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RG scaling at chiral phase transition in two-flavor QCD

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Objective

Clarify the nature of the chiral phase transition for Nf=2
first order or second order ?
If second order, estimate critical exponents

Strategy

derive new RG scaling relations
verify whether the scaling is satisfied for data
on lattice with Ns =16, 24, 32

Results

the data excellently satisfy the scaling relations
It implies the transition is second order
the anomalous mass dimension $0.67 \sim 1.1$

Stage and Tools

SU(3) gauge theories with degenerate two quarks

Action: RG improved gauge action + Wilson fermion action

Lattice size: 32^3 x 16, 24^3 x 12, 16^3 x 8

Boundary conditions: periodic boundary conditions except for an anti-periodic boundary conditions in the t direction for fermions

Algorithm: Blocked HMC for 2N and RHMC for 1 : Nf=2N + 1

Statistics: 1,000 +1,000 ~5000 trajectories

Computers: U. Tsukuba: CCS HA-PACS; KEK: SR16000

Chiral phase transition

- $\beta c \simeq 2.8$; $Kc = 0.1455$ on the $32^3 \times 16$ lattice;
- $\beta c \simeq 2.6$; $Kc = 0.149$ on the $24^3 \times 12$ lattice;
- $\beta c \simeq 2.3$; $Kc = 0.1547$ on the $16^3 \times 8$ lattice.

estimated by "on Kc method",

monitoring the number of iteration of CG inversion

From the results of spectroscopy on $16^3 \times 64$ lattice we obtain

$T_c \sim 163\ (+11; -1)$ MeV

RG equations

At the vicinity of a critical point where the correlation length becomes infinity, the following RG equations hold.
note: only relevant operators included

Temporal propagator
Gt(nt; g, mq, Ns, Nt, μ) = $(\frac{N'}{N})^{-2\gamma}$ Gt(nt';g',m'q,Ns',Nt',μ').

Spatial propagator
Gs(ns; g, mq, Ns, Nt, μ) = $(\frac{N'}{N})^{-2\gamma}$ Gs(ns';g',m'q,Ns',Nt',μ')

may use $r = N_s/N_t$ $N = N_t$

Scaling at IR fixed point

g=g'=g*; mq=mq'=0;

Temporal propagator

Gt(nt; g*, mq=0, N, μ) = $(\frac{N^{'}}{N})^{3-2\gamma}$ Gt(nt';g*,m'q=0,N',μ).

