YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

BROOKHAVEN NATIONAL LABORATORY

Quark masses and coupling constant with Highly-Improved Staggered Quarks

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arXiv:1606.08798

Heavy meson correlators

Long distance
+ experiments → precise m_c^{lat} & m_c/m_s , m_b/m_c

Short distance
+ perturbation → $\alpha_s(\mu)$ and $m_c(\mu)$ at $\mu = m_c$
in the lowest energy scale

Lattice2016, Southampton, England, Sep. 24--30

Introduction

Fundamental parameters of QCD: coupling constant & quark masses

$\alpha_s \quad m_u, \ m_d, \ m_s, \ m_c, \ m_b, \ m_t$

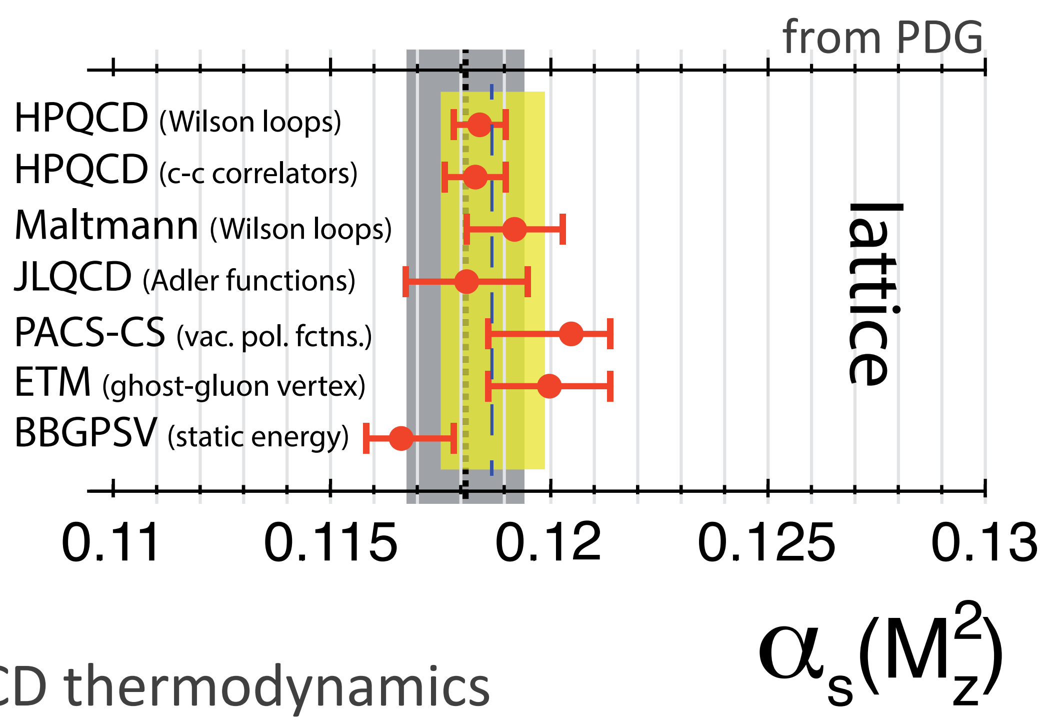
Determination: most elemental quest in the lattice QCD simulations

The lattice provides certain results?

...not enough

not all error sources under control

Further study: necessary



Low energy scale: important

e.g. applications to QCD thermodynamics

Lattice results (EoS, etc.) vs. weak coupling predictions

at typical scale $\simeq \pi T \lesssim 1.0$ GeV

Lattice simulations

HotQCD gauge configurations

PRD90 (2014) 094503

- ✓ Highly Improved Staggered Quarks
- ✓ Tree-level improved gluons
- ✓ (2+1)-flavor QCD
  - ➢ Strange quark m_s: physical
  - ➢ Light quark: $m_l = m_s/20$
  - ➢ Charm quark: valence
- ➢ Scale: $r_1 = 0.3106(18)$ fm

  via pion decay constant

β	Lattice	TU*	a^{-1} [GeV]
6.740	48^4	8K	1.81
6.880	48^4	8K	2.07
7.030	48^4	10K	2.39
7.150	$48^3 \times 64$	8K	2.67
7.280	$48^3 \times 64$	8K	3.01
7.373	$48^3 \times 64$	9K	3.28
7.596	64^4	9.5K	4.00
7.825	64^4	9.5K	4.89

