Absence of bilinear condensate in QED$_3$

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1 Motivation and Method

2 Parity-invariant Lattice Formulations

3 Results

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Non-compact parity-invariant QED$_3$ on $\ell^3$ Euclidean torus

$$L = \sum_{i=1}^{N_f} \left\{ \bar{u}_i C_{\text{reg}} u_i - \bar{d}_i C\dagger_{\text{reg}} d_i \right\} + \frac{1}{4g^2} (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu})^2$$

- $u, d \rightarrow$ 2-component fermion field.
- Massless regulated Dirac operator: $C_{\text{reg}}$
- $g^2 \rightarrow$ Coupling constant of dimension [mass]$^1$
  Scale setting: $g^2 = 1 \iff$ specify $\ell$.
- $\text{U}(2N_f)$ flavor symmetry in the continuum limit since $C = -C\dagger$.
- $N_f \rightarrow \infty$ has an IR fixed point. What is the effect of finite $N_f$ corrections? Spontaneously break $\text{U}(2N_f) \rightarrow \text{U}(N_f) \times \text{U}(N_f)$?
Transition from massive to conformal phase *plausible*

<table>
<thead>
<tr>
<th>$N_f$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\Sigma$ as $m \rightarrow 0$ (Hands et al)</td>
</tr>
<tr>
<td>1</td>
<td>Running coupling (Raviv et al)</td>
</tr>
<tr>
<td>2</td>
<td>$1/N_f$ expansion (Pufu et al)</td>
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<td>3</td>
<td>Free energy (Appelquist et al)</td>
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<td>4</td>
<td>$\epsilon$-expansion (Pietro et al)</td>
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<tr>
<td>5</td>
<td>Gap eqn (Appelquist et al)</td>
</tr>
<tr>
<td>$\infty$</td>
<td>IRFP</td>
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</tbody>
</table>
Tell-tale signs for bilinear condensate $\Sigma$

*Shuryak and Verbaarschot ’93* Spontaneous flavor symmetry breaking $\Rightarrow$ Chiral lagrangian at finite $\ell \Rightarrow$ Random matrix theory for low eigenvalues ($z = \Sigma \lambda \ell^3$)

- Finite-size scaling of low-lying eigenvalues of the Dirac operator:
  \[
  \lambda \ell \sim \frac{1}{\ell^2}.
  \]

- IPR: eigenvectors $\Psi_{\lambda}$ of the Dirac operator are completely delocalized $\Rightarrow$
  \[
  l_2 \equiv \int \langle |\psi(x)|^4 \rangle d^3x \sim \frac{1}{\ell^3}
  \]

- Ergodic behaviour of number-variance $\Sigma_2(n)$, the variance in the number of eigenvalues $n$ below a value $\lambda$.
  \[
  \Sigma_2(n) \sim \log(n)
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Parity-invariant Wilson-Dirac fermions

Regularize at the level of two-component fermions (as opposed to the equivalent four component fermions):

\[ L = \bar{u} C_w u - \bar{v} C_w^\dagger v; \quad C_w = C_n + B - m \]

Corresponding 4-component Hermitian Wilson-Dirac operator:

\[
H_w = \begin{bmatrix}
0 & C_w(m) \\
C_w^\dagger(m) & 0
\end{bmatrix} \longrightarrow \text{eigenvalues } \lambda.
\]

\( m \to \) tune mass to zero as Wilson fermion has additive renormalization.

Advantage: All even flavors \( 2N_f \) can be simulated without involving square-rooting.
Parity-invariant overlap fermions

Start from multi-particle Hamiltonians $\mathcal{H}_\pm = -a^\dagger H_\pm a$ where $H_+ = H_w$; $H_- = \gamma_5$.

With one choice of phase, the gauge-invariant overlap has an explicit formula in 3d:

$$\langle +| - \rangle = \det \left( \frac{1 + V}{2} \right); \quad V = \frac{1}{\sqrt{C_w C_w^\dagger}}.$$

Parity-invariant fermion determinant: $\{\langle +| - \rangle\}_u \{\langle -| + \rangle\}_v$.