Gt(τ, N) = $(\frac{N^{'}}{N})^{3-2\gamma}$ Gt(τ, N'). $\tau = n_t/N$

effective mass

$$m(n_t, N) = \ln \frac{G(n_t, N)}{G(n_t + 1, N)}$$

scaled
effective mass

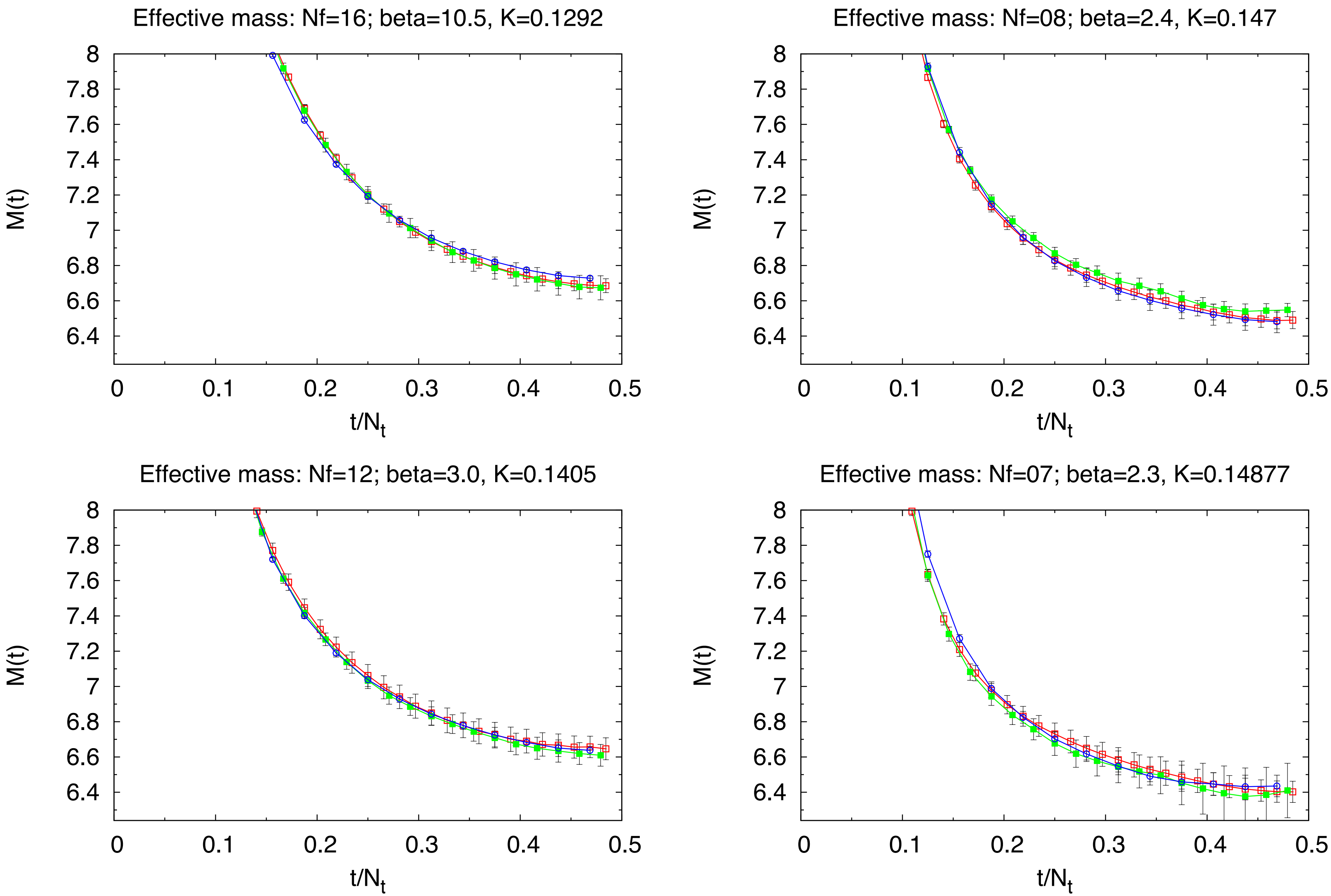
$$\mathfrak{m}(n_t, N) = N m(n_t, N)$$

$$\mathfrak{m}(\tau, N) = -\partial_\tau \ln \tilde{G}(\tau, N)$$

Scaling at IR fixed point

$$\mathfrak{m}(\tau, N) = \mathfrak{m}(\tau, N')$$

Phys.Lett.B 748 (2015) 289;arXiv: 1503.02359



SCALING AT CHIRAL PHASE TRANSITION

temporal $$\mathfrak{m}_t(\tau, g, N) = \mathfrak{m}_t(\tau, g^{'}, N^{'})$$

spatial $$\mathfrak{m}_s(\tau, g, N) = \mathfrak{m}_s(\tau, g^{'}, N^{'})$$

Numerical data scale

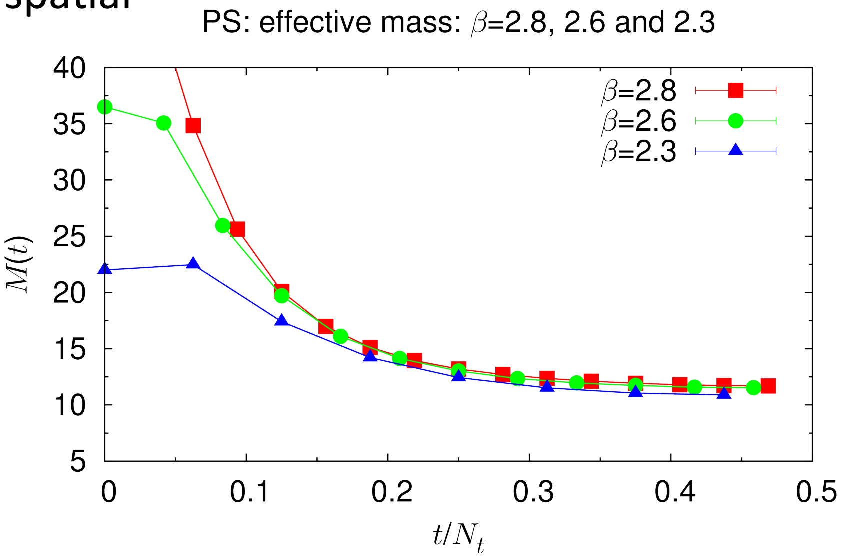
the data excellently scale both for the spatial and temporal effective masses, in the pseudo-scalar and vector channel, respectively, except only for short distance at n=0, 1, 2