*molecular dynamic time units

(c.f. covers at T=150--400 MeV on Nt=12)

Our approach

1. Tuning the charm mass via meson correlators with wall sources

   ➡ $m_c/m_s,\ m_b/m_c$

2. Measuring the moments via meson correlators with point sources

   ➡ $\alpha_s(m_c),\ m_c(m_c)$

Quark masses from meson masses

Quark masses on the lattice ⟷ Meson masses in experiments

Unmixed $s\bar{s}$ eta

$$M_{\eta_{s\bar{s}}} = \sqrt{2M_K^2 - M_\pi^2}$$

Ground state charmonia

$$\overline{M} = \frac{1}{4}\left(3M_{J/\psi} + M_{\eta_c}\right)$$

Bottomonium

$$M_{\eta_b}$$

Tuning quark masses

$$M_{\eta_{s\bar{s}}}^2 = B\, m_s$$

$$\overline{M} = d + b\, m_c$$

$$M_{\eta_h} = d^h + b^h\, m_h$$

$$(m_c < m_h \lesssim m_b)$$

Experimental values from PDG

$$M_{\eta_{s\bar{s}}} = 686.00(92)\ \mathrm{MeV}$$

PRD70, 114501

$$\overline{M} = 3.067(3)\ \mathrm{GeV}$$

PRD83, 074504

$$M_{\eta_b} = 9.398(3)\ \mathrm{GeV}$$

uncertainties: absence of EM effects and disconnected diagram

Determination m_s^{lat} done by HotQCD

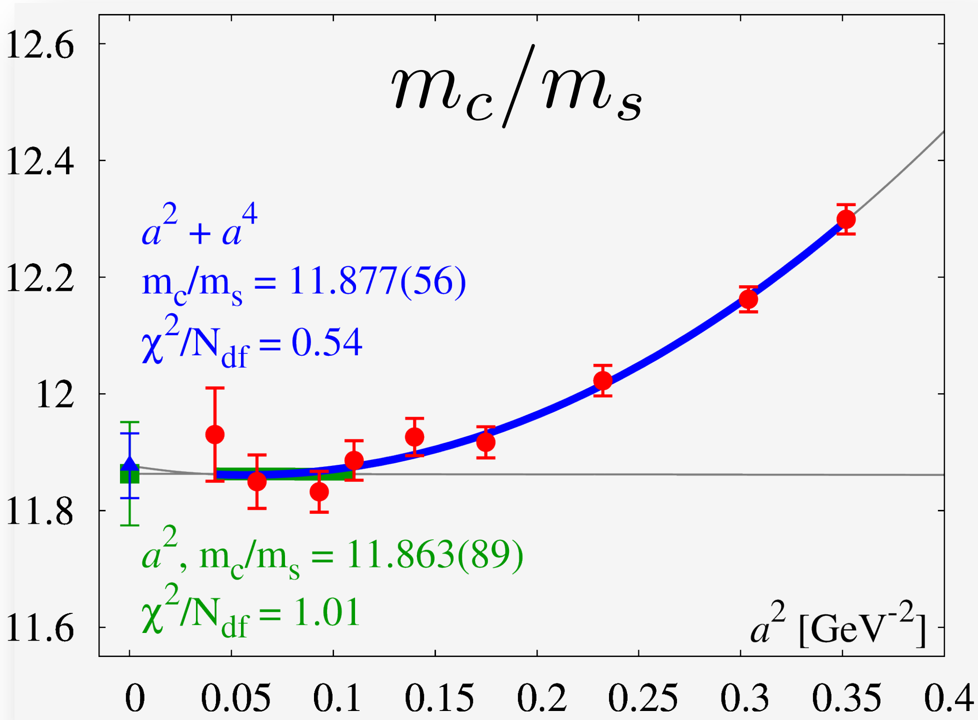
Determine m_c^{lat} within 0.4--0.7% errors

