Kaon semileptonic decays with $N_f = 2+1+1$ HISQ fermions and physical light quark masses

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1. Introduction: CKM unitarity in the first row

Check unitarity in the first row of CKM matrix

$$\Delta_u \equiv |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1$$

* $V_{ud} = 0.97417(21)$ from superallowed nuclear $\beta-$decays **Hardy & Towner**, 1411.5987 and $V_{ub} \sim 0$.

* V_{us} from photon-inclusive decay rate for all $K \to \pi l \nu$ decay modes

$$\Gamma_{K_{l3(\gamma)}} = \frac{G_F^2 M_K^5 C_K^2}{128\pi^3} S_{\rm EW} |V_{us}|^2 f_+^{K^0\pi^-}(0)^2 I_{Kl}^{(0)} \left(1 + \delta_{\rm EM}^{Kl} + \delta_{\rm SU(2)}^{K\pi}\right)$$

with $C_K = 1(1/\sqrt{2})$ for neutral (charged) K, $S_{EW} = 1.0223(5)$, $I_{Kl}^{(0)}$ a phase integral depending on shape of $f_\pm^{K\pi}$, and $\delta_{\rm EM}^{Kl}$, $\delta_{\rm SU(2)}^{K\pi}$ are long-distance em and strong isospin corrections respectively

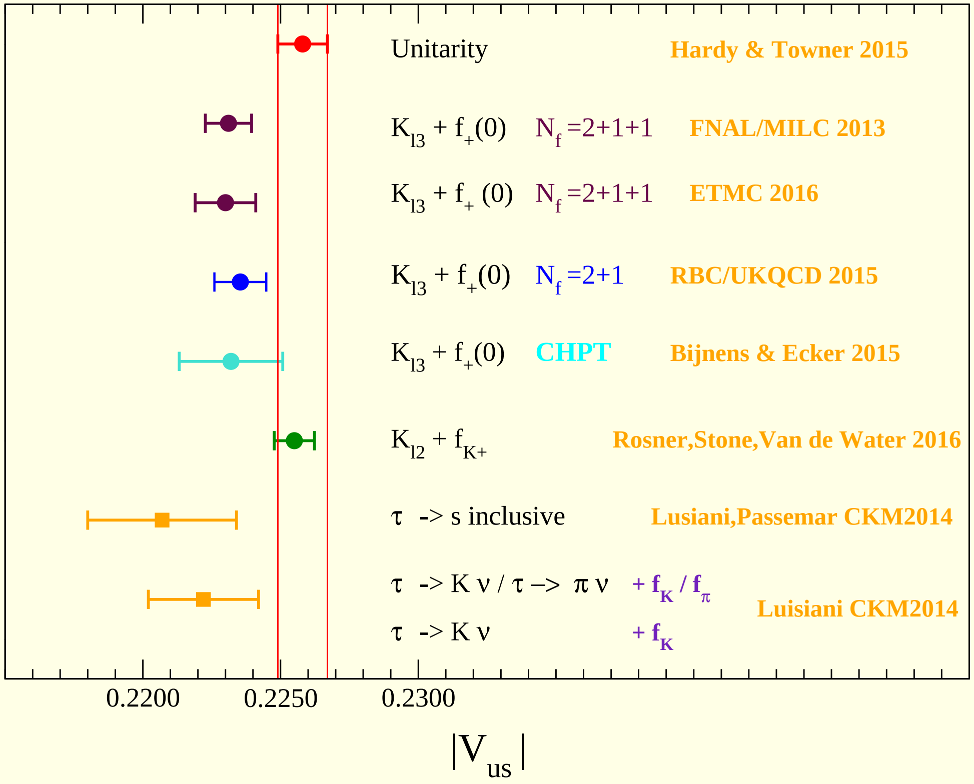
** Using experimental average from **Moulson**, 1411.5252 (CKM2014) (includes uncertainties from $\delta_{\rm EM}^{Kl}$, $\delta_{\rm SU(2)}^{K\pi}$.) and $N_f = 2+1+1$ **FNAL/MILC** (1312.1228)

$$|V_{us}| f_+^{K\pi}(0)\Big|_{exp} = 0.2165(\pm 0.18\%) \qquad f_+^{K\pi}(0)_{FNAL/MILC} : 0.9704(\pm 0.33\%)$$