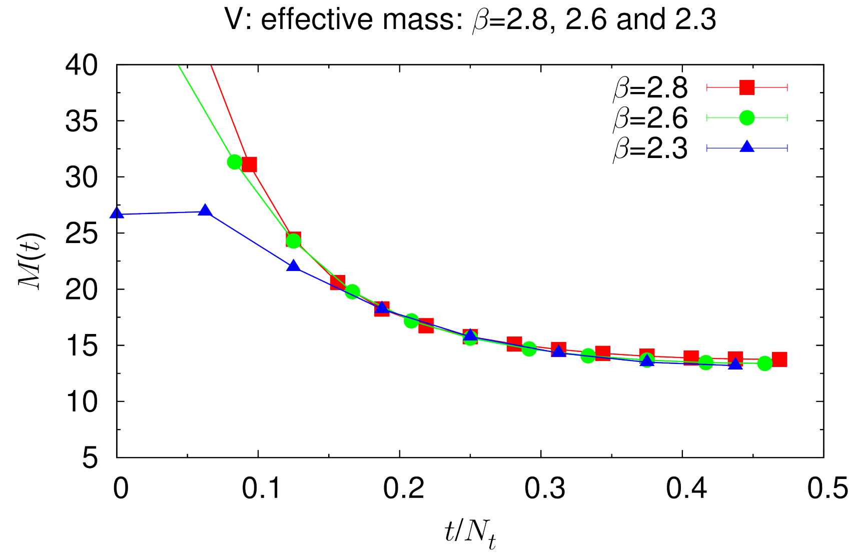
scaling curves give continuum limit

beta function is common both to spatial and temporal effective masses

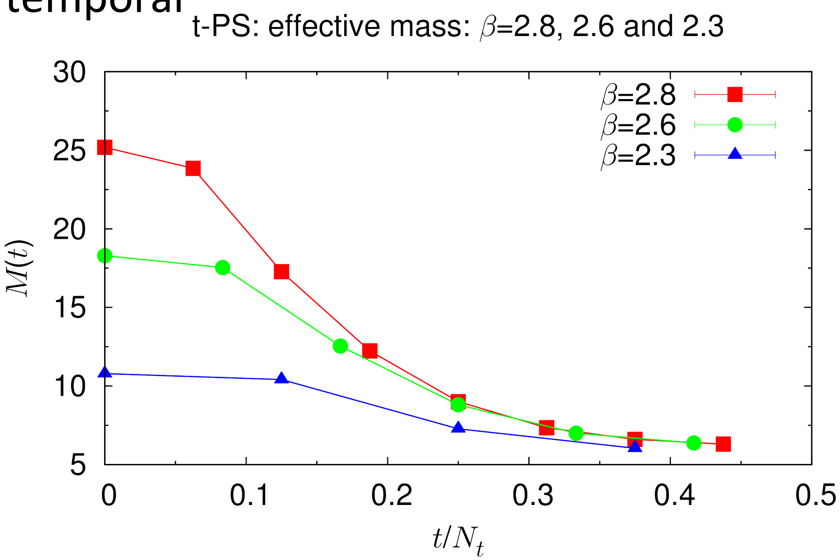
Numerical data scale

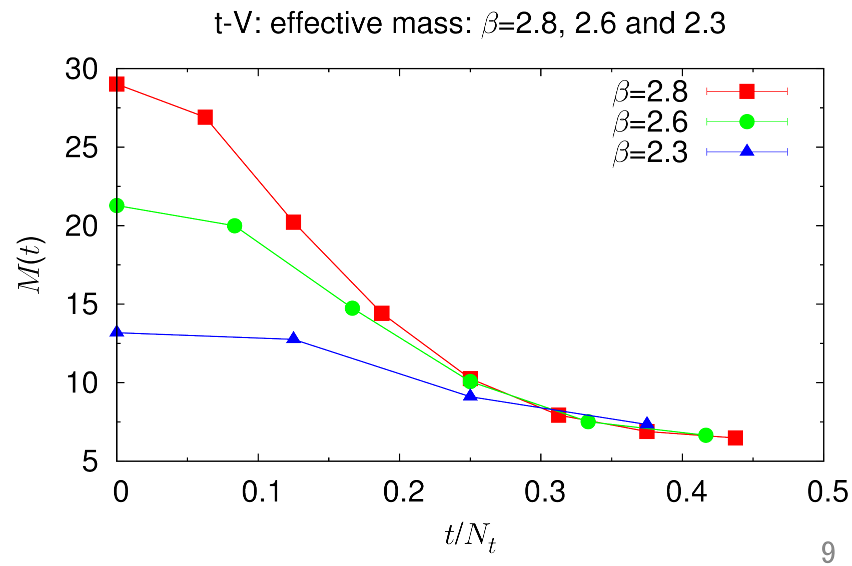
spatial





