Partially conserved axial vector current and applications

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QCDSF collaboration

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Outline

Introduction

- In 2014 Horsley et al. (2015) we started to investigate the point split axial vector current
- Its divergence exactly satisfies a lattice Ward identity, involving the pseudoscalar density and a number of irrelevant operators
- Such operators naturally appear in derivation of lattice Ward identities

  see e.g. Bochicchio, Maiani, Martinelli, Rossi, Testa NPB262 (1985), also Reisz, Rothe PRD62 (2000), Bhattacharya et al. PRD92 (2015)
- We proved such an axial Ward identity for clover fermions and check it both perturbatively and nonperturbatively (Schiller Lattice2015)

AWI

- Point split axial vector current

$$A_\mu^{ps}(x) = \frac{1}{2}\left[\bar{\psi}_x \gamma_\mu \gamma_5 U_\mu(x) \psi_{x+a\hat{\mu}} + \bar{\psi}_{x+a\hat{\mu}} \gamma_\mu \gamma_5 U_\mu^\dagger(x) \psi_x\right]$$

- Axial Ward identity

$$\langle \partial_\mu A_\mu^{ps} \rangle = 2M_0 \langle P \rangle + X$$

with

$$M_0 = \frac{1}{2\kappa_l} - 4$$

$$X = 2\langle O_C \rangle + \langle O_W \rangle$$

$$P = \bar{\psi}_x \gamma_5 \psi_x \,, \quad aO_C = \bar{\psi}_x \gamma_5 C_{xx} \psi_x$$

$$aO_W = 8P - \frac{1}{2}\sum_\mu \left[\bar{\psi}_x \gamma_5 U_\mu(x) \psi_{x+a\hat{\mu}} + \bar{\psi}_{x+a\hat{\mu}} \gamma_5 U_\mu^\dagger(x) \psi_x + (x \to x - a\hat{\mu})\right]$$

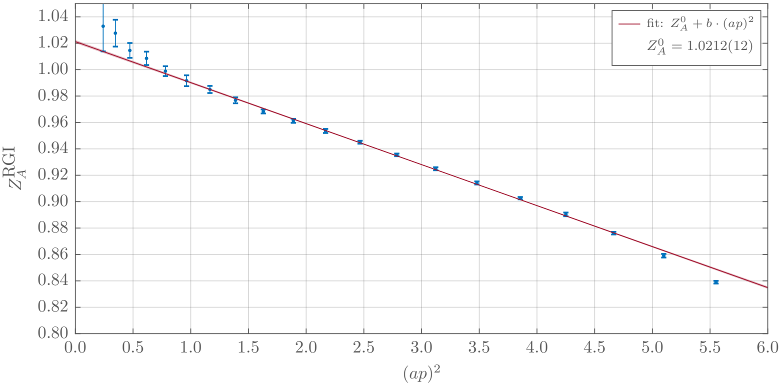
ZA

- RI′-MOM scheme with subsequent trafo into RGI and $\overline{\mathrm{MS}}$
- $\Gamma_B^{\mathrm{ps}} = \gamma_\mu \gamma_5 \cos\left(p_\mu \frac{2\pi}{L_\mu}\right)$
- Lattice setup:
  Gluon action: tree-level Symanzik improved
  Fermion action: $n_f = 2 + 1$ Wilson fermions with clover term
  analytic stout smeared links in the Dirac kinetic and mass terms, no smearing in the clover term
  $32^3 \times 64$, $c_{sw} = 2.65$, $\omega = 0.1$, $\beta = 5.5$ [$a = 0.074(2)$ fm]
- (κ_l, κ_s) choices: Flavor symmetric line $(\kappa_l = \kappa_s)$ corresponding to pion masses

$$M_\pi = 470,\ 438,\ 402,\ 342,\ 290 \text{ MeV}$$

Z_A

Transformation from RI′-MOM into RGI scheme (chiral limit)