1. Introduction

$$\Delta_u = -0.00126(37)_{V_{us}}(41)_{V_{ud}} \quad \rightarrow \quad \sim 2\sigma \text{ tension}$$

(similar results with $f_+(0)_{RBC/UKQCD} = 0.9685(34)(14)$ **1504.01692** and $f_+(0)_{ETMC} = 0.9709(46)$ **1602.04113**)



checking for SM consistency

Probe the `W-boson` coupling to u and d quarks via the vector current (semilept.) and the axial current (leptonic)

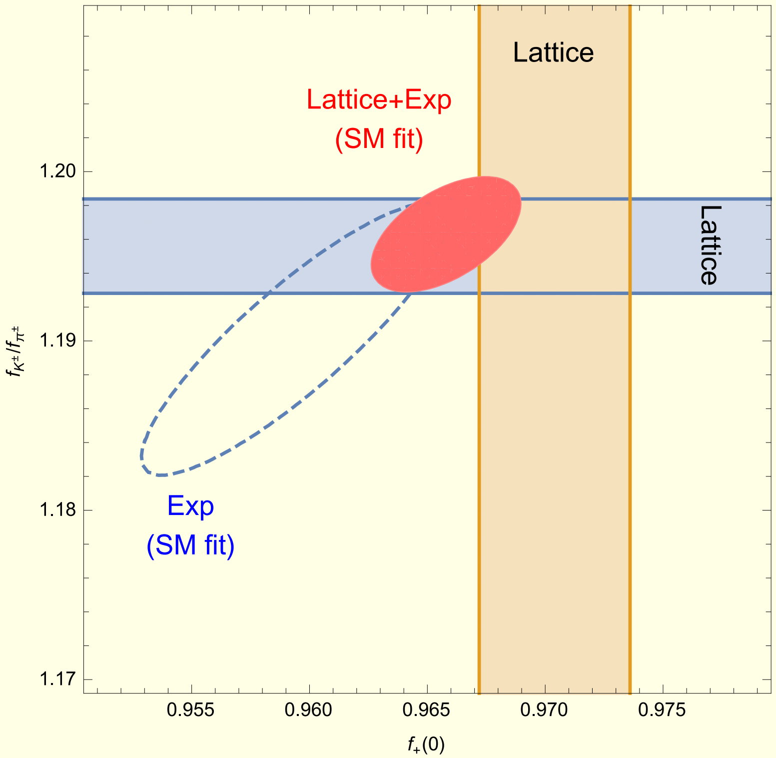
$\sim 2.2\sigma$ tension

1. Introduction

Probe the W-boson coupling to u and d quarks via the vector current (semileptonic) and the axial current (leptonic)