temporal





Scaling and Continuum limit

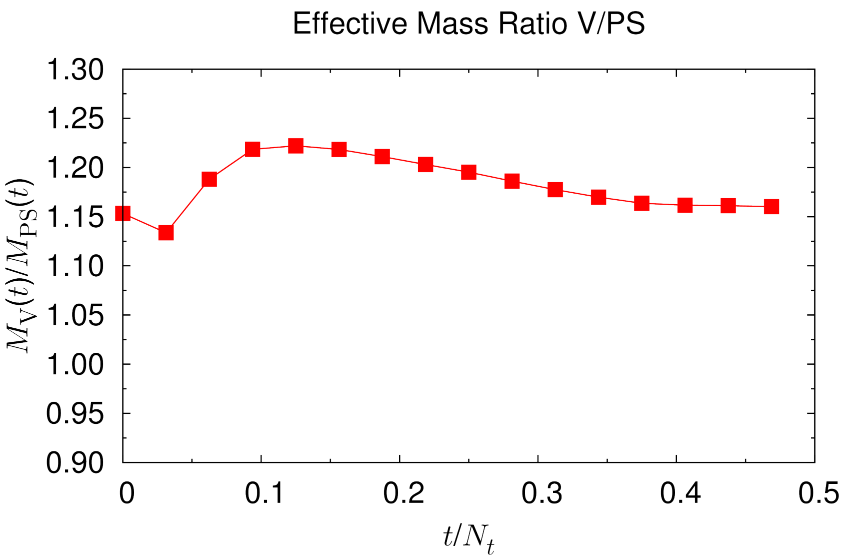
Scaling implies

$$\mathfrak{m}_{\mathfrak{s}}(\tau, g, N) = \mathfrak{F}_{\mathfrak{s}}(\tau)$$

independent from g and N

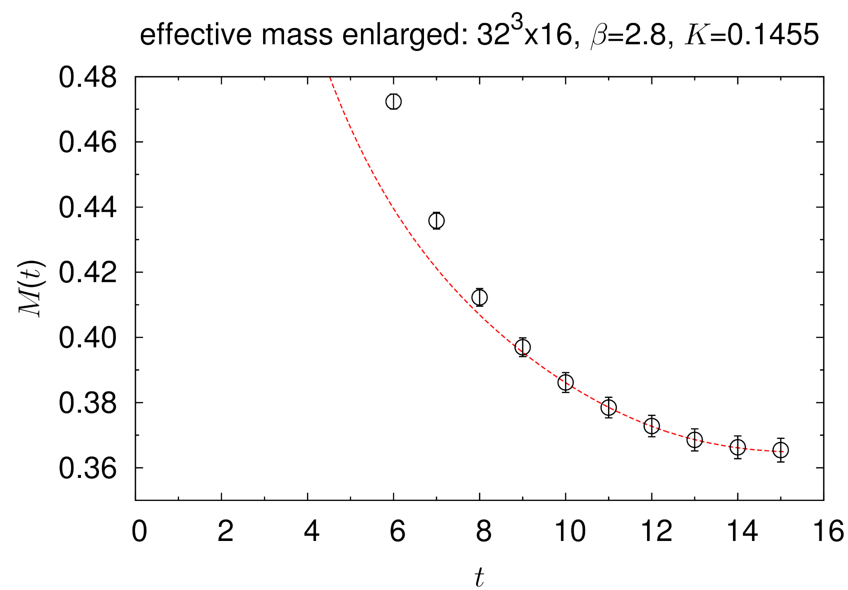
this is the continuum limit