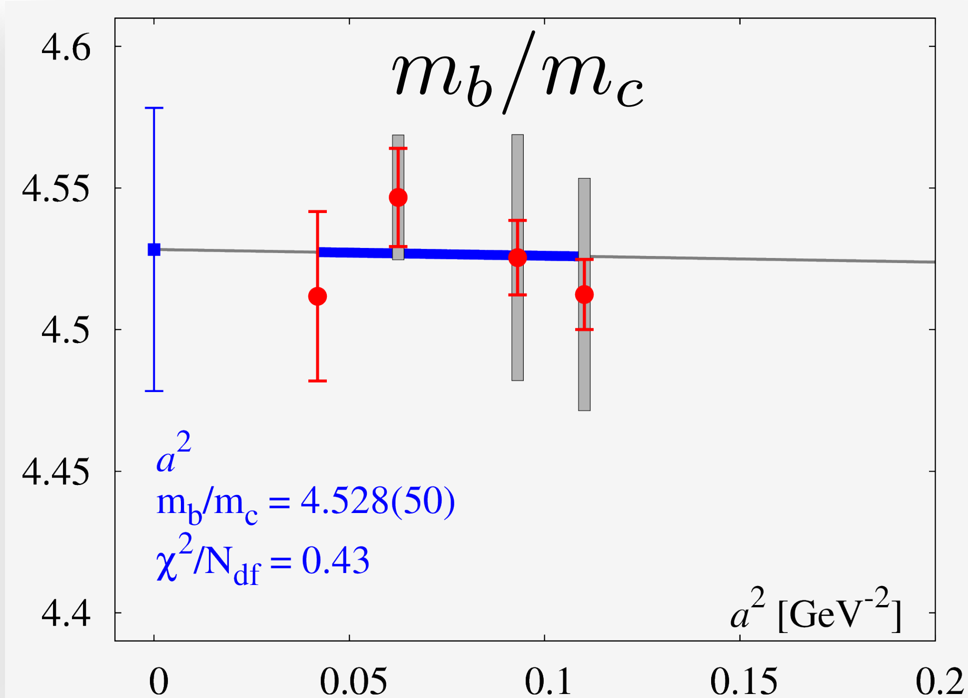
→ hyper-fine splitting reproduced in continuum

Estimate m_b^{lat} partly using extrapolation from $am_h < 1.0$ to m_b

Quark masses ratios

Scheme and scale independent: $m_c^{\rm lat}/m_s^{\rm lat} = m_c/m_s$





- ✓ a^2 and a^2+a^4 fits well done and show similar results

$$\frac{m_c}{m_s} = 11.877(56)_{\rm stat.}(69)_{r_1}(15)_{\eta_{s\bar{s}}}(12)_{\overline{M}}$$

$$\frac{m_c}{m_s} = 11.877(91)$$

- ✓ only four finest β points
- ✓ systematic error due to extrapolation to M_{η_b}: gray shadow

$$\frac{m_b}{m_c} = 4.528(50)_{\rm stat.}(26)_{r_1}(04)_{\overline{M}}(02)_{M_{\eta_b}}$$

$$\frac{m_b}{m_c} = 4.528(57)$$

Moments

Pioneered by HPQCD and Karlsruhe groups:

$$G_n = \sum_t (t/a)^n G(t): \text{ } n\text{-th order with } G(t) = a^6 \sum_{\mathbf{x}} (am_{c0})^2 \langle j_5(\mathbf{x},t) j_5(0,0)\rangle$$

Perturbation theory in $\overline{\text{MS}}$ scheme

$$G_n = \frac{g_n(\alpha_s(\mu), \mu/m_c)}{am_c^{n-4}(\mu)},$$

g_n known up to 4-loop (α_s^3)

Reduced moments: useful to cancel lattice artifacts

$$R_n = \left(\frac{G_n}{G_n^{(0)}}\right)^{1/(n-4)},$$

with moments of free correlators: $G_n^{(0)}$

$$R_n = \begin{cases} r_4 & (n=4) \\ r_n \cdot (m_c^{\text{lat}}/m_c(\mu)) & (n \geq 6) \end{cases}, \quad r_n = 1 + \sum_{j=1}^{3} r_{nj}(\mu, m_c)\alpha_s^j(\mu)$$

