Finite Size Scaling of the Higgs-Yukawa Model near the Gaussian Fixed Point

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Outline

Introduction

Strategy -- 1. Finite Size Scaling (FSS)

2. Scaling Functions

3. Renormalisation

Test case -- O(4) Model

Conclusion and Outlook

Introduction

FSS as a way to identify the universality of a fixed point.

$$\frac{X_L(t)}{X_\infty(t)} = f\left(\frac{L}{\xi_\infty(t)}\right), \quad t = \frac{T - T_c}{T_c}$$

M. E. Fisher, M. N. Barber, (1972)

$$\xi_\infty(t) \sim |t|^{-\nu}, X_\infty(t) \sim |t|^{-\gamma_X} \implies X_L(t) \propto L^{\gamma_X/\nu} g(tL^{1/\nu})$$

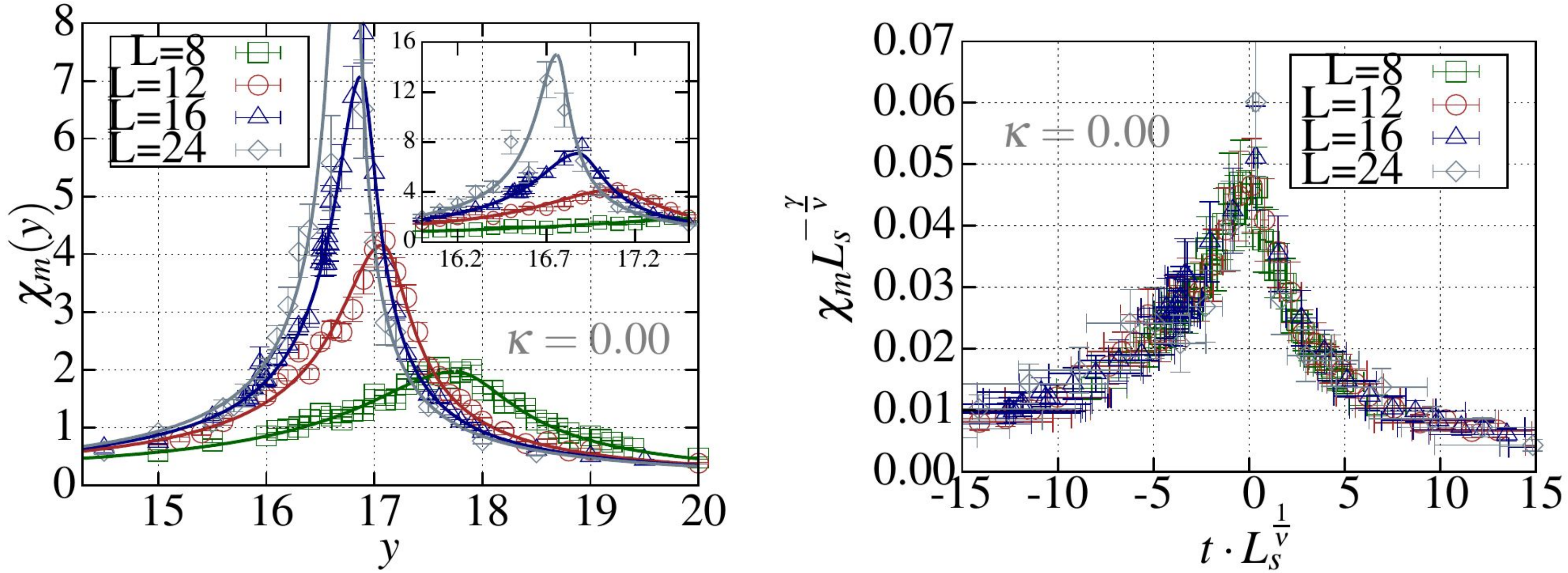
E. Brezin, (1982)

Scaling corrections :

Type of fixed point	Non-trivial	Trivial R. Kenna *et al.*, (2006)
Correction	$\xi_\infty(t) \sim \lvert t\rvert^{-\nu}$	(in D=4) $\xi_\infty(t) \sim \lvert t\rvert^{-1/2} \log(\lvert t\rvert)^{\hat{\nu}}$

Large bare Yukawa coupling

J. Bulava *et al.*, AHEP (2013)
J. Bulava *et al.*, PoS (2013)



$$\nu = 0.541(22),\ \gamma = 0.996(15).$$

$$t = \frac{T}{T_c^{(L=\infty)} - CL^{-b}} - 1$$

Trivial or not?