ratio of vector to pseudo-scalar



Effective mass decay with a power modified Yukawa type

an example below
fit m=0.305(7)
power=0.87(6)



Thermodynamic Limit

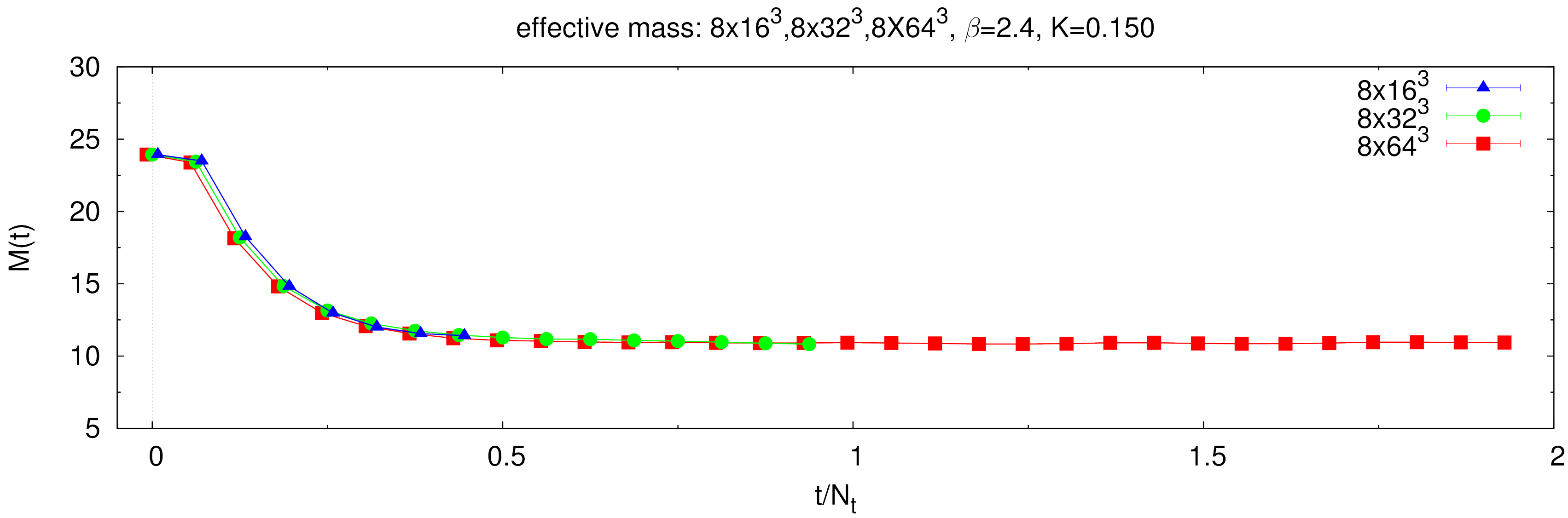
Effective mass decay with a power modified Yukawa type