Z_A

Result:

$$Z_{A^{\mathrm{ps}}}^{\mathrm{RGI}} = Z_{A^{\mathrm{ps}}}^{\overline{\mathrm{MS}}} = 1.0212(12)$$

compare with

-*local* axial vector current (Constantinou et al.):

$$Z_{A^{\mathrm{loc}}}^{\mathrm{RGI}} = Z_{A^{\mathrm{loc}}}^{\overline{\mathrm{MS}}} = 0.8728(27)$$

-one-loop PT (Horsley et al.) (*Symanzik gauge action* $\rightarrow$ *plaquette gauge action!*):

$$Z_{1-\mathrm{loop},A^{\mathrm{ps}}}^{\overline{\mathrm{MS}}} = 0.996$$

WI masses

- Remember AWI

$$\langle \partial_\mu A_\mu^{\rm ps} \rangle = 2M_0 \langle P \rangle + 2\langle O_C \rangle + \langle O_W \rangle$$

$\Rightarrow$ under renormalization

$$\overline{X} = 2\langle O_C \rangle + \langle O_W \rangle + 2\overline{M}\langle P \rangle + (Z_A - 1)\langle \partial_\mu A_\mu^{\rm ps} \rangle$$

- Is it possible to find $\overline{M}$ to make $\overline{X}$ vanish for all quark masses?

$$\Rightarrow Z_A \langle \partial_\mu A_\mu^{\rm ps} \rangle = 2(M_0 - \overline{M})\langle P \rangle \equiv 2m_I^{WI} \langle P \rangle$$

In the following we set $Z_A = 1$

WI masses

- Lattice setup ($32^3 \times 64, \beta = 5.50$):

κ_l	κ_s
0.120900	0.120900
0.121040	0.120620
0.121095	0.120512
0.120990	0.120990

- $\overline{m} = 1/3(m_u + m_d + m_s) = \mathrm{const.}$ line

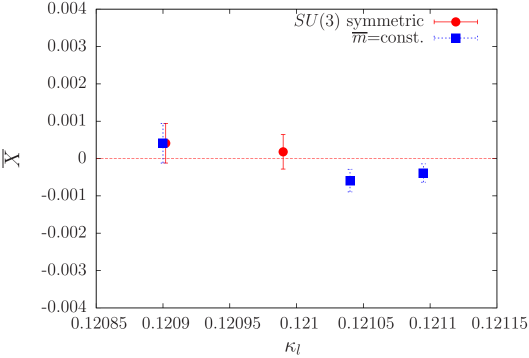
$$\overline{M} = M_0 - \frac{c}{1+\alpha_Z}\left(\frac{1}{2\kappa_l} - \frac{1}{2\kappa_c}\right), \alpha_Z = \frac{Z_m^S}{Z_m^{NS}} - 1 \approx 0.767$$

- $SU(3)$ symmetric line

$$\overline{M} = M_0 - c\left(\frac{1}{2\kappa_l} - \frac{1}{2\kappa_{0,c}}\right)$$

WI masses

We find $c = 1.55(4)$ to give $\overline{X} \approx 0$



WI masses

$\overline{m} = $ const. line:

- $m_l^{WI} = \frac{c}{1+\alpha_Z}\tilde{m}_l \approx 0.86(m_l + \alpha_Z \overline{m})$
- The renormalized vector and WI quark masses should be equal

$$m_l^{V,R} = Z_m^{NS} \Delta Z_m^{sea} \tilde{m}_l$$

$$m_l^{WI,R} = \frac{1}{Z_P} 0.86 \tilde{m}_l$$

$$R = \frac{m_l^{V,R}}{m_l^{WI,R}} = Z_m^{NS} \Delta Z_m^{sea} Z_P \frac{1}{0.86} \approx 1.01$$

- The same result is obtained for the $SU(3)$ symmetric case

g_A and the axial vector current A_μ

Relation between A_μ and g_A given by the forward matrix element

$$\langle p, s | A_\mu^{\mathrm{u-d}} | p, s \rangle = 2\, g_A\, s_\mu \,,$$

with

$$A_\mu^{\mathrm{u-d}} = A_\mu^{\mathrm{ps,u}} - A_\mu^{\mathrm{ps,d}}$$

and

$|p, s\rangle$: proton state with momentum p and spin s.