Finite Size Scaling

E. Brezin, J. Zinn-Justin, (1985)

Observable :

$$\hat{X}_b\left[M_b^2,\{g_i^{(b)}\};a^{-1},L\right]Z_X(a^{-1},m_P)$$

$$=\ \hat{X}\left[M^2(m_P),\{g_i(m_P)\};m_P,L\right]$$

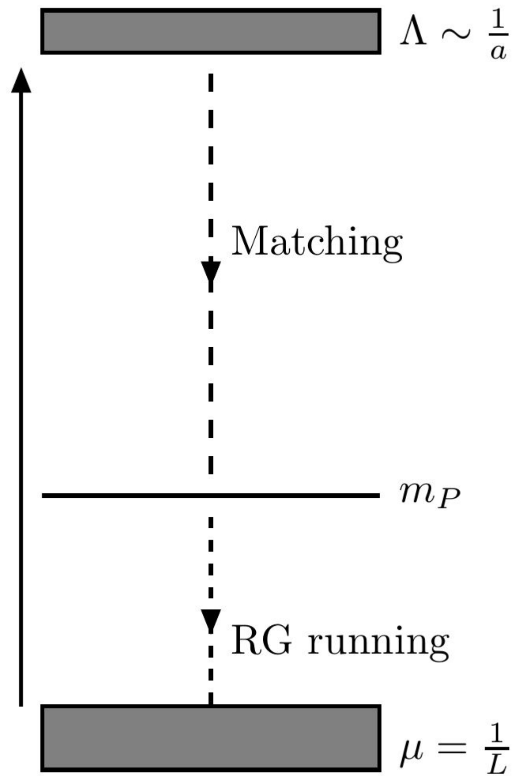
$$=\ \zeta_X(m_P L)\hat{L}^{-D_X}\hat{X}\left[\hat{M}^2(L^{-1})\hat{L}^2,\{g_i(L^{-1})\}\right]$$

$$\zeta_X(m_P L)=\exp\left(\int_{m_P}^{L^{-1}}\gamma_X(t)d\log(t)\right)$$

Criterions :

$$am_P \gg 1,\ \frac{L}{a}\gg 1,\ \text{but } m_P L \gtrsim 1.$$

Change of scales :



Scaling Functions (scalar models)

E. Brezin, J. Zinn-Justin, (1985)

Scalar Action :

$$\Phi = (\phi_0, \phi_1, \cdots, \phi_N), \quad \varphi_a = \frac{1}{V}\sum_V \phi_a, \quad \varphi = \sqrt{\sum_{a=1}^{N} \varphi_a^2},$$

$$S = \int d^4x \left\{ \frac{1}{2}\left(\partial_\mu \phi_a\right)^2 + \frac{1}{2} M_b^2 \left(\phi_a \phi_a\right) + \lambda_b \left(\phi_a \phi_a\right)^2 \right\}$$

The scaling variable :

$$z = \sqrt{s}\hat{M}^2(L^{-1})\hat{L}^2\lambda(L^{-1})^{-1/2}$$

$$L_t = sL$$

The expectation value :

$$\bar{\varphi}_N(z) = \int_0^\infty \mathrm{d}\varphi\, \varphi^N \exp\left[-\frac{1}{2}z\varphi^2 - \varphi^4\right]$$

$$\langle \varphi^n \rangle = \left(sL^4\lambda\left(L^{-1}\right)\right)^{-n/4} \left(\frac{\bar{\varphi}_{N+n-1}(z)}{\bar{\varphi}_{N-1}(z)}\right)$$

Scaling variable (Higgs-Yukawa) 2 fit parameters.