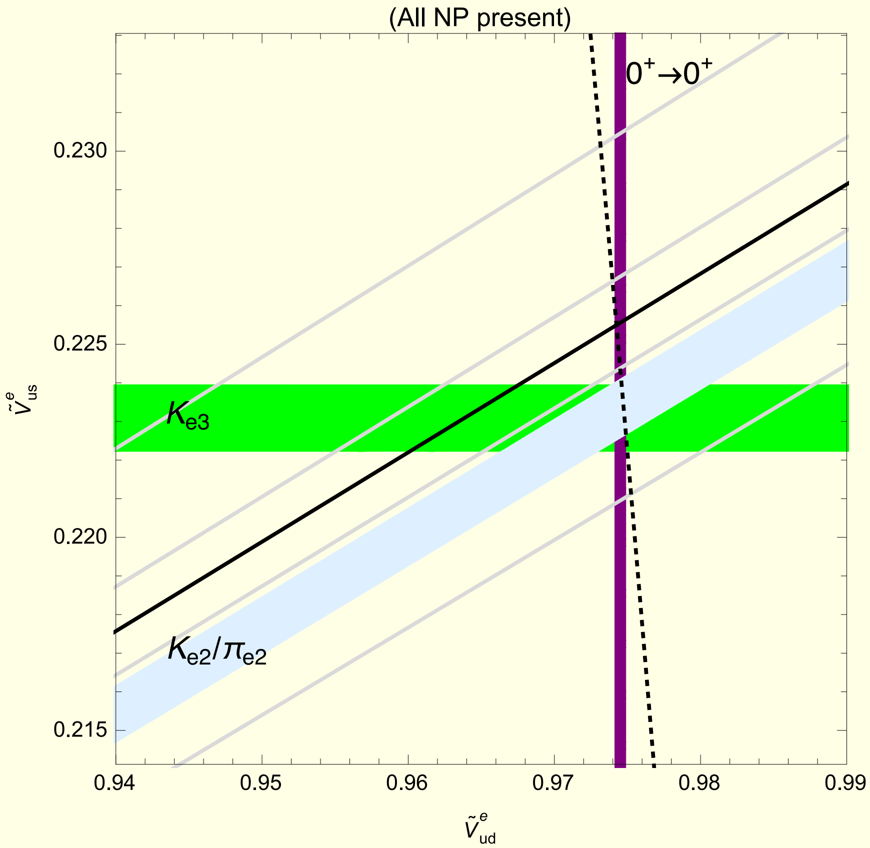
$\rightarrow$ checking for consistency.

$\sim 2\sigma$ tension



Plot from **M.González-Alonso** with f_K^+/f_π and $f_+(0)$ from **FNAL/MILC**, 1407.3772 and 1312.1228

1. Introduction



Plot from **M.González-Alonso** with f_K^+/f_π and $f_+(0)$ from **FNAL/MILC**, 1407.3772 and 1312.1228

* Fit allows NP contributions to leptonic and semileptonic decays (electron chanel)

Decays sensitive to NP: probes $\mathcal{O}(100)$ TeV scales.

Black solid line: SM leptonic

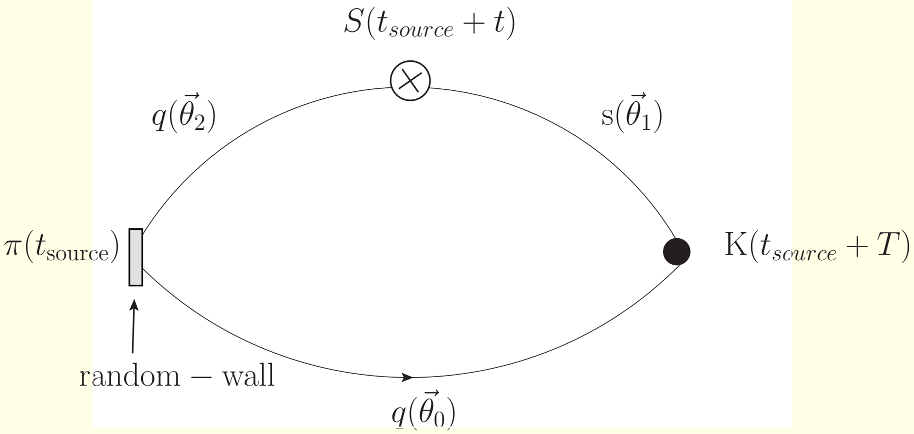
Black dashed line: CKM unitarity

Light blue band: Best fit with NP not absorbed in $\tilde{V}_{ud}$

2. FNAL/MILC Methodology

From the vector Ward-Takahashi identity

$$f_+^{K\pi}(0) = f_0^{K\pi}(0) = \frac{m_s - m_l}{m_K^2 - m_\pi^2} \langle \pi | S | K \rangle_{q^2=0}$$



* **Twisted boundary conditions** → allow generating correlation functions with non-zero external mom. such that $q^2 \simeq 0$

Twisted boundary conditions: $\psi(x_k + L) = e^{i\theta_k}\psi(x_k)$
(with k a spatial direction and L the spatial length of the lattice).

→ the propagator carries a momentum $p_k = \pi \frac{\theta_k}{L}$

* We inject momentum in the π (moving K give noisier data).