Lattice in continuum

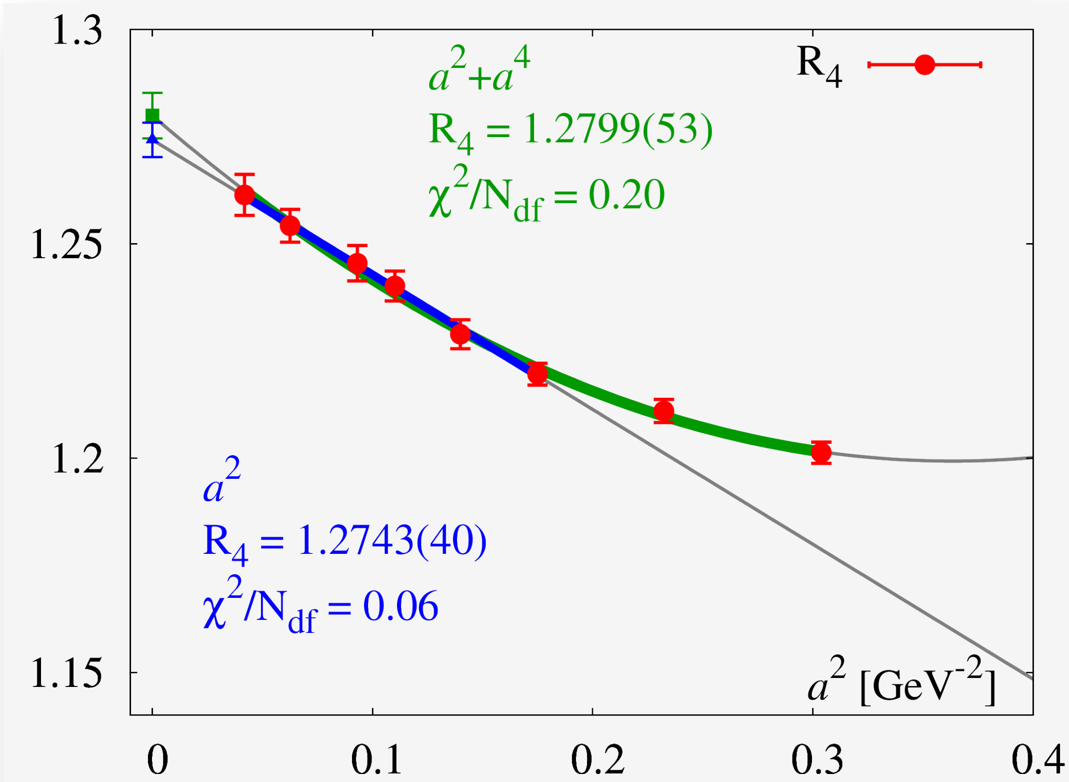
Perturbation

Numerical numbers at $\mu = m_c$ ($N_f = 4$)

Effects of charm loops: 0.7% for R_4 estimated in perturbation theory PRD78, 054513

→ correct lattice R_4 in our 2+1 flavor simulations

4-th Moments and coupling constant



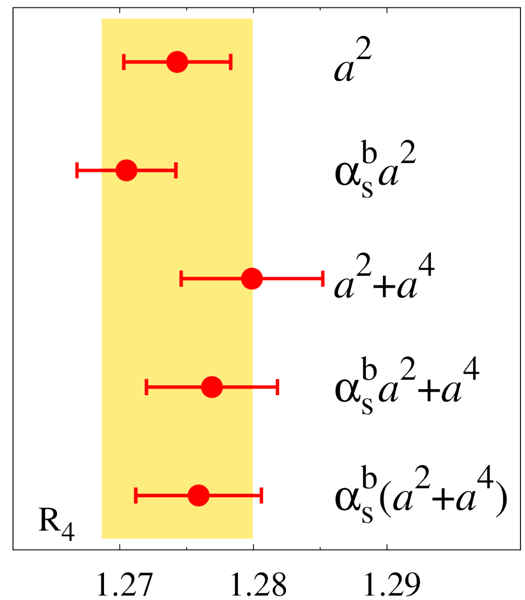
✓ data well extrapolated by a^2 and a^2+a^4 fits

✓ discretization in HISQ: $\alpha_s a^2$

Several fits with boosted coupling

$$\alpha_s^b(1/a) = \frac{1}{4\pi}\frac{g_0^2}{u_0^4}$$

uncertainty due to continuum extrap.