$$z = \hat{M}^2(m_P)\hat{L}^2\left(2s\beta_{\lambda\lambda^2}^{(N,N_f)}\right)^{1/2}\left(\beta_+^{(N,N_f)}-\beta_-^{(N,N_f)}\right)^{\frac{2\gamma_\lambda^{(N,N_f)}}{\beta_{\lambda\lambda^2}^{(N,N_f)}}}\left[Y(m_P)\right]^{\frac{2\gamma_\lambda^{(N,N_f)}}{\beta_{\lambda\lambda^2}^{(N,N_f)}}-\frac{2\gamma_Y^{(N,N_f)}}{\beta_{YY^2}^{(N,N_f)}}-\frac{\beta_-^{(N,N_f)}\gamma_\lambda^{(N,N_f)}}{\beta_{YY^2}^{(N,N_f)}\beta_{\lambda\lambda^2}^{(N,N_f)}}}$$

$$\times\left[Y\left(L^{-1}\right)\right]^{-\frac{1}{2}+\frac{2\gamma_Y^{(N,N_f)}}{\beta_{YY^2}^{(N,N_f)}}+\frac{\beta_-^{(N,N_f)}\gamma_\lambda^{(N,N_f)}}{\beta_{YY^2}^{(N,N_f)}\beta_{\lambda\lambda^2}^{(N,N_f)}}}\frac{\left\{B_+^{(N,N_f)}-B_-^{(N,N_f)}\left[\frac{Y(m_P)}{Y(L^{-1})}\right]^{\frac{\beta_+^{(N,N_f)}-\beta_-^{(N,N_f)}}{2\beta_{YY^2}^{(N,N_f)}}}\right\}^{\frac{1}{2}-\frac{2\gamma_\lambda^{(N,N_f)}}{\beta_{\lambda\lambda^2}^{(N,N_f)}}}}{\left\{B_+^{(N,N_f)}\beta_-^{(N,N_f)}-B_-^{(N,N_f)}\beta_+^{(N,N_f)}\left[\frac{Y(m_P)}{Y(L^{-1})}\right]^{\frac{\beta_+^{(N,N_f)}-\beta_-^{(N,N_f)}}{2\beta_{YY^2}^{(N,N_f)}}}\right\}^{\frac{1}{2}}}$$

$$B_\pm^{(N,N_f)} = Y(m_P)\beta_\pm^{(N,N_f)} - 2\lambda(m_P)\beta_{\lambda\lambda^2}^{(N,N_f)}$$

$$\beta_\pm^{(N,N_f)} = \left(\beta_{YY^2}^{(N,N_f)}-\beta_{\lambda\lambda Y}^{(N,N_f)}\right)\pm\sqrt{\left(\beta_{YY^2}^{(N,N_f)}-\beta_{\lambda\lambda Y}^{(N,N_f)}\right)^2-4\beta_{\lambda\lambda^2}^{(N,N_f)}\beta_{\lambda Y^2}^{(N,N_f)}}$$

Renormalisation -- On-shell Scheme

Propagator on lattice :

$$\hat{G}_{\phi}^{(\mathrm{lat})}(p) = \frac{Z_{\phi}}{\hat{p}^2 + a^2 m_P^2(\hat{L})},$$

Lattice momenta :

$$\hat{p}^2 = 4 \sum_{\mu} \sin\left(\frac{ap_{\mu}}{2}\right),$$

Infinite-volume extrapolation :

$$am_P = am_P(\hat{L}) + \frac{Aa^2}{L^2} + \cdots ,$$

Phase separation :

$$\begin{aligned} M^2(m_P^{\mathrm{s.}}) &= (m_P^{\mathrm{s.}})^2 , \\ M^2(m_P^{\mathrm{b.}}) &= -\frac{1}{2}\left(m_P^{\mathrm{b.}}\right)^2 . \end{aligned}$$

