Two-Colour QCD at Finite Density with Two Flavours of Staggered Quarks

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Outline

Introduction

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Introduction

\[ S_f = \bar{\psi} D(\mu) \psi \]
\[ + \frac{\lambda}{2} \left( \psi^T (C\gamma_5) \tau_2 \psi + \bar{\psi} (C\gamma_5) \tau_2 \bar{\psi}^T \right) \]

- \( D = \) rooted staggered kernel
- \( N_f = 2 \)

- study diquark condensation transition, \( \lambda \rightarrow 0 \)
Previous studies

[Kogut, Toublan, PRD68, 054507 (2003)]
Previous studies

The Bulk Phase

\[ \langle z \rangle = 1 - \frac{1}{N_C} \sum_C \prod_{P \in \partial C} \text{sgn} \text{ tr } P \]

- \( \beta = 1.5 \rightarrow \langle z \rangle \approx 0.95 \)
- gauge action Symanzik improvement
The Bulk Phase

unimproved

improved

[D. Scheffler, PhD Thesis, Technische Universität Darmstadt (2015)]
Parameters

- Compromise:
  - $\beta = 1.7$, $\frac{m_\pi}{m_\rho} = 0.5816(27)$
  - $N_s = 16$, $N_t = 32$
  - standard rooted staggered quarks ($N_f = 2$), improved gauge action

[D. Scheffler, PhD Thesis, Technische Universität Darmstadt (2015)]
The diquark onset

- condensation of diquarks for $\mu \geq \frac{\pi}{2}$
- $\langle \bar{q}q \rangle$ rotates to $\langle qq \rangle$
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Results at High Density

- The diquark onset

The diquark onset

![Graph showing diquark and chiral condensates at different densities](image)

Diquark condensate $\langle qq \rangle$

Chiral condensate $\langle q - q \rangle$

- $\lambda = 0.005$
- $\lambda = 0.0025$
- $\lambda = 0.001$
- $\lambda \to 0$

![Graph showing diquark and chiral condensates at different densities](image)
The diquark onset

from chiral effective Lagrangian:

\[
\langle qq \rangle = \langle \bar{q}q \rangle_0 \sqrt{1 - \left( \frac{\mu_c}{\mu} \right)^4}
\]

\[
\langle \bar{q}q \rangle_0 = 0.00490(65)
\]

\[
a\mu_c = 0.1356(86)
\]

\[
am_\pi/2 = 0.1428(26)
\]
Renormalization of

\[
\langle \bar{q}q \rangle_{m_q} = \langle \bar{q}q \rangle_0 + c_2 m_q + \frac{c_{UV}}{a^2} m_q + O(m_q^3)
\]

\[
\chi_{m_q} = c_2 + \frac{c_{UV}}{a^2} + O(m_q^2)
\]

\[
\Rightarrow \Sigma := \langle \bar{q}q \rangle_{m_q} - m_q \chi_{m_q}
\]
Renormalization of

\[ \langle \overline{q}q \rangle_{m_q} = \langle \overline{q}q \rangle_0 + c_2 m_q + \frac{c_{UV}}{a^2} m_q + O(m_q^3) \]

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\[ \Rightarrow \Sigma := \langle \overline{q}q \rangle_{m_q} - m_q \chi_{m_q} \]
Renormalization of

\[ \langle \bar{q}q \rangle_m = \langle \bar{q}q \rangle_0 + c_2 m_q + \frac{c_{UV}}{a^2} m_q + \mathcal{O}(m^3) \]

\[ \chi_{m_q} = c_2 + \frac{c_{UV}}{a^2} + \mathcal{O}(m_q^2) \]

\[ \Rightarrow \Sigma := \langle \bar{q}q \rangle_m - m_q \chi_{m_q} \]
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Results at High Density

Lattice saturation

Lattice saturation

\[ n_s = \frac{N_f}{N_c} \]

\[ \mu / m_\pi \]

\[ \lambda \rightarrow 0 \]
Lattice saturation

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Results at High Density

Lattice saturation

- Quark number density $<n>/n_s$
- Polyakov loop $<l>$

Parameters:
- $\lambda = 0.005$
- $\lambda = 0.0025$
- $\lambda = 0.001$
- $\lambda \to 0$
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Results at High Density

... with Wilson Quarks

... with Wilson Quarks

![Graphs showing quark number density and Polyakov loop against chemical potential for different values of \( \lambda \)]
Quenching

- lattice saturation $\rightarrow$ re-quenching?
  - $\langle qq \rangle \rightarrow 0$
  - $\langle I \rangle \rightarrow 0$

Staggered quarks:

<table>
<thead>
<tr>
<th>$\mu / m_\pi$</th>
<th>Z(2) monopole density $&lt;z&gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.22</td>
</tr>
<tr>
<td>0.5</td>
<td>0.24</td>
</tr>
<tr>
<td>1</td>
<td>0.26</td>
</tr>
<tr>
<td>1.5</td>
<td>0.28</td>
</tr>
<tr>
<td>2</td>
<td>0.30</td>
</tr>
<tr>
<td>2.5</td>
<td>0.32</td>
</tr>
<tr>
<td>3</td>
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Wilson quarks:

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$\lambda = 0.005$

$\lambda = 0.0025$

$\lambda = 0.001$

$\lambda \rightarrow 0$
The Goldstone modes

- $qq \rightarrow$ true Goldstone boson
- $\bar{q}q$ constant until onset
Conclusion

- study diquark condensation at $\mu = \frac{m_\pi}{2}$
  - severe effects of the bulk phase
  - gauge improvement
    - $\rightarrow$ finite size effects
  - renormalization effects
- Polyakov loop insensitive to $\mu$ for staggered fermions