Non-perturbative determination of improvement coefficients using coordinate space methods in $N_f = 2 + 1$ lattice QCD

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Introduction

Improvement of LQCD with Wilson fermions

Wilson fermions introduce cut-off effects linear in the lattice spacing a.

One can account for them following Symanzik improvement programme

$$S_{\mathrm{QCD}}\big(a(\beta)\big) = S_{\mathrm{continuum}} + aS_1 + a^2 S_2 + \ldots$$

Improvement of the action requires knowledge of the c_{SW} coefficient and of b_g and b_m which are proportional to the quark mass.
Apart of the action itself we want to use improved operators. We usually define

$$A_\mu^{jk,\mathrm{I}}(x) = \bar{\psi}_j(x)\gamma_\mu\gamma_5\psi_k(x) + a c_A \partial_\mu^{\mathrm{sym}} P^{jk}(x)$$

and

$$A_\mu^{jk,\mathrm{R}}(x) = Z_A\big(1 + a b_A m_{jk} + a3\tilde{b}_A\overline{m}\big)A_\mu^{jk,\mathrm{I}}(x)$$

$\Rightarrow$ full, non-perturbative improvement of Wilson fermions needs c_{SW}, b_g, c_J, b_J.

Introduction

CLS ensembles

The CLS initiative is currently generating ensembles with $N_f = 2 + 1$ flavours of non-perturbatively improved Wilson Fermions and the tree-level Lüscher-Weisz gauge action at $\beta = 3.4, 3.46, 3.55$ and 3.7. This corresponds to lattice spacings of $a \in [0.05, 0.09]$ fm.

Improvement coefficients

- $c_{\mathrm{SW}} \rightarrow$ Bulava, Schaefer, '13
- $c_A \rightarrow$ Bulava, Della Morte, Heitger, Wittemeier '15
- b_Γ, $\tilde{b}_\Gamma \rightarrow$ this talk

Mass dependent improvement coefficients

We use coordinate space method proposed by Martinelli *et al.* (Phys. Lett. B 411, 141 (1997)) to determine b_J, $\tilde{b}_J$ for flavour non-singlet scalar, pseudoscalar, vector and axialvector currents.

Notation

We denote quark mass averages as

$$m_{jk} = \frac{1}{2}(m_j + m_k)\,,$$

where

$$m_j = \frac{1}{2a}\left(\frac{1}{\kappa_j} - \frac{1}{\kappa_{\rm crit}}\right).$$

The mass dependence of physical observables can be parameterized in terms of the average quark mass

$$\overline{m} = \frac{1}{3}\left(m_s + 2m_\ell\right)\,.$$

We define connected Euclidean current-current correlation functions in a continuum renormalization scheme R, e.g., $R = \overline{\rm MS}$, at a scale μ:

$$G^R_{J^{(jk)}}(x, m_\ell, m_s; \mu) = \left\langle \Omega \middle| T\, J^{(jk)}(x)\overline{J}^{(jk)}(0) \middle| \Omega \right\rangle^R .$$

Method

Two observations

1) The continuum correlation function differs from that of the massless case by mass dependent terms

$$G^R_{J(jk)}(x, m_\ell, m_s; \mu) = G^R_{J(jk)}(x, 0, 0; \mu)$$
$$\times \left[1 + \mathcal{O}\left(m^2x^2, m^2\langle FF\rangle x^6, m\langle\overline{\psi}\psi\rangle x^4, m\langle\overline{\psi}\sigma F\psi\rangle x^6\right)\right] ,$$

2) The continuum Green function G^R above can be related to the corresponding Green function G obtained in the lattice scheme at a lattice spacing $a = a(g^2)$ as follows:

$$G^R_{J(jk)}(x, m_\ell, m_s; \mu) = \left(Z^R_J\right)^2(\tilde{g}^2, a\mu)$$
$$\times \left(1 + 2b_J am_{jk} + 6\overline{b}_J a\overline{m}\right) G_{J(jk),I}(n, am_{jk}, a\overline{m}; g^2) ,$$

Method

$$\frac{G_{J^{(jk)}}\left(n, am_{jk}^{(\rho)}, a\overline{m}^{(\rho)}; g^2\right)}{G_{J^{(rs)}}\left(n, am_{rs}^{(\sigma)}, a\overline{m}^{(\sigma)}; g^2\right)} = 1 + 2b_J a\left(m_{rs}^{(\sigma)} - m_{jk}^{(\rho)}\right) + 6\tilde{b}_J a\left(\overline{m}^{(\sigma)} - \overline{m}^{(\rho)}\right) + \mathcal{O}\left(a^2, x^2\right),$$

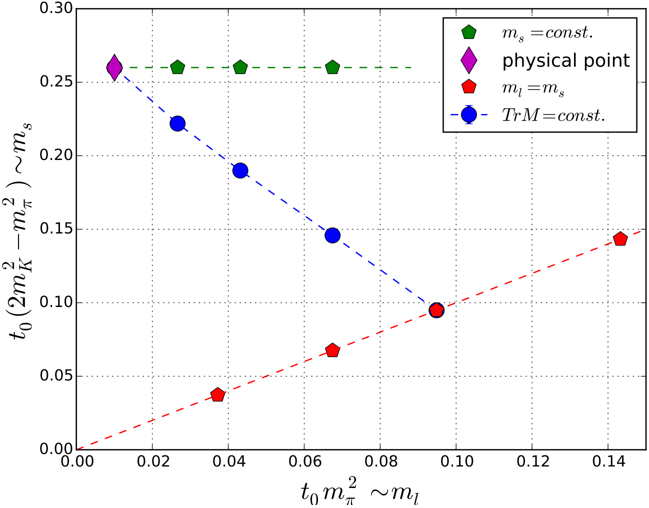
We can construct two useful observables:

$$R_J(x, \delta m) \equiv \frac{G_{J^{(12)}}\left(n, am_{12}^{(\rho)}, a\overline{m}^{(\rho)}; g^2\right)}{G_{J^{(13)}}\left(n, am_{13}^{(\rho)}, a\overline{m}^{(\rho)}; g^2\right)} = 1 + 2b_J a\delta m$$

$$\widetilde{R}_J(x, \delta\overline{m}) \equiv \frac{G_{J^{(12)}}\left(n, a\overline{m}^{(\rho)}, a\overline{m}^{(\rho)}; g^2\right)}{G_{J^{(12)}}\left(n, a\overline{m}^{(\sigma)}, a\overline{m}^{(\sigma)}; g^2\right)} = 1 + (2b_J + 6\tilde{b}_J)a\delta\overline{m}$$

$\Rightarrow$ Let's see how this works in practice!

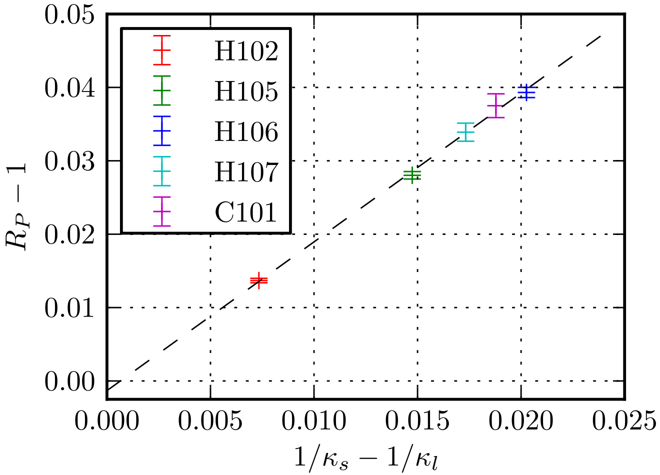
Overview of CLS ensembles at $\beta = 3.4$