O(4) Model

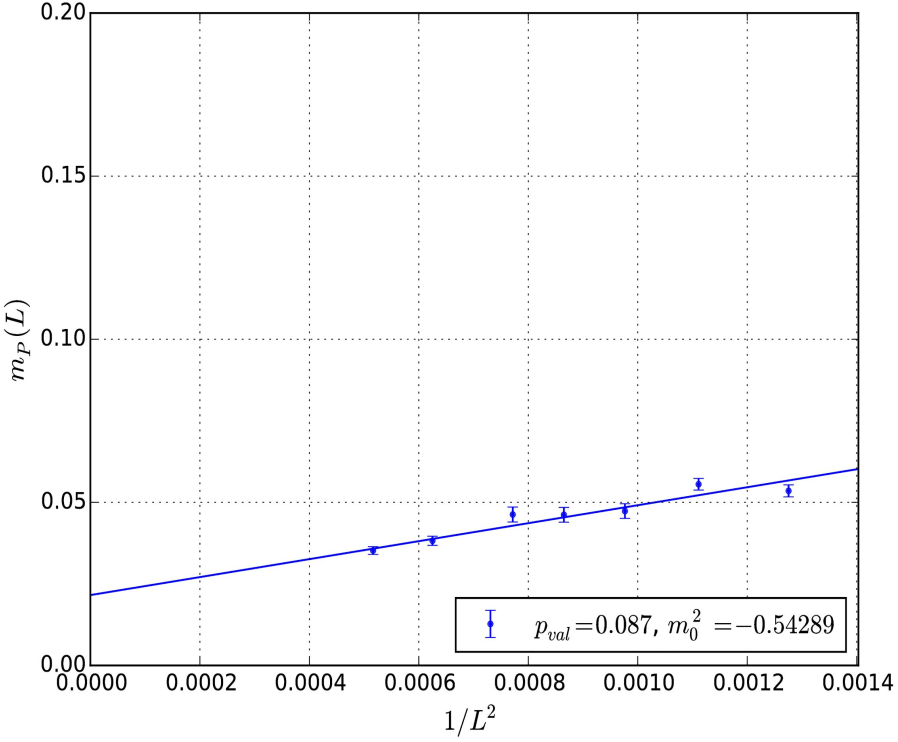
Simulation performed at $\lambda_b = 0.15\,, \;\; L_t = 2L$

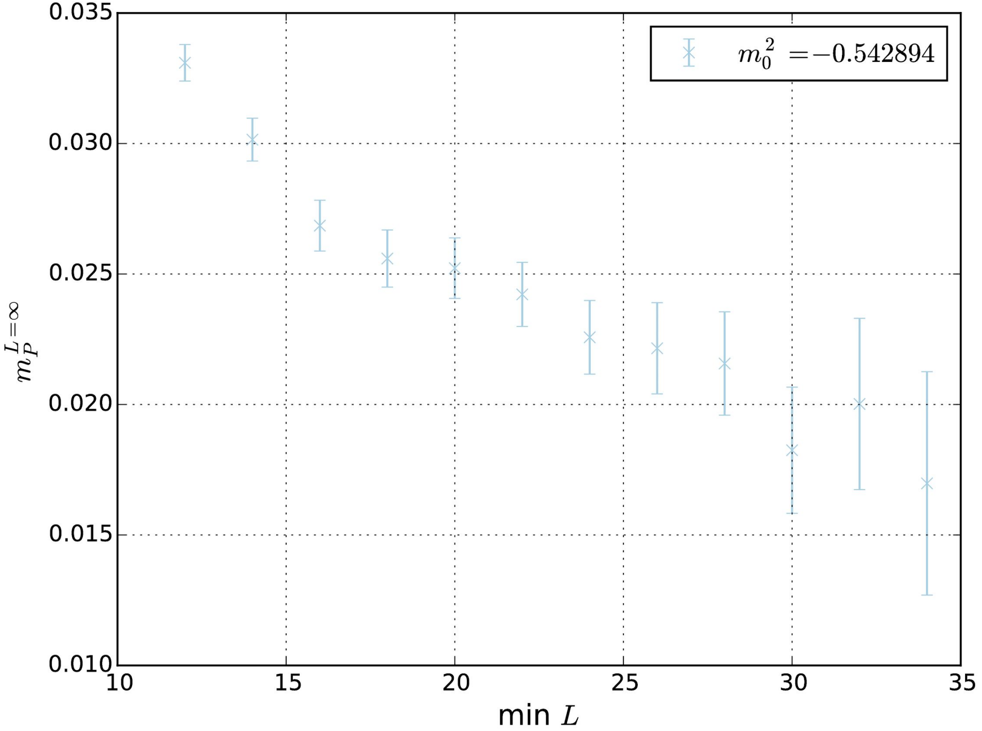
Advantages :

1. Well studied, Trivial
2. Efficient algorithms (cluster, worm)
3. Large lattices possible ($\hat{L} = 36, 40, 44, \ldots$)
4. Scaling variable : $z(\lambda) = \sqrt{2} M^2(m_P) L^2 \lambda^{-1/2}(m_P)$

