Nucleon structure from 2+1-flavor dynamical DWF ensembles

Shigemi Ohta *†‡ for RBC and UKQCD Collaborations

Lattice 2016, Southampton, UK, July 2016

RBC and UKQCD have been generating dynamical DWF ensembles with good chiral and flavor symmetries. We have been at physical mass for a while now, with a range of momentum cuts off, 1-3 GeV, and volumes $m_\pi L >\sim 4$ and producing a lot of good physics in pion and kaon

In nucleon: we observed puzzling and persistent deficit in the isovector axial charge, g_A, while vector-current form factors are well-behaved, and low structure-function moments are trending toward experiments.

But during the past couple of years,

- nucleon structure calculations at physical mass, jointly with LHP, did not gain any new statistics.
- We improved statistics for some earlier RBC+UKQCD ensembles, driven by Michael Abramczyk,

and with Tom Blum, Taku Izubuchi, Chulwoo Jung, Meifeng Lin, SO, and Eigo Shintani.

*Institute of Particle and Nuclear Studies, High-Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan
†Department of Particle and Nuclear Physics, Sokendai Graduate University of Advanced Studies, Hayama, Kanagawa 240-0193, Japan
‡RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu F_V(q^2) + \frac{i\sigma_{\mu\lambda}q_\lambda}{2m_N}F_T(q^2)\right]u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_5\gamma_\mu F_A(q^2) + \gamma_5 q_\mu F_P(q^2)\right]u_n e^{iq\cdot x}.$$

$$F_V = F_1, F_T = F_2; G_E = F_1 - \frac{q^2}{4m_N^2}F_2, G_M = F_1 + F_2.$$

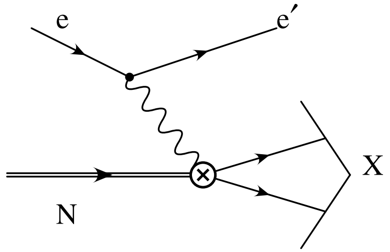
Related to mean-squared charge radii, anomalous magnetic moment, $g_V = F_V(0) = G_{\rm Fermi}\cos\theta_{\rm Cabibbo}$, $g_A = F_A(0) = 1.2701(25)g_V$, Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, ... determine much of nuclear physics.

On the lattice, with appropriate nucleon operator, for example, $N = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$, ratio of two- and three-point correlators such as $\dfrac{C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t)}{C_{\rm 2pt}(t_{\rm sink})}$ with

$$C_{\rm 2pt}(t_{\rm sink}) = \sum_{\alpha,\beta}\left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta}\langle N_\beta(t_{\rm sink})\bar{N}_\alpha(0)\rangle,$$

$$C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t) = \sum_{\alpha,\beta}\Gamma_{\alpha\beta}\langle N_\beta(t_{\rm sink})O(t)\bar{N}_\alpha(0)\rangle,$$

give a plateau in t for a lattice bare value $\langle O\rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

Deep inelastic scatterings 

$: \left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu},\ W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$

- unpolarized: $W^{\{\mu\nu\}}(x, Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x, Q^2) + \left(P^\mu - \frac{\nu}{q^2} q^\mu\right)\left(P^\nu - \frac{\nu}{q^2} q^\nu\right) \frac{F_2(x, Q^2)}{\nu}$,
- polarized: $W^{[\mu\nu]}(x, Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x, Q^2) + g_2(x, Q^2)) - \frac{q \cdot S P_\sigma}{\nu^2} g_2(x, Q^2)\right)$,

with $\nu = q \cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

Moments of the structure functions are accessible on the lattice:

$$
\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x, Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\ \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2), \\
\int_0^1 dx x^{n-2} F_2(x, Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\ \langle x^n \rangle_q(\mu) + \mathcal{O}(1/Q^2), \\
2\int_0^1 dx x^n g_1(x, Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\ \langle x^n \rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2), \\
2\int_0^1 dx x^n g_2(x, Q^2) &= \frac{1}{2}\frac{n}{n+1} \sum_{q=u,d} [e_{2,n}^q(\mu^2/Q^2, g(\mu))\ d_n^q(\mu) - 2e_{1,n}^q(\mu^2/Q^2, g(\mu))\ \langle x^n \rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\end{aligned}
$$

- c_1, c_2, e_1, and e_2 are the Wilson coefficients (perturbative),
- $\langle x^n \rangle_q(\mu)$, $\langle x^n \rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is $\langle 1 \rangle_{\delta q}(\mu) = \langle P, S | \bar{\psi} i \gamma_5 \sigma_{\mu\nu} \psi | P, S \rangle$ which may be measured by polarized Drell-Yan and RHIC Spin.

