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The Calculation of Parton Distributions from Lattice QCD.

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Collaborators.

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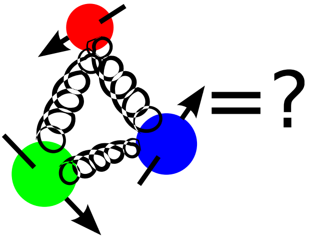
Parton distributions.

- powerful tool to describe the structure of a nucleon: parton distribution functions (PDFs)
- set of three PDFs
  - momentum distribution $q(x) = q^{\uparrow}(x) + q^{\downarrow}(x)$
  - helicity distribution $\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$
  - transversity distribution $\delta q(x) = q^{\top}(x) - q^{\perp}(x)$

- precise quark and gluon distributions are necessary for the analysis of scattering data (LHC)
- in addition: still open questions for the nucleon structure

→ how do quarks and gluons compose the proton spin?

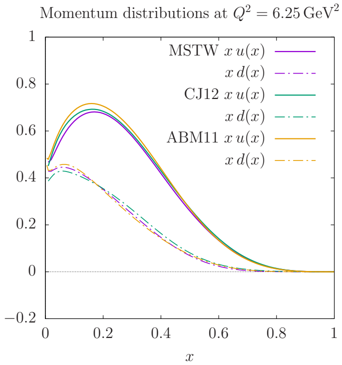


Obtaining PDFs.

- world-wide effort to measure relevant structure functions (CERN, SLAC, DESY, JLab)
- inclusive and semi-inclusive, different polarizations
- computation of Wilson coefficients
- fit distributions to the data
- results depend on the fitting scheme and selected data

→ we need a prediction of quark distributions from first principles

→ crucial test of QCD



Quark distributions from lattice QCD.

- why is it difficult to access PDFs on the lattice?
- definition via matrix elements of light cone operator

$$q(x) = \int_{-\infty}^{\infty} \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-, 0)\psi(0)|P\rangle$$

- dominated by area close to light cone $\xi^2 = 0$

→ issue on an Euclidean lattice ($\xi^2 = \mathbf{x}^2 + t^2$)

- new proposal: compute spatial quasi distribution (Ji, 2013)

$$\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P|\bar{\psi}(z)\gamma_3 W_3(z, 0)\psi(0)|P\rangle$$

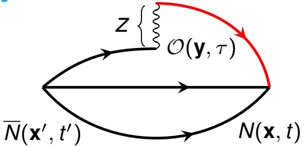
- has to be computed at sufficiently large momentum P_3

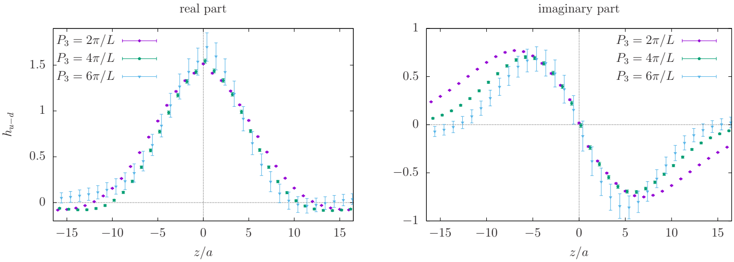
→ nucleon boosted in the same direction as the Wilson line

- in this limit a connection to the light-cone distribution can be made

Necessary form factors.

- form factors:
  $\langle P|\overline{\psi}(z)\gamma_3 W_3(z,0)\psi(0)|P\rangle =$
  $\bar{u}(P)\gamma_3 h(P_3,z) u(P)$





- quasi-distribution $\tilde{q}(x,P_3) = 2P_3 \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dz}{4\pi} e^{-izxP_3} h(P_3,z)$
- apply one-loop matching (Xiong, Ji, Zhang, Zhao 2014)
- apply target mass corrections

Latest results.

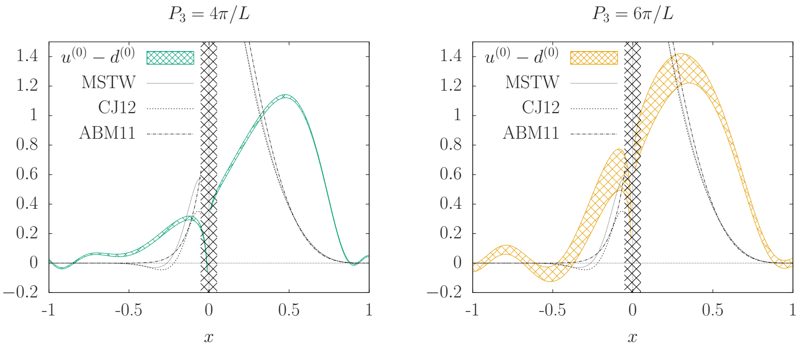
- latest results are computed on the B55.32 ETMC ensemble
- $N_f = 2 + 1 + 1$, $V = 32^3 \times 64$, $m_{PS} \approx 370\,\mathrm{MeV}$, $a \approx 0.082\,\mathrm{fm}$
- 1000 gauge configurations with 15 source positions each and 2 sets of stochastic samples

