Heavy-heavy current improvement for calculation of $B \to D^{(*)}\ell\nu$ semi-leptonic form factors using the Oktay-Kronfeld action

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$B \to D^{(*)}\ell\nu$ form factors and flavor physics

- $|V_{cb}|$ normalizes Unitarity Triangle, dominates uncertainty in many observables $\sim$ searches for new physics
  - Rare decays $K \to \pi\nu\overline{\nu}$, $B_s^0 \to \mu^+\mu^-$
  - Indirect CP violation $\sim \epsilon_K$
  - $|V_{cb}|$ from inclusive decays, exclusive decays
- Ratios $R(D), R(D^*)$ from SM, experiment

$$R(D^{(*)}) = \frac{\mathscr{B}(B \to D^{(*)}\tau\nu)}{\mathscr{B}(B \to D^{(*)}\ell\nu)}$$

$|V_{cb}|$ from $B \to D^{(*)}\ell\nu$

$$\frac{d\Gamma}{d\omega}(B \to D\ell\nu) = \frac{G_F^2|V_{cb}|^2 m_b^5}{48\pi^3}(\omega^2-1)^{3/2} r^3 (1+r)^2 F_D^2(\omega)$$

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\nu) = \frac{G_F^2|V_{cb}|^2 m_b^5}{4\pi^3}|\eta_{\mathrm{EW}}|^2(1+\pi\alpha)\times$$
$$(\omega^2-1)^{1/2} r^{*3}(1-r^*)^2 \chi(\omega) F_{D^*}^2(\omega)$$

- Decay rates from experiments $\Rightarrow$ form factor shapes
- Kinematic factors, perturbative corrections, $|V_{cb}|$
  - Velocity transfer $\omega = v_{D^{(*)}} \cdot v_B$, ratios $r^{(*)} = m_{D^{(*)}}/m_B$,

    $$\chi(\omega) = \frac{\omega+1}{12}\left(5\omega + 1 - \frac{8\omega(\omega-1)r^*}{(1-r^*)^2}\right)$$

  - Higher order electroweak corrections: $\eta_{\mathrm{EW}} \sim$ NLO box diagrams, $\pi\alpha \sim$ Coulomb attraction for charged D^*
- Form factors $\sim$ hadronic matrix elements, flavor-changing currents between initial, final mesons

Hadronic matrix elements

$$\frac{\langle D(p_D)|V^\mu|B(p_B)\rangle}{\sqrt{m_D m_B}} = (v_B + v_D)^\mu h_+(\omega) + (v_B - v_D)^\mu h_-(\omega)$$

$$\frac{\langle D^*(p_{D^*},\epsilon)|V_\mu|B(p_B)\rangle}{\sqrt{m_{D^*} m_B}} = \varepsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu} v_B^\rho v_{D^*}^\sigma h_V(\omega)$$

$$\frac{\langle D^*(p_{D^*},\epsilon)|A^\mu|B(p_B)\rangle}{\sqrt{m_{D^*} m_B}} = i\epsilon^{*\mu}(\omega+1)h_{A_1}(\omega) - i(\epsilon^*\cdot v_B)\times \left[v_B^\mu h_{A_2}(\omega) + v_{D^*}^\mu h_{A_3}(\omega)\right]$$

- Form factors h_i in HQET $\longrightarrow F_D,\ F_{D^*}$
- Lorentz symmetry, constraints from P, T
- HQ symmetry $\Longrightarrow$ constraints on h_i
- $B \to D^*$ at zero recoil $\sim F_{D^*}(1) = h_{A_1}(1) \approx 1$

Heavy quarks on the lattice

- Discretization errors $\sim am_q$ for traditional Wilson, Kogut-Susskind fermions $\Longrightarrow$ uncontrolled systematics for $am_q \sim 1$
- Different approaches for controlling heavy quark discretization errors
  - Extrapolation from light to heavy
  - Step scaling with static limit anchor point
  - Effective field theories $\sim$ HQET, NRQCD (nonrenormalizable, no continuum limit $\sim$ heavy quarks only)
  - Fermilab method, RHQ $\sim$ lift symmetry interchanging time and spatial axes, retain Wilson time derivative, include irrelevant operators $\Longrightarrow$ tune to renormalized trajectory for arbitrary quark masses am_q

Improvement for heavy quarks

[El-Khadra, Kronfeld, Mackenzie, PRD 1997; Kronfeld, PRD 2000]

- Effective (continuum) field theories $\sim$ convenient, practical tools for quantifying discretization errors, assessing and systematizing improvement
  - Symanzik EFT $\sim$ light quarks, $am_q \ll 1$; generalization for arbitrary quark masses

$$S_{\mathrm{LGT}} \doteq S_{\mathrm{QCD}} + aS_5 + a^2 S_6 + a^3 S_7 + \dots$$
$$Z_J J_{\mathrm{LGT}} \doteq J_{\mathrm{QCD}} + J' + J'' + J''' + \dots$$

  - Nonrelativistic EFTs $\sim$ (continuum) HQET, NRQCD at energy scales $\ll m_q$
- Match correlation functions $\sim$ on-shell matrix elements, other convenient quantities; power counting $\sim$ momentum, coupling dependence
- Quantify errors in desired quantities $\sim$ assess improvement, estimate systematics