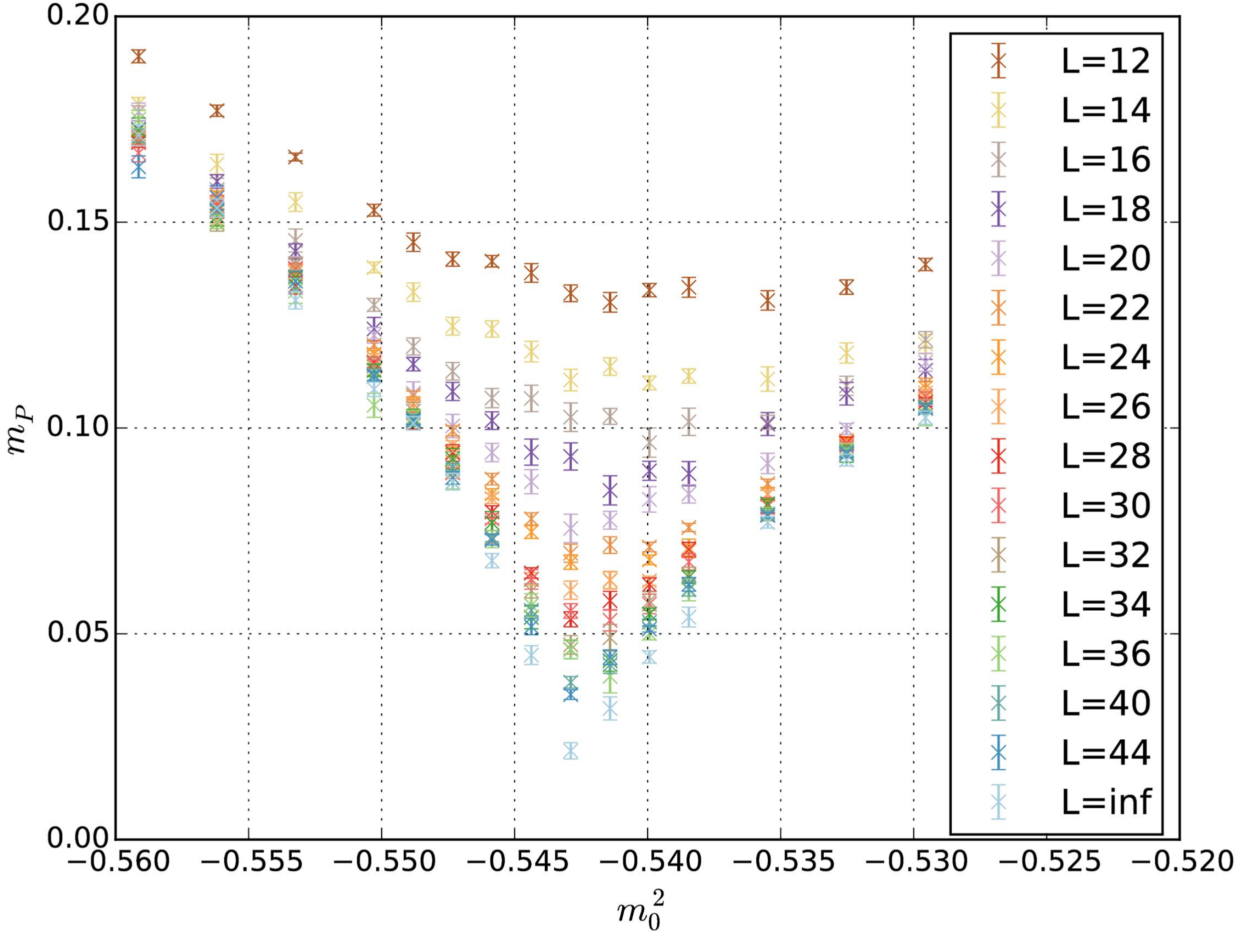
One fit parameter!