3.1. 2013 FNAL/MILC: Simulation details

HISQ valence quarks on MILC $N_f = 2+1+1$ HISQ configurations

$a(\mathrm{fm})$	m_l/m_s	`Volume`	$N_{conf.} \times N_{t_s}$	am_s^{sea}	am_s^{val}	
0.15	0.035	$32^3 \times 48$	1000×4	0.0647	0.0691	
0.12	0.2	$24^3 \times 64$	1053×8	0.0509	0.0535	
	0.1	$32^3 \times 64$	993×4	0.0507	0.053	
	0.1	$40^3 \times 64$	391×4	0.0507	0.053	FV check
	0.035	$48^3 \times 64$	945×8	0.0507	0.0531	
0.09	0.2	$32^3 \times 96$	775×4	0.037	0.038	
	0.1	$48^3 \times 96$	853×4	0.0363	0.038	
	0.035	$64^3 \times 96$	625×4	0.0363	0.0363	
0.06	0.2	$48^3 \times 144$	362×4	0.024	0.024	

* Physical quark mass ensembles

* HISQ action on the valence and sea: small discret. effects.

* Charm quarks on the sea.

* Very well tuned strange quark mass on the sea.

3.2. 2013 FNAL/MILC: Chiral and continuum extrapolation

The form factor $f_+(0)$ can be written in ChPT as

$$f_+(0) = 1 + f_2 + f_4 + f_6 + ... = 1 + f_2 + \Delta f$$

$f_+(0)$ goes to 1 in the $SU(3)$ limit due to vector current conservation

Ademollo-Gatto theorem $\rightarrow$ SU(3) breaking effects are second order in $(m_K^2 - m_\pi^2)$ and f_2 is completely fixed in terms of experimental quantities.

* At finite lattice spacing systematic errors can enter due to violations of the dispersion relation needed to derive

$$f_+(0) = f_0(0) = \frac{m_s - m_q}{m_K^2 - m_\pi^2} \langle S \rangle_{q^2=0}$$

Dispersion relation violations in our data are $\leq 0.1\%$.

3.2. 2013 FNAL/MILC: Chiral and continuum extrapolation

* One-loop (NLO) partially quenched `Staggered ChPT` + Two-loop (NNLO) continuum `ChPT`.

Bernard, Bijnens, E.G., 1311.7511; **Bijnens & Talavera**, 0303103

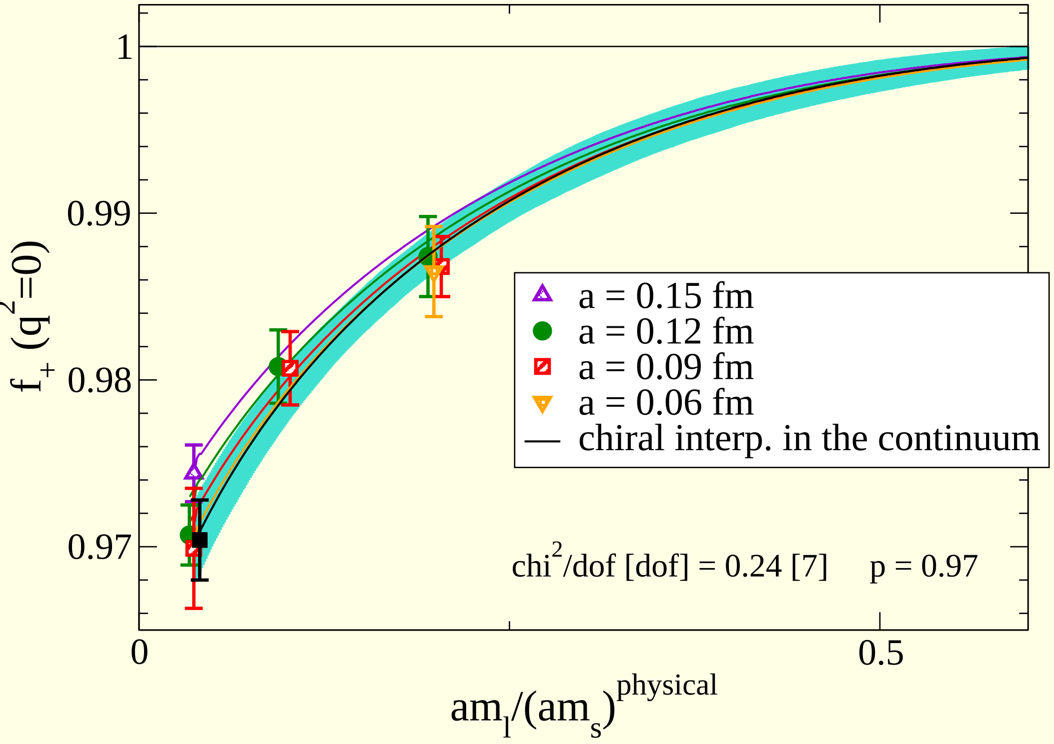
$$
\begin{aligned}
f_+(0) \quad &= \quad 1 + f_2^{\mathrm{PQS}\chi\mathrm{PT}}(a) + f_4^{\mathrm{cont}} + K_1\, a^2\sqrt{\bar\Delta} + K_3\, a^4 \\
&+ \quad (m_\pi^2 - m_K^2)^2 \left[C_6 + K_2\, a^2\sqrt{\bar\Delta} + K_2'\, a^2\bar\Delta + C_8\, m_\pi^2 + C_{10}\, m_\pi^4 \right]
\end{aligned}
$$