$$R_4 = 1.2743(40)(40)$$

c.f.) R_4=1.281(5) HPQCD'08

R_4=1.282(4) HPQCD'10

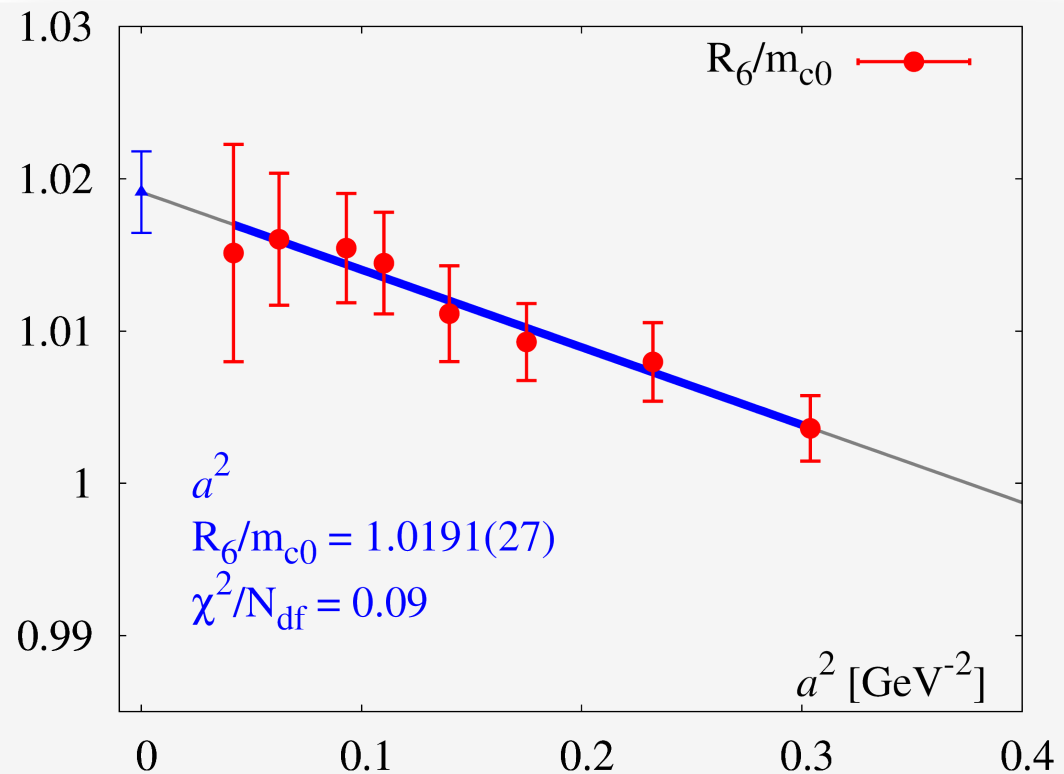
From $R_4 = r_4(\alpha_s;\mu = m_c)$ we obtain coupling constant:

$$\alpha_s(\mu = m_c, n_f = 3) = 0.3697(54)_{\text{stat.}}(50)_{\text{cont.}}(15)_{\text{trun.}}$$

in the lowest energy scale so far

"trun.": truncation errors coming from α_s^4 terms

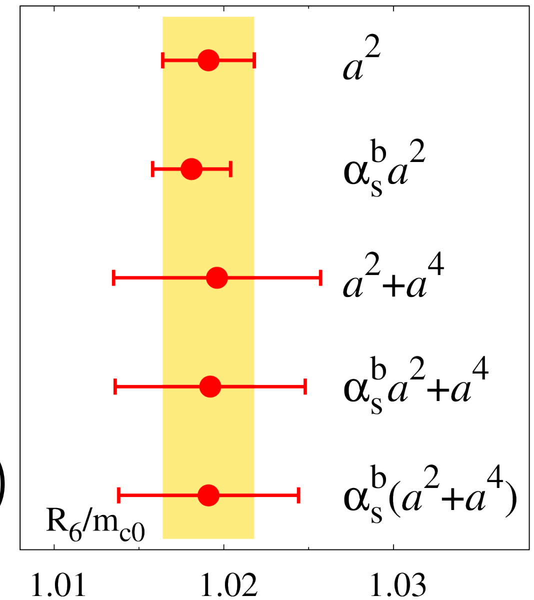
6-th Moments and charm quark mass



- ✓ Linear-like behavior
- ✓ uncertainty due to continuum extrap.

no significant dependence

$$\frac{R_6}{m_c^{\rm lat}} = 1.0191(27)$$



From $$m_c(m_c) = \frac{r_6(\alpha_s; \mu = m_c)}{R_6/m_c^{\rm lat}}$$

we obtain charm quark mass:

$$m_c(\mu = m_c, n_f = 3) = 1.2668(33)_{\rm stat.}(34)_{\rm trun.}(70)_{\alpha_s}(73)_{\rm scale}\ \text{GeV}$$