Propagator with the full $U(2N_f)$ symmetry:

$$\begin{bmatrix} 0 & \frac{1 - V}{1 + V} \\ 1 - V & \frac{1}{1 + V} \end{bmatrix} \rightarrow \text{eigenvalues} \frac{1}{i\lambda}.$$
Continuum limit at fixed $\ell$

- $L^3$ periodic lattice with physical volume $\ell^3$.

- Non-compact gauge-action: $S_g = \frac{L}{\ell} \sum_n \sum_{\mu \neq \nu} \left( \Delta_\mu \theta_\nu(n) - \Delta_\nu \theta_\mu(n) \right)^2$

- Continuum limit at fixed $\ell$ by taking $L \to \infty$.

- $L = 12, 14, 16, 20, 24$ at different $4 < \ell < 250$.

- HYP smeared $\theta$ in Dirac operator.

- Dynamical fermion simulation using standard HMC with both massless Wilson and overlap fermions.
Continuum limit at fixed $\ell$

$\lambda_j \rightarrow$ eigenvalues of Hermitian Dirac operator
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If bilinear condensate: \( \lambda \ell \sim \ell^{-2} \)

\[ \lambda \ell \sim \ell^{-p} F\left(\frac{1}{\ell}\right) \rightarrow \text{approximate by } [1/1] \text{ Padé} \]
If bilinear condensate: $\lambda \ell \sim \ell^{-2}$

Likelihood of different values of $p$ as $\ell \to \infty$

Expectation when condensate: $p = 2$ $\longrightarrow$ seems to be ruled out.
Agreement between Wilson and overlap fermion formulations

On lattice, Wilson fermions break $U(2N_f) \rightarrow U(N_f) \times U(N_f)$. Overlap has exact $U(2N_f)$. The agreement shows continuum limits are under control.
$p$ decreases with $N_f$

$p \approx 1/N_f$

Finite Size Scaling of Dirac Spectrum

$\lambda \ell \sim \ell^{-2}$
Fractal behavior of Inverse Participation Ratio (IPR)

Condensate: $I_2 \sim \ell^{-3}$; Critical: $I_2 \sim \ell^{-3+\eta}$

$N_f = 1$

$\eta = 0.38(1)$  (Critical!)
Number variance $\Sigma_2$ shifts away from RMT expectation

(Altshuler et al. '88) Critical relation: $\Sigma_2 \sim \frac{\eta}{6} n \quad \rightarrow \quad \eta$ from IPR
Further evidence for scale-invariance: Absence of mass-gap

Scalar: $\bar{u}u(t) - \bar{v}v(t)$

$$M(t) \ell^N$$

$\Rightarrow M \sim \ell^{-1}$
Further evidence for scale-invariance: Absence of mass-gap

Vector: $\bar{u}\sigma_i u(t) - \bar{v}\sigma_i v(t)$

\[ M(t) \propto \frac{M}{\ell^N} \]

$\Rightarrow M \sim \ell^{-1}$
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Agreement with Non-chiral random matrix model in large-$N_c \Rightarrow$ condensate. $\Sigma/\sigma = 0.10(1)$. 
Conclusions

- Even for $N_f = 1$, the low-lying eigenvalues of the Dirac operator do not scale as $\ell^{-3}$ $\Rightarrow$ No bilinear condensate.

- Converse: $\lambda \sim \ell^{1+\gamma_m}$ for a scale-invariant theory $\Rightarrow$ $\gamma_m \approx 1$ for $N_f = 1$ (upper bound for CFTs).

- Inverse Participation Ratio does not scale as $\ell^{-3}$.

- The number variance $\Sigma_2(n)$ does not agree with the ergodic random matrix theory behavior. Instead, the behavior is critical.

- No mass scale in the long-distance behavior of scalar and vector correlators.

- We also established the presence of condensate using the same methods in the large-$N_c$ theory. Exploring the $(N_f, N_c)$-plane for a line of transition from scale-invariant to broken phase seems to the interesting next step.