$G(\tau) = A\exp(-m\,\tau)/\tau\uparrow\alpha$

$$\mathfrak{m}(\tau; N_x) \simeq \hat{\mathfrak{m}} + \alpha/\tau$$

$\hat{\mathfrak{m}}$: thermodynamic limit

Increasing volume, modified Yukawa type as shown does not change

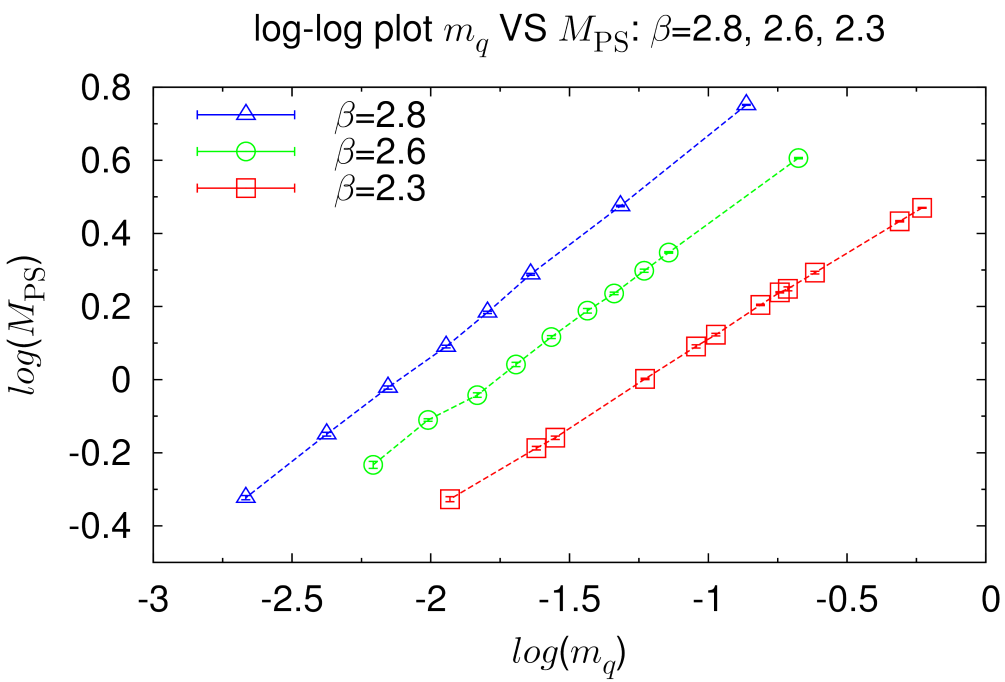


Hyper Scaling relations and anomalous mass dimension

Fit $\ln m_\pi = \alpha \ln m_q + c$

$$m_\pi = m_q^{1/1+\gamma^*}$$