Summary

arXiv:1606.08798

via meson masses: $m_c/m_s = 11.877(91)\,, \quad m_b/m_c = 4.528(57)$

via moments: $\alpha_s(\mu = m_c, n_f = 3) = 0.3945(75)\,, \quad m_c(\mu = m_c, n_f = 3) = 1.267(11)\ \mathrm{GeV}$

evolving with 4-loops PT in $\overline{\mathrm{MS}}$ scheme:
RunDeC package

$$\begin{cases} \alpha_s(M_Z, n_f = 5) = 0.11622(75) \\ m_s(\mu = 2\,\mathrm{GeV}, n_f = 3) = 92.0(1.5)\ \mathrm{MeV} \\ m_b(\mu = m_b, n_f = 5) = 4.184(83)\ \mathrm{GeV} \end{cases}$$

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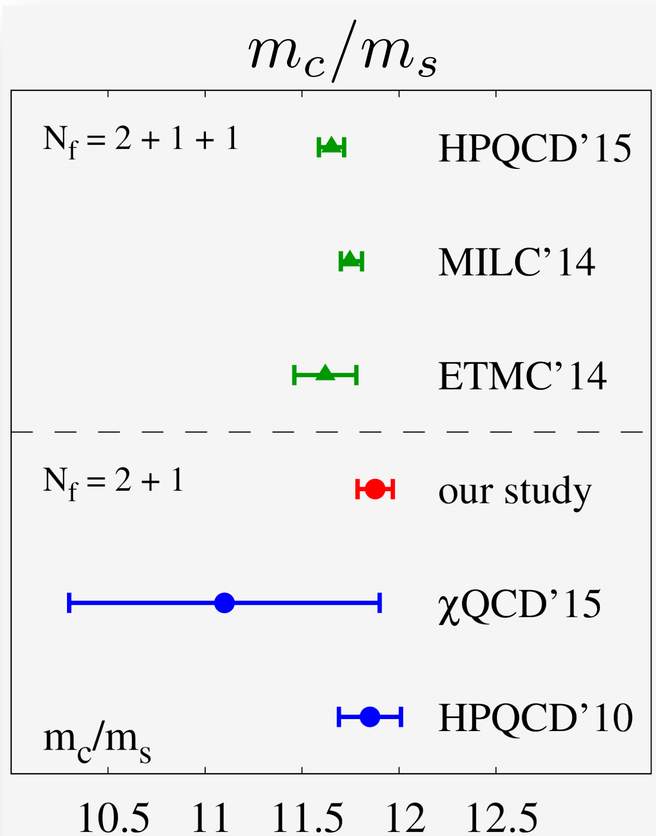
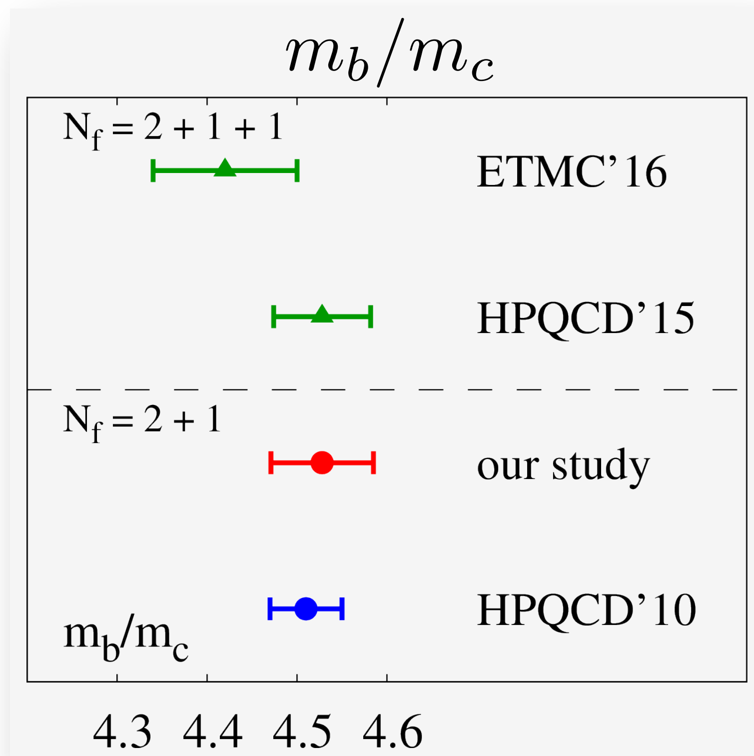