Unpolarized (F_1/F_2): on the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{trace})\right]q$$

Polarized (g_1/g_2): on the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_\sigma\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1}-S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_{[\sigma}\overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

and transversity (h_1):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})]q$$

Higher moment operators mix with lower dimensional ones: Only $\langle x\rangle_q$, $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$.

What I am reporting today are from our "ID32" ensembles,

- with Iwasaki × dislocation-suppressing-determinatn-ratio (DSDR) gauge action at $\beta = 1.75$, $a^{-1} = 1.378(7)$ GeV, and pion mass of about 250 and 172 MeV.

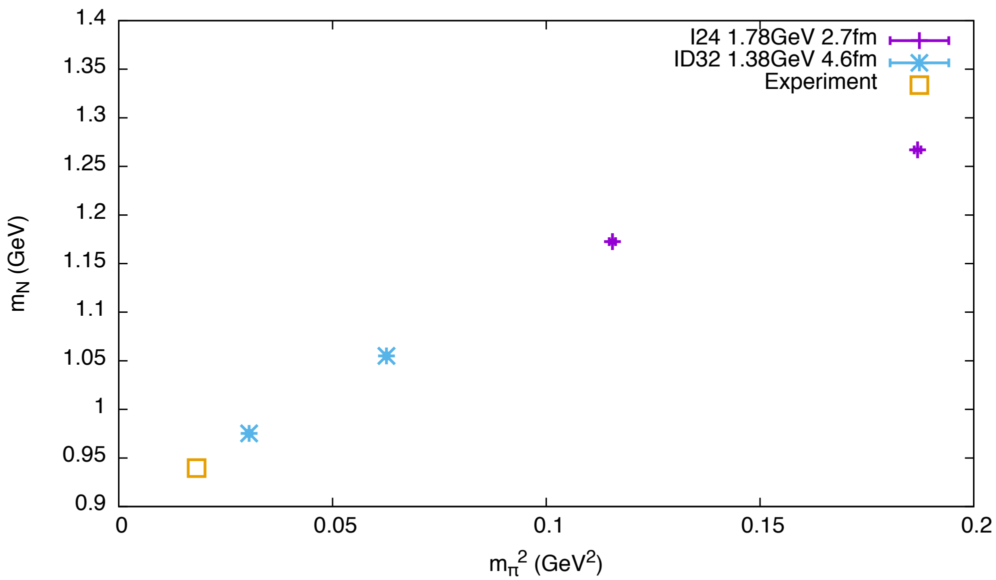
We are also improving AMA statistics for "I24" ensembles

- with Iwasaki gauge action at $\beta = 2.13$, corresponding the inverse lattice spacing of $a^{-1} = 1.7848(5)$ GeV, and pion mass values of about 432 and 340 MeV.

Both are driven by Michael Abramczyk.

From these we estimate the nucleon mass:

a^{-1}[GeV]	$m_q a$	$m_N a$	m_N [GeV]
1.378(7)	0.001	0.7077(08)	0.9752(11)
	0.0042	0.76557(16)	1.0550(20)
1.7848(5)	0.005	0.6570(9)	1.1726(16)
	0.01	0.7099(5)	1.2670(09)



Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

No source or sink is purely ground state:

$$e^{-E_0 t}|0\rangle + A_1 e^{-E_1 t}|1\rangle + ...,$$

resulting in dependence on source-sink separation, $t_{\rm sep} = t_{\rm sink} - t_{\rm source}$,

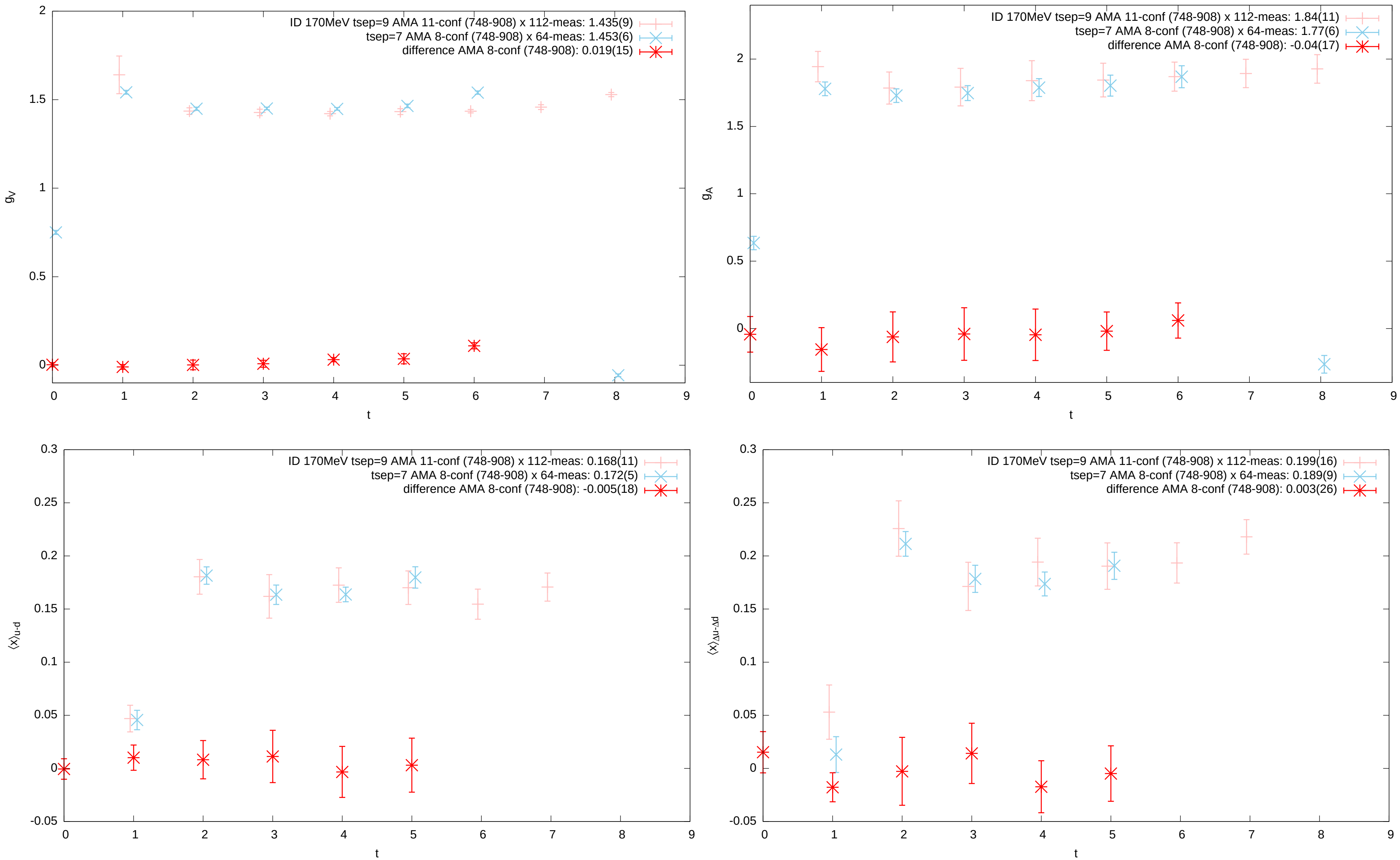
$$\langle 0|O|0\rangle + A_1 e^{-(E_1-E_0)t_{\rm sep}}\langle 1|O|0\rangle + ...$$

Any conserved charge, $O = Q$, $[H, Q] = 0$, is insensitive because $\langle 1|Q|0\rangle = 0$.

- g_V is clean,
- g_A does not suffer so much, indeed we never detected this systematics,
- structure function moments are not protected, so we saw the problem.

We can optimize the source so that A_1 is small, and we take sufficiently large $t_{\rm sep}$: Indeed with AMA we established there is no excited-state contamination present in any of our 170-MeV calculations.

With the AMA we established no excited-state contamination is present in any of our 170-MeV calculations:

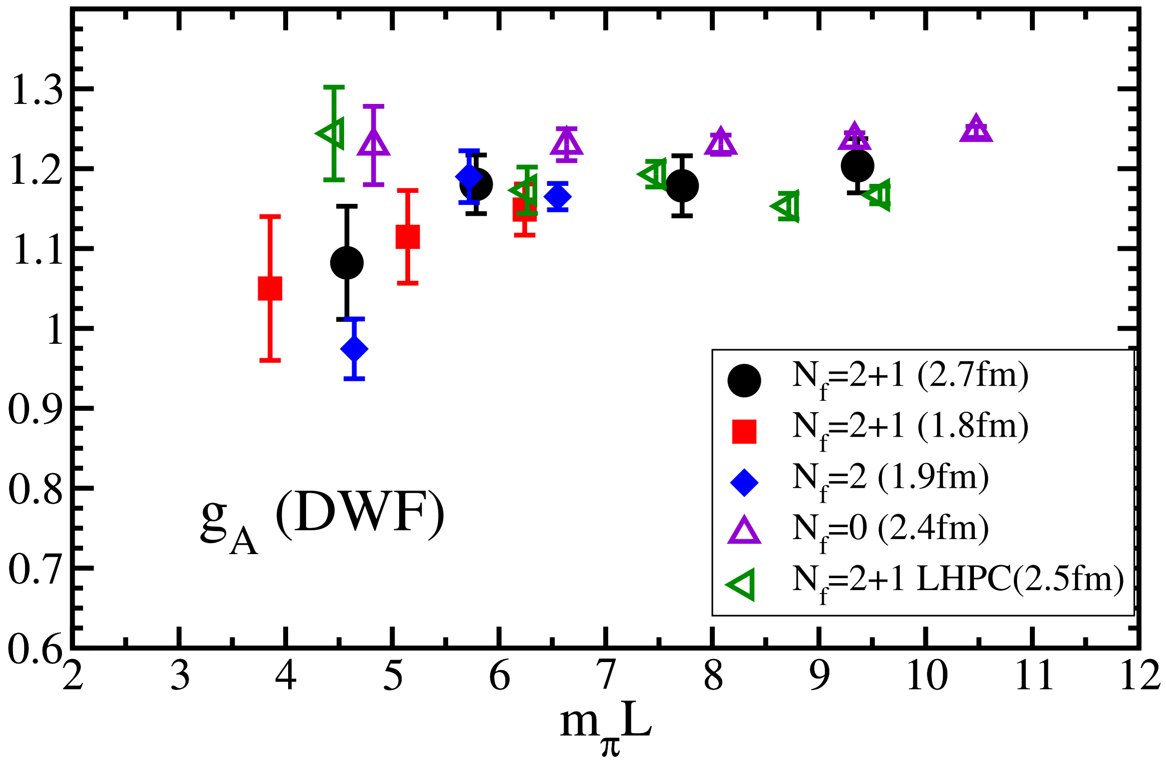


When compared with the same configurations, the difference is always consistent with 0.
$A_1\langle 1|O|0\rangle \sim 0$ for any observable we look at: A_1 is negligible for these small $\langle 1|O|0\rangle$.

In agreement with many other groups' experiences in controlling this systematics.

Spatial volume. In Lattice 2007 Takeshi Yamazaki reported unexpectedly large finite-size effect:

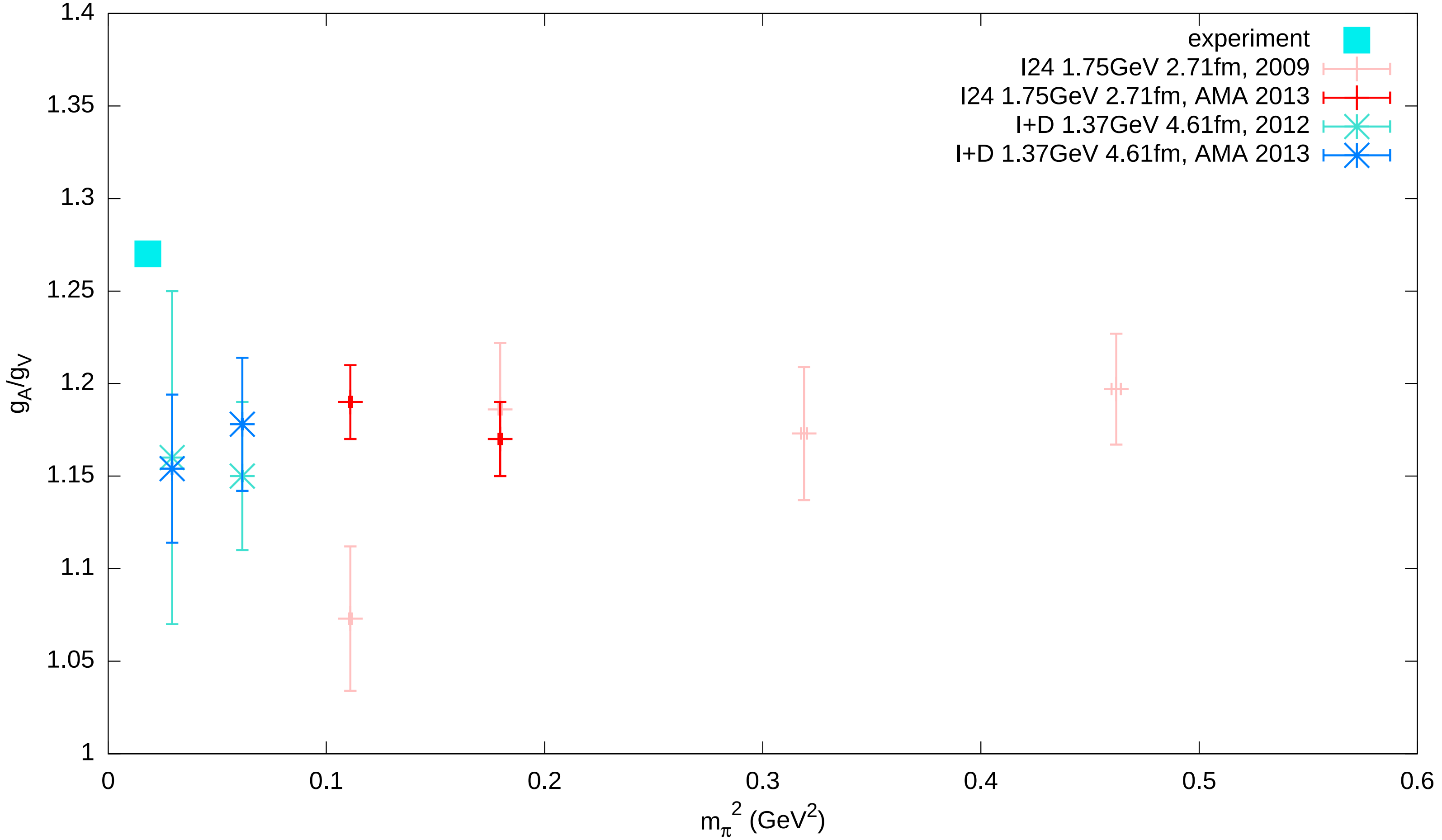
- in axial charge, $g_A/g_V = 1.2701(25)$, measured in neutron β decay, decides neutron life.



- Heavier quarks: almost consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
- If confirmed, first concrete evidence of pion cloud surrounding nucleons.

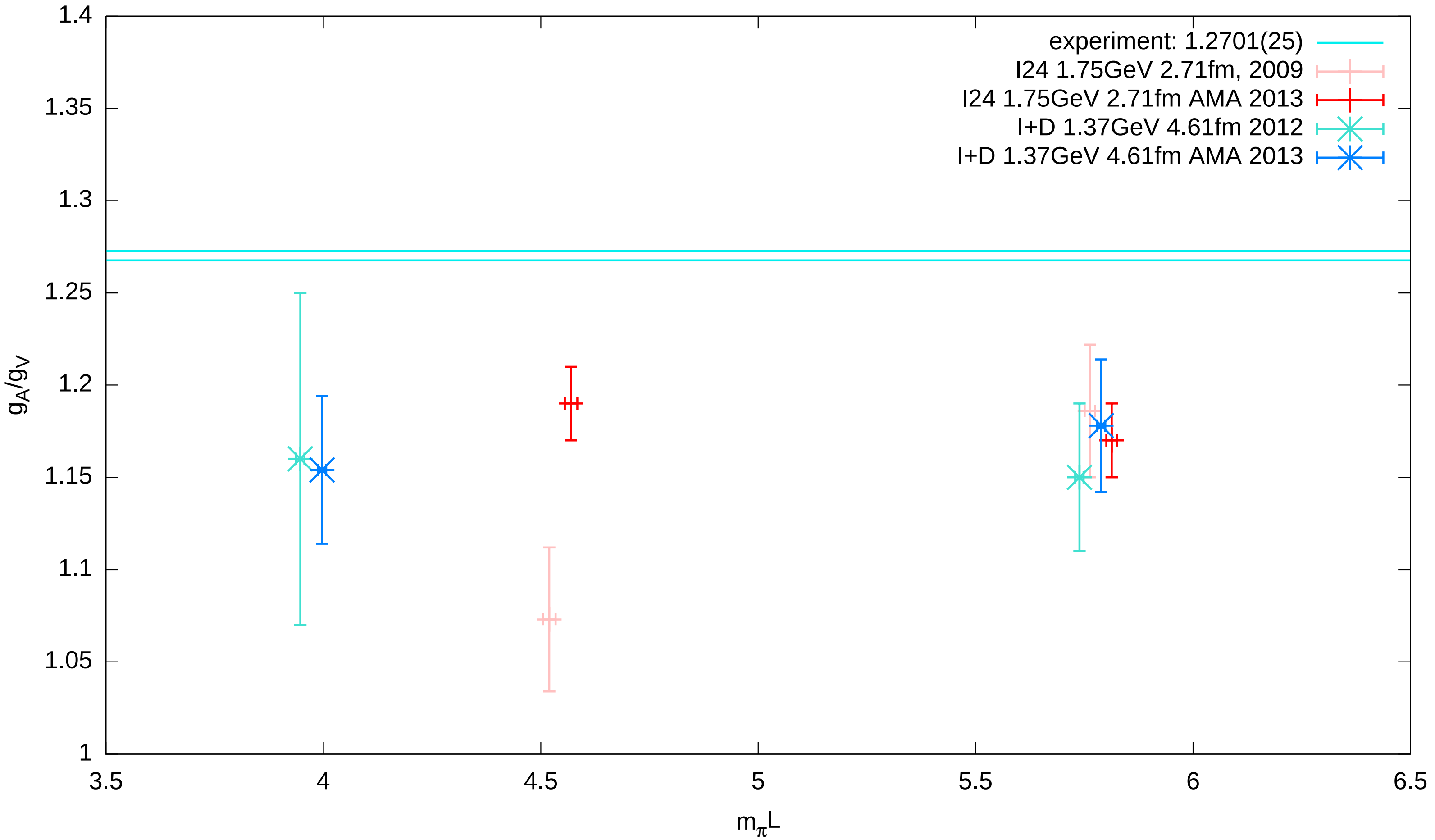
Many in the past pointed out this is a fragile quantity as pion mass is set light: Adkins+Nappi+Witten, Jaffe, Kojo+McLerran+Pisarski, ...