→ 30 000 measurements

- for now rather small source-sink separation of $t_s = 8a$
- we apply 5 steps of HYP smearing to the gauge links in the operator

→ preliminary way to reduce the linear divergence from the Wilson line (ongoing effort to treat this rigorously)

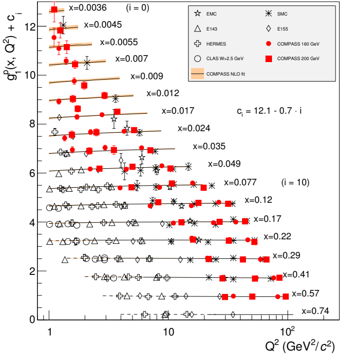
Results for momentum distribution.



- antiquark distribution can be related to the negative x region by the crossing relation
- $\bar{q}(x) = -q(-x)$
- negative $x \Rightarrow \bar{d} - \bar{u}$

The helicity distribution.

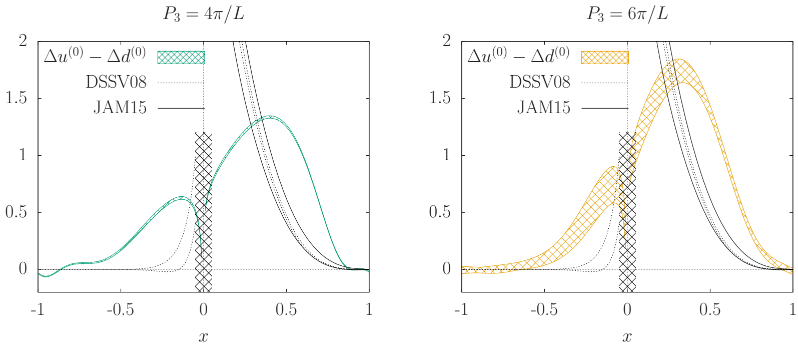
- important tool to understand the spin structure of the nucleon
- $\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$
- contains crucial form factors, e.g. $\int dx \Delta u(x) - \Delta d(x) = g_A^{(3)}$
- obtained by polarized scattering experiments
- on the lattice, we can use the same tools as for the momentum distribution



(COMPASS Collaboration, arXiv:1503.08935)

$$\Delta\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P|\bar{\psi}(z)\gamma_5\gamma_3 W_3(z,0)\psi(0)|P\rangle$$

Results for the helicity distribution.



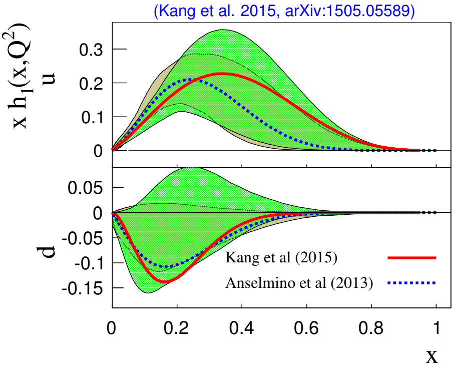
- crossing relation: $\Delta\bar{q}(x) = \Delta q(-x)$
- negative x region $\Rightarrow \Delta\bar{u} - \Delta\bar{d}$

The transversity distribution.

- $\delta q(x) = q^{\top}(x) - q^{\perp}(x)$
- purely non-singlet quantity

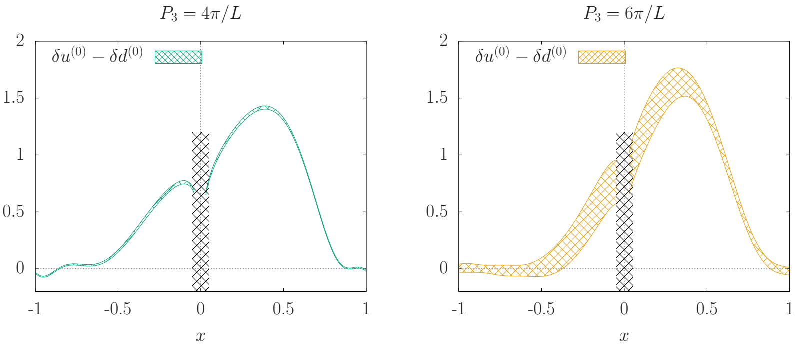
$\rightarrow$ no gluon contribution

- so far only parametrization with large uncertainties
- no parametrization for antiquark distribution available



$$\delta\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P|\bar{\psi}(z)\gamma_j\gamma_3 W_3(z,0)\psi(0)|P\rangle$$

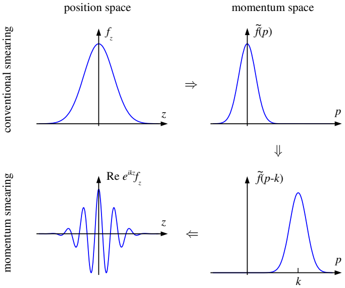
Results for the transversity distribution.