✓ Quark mass ratio: no significant deviations

slight deviation? $m_c/m_s(N_f = 4) \lesssim m_c/m_s(N_f = 3)$

Summary

arXiv:1606.08798

✓ Values similar to "Bazavov'14"

same gauge configurations

"Bazavov'14": static $q\bar{q}$ energy PRD90, 074038

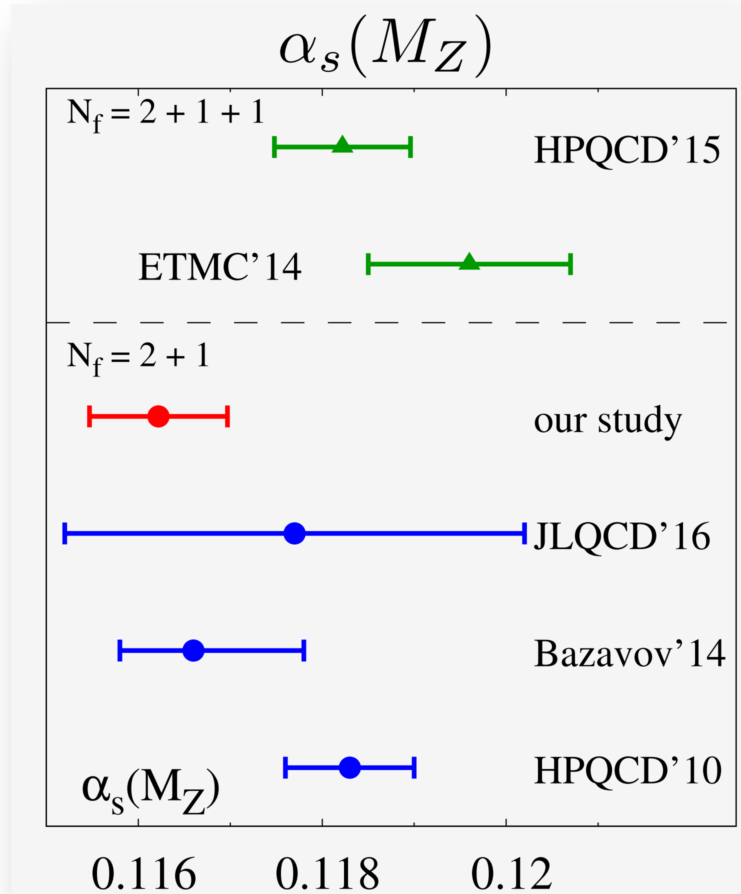
our study: moments

➡ consistency in different approaches

✓ Deviation from HPQCD (~1.7%)

both using HISQ and moments

Our study	HPQCD PRD91, 054508
2+1 flavor + c-loop	2+1+1 flavor
Tree level imp. gauge	1-loop tadpole imp. gauge
r_1 scale	w_0 scale
$a^{-1} \lesssim 4.98$ GeV	$a^{-1} \lesssim 3.33$ GeV
direct extrap. of R_4 and R_6	Bayesian fit of $R_{4,6,8,10}$ ~90 data by ~10 param.



⬇ higher moments: enable cross checks

crucial uncertainties due to finite volume in our study,

as well as perturbative truncation

Further investigation: higher moments, different scale, different approach, etc...

Backup slides…

Unmixed $s\bar{s}$ eta

$$M_{\eta_{s\bar{s}}} = \sqrt{2M_K^2 - M_\pi^2}$$

Assign errors for EM effects and absence of disconnected diagram

$$M_\pi^2 = M_{\pi^0}^2$$

$$M_K^2 = \frac{1}{2}(M_{K^0}^2 + M_{K^+}^2 - (1+\Delta_E)(M_{\pi^+}^2 - M_{\pi^0}^2))$$

$$M_{\eta_{s\bar{s}}} = 686.00(92)\ \mathrm{MeV} \quad (\Delta_E = 0\text{–}2)$$

Aubin et al (MILC), PRD 70 (2004) 114501

c.f.) HPQCD: 0.6858(38)(12), 0.6885(22) GeV

Meson masses to quark masses

Charmonium correlators with wall sources

Spin averaged mass

$$\bar{M} = \frac{1}{4}\left(3M_{J/\psi} + M_{\eta_c}\right)$$

Experiments:

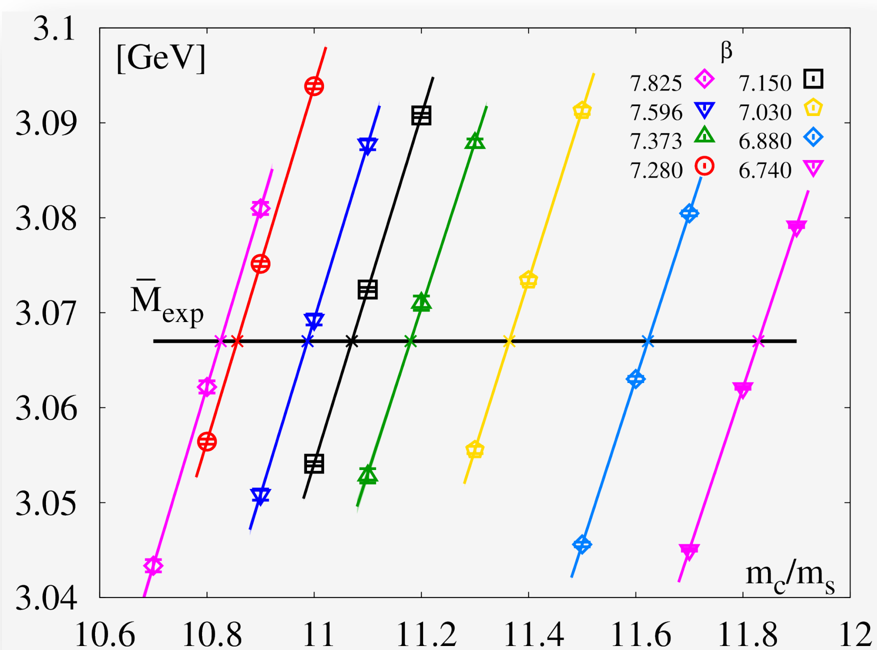
$$\bar{M} = 3.067(3)\ \mathrm{GeV}$$

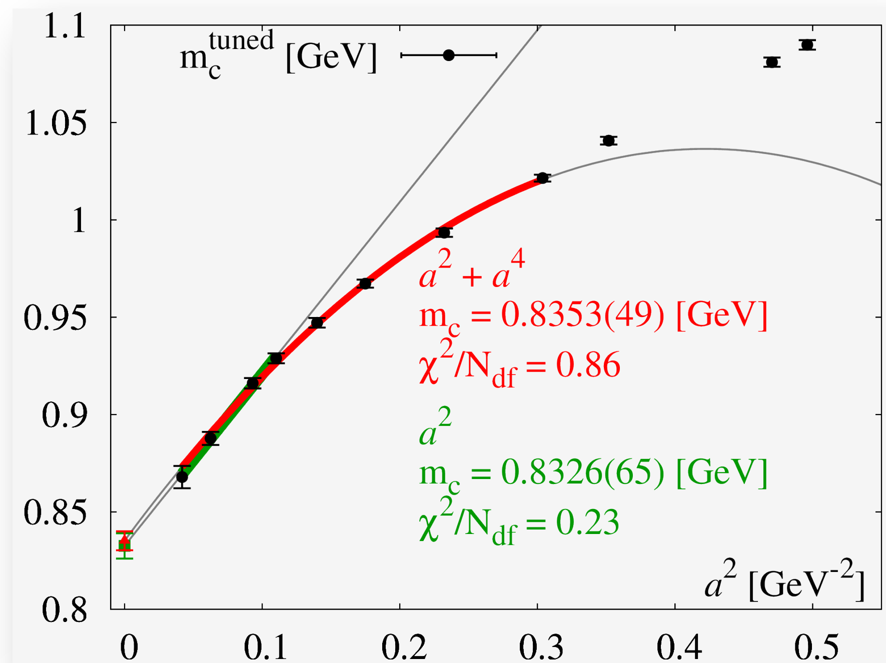
uncertainties:

neglecting disconnected and EM

Quark mass

$$\bar{M} = d + b\, m_c$$





Precise determination of mc within 0.4--0.7% uncertainties

Charmonia

Charmonium states in continuum

$$M_{\mathrm{PS}} = 2.982(12)\ \mathrm{GeV}$$
$$M_{\mathrm{V}} = 3.095(12)\ \mathrm{GeV}$$

experiments:

