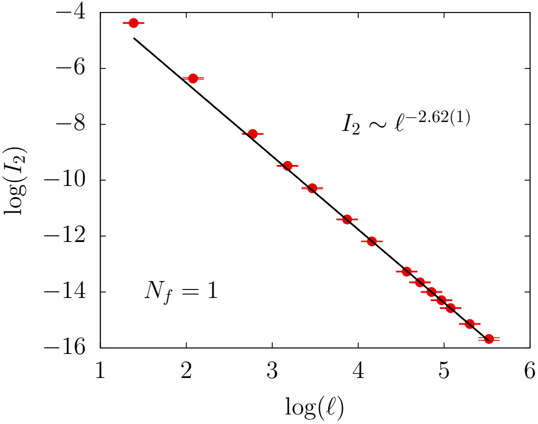
p decreases with N_f



$$p \approx 1/N_f$$

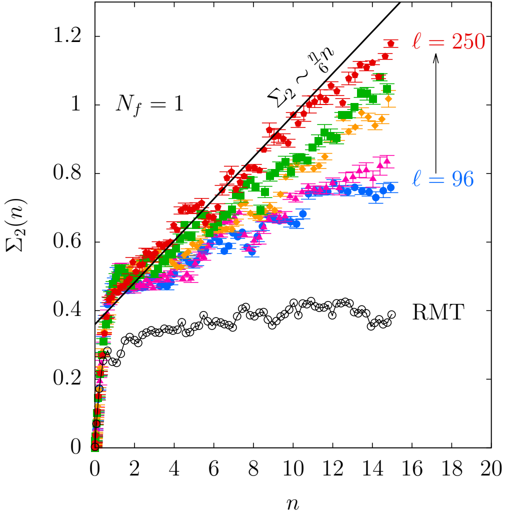
Fractal behavior of Inverse Participation Ratio (IPR)

Condensate: $I_2 \sim \ell^{-3}$; Critical: $I_2 \sim \ell^{-3+\eta}$



$\eta = 0.38(1)$ (Critical!)

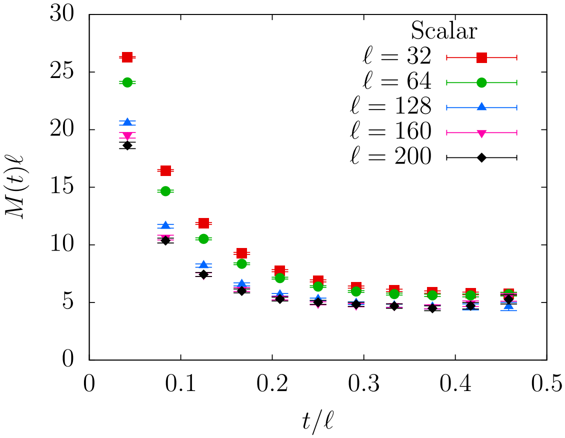
Number variance Σ_2 shifts away from RMT expectation



(Altshuler *et al.* '88) Critical relation: $\Sigma_2 \sim \dfrac{\eta}{6} n \quad \longrightarrow \quad \eta$ from IPR

Further evidence for scale-invariance: Absence of mass-gap

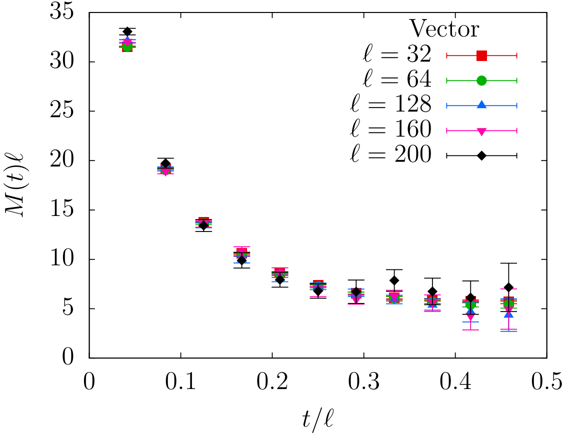
Scalar: $\overline{u}u(t) - \overline{v}v(t)$



$$\Rightarrow \quad M \sim \ell^{-1}$$

Further evidence for scale-invariance: Absence of mass-gap

Vector: $\overline{u}\sigma_i u(t) - \overline{v}\sigma_i v(t)$



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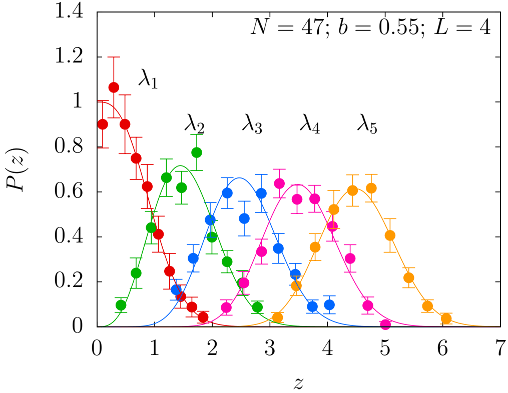
Exploring the (N_f, N_c) plane as a possibility



Agreement with Non-chiral random matrix model in large-N_c $\Rightarrow$ condensate. $\Sigma/\sigma = 0.10(1)$.

Conclusions

- Even for $N_f = 1$, the low-lying eigenvalues of the Dirac operator do not scale as ℓ^{-3} $\Rightarrow$ No bilinear condensate.
- Converse: $\lambda \sim \ell^{1+\gamma_m}$ for a scale-invariant theory $\Rightarrow$ $\gamma_m \approx 1$ for $N_f = 1$ (upper bound for CFTs).
- Inverse Participation Ratio does not scale as ℓ^{-3}.
- The number variance $\Sigma_2(n)$ does not agree with the ergodic random matrix theory behavior. Instead, the behavior is critical.
- No mass scale in the long-distance behavior of scalar and vector correlators.
- We also established the presence of condensate using the same methods in the large-N_c theory. Exploring the (N_f, N_c)-plane for a line of transition from scale-invariant to broken phase seems to the interesting next step.