Action improvement

[El-Khadra, Kronfeld, Mackenzie, PRD 1997; Oktay and Kronfeld, PRD 2008]

- Clover fermions without time-space axis interchange symmetry $\sim$ tune to renormalized trajectory
- Standard clover fermions + nonrelativistic interpretation $\Longrightarrow$ renormalized trajectory contains particles with different rest, kinetic masses
  - rest mass = energy at zero momentum
  - kinetic mass $\sim$ characterizes inertial response
- Unphysical effect completely understood, quarantined if kinetic masses tuned to physical masses $\Longrightarrow$ correctly tuned clover action yields QCD matrix elements $\longrightarrow$ phenomenological success for charm and bottom physics
- Charm quark discretization errors $\longrightarrow$ extend Fermilab method, action $\Longrightarrow$ Oktay-Kronfeld action, spectrum tests of improvement

Current improvement

[El-Khadra, Kronfeld, Mackenzie, PRD 1997; Kronfeld, PRD 2000; Harada *et al.*, PRD 2002]

- Include operators J_i with quantum numbers of desired operator $\sim$ power counting, enumeration

$$J_{\rm QCD} = Z_J(am_q, g^2)\left[J_0 + \sum_i C_i(am_q, g^2) J_i\right]$$

- Correct deviations from continuum limit $\sim$ suppression, enhancement by powers of lattice spacing
- Fix renormalization factor(s) Z_J, coefficients C_i of improvement operators $\sim$ matching matrix elements
- Generalized Symanzik improvement $\sim$ no expansion in am_q $\Longleftrightarrow$ arbitrary am_q

Improved heavy quark field

[El-Khadra, Kronfeld, Mackenzie, PRD 1997; Harada *et al.*, PRD 2002]

Currents improved through lowest nontrivial order in HQET ∼ order $\boldsymbol{p}$ in two-quark current matrix elements

$$\psi(x) \to \Psi_I(x) \equiv e^{M_1/2}\left[1 + d_1 \boldsymbol{\gamma}\cdot\boldsymbol{D}\right]\psi(x)$$

$$\overline{\psi}_c \Gamma \psi_b \to \overline{\Psi}_{Ic} \Gamma \Psi_{Ib}$$

$$\langle c(\xi_c, \boldsymbol{p}_c)|\overline{c}\Gamma b|b(\xi_b, \boldsymbol{p}_b)\rangle \to \sqrt{\frac{m_c}{E_c}}\,\overline{u}_c(\xi_c, \boldsymbol{p}_c)\Gamma\sqrt{\frac{m_b}{E_b}}\,u_b(\xi_b, \boldsymbol{p}_b)$$

$$\langle q_c(\xi_c, \boldsymbol{p}_c)|\overline{\Psi}_{Ic}\Gamma\Psi_{Ib}|q_b(\xi_b, \boldsymbol{p}_b)\rangle \to$$
$$\mathscr{N}_c(\boldsymbol{p}_c)\overline{u}_c^{\mathrm{lat}}(\xi_c, \boldsymbol{p}_c)R_c^{(0)}(\boldsymbol{p}_c)\Gamma R_b^{(0)}(\boldsymbol{p}_b)\mathscr{N}_b(\boldsymbol{p}_b)u_b^{\mathrm{lat}}(\xi_b, \boldsymbol{p}_b)$$

$$R^{(0)}(\boldsymbol{p}) = e^{M_1/2}\left[1 + i d_1 \gamma_k \sin p_k\right]$$

Matching two-quark matrix elements

- Third order in HQET $\sim$ order $\boldsymbol{p}^3$ in two-quark matrix elements
- OK action $\Longrightarrow$ corrections to lattice spinors, normalization factors
- Lattice matrix elements differ from continuum $\sim$ mismatch between lattice, continuum spinors, normalization factors $\Longrightarrow \Psi_I$ exists
- Ansatz: operators $\sim$ mass dimension 6 in OK action

$$\begin{aligned}\Psi_I(x) = e^{M_1/2}\Big[&1 + d_1\boldsymbol{\gamma}\cdot\boldsymbol{D} + \tfrac{1}{2}d_2\triangle^{(3)} + \tfrac{1}{2}id_B\boldsymbol{\Sigma}\cdot\boldsymbol{B} + \tfrac{1}{2}d_E\boldsymbol{\alpha}\cdot\boldsymbol{E} \\ &+ d_{r_E}\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \boldsymbol{\alpha}\cdot\boldsymbol{E}\} + d_{z_E}\gamma_4(\boldsymbol{D}\cdot\boldsymbol{E} - \boldsymbol{E}\cdot\boldsymbol{D}) \\ &+ \tfrac{1}{6}d_3\gamma_k D_k\triangle_k + \tfrac{1}{2}d_4\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \triangle^{(3)}\} + d_5\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}\} \\ &+ d_{EE}\{\gamma_4 D_4, \boldsymbol{\alpha}\cdot\boldsymbol{E}\} + d_{z_3}\boldsymbol{\gamma}\cdot(\boldsymbol{D}\times\boldsymbol{B} + \boldsymbol{B}\times\boldsymbol{D})\Big]\psi(x)\end{aligned}$$

