Landau levels in QCD

G. Endrődi, F. Bruckmann, M. Giordano, S.D. Katz,
T.G. Kovács, F. Pittler, J. Wellnhofer

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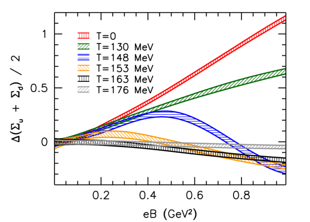
Motivation

Magnetic (inverse) catalysis in QCD

- Below T_c: Condensate grows as a function of $B \Longrightarrow$ catalysis
- Around T_c: Condensate decreases as a function of $B \Longrightarrow$ inverse catalysis



- Aim: understanding the mechanism leading to these phenomena

Effective models

- Work in Landau level basis
- ? Some of them take into account only Lowest Landau

- Work is in progress

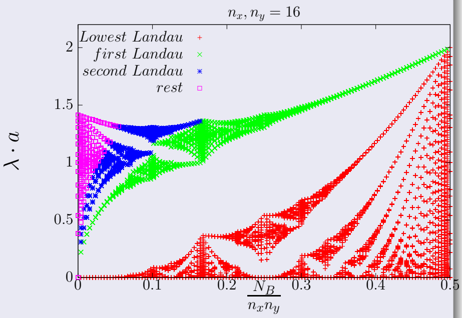
Landau levels in the Dirac spectrum($2d$)

Free fermions in uniform B field, continuum

- Degeneracies in the spectrum of Dirac operator
- $\lambda^2 = |qB| 2k$, where $k \in \mathbb{N}$

On the lattice: Hofstadter's butterfly Endrődi[LAT2014]

- Magnetic flux is quantized: N_b (finite volume)
- Lowest Landau: first $3N_B$ modes
- First Landau: next $2 \cdot 3N_B$ modes
- Free case
- Hofstadter's butterfly



Landau levels in the Dirac spectrum($2d$)

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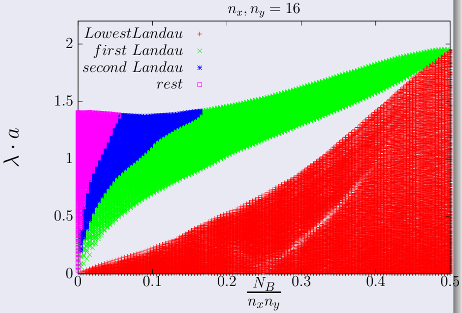
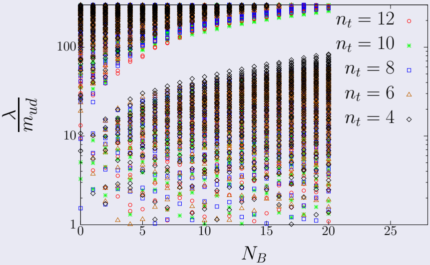
- Magnetic flux is quantized: N_b (finite volume)
- Lowest Landau: first $3N_B$ modes
- First Landau: next $2 \cdot 3N_B$ modes
- Interacting case
- Lowest Landau level is protected by topology



Landau levels in the Dirac spectrum(2d)

Continuum limit in $2d$

- The gap in $2d$ is physical

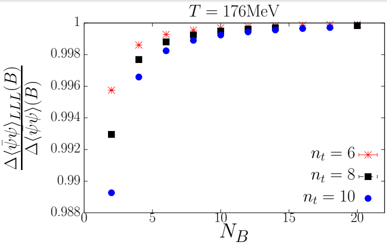


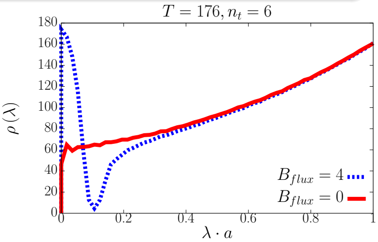
- 2+1 flavors of staggered quarks with physical masses
- Measure λ in units of bare mass
- Keep the physical magnetic field fixed
- Configurations are generated at $B = 0 \Longrightarrow$ valence effect

How to measure Lowest Landau Level (LLL) dominance?