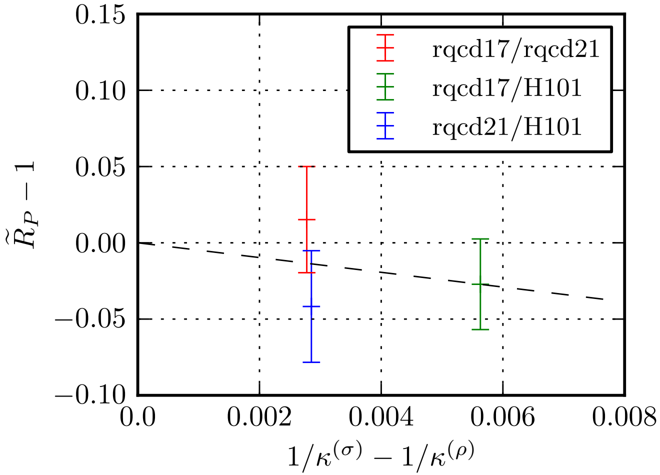
List of ensembles

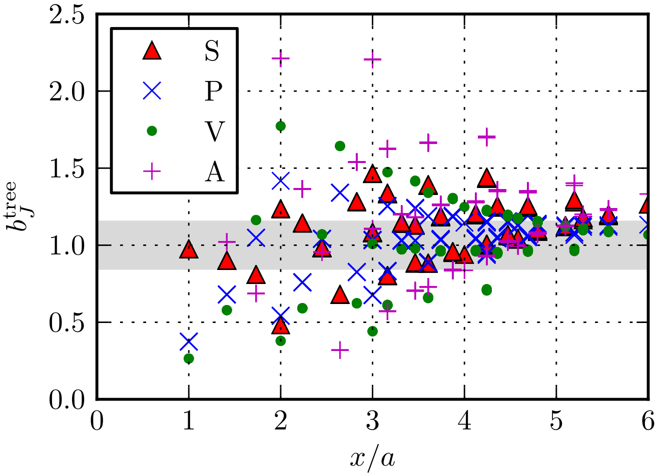
β	name	κ_l	κ_s	# conf.	step
3.4	H101	0.136759	0.136759	100	10
3.4	H102	0.136865	0.136549339	100	10
3.4	H105	0.136970	0.13634079	103	5
3.4	H106	0.137016	0.136148704	57	5
3.4	H107	0.136946	0.136203165	49	5
3.4	C101	0.137030	0.136222041	59	10
3.4	C102	0.137051	0.136129063	48	10
3.4	rqcd17	0.1368650	0.1368650	150	10
3.4	rqcd19	0.13660	0.13660	50	10
3.46	S400	0.136984	0.136702387	83	10
3.55	N203	0.137080	0.136840284	74	10
3.7	J303	0.137123	0.1367546608	38	10

Example: $R_P - 1$ for $n = (0, 1, 1, 1)$



Example: $\widetilde{R}_P - 1$ for $n = (0, 1, 2, 2)$





2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
6
S
P
V
A
b_J^{tree}
x/a

Applying OPE to the ratio of correlation functions one gets

$$\frac{G_{J(12)}(x)}{G_{J(34)}(x)} = 1 + (A^J_{12} - A^J_{34})x^2 + \left[\left(A^J_{34}\right)^2 - A^J_{12}A^J_{34} + B^J_{12} - B^J_{34} \right] x^4 + \cdots ,$$

with the mass dependent coefficients

$$A^J_{jk} = -\frac{1}{4}\left(m_j^2 + m_k^2 + \frac{m_j m_k}{s_J} \right) ,$$

$$B^J_{jk} = \frac{\pi^2}{32N}\langle FF \rangle + \frac{m_j^2 m_k^2}{16} + \frac{\pi^2}{8N}\frac{2+s_J}{s_J}(m_j + m_k)\langle \overline{\psi}\psi \rangle .$$

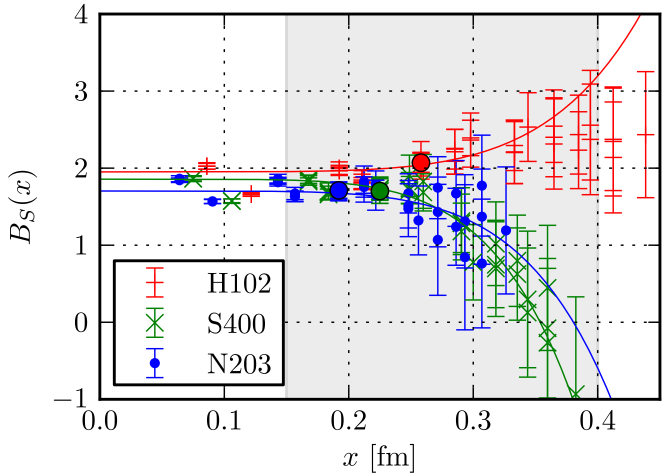
We correct our observables R_J and $\widetilde{R}_J$ by subtracting the leading continuum corrections.