Improvement parameters from tree-level two-quark matrix elements

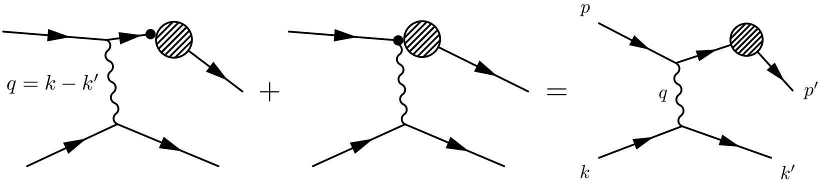
- Operators without explicit $\boldsymbol{E}$, $\boldsymbol{B}$ fields $\propto d_1, d_2, d_3, d_4$

$$R^{(0)}(\boldsymbol{p}) \Rightarrow e^{M_1/2}\Big[1 + id_1\gamma_k \sin p_k - 2d_2 \sum_l \sin^2 \tfrac{1}{2}p_l - \tfrac{2}{3} id_3\gamma_k \sin p_k \sin^2 \tfrac{1}{2}p_k - 4id_4\gamma_k \sin p_k \sum_l \sin^2 \tfrac{1}{2}p_l\Big]$$

- $\mathscr{O}(\boldsymbol{p})$ terms $\to d_1$
- $\mathscr{O}(\boldsymbol{p}^2)$ terms $\to d_2$
- Rotation-breaking $\mathscr{O}(\boldsymbol{p}^3)$ terms $\to d_3$
- Rotation-invariant $\mathscr{O}(\boldsymbol{p}^3)$ terms $\to d_4$

Matching four-quark matrix elements

- Tree-level dependence on improvement parameters, operators with explicit $\boldsymbol{E}$, $\boldsymbol{B}$ fields



- One-gluon vertex from OK action $\sim$ OK action tuning $\Longrightarrow$ match heavy-quark part of diagrams; no gluon propagator, gauge dependence
- Consider $b \to u$ transition $\Longrightarrow$ one heavy quark, set of improvement parameters

Momentum expansions

- $\boldsymbol{p},\ \boldsymbol{q},\ q_4 \ll 1/a,\ m_b,\ M_1,\ M_2,\ M_X,\ M_Y,\ \ldots$
- $\boldsymbol{p}$ on shells, expand first in $\boldsymbol{p} \sim$ full dependence on q, no missing terms in $\boldsymbol{p}$
- OK action vertex, field improvement vertices $\sim$ expansions in lattice spacing
- Continuum propagator $\sim$ expansion in $\boldsymbol{p}/m_b$, q_μ/m_b
- Lattice propagator $\sim$ joint expansion in $a\boldsymbol{p}$, $\boldsymbol{p}/M_i$, aq_μ, $q_\mu/M_i,\ \ldots$

Improvement parameters from four-quark matrix elements

- At zeroth order in $\boldsymbol{p}$, spatial and temporal components of matching condition, several independent terms (in general) at each formal order in $q \Longrightarrow$ sufficient constraints to extract all improvement parameters in ansatz?
- Algebra of expansions, solving constraints, reducing results $\longrightarrow$ *Mathematica*
  - Spatial $\mathscr{O}(\boldsymbol{q}/q_4)$ terms $\rightarrow d_1$ (consistent)
  - Spatial $\mathscr{O}(\boldsymbol{q})$ terms $\rightarrow d_2$ (consistent)
  - $\cdots \rightarrow d_3,\ d_4,\ d_B$ (consistent)
  - Temporal $\mathscr{O}(\boldsymbol{q})$ terms $\rightarrow d_E$ (two calculations)
  - Temporal $\mathscr{O}(\boldsymbol{q}^2)$ terms $\rightarrow d_{r_E} - d_{z_E}$ (two calculations)
  - Temporal $\mathscr{O}(\boldsymbol{q}q_4)$ terms $\rightarrow d_{EE}$ (two calculations)
  - Spatial $\mathscr{O}(\boldsymbol{q}^2)$ terms $\rightarrow d_5 - d_{z_3}$ (one calculation)
- Multiple equivalent constraints $\rightarrow d_i$
- Consistent with OK action improvement parameters $c_1,\ c_2,\ c_3,\ c_4,\ c_5,\ c_E,\ c_B$

Summary

- $B \to D^{(*)}\ell\nu$ form factors $\Rightarrow |V_{cb}|$, tests of SM
- Discretization errors significant $\sim$ highly improved actions, currents
- Improved quark field $\sim$ current improvement
  - Ansatz: Operators $\sim$ mass dimension 6 in improved action
  - Matching two-quark, four-quark current matrix elements; preliminary results for 7 of 11 improvement parameters, consistent with previous results for currents, action
  - Checking equality of lattice, continuum four-quark matrix elements through third order in momentum expansions
- Demonstrate systematic improvement $\longrightarrow$ operator enumeration, HQET matching analysis