Infinite volume extrapolation -- systematic



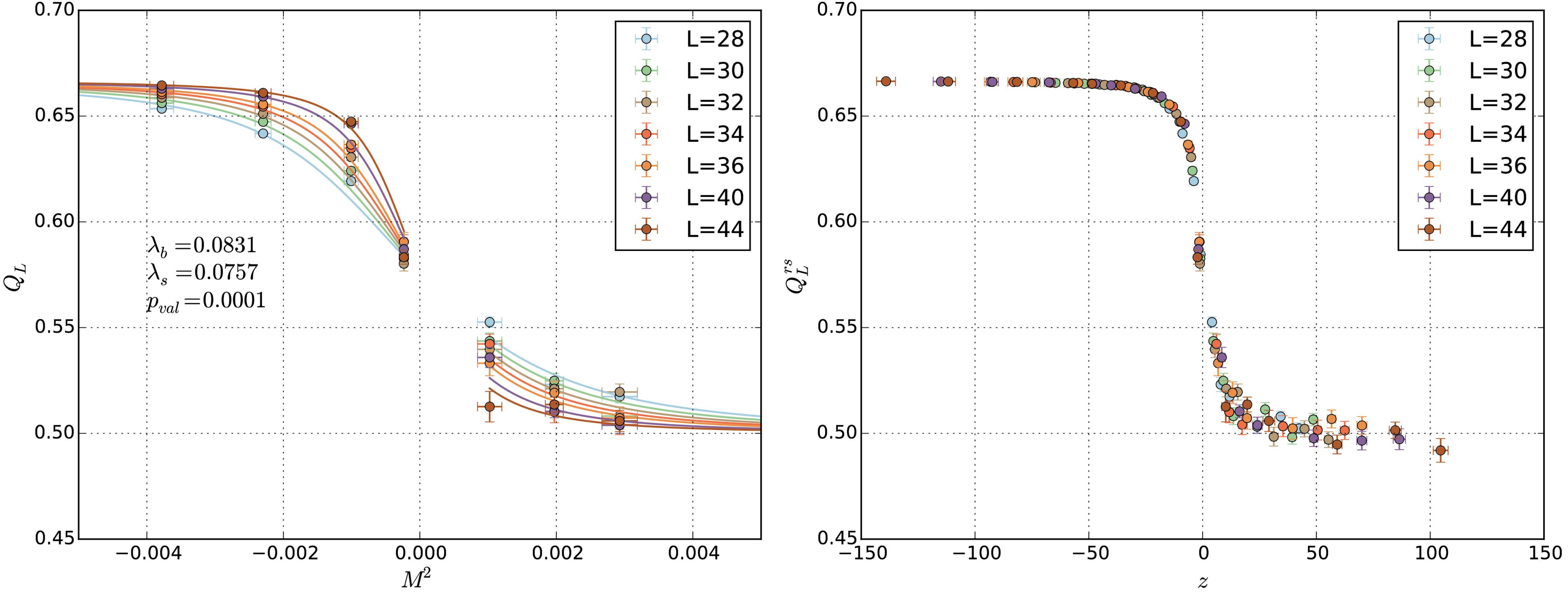


Infinite volume extrapolation



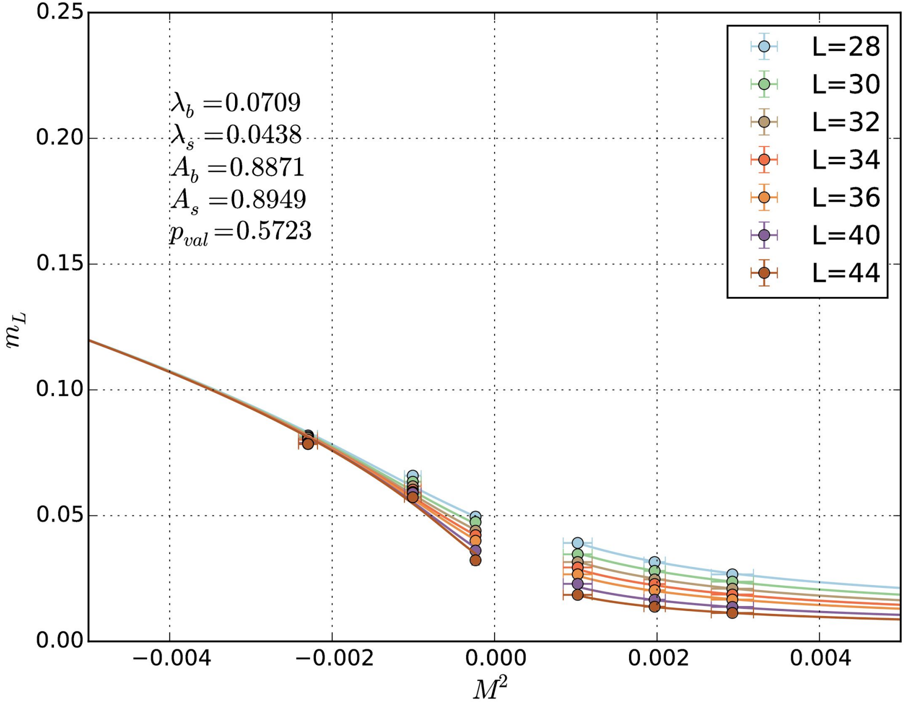
Binder's Cumulant

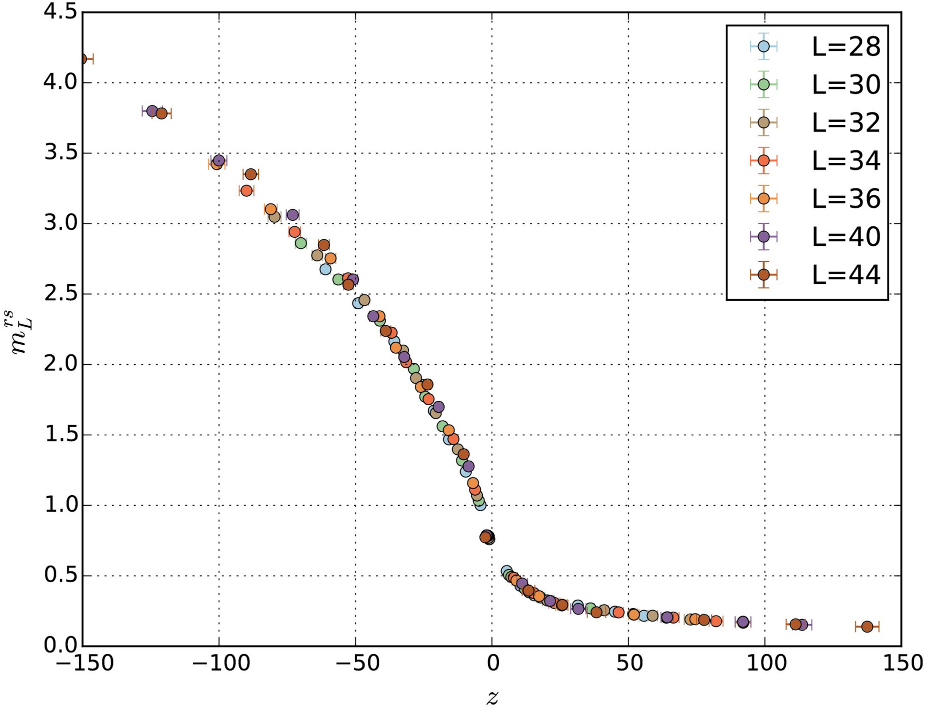
$$Q_L = 1 - \frac{\langle \varphi^4 \rangle}{3\langle \varphi^2 \rangle^2} = 1 - \frac{\bar{\varphi}_7\left(z(\lambda)\right)\bar{\varphi}_3\left(z(\lambda)\right)}{3\bar{\varphi}_5\left(z(\lambda)\right)^2}$$