Improved observables

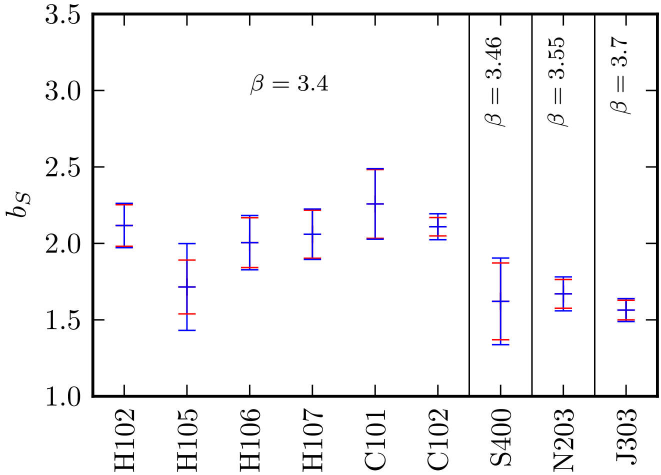
$$B_J(x,\delta m) \equiv \left[R_J(x,\delta m) - R_J^{\text{tree}}(x,\delta m) + \frac{\pi^2}{8N}\frac{2+s_J}{s_J}\left(M_\pi^2 - M_K^2\right) F_0^2 x^4 \right] \times \left(\frac{1}{\kappa_s} - \frac{1}{\kappa_\ell} \right)^{-1} = b_J + \mathcal{O}(x^6) + \mathcal{O}(g^2 a^2) + \cdots ,$$

$$\widetilde{B}_J(x,\delta\overline{m}) \equiv \left[\widetilde{R}_J(x,\delta\overline{m}) - \widetilde{R}_J^{\text{tree}}(x,\delta\overline{m}) + \frac{\pi^2}{8N}\frac{2+s_J}{s_J}\left(\delta M_\pi{}^2\right) F_0^2 x^4 \right] \times \left(\frac{1}{\kappa^{(\sigma)}} - \frac{1}{\kappa^{(\rho)}} \right)^{-1} = b_J + 3\tilde{b}_J + \mathcal{O}(x^6) + \mathcal{O}(g^2 a^2) + \cdots .$$

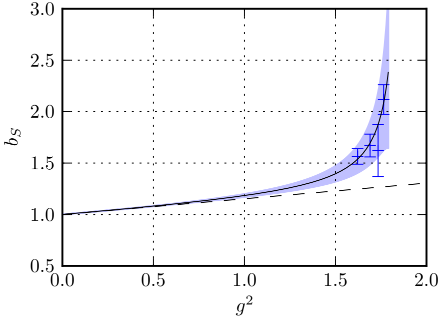
Example of numerical data



Results in the scalar channel



Final results: rational parametrizations



Final results

$$b_S(g^2) = 1 + 0.11444(1)C_F(1 - 0.439(50)g^2)(1 - 0.535(14)g^2)^{-1}$$
$$b_P(g^2) = 1 + 0.0890(1)C_F(1 - 0.354(54)g^2)(1 - 0.540(11)g^2)^{-1}$$
$$b_V(g^2) = 1 + 0.0886(1)C_F(1 + 0.596(111)g^2)$$
$$b_A(g^2) = 1 + 0.0881(1)C_F(1 - 0.523(33)g^2)(1 - 0.554(10)g^2)^{-1}$$

Conclusions

$\tilde{b}_J$ improvement coefficients at $\beta = 3.4$

$$
\begin{aligned}
\tilde{b}_S &= 2.0 \quad (1.3)_{\mathrm{stat}} \quad (0.3)_{\mathrm{sys}} \\
\tilde{b}_P &= -3.4 \quad (1.3)_{\mathrm{stat}} \quad (0.6)_{\mathrm{sys}} \\
\tilde{b}_V &= -0.1 \quad (0.4)_{\mathrm{stat}} \quad (0.1)_{\mathrm{sys}} \\
\tilde{b}_A &= 1.4 \quad (0.4)_{\mathrm{stat}} \quad (0.9)_{\mathrm{sys}}
\end{aligned}
$$

Conclusions

- We implemented a coordinate space method to determine improvement coefficients proportional to quark mass
- With a negligible numerical effort we can achieve a 5% - 10% precision on b_J
- Improvement coefficients proportional to the trace of the mass matrix are accessible, but need a better statistical precision