On the lattice g_A is computed via the ratio of the 3-pt to the 2-pt functions

$$g_A = R(t_i, t_f, \tau) = \frac{G_3(t_i, t_f, \tau)}{G_2(t_i, t_f)}$$

with $(t_f - t_i)$ - source-sink distance, τ - source-operator insertion distance

Contributing diagrams

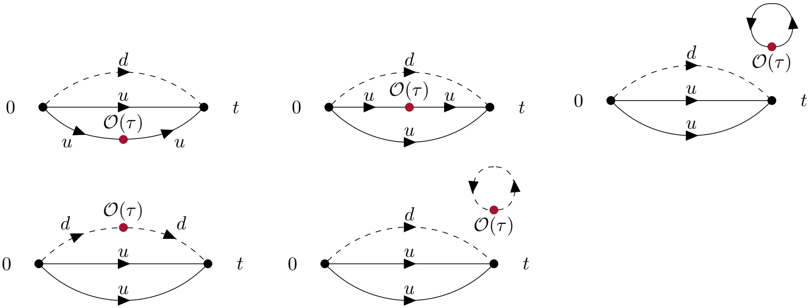


Figure: Connected and disconnected diagrams contributing to the 3-pt-function.

Lattice setup

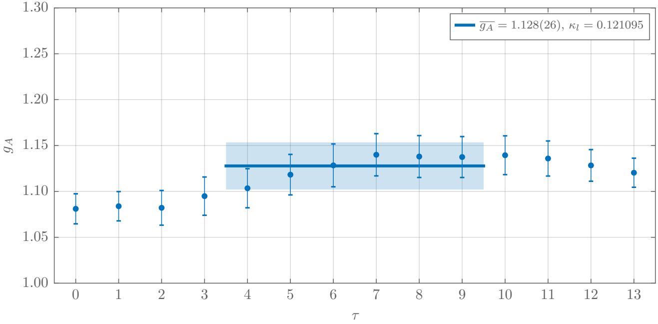
- Gluon action: tree-level Symanzik improved
  Fermion action: $n_f = 2 + 1$ Wilson fermions with clover term
  analytic stout smeared links in the Dirac kinetic and mass terms, no smearing in the clover term
  $32^3 \times 64$, $c_{sw} = 2.65$, $\omega = 0.1$, $\beta = 5.5$ [$a = 0.074(2)$ fm]
  $48^3 \times 96$, $c_{sw} = 2.34$, $\omega = 0.1$, $\beta = 5.8$ [$a = 0.059(3)$ fm]
- (κ_l, κ_s) choices: Flavor symmetric line corresponding to pion masses

$$\beta = 5.5\text{: } M_\pi\text{=470, 360, 310 MeV}$$
$$\beta = 5.8\text{: } M_\pi\text{=427 MeV}$$

Plateau method I

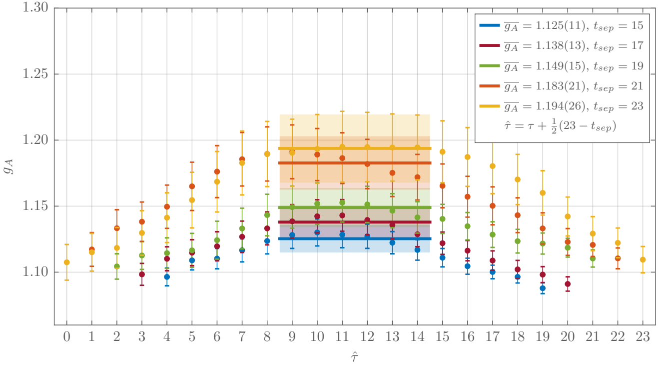
$\beta = 5.5$: three different pion masses as function of operator insertion time τ