with $\bar\Delta$ used as a proxy of α_s^2 and $C_6 \propto \left(C_{12} + C_{34} - L_5^2\right)(\mu)$.

** We include leading isospin corrections in `CHPT` **Gasser & Leutwyler**, NPB250, 517 (1985).

** And estimate remaining isospin effects from two-loop continuum `CHPT` calculation **Bijnens & Ghorbani**, 0711.0148

3.3. 2013 FNAL/MILC: Results



Source of uncertainty	Error $f_+(0)$ (%)
Statistics+Discretization +Chiral interpolation	0.24
$m_s^{\rm sea} \neq m_s^{\rm val}$	0.03
Scale r_1	0.08
Finite volume	0.2
Isospin	0.016
Total Error	0.33

We do not see discretization effects except in the $a \approx 0.15\ fm$ ensemble.

$$\boxed{f_+(0) = 0.9704(24)(22) = 0.9704(32)}$$

4.1. New data

Improved statistical errors and data at smaller lattice spacing.

$a(\mathrm{fm})$	m_l/m_s	Volume	$N_{conf.} \times N_{t_s}$	am_s^{sea}	am_s^{val}
0.15	0.035	$32^3 \times 48$	1000×4	0.0647	0.0691
0.12	0.2	$24^3 \times 64$	1053×8	0.0509	0.0535
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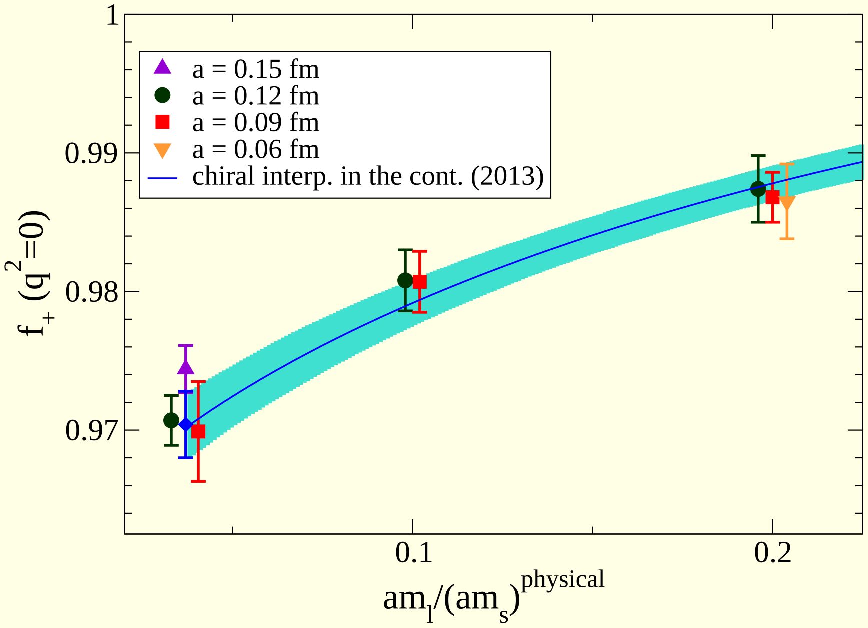
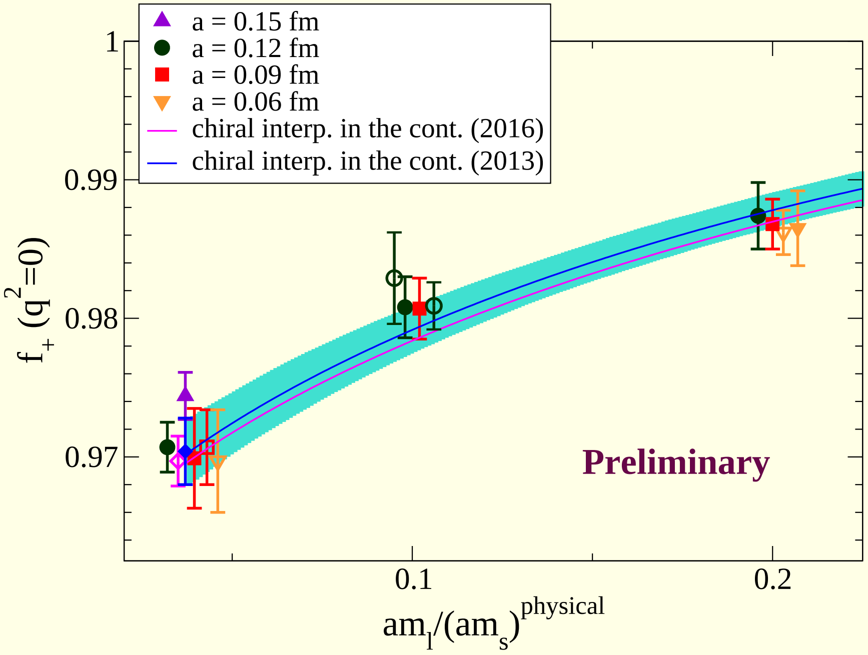