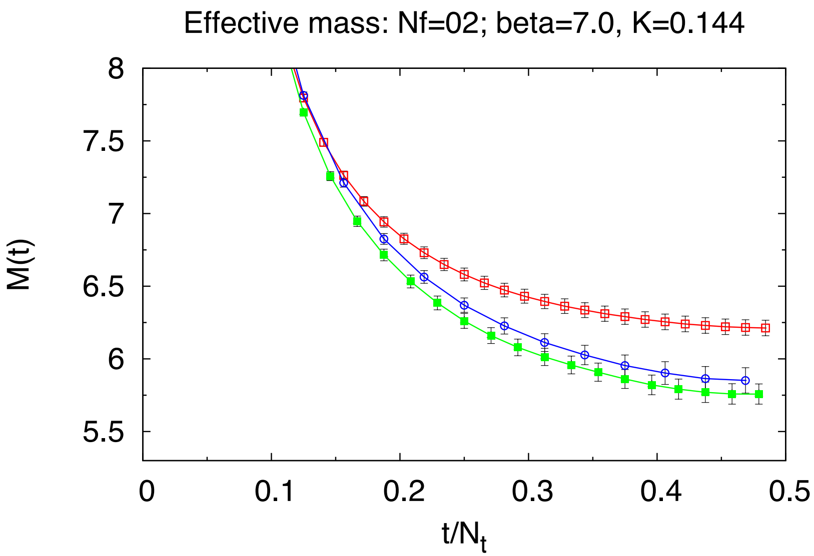
$$\gamma^* = 1/\alpha - 1$$

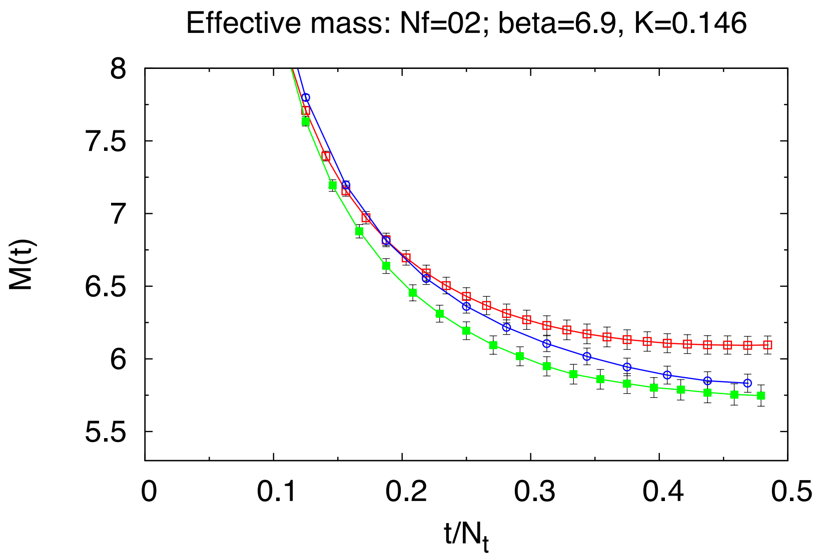


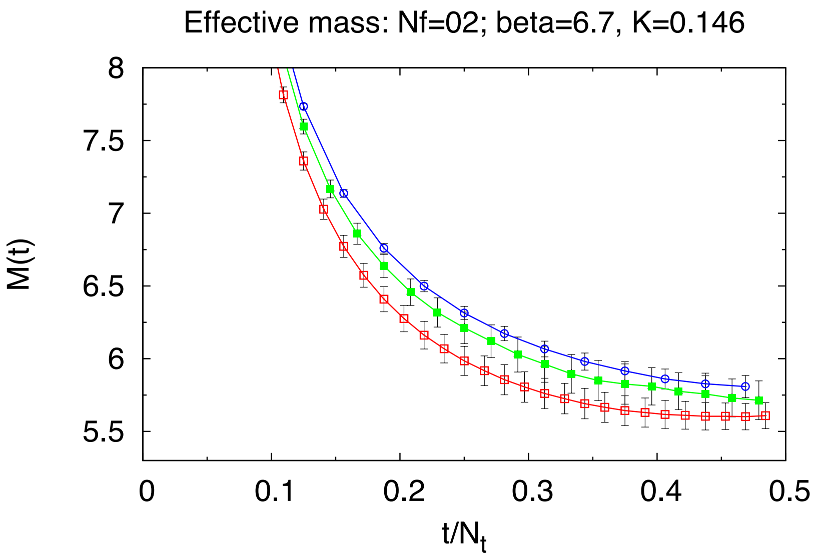
β	α	γ^*
2.3	0.468(1)	1.14(4)
2.6	0.545(2)	0.83 (7)
2.8	0.600(2)	0.667(6)

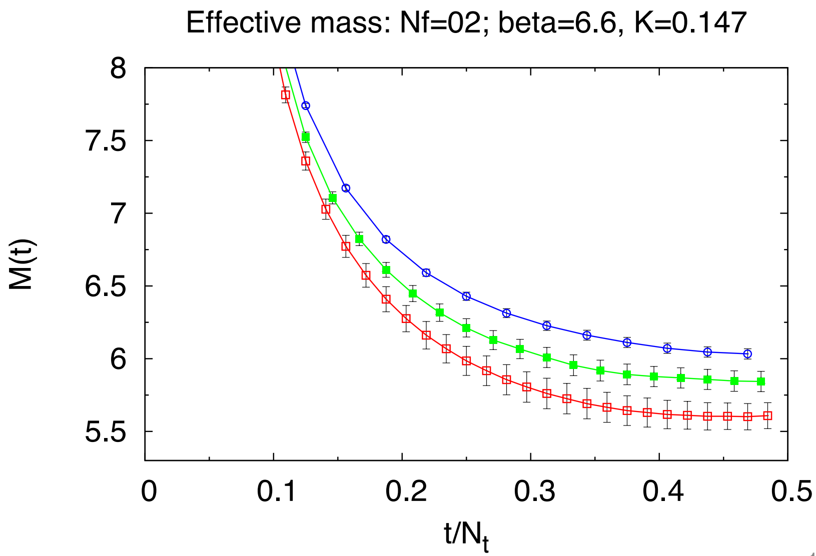
at each lattice the hyper scaling relation excellently is satisfied
However, the anomalous mass dimension does not scale well.