- crossing relation: $\delta\bar{q}(x) = -\delta q(-x)$
- negative x region $\Rightarrow \delta\bar{d} - \delta\bar{u}$

Momentum smearing.

- we would like to study the behavior for larger momenta
  - problem: for the standard methods the signal is lost in noise for $P_3 > 6\pi/L$
  - extrapolation is rather unfeasible
- possible solution presented by Bali *et al.* in arXiv:1602.05525
- idea: alter Gaussian smearing so that the used momentum is modeled
  - in practice we use the standard Gaussian smearing with a modified Gauge field $\tilde{U}_j(x,k) = e^{ik\hat{\jmath}} U_j(x)$

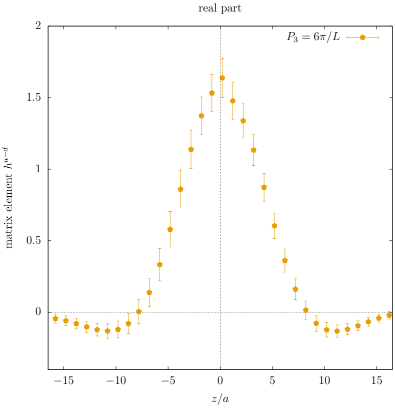


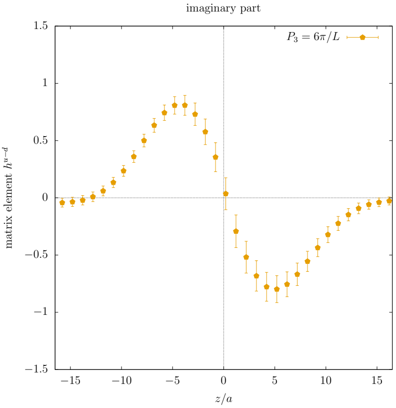
[arXiv:1602.05525]

Results for momentum smearing.

- Results for momentum smearing on 50 configurations

$\rightarrow$ 150 measurements





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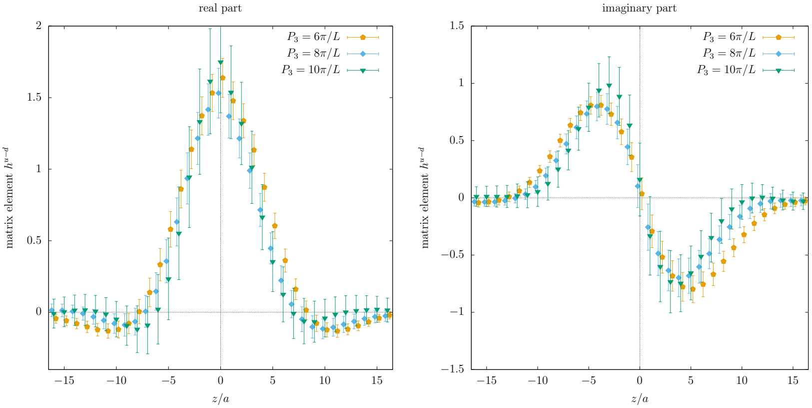

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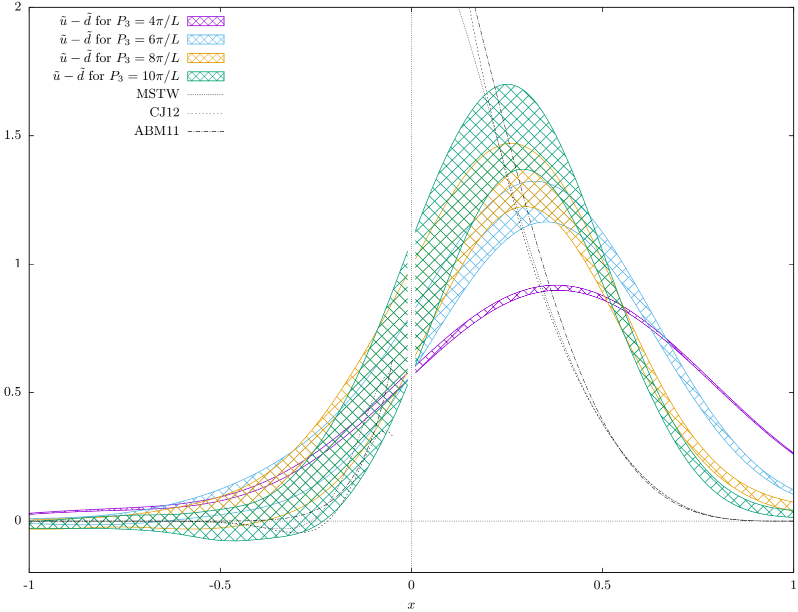

Summary & Outlook.

Achievements

- we successfully employed Ji's method to obtain quark quasi-distributions from lattice QCD calculations and related them to quark distributions via a matching procedure and target mass corrections
- the obtained distributions show a qualitative, not yet quantitative agreement with phenomenological distributions
- momentum smearing is a powerful tool that allows us to access larger momenta

Challenges

- identify and remove further systematic effects

e.g. compute distributions at a physical pion mass

- proper renormalization of the operator