With AMA and other statistical improvements, g_A/g_V vs m_π^2 then looked like the following:



Moves away from the experiment as m_π approaches the experimental value.

About 10-% deficit in g_A/g_V seems solid except perhaps for $O(a^2)$ error:



Excited-state contamination now is unlikely the cause.

Appears like monotonically decreasing with $m_\pi L$.

In agreement with the great majority of other groups.

Why?

There appear long-range autocorrelations in axial charge but not in others:

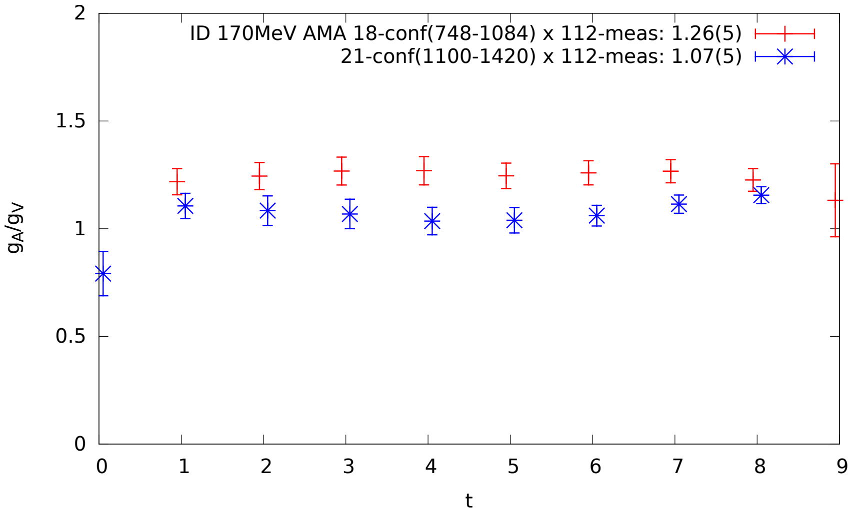
Blocked jackknife analysis

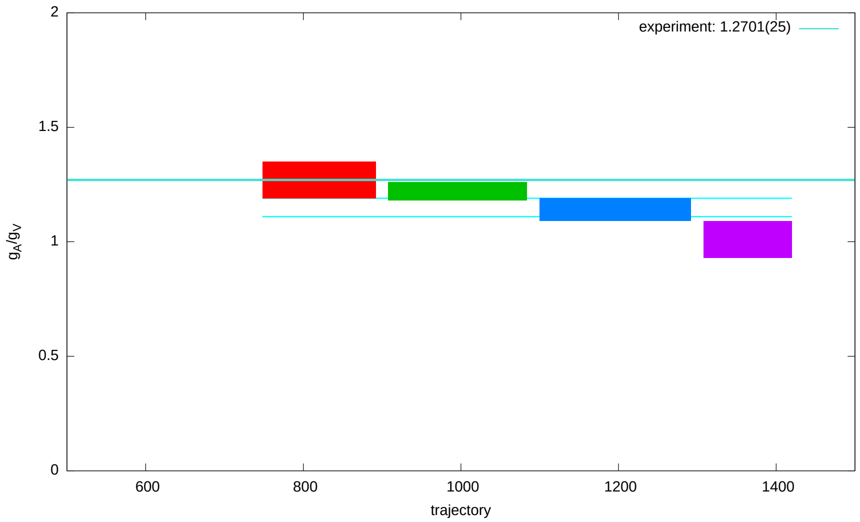
	bin size			
	1	2	3	4
g_V	1.447(8)	1.447(6)	-	-
g_A	1.66(6)	1.66(7)	1.71(8)	1.65(4)
g_A/g_V	1.15(4)	1.15(5)	1.15(6)	1.14(3)
$\langle x\rangle_{u-d}$	0.146(7)	0.146(8)	0.146(8)	-
$\langle x\rangle_{\Delta u-\Delta d}$	0.165(9)	0.165(11)	0.165(10)	-
$\langle x\rangle_{u-d}/\langle x\rangle_{\Delta u-\Delta d}$	0.86(5)	0.86(4)	-	-