Search a critical point on lattices with r=1/4







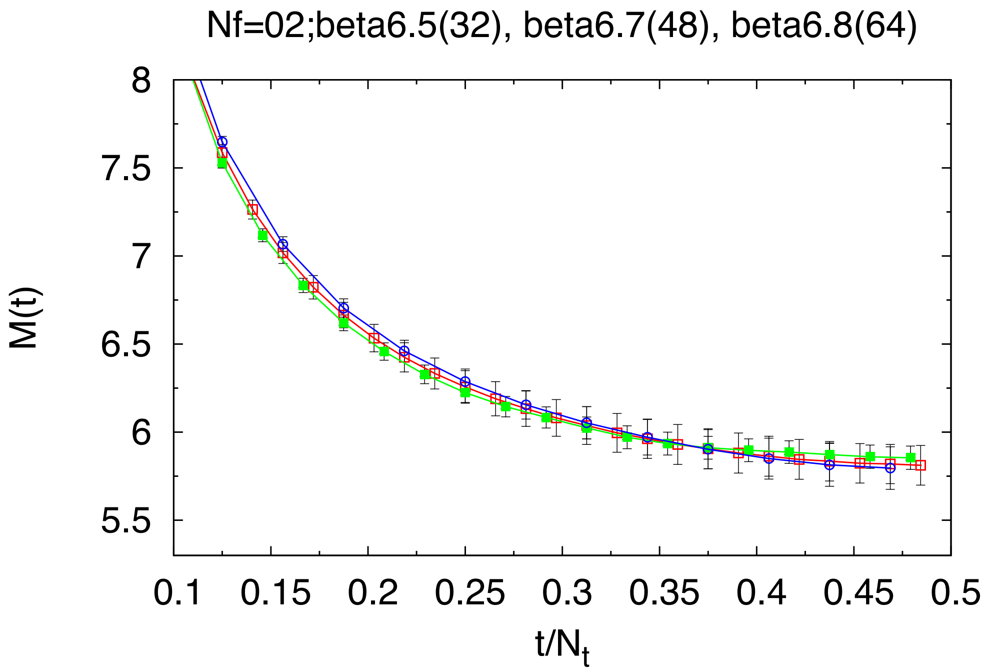


Search a critical point on lattices with r=1/4

Action = one plaquette gauge action

Size = 16^3x64, 12^3x48, 8^3X32

data at beta=6.8 on16^3x64, beta=6.7 on 12^3x48, beta=6.5 on 8^3X32
scale =>critical point (preliminary)



Summary and Discussion

Wilson's basic idea:

in the vicinity of a critical point RG equations are satisfied

Usually it is considered that a critical point is an IR fixed point

Scaling relation at an IR fixed point

$$\mathfrak{m}(\tau, N) = \mathfrak{m}(\tau, N^{'})$$

If the chiral phase transition is second order, it is a critical point

Numerical data excellently satisfy RG scaling relations

$$\mathfrak{m}_t(\tau, g, N) = \mathfrak{m}_t(\tau, g^{'}, N^{'}) \qquad \mathfrak{m}_s(\tau, g, N) = \mathfrak{m}_s(\tau, g^{'}, N^{'})$$

It implies the chiral transition is second order

may suppress g, since T and aspect ratio r given, g(a) are fixed

$$\mathfrak{m}_s(\tau, N) = \mathfrak{m}_s(\tau, N^{'}) \qquad \mathfrak{m}_s(\tau, N) = \mathfrak{m}_s(\tau, N^{'})$$

The same form as the scaling at IR fixed point

They imply "scale invariance"