Example: $\kappa_l = 0.121095$ ($M_\pi = 310$ MeV)



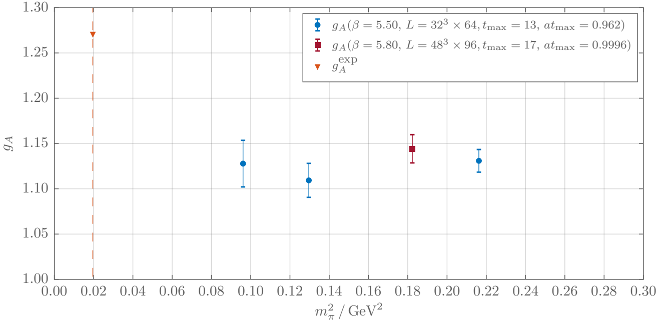
Plateau method II

$\beta = 5.8$: four different source-sink distances as function of operator insertion time τ



Plateau method - *a* dependence

Comparison for $\beta = 5.5$ and $\beta = 5.8$ at comparable physical source-sink distances



We recognize a very weak dependence on β, or *a* resp.

Excited states,...

It is obvious that excited states contaminate the measured result

There are several methods which try to handle this issue, e.g.

- Exponential fit (Capitani et al., Bali et al., Bhattacharya et al., Dragos et al.)

$$
\begin{aligned}
C_{2pt}(t_i, t_f) &= \sum_{i=0}^{k} A_i^2 \, e^{-M_i\,(t_f - t_i)} \\
C_{3pt}(t_i, \tau, t_f) &= A_0^2 \, g_A \, e^{-M_0\,(t_f - t_i)} + \sum_{i=1}^{k} A_i^2 \, C_i \, e^{-M_i\,(t_f - t_i)} + \\
&\quad \text{mixed terms containing functions of } (\tau, \mathrm{t_i}, \mathrm{t_f})
\end{aligned}
$$

- Summation method (Capitani et al., Dragos et al.)
  Sum of the ratio $R(t_i, t_f, \tau)$
- Variational method (Owen et al., Dragos et al., Bhattacharya et al.)
  Basis of states that couple differently to different energy levels

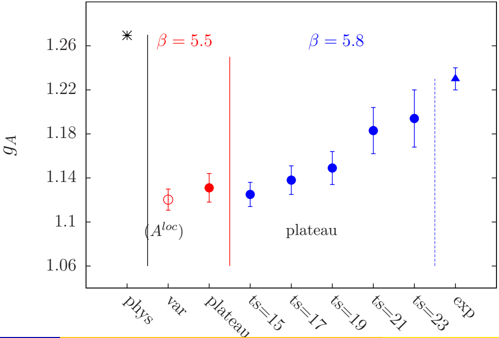
Some numbers,...

- At the moment we can realistically compare with computation based on the same action, lattice and at the same β and κ values ($32^3 \times 64, \beta = 5.5, M_\pi = 470$ MeV)

$$A^{\rm loc} : \; g_A^{var} = 1.1203(96) \; (\text{Dragos et al., arXiv : 1606.03195})$$

$$A^{\rm ps} : \; g_A^{plat} = 1.131(13)$$

- g_A summary



Short summary and outlook

- We have proposed to use an point-split axial vector present in a lattice axial Ward identity
- We found the corresponding nonperturbative Z_A factor very near to one
- We found AWI masses which lead to a continuum-like form of the axial Ward identity
- We have used A_μ^{ps} to compute the axial charge g_A inside the proton for two β values. The resulting $g_A(\beta = 5.5)$ do not differ within the errors from the corresponding result for the same action using the local axial vector current.
- $g_A(\beta = 5.8)$ is somewhat larger if we use methods like 2-exp or summation.

Short summary and outlook

Outlook (g_A)

- More detailed investigation of the influence of the excited states (variational method)
- Investigation of volume dependence
- Computation of g_A for lighter pion masses
- Continuum limit