$\langle 1\rangle_{\delta u-\delta d}$	1.42(4)	1.42(6)	1.42(6)	1.41(3)

except in perhaps transversity.

But the difference may be hard to notice by standard blocked jackknife analysis.

Long-range auto-correlation seen in g_A/g_V:





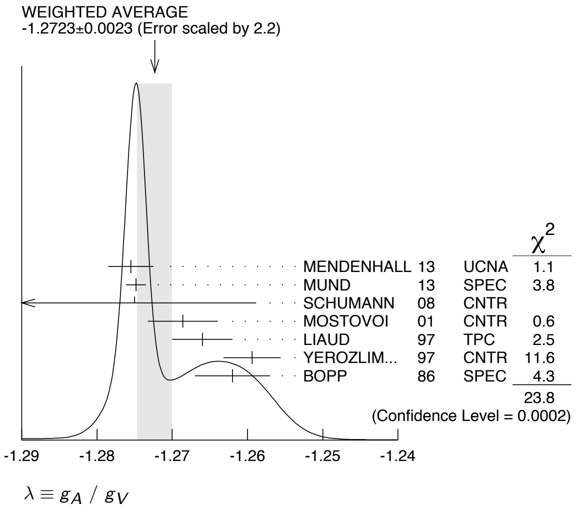
Non-AMA analyses are much noisier but not inconsistent with these:

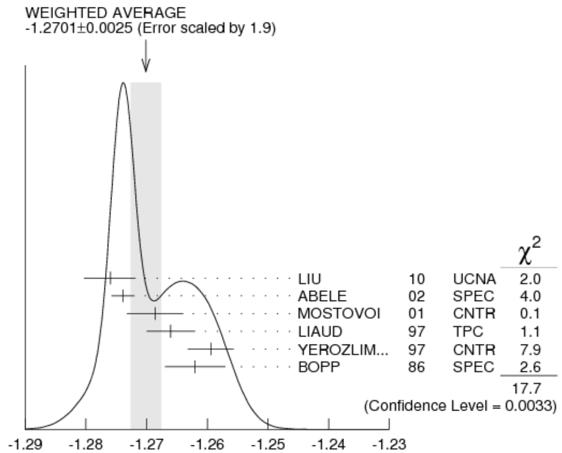
Indicative of inefficient sampling, but only in g_A and g_V.

Why?

Why?

Difficult history:





Experimental value has been almost monotonically increasing since Maurice Goldhaber's first measurement.

Lattice calculations appeared to follow the same path.

Why?

Difficult history:

Non-relativistic quark model: 5/3. Very bad, but some "large-N_c" conform?
And with absurd "relativistic" correction: 5/4, really?

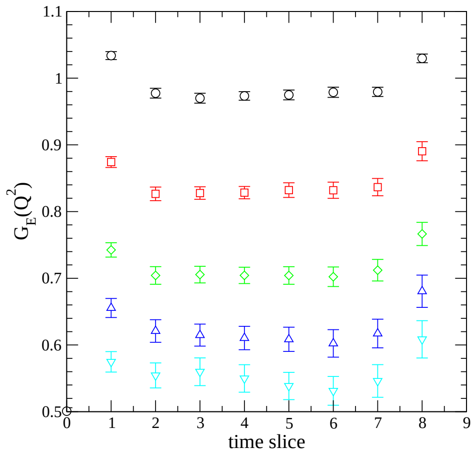
Without pion,
MIT bag model: 1.09, as good(!) as lattice but when experiment was 1.22.[1]