Condensate in 2*d*

- Condensate from LLL: $\langle\bar\psi\psi\rangle_{LLL} = \sum_{i=1\cdots 3N_b} \frac{2m}{m^2+\lambda_i^2}$
- Full condensate: $\langle\bar\psi\psi\rangle = \sum_i \frac{2m}{m^2+\lambda_i^2}$





Catalysis comes entirely from the LLL in 2*d*

- Modes below the gap are enhanced
- Modes above the gap remain the same

Landau levels in $4d$

- More complicated

Free fermions in uniform B field, continuum

$$\lambda^2 = |qB|\left(2n+1+s\,\mathrm{sgn}(qB)\right)+p_z^2+\omega^2$$

where $n \in \mathbb{N}$ and $s = \pm 1$

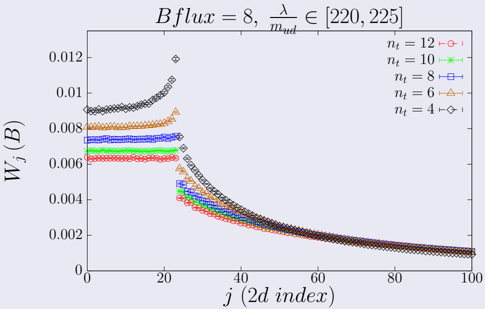
What happens with the gap on the lattice?

- p_z, ω will be quantized
- They fill in the gap even in the free case
- Landau levels can have effect through the $2d$ modes!

Decompose 4*d* modes in terms of 2*d* ones

Weight of the *j*-th 2d mode: $W_j(B) = \sum_{z,t} |(\phi_{t,z,j}(B), \psi)|^2$

- 2*d* mode: ϕ
- 4*d* mode: ψ



- Sharp jump at $3 \cdot B_{flux}$ even in the continuum
- Still large contribution from the higher 2*d* modes

Contribution of Landau levels to observables in $4D$

- How these 2d subspaces contribute to 4d observables?

√ Compute 4d observables on these subspaces only

- Projection to these modes:

$$C_i(B) = \sum_{j=1}^{3N_B} W_j(B) = \sum_{z,t} \sum_{j=1}^{3N_B} |\left(\phi_{t,z,j}(B), \psi_i\right)|^2$$

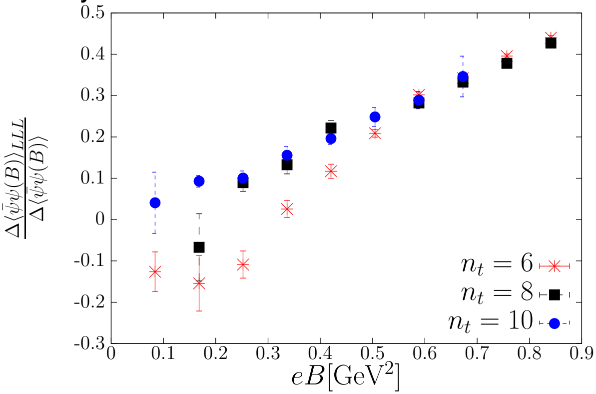
- The observable:

$$\langle \bar{\psi}\psi(B) \rangle_{LLL} = \left\langle \sum_i \frac{2m}{\lambda_i(B)^2 + m^2} C_i(B) \right\rangle$$

- Renormalization is not trivial

Lowest Landau Level dominance in 4*d*?

- Preliminary



- The lowest Landau approximation works where *B* is the dominant scale

Conclusion, outlook

- Lowest Landau is physical in $2d$
- √ Measuring the effect of Lowest Landau Level through the projection to the first $3N_B$ $2d$ modes
- ⊗ Valence catalysis not only due to the Lowest Landau (4d)

Still to be done

- Checking goodness of Lowest Landau approximation as the magnetic field increases
- Including also sea effects

Thank you for your attention!