Magnetisation

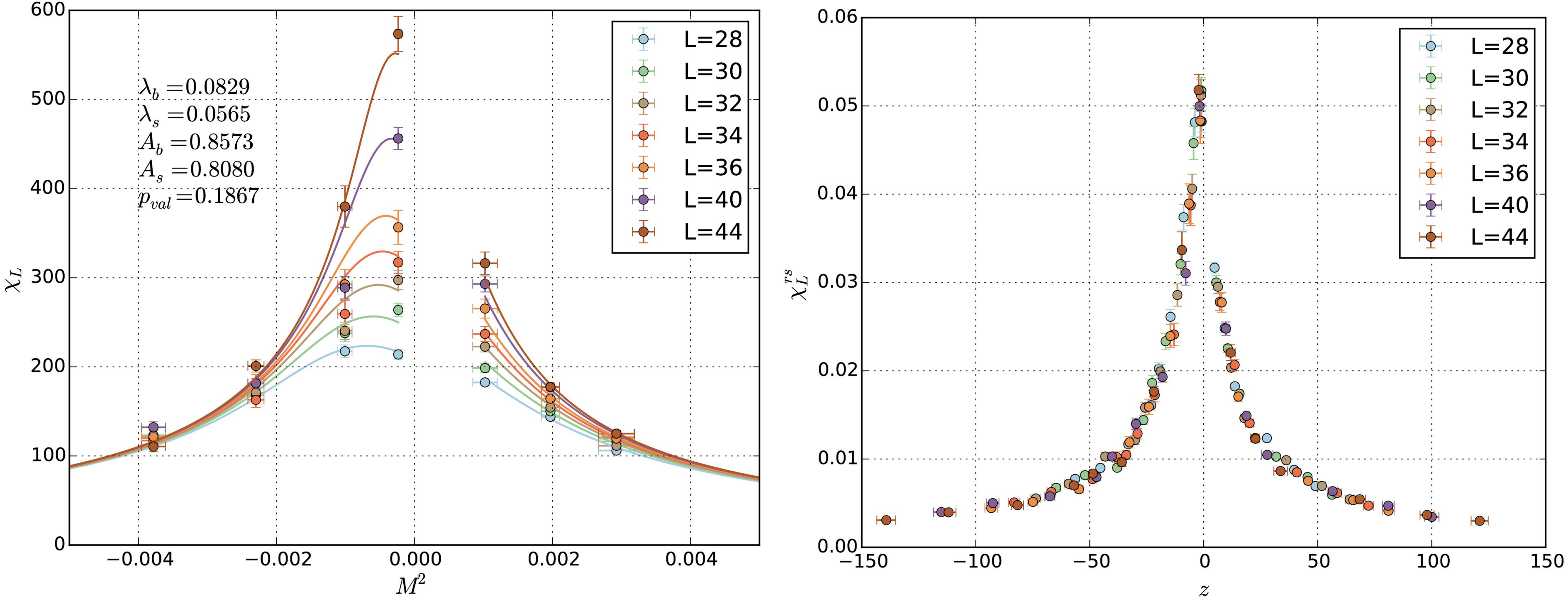
$$m_L = 2^{-1/4} A \hat{L}^{-1} \left[\frac{1 + \frac{6}{\pi^2} \log(m_P L)}{\lambda(m_P)} \right]^{1/4} \frac{\bar{\varphi}_4(z(\lambda))}{\bar{\varphi}_3(z(\lambda))}$$





Susceptibility

$$\chi_L = 2^{1/2} A \hat{L}^2 \left[\frac{1 + \frac{6}{\pi^2} \log(m_P L)}{\lambda(m_P)} \right]^{1/2} \left[\frac{\bar{\varphi}_5(z(\lambda))}{\bar{\varphi}_3(z(\lambda))} - \left(\frac{\bar{\varphi}_4(z(\lambda))}{\bar{\varphi}_3(z(\lambda))} \right)^2 \right]$$



Conclusion

Derived scaling formulae for generic Higgs-Yukawa model near Gaussian fixed-point.

Proposal on the renormalisation strategy.

On-going numeric study of logarithmic scaling of O(4) model.

⇨ Check with data from larger lattices?

⇨ More precise determination of the pole mass?

⇨ More precise data at all?

⇨ Statistical and systematic analysis of the data!

Outlook

FSS study of logarithmic corrections is useful for determining the phase structure of Higgs-Yukawa models.

Scaling test of our Higgs-Yukawa model at small bare couplings.

Apply the scaling formula to test the data at large Yukawa couplings.

Backup -- Integrals for Scaling Functions

$$\bar{\varphi}_N(z) = \int_0^\infty \mathrm{d}\varphi\, \varphi^N \exp\left[-\frac{1}{2} z\varphi^2 - \varphi^4\right]$$

$$\begin{aligned}
\bar{\varphi}_0(z) &= \frac{\pi}{8} \exp\left(\frac{z^2}{32}\right) \sqrt{|z|} \left[I_{-1/4}\left(\frac{z^2}{32}\right) - \mathrm{Sign}(z) I_{1/4}\left(\frac{z^2}{32}\right)\right], \\
\bar{\varphi}_1(z) &= \frac{1}{4} \exp\left(\frac{z^2}{16}\right) \left[1 - \mathrm{Sign}(z) \mathrm{Erf}\left(\frac{|z|}{4}\right)\right], \\
\bar{\varphi}_{N+2}(z) &= -2 \times \frac{\mathrm{d}\bar{\varphi}_N(z)}{\mathrm{d}z}.
\end{aligned}$$