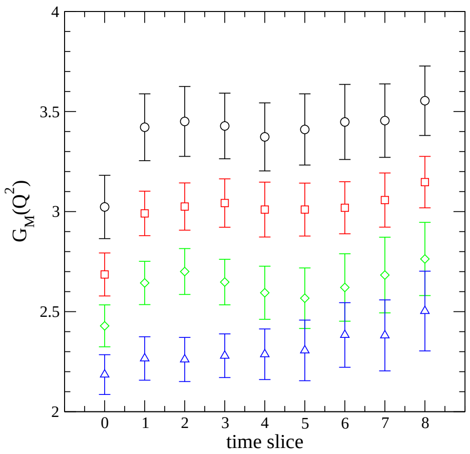
With only pion,
Skyrmion: 0.61(!) with a peculiar geometry but when experiment was 1.23.

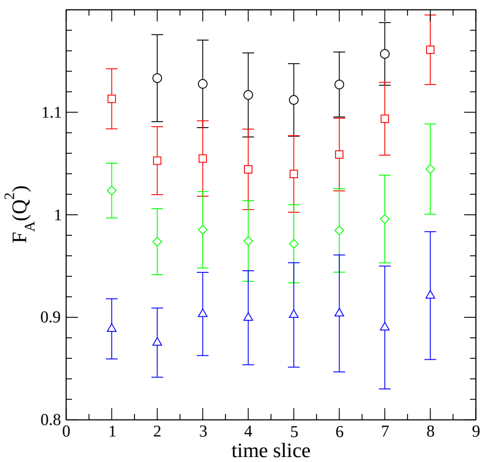
Accurate reproduction of the 'pion cloud' geometry seems essential.

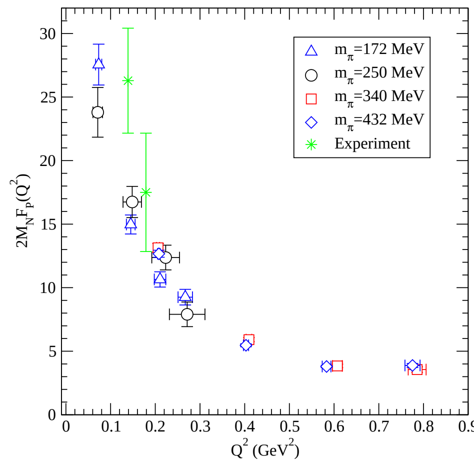
[1]Assuming a growth rate of 0.001 per year.

Now we are ready to report some preliminary nucleon isovector form factors from the ID32 1.378(7)-GeV ensembles:







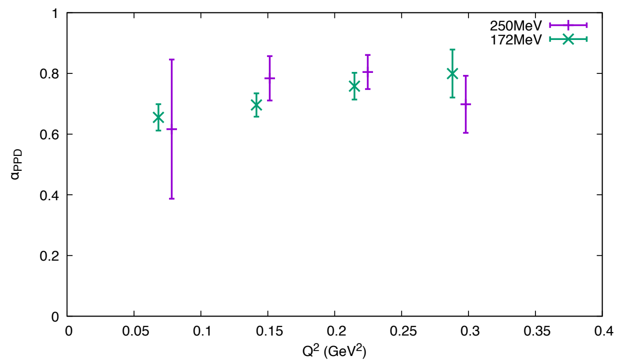


m_π (GeV)	M_1 (GeV)	r_1 (fm)
172	1.0898(0.0787)	0.6273(0.0453)
250	1.1040(0.1974)	0.6192(0.1107)

m_π (GeV)	M_2 (GeV)	r_2 (fm)
172	0.7236(0.0469)	0.9448(0.0612)
250	0.7109(0.0813)	0.9617(0.1100)

Dipole fits to isovector Dirac and Pauli form factors.

m_π (GeV)	M_A (GeV)	r_A (fm)
172	1.2347(0.1389)	0.5537(0.0623)
250	1.2643(0.1476)	0.5407(0.0632)



Dipole fit to axialvector form factor and pion pole dominance, $\frac{(m_\pi^2+q^2)F_P(q^2)}{2m_NF_A(q^2)}$, in pseudoscalar form factor.

Summary

We improved statistics for 1.378(7)-GeV ensembles with pion mass of 172 and 250 MeV, and 1.7848(5)-GeV ensembles at pion mass of 432 and 340 MeV.

Form factor analyses are nearly complete for the former two, and ongoing for the latter two:

- both vector- and axialvector-current form factors appear behaving well as were seen before.

Analyses for low structure-function moments are almost complete, as were reported previously, except for some NPRs for the former two.