0.09	0.2	$32^3 \times 96$	773×4	0.037	0.038
	0.1	$48^3 \times 96$	853×4	0.0363	0.038
	0.035	$64^3 \times 96$	950×8	0.0363	0.0363
0.06	0.2	$48^3 \times 144$	1000×8	0.024	0.024
	0.035	$96^3 \times 192$	692×6	0.022	0.022
0.042	0.2	$64^3 \times 192$	432×12	0.0158	0.0158

* New (since 2013) data in red.

4.1. New data



FNAL/MILC 2013 (1312.1228)



FNAL/MILC 2016

Updated values of: r_1/a, `taste-splittings`, f_π

Open symbols: New data.

Not included yet: ensemble with ≈ 0.042 fm and $m_l/m_s = 0.2$.

Preliminary: Reduction of Statistics + discretization + chiral interpolation error under 0.2%.

4.2. Finite volume effects

2013 estimate: Compare results for $L = 32, 40$ ensembles and see no difference within statistics: took statistical error of most precise value as our FV uncertainty: 0.2%

Current values with different volumes for $a \approx 0.12$ fm and $m_l = 0.1 m_s$

Preliminary

L	24	32	40
$f_+(0)$	0.9829(33)	0.9808(22)	0.9809(17)
`variation`	+0.2%	0%	∼0%

Issue: Finite volume effects are modified when using twisted boundary conditions. They could be larger than with periodic boundary conditions.

Use `CHPT` to remove leading **FV** effects

Sachrajda & Villadoro, 2004, **Jiang & Tiburzi**, 2007, **Bijnens & Relefors**, 2014

See **J. Bijnens talk**

Partial twisting: At FV there are more form-factors since Lorentz-invariance is broken.

$$\langle \pi^+(p') | V_\mu | K^0(p) \rangle = f_+ (p_\mu + p'_\mu) + f_- q_\mu + h_\mu$$

4.2. Finite volume effects

Partial twisting: At FV there are more form-factors since Lorentz-invariance is broken

$$\langle \pi^-(p')|V_\mu|K^0(p)\rangle = f_+(p_\mu + p'_\mu) + f_- q_\mu + h_\mu$$

$\rightarrow$ Ward identity gets modified at FV

$$\rho \equiv (m_s - m_l)\langle \pi|S|K\rangle_{q^2} = (\tilde{m}^2_{K^0} - \tilde{m}^2_{\pi^-})f_+(q^2) + q^2 f_-(q^2) + q_\mu h_\mu$$

with $\tilde{m}^2_{K^0} - \tilde{m}^2_{\pi^-}$ including one-loop FV corrections.

Need FV corrections to $\frac{\rho}{\tilde{m}^2_{K^0} - \tilde{m}^2_{\pi^-}}$ (output of the correlator fits) with:

- * Twisted boundary conditions for the valence u quark: $\vec{\theta}_u = (\theta, \theta, \theta)$
- * Staggered fermions.

In progress: NLO (p^4) ChPT calculation of f_+, f_- and h_μ including finite volume, arbitrary twisted boundary conditions, partial quenching and staggered effects (also in the continuum)

C. Bernard, J. Bijnens, E.G., J. Relefors, see J.Bijnens talk

4.2. Finite volume effects

Preliminary

FV corrections in our data are $< 0.05\%$

J. Bijnens talk

(for the smallest volume the correction is $\sim 0.08\%$, but we only use it for checking purposes)

→ incorporating FV corrections at NLO in SChPT in our chiral-continuum extrapolation/interpolation will eliminate the dominant effects ($\leq 0.05\%$) and make negligible remaining effects.

5. Conclusions and outlook

FNAL/MILC $N_f = 2+1+1$ calculation with HISQ valence and sea quarks and four lattice spacings

$$f_+(0) = 0.9704 \pm 0.0024 \pm 0.0022$$

(this gives $|V_{us}| = 0.22311 \pm 0.00074_{f_+(0)} \pm 0.00040_{V_{ud}}$, $\sim 2.2\sigma$ lower than unitarity value and leptonic extraction of $|V_{us}|$)

- Physical light quark masses
- Same action on the valence and sea sectors.
- Very good tuning of sea quark masses
- Include sea charm quark effects

Error dominated by statistical+discretization+chiral and finite volume corrections.

5. Conclusions and outlook

* Statistical+discretization+chiral: increase statistics in key ensembles and data on new ensembles (in particular, $a \approx 0.06$ fm with phys. quark masses)

$$0.24\% \quad \rightarrow \quad < 0.2\%$$

* Finite Volume: NLO partially quenched `SChPT` finite size effects incorporated to chiral-continuum extrap./interp.

C. Bernard, J. Bijnens, E.G., J. Relefors

$$0.2\% \quad \rightarrow \quad \sim 0\%$$

\# Work to do:

* Optimize correlator fits for new data and include ≈ 0.042 fm ensemble

* Include leading FV corrections → extrapolation to infinite volume

Final goal: total error $\sim 0.2\%$ to match experimental error

* Study $f_+(0)/(f_K/f_\pi)$ (dependence on $|V_{us}|$ cancels) → Consistency check of SM/$|V_{ud}|$ **N. Brown [MILC]**, work in progress

3.2. 2013 FNAL/MILC: Chiral and continuum extrapolation

Even when simulations at physical quark masses are available, use of CHPT for the extrapolation is useful because

- * Allow the inclusion of data at $m_\pi > m_\pi^{\rm phys}$ (computationally cheaper and thus typically more precise) → reduce final statistical errors.
- * Correction of small mass mistunings and partially quenched effects ($m_s^{val} \neq m_s^{sea}$)
- * Dominant discretization effects can be analytically incorporated to CHPT expressions and thus removed in the extrapolation.
- * Ideal framework to analytically incorporate (and remove) dominantfinite volume effects.

Combined chiral+continuum+infinite volume extrapolation.

\# CHPT interpolation found to give smaller errors also in f_D, f_{D_s} calculation FNAL/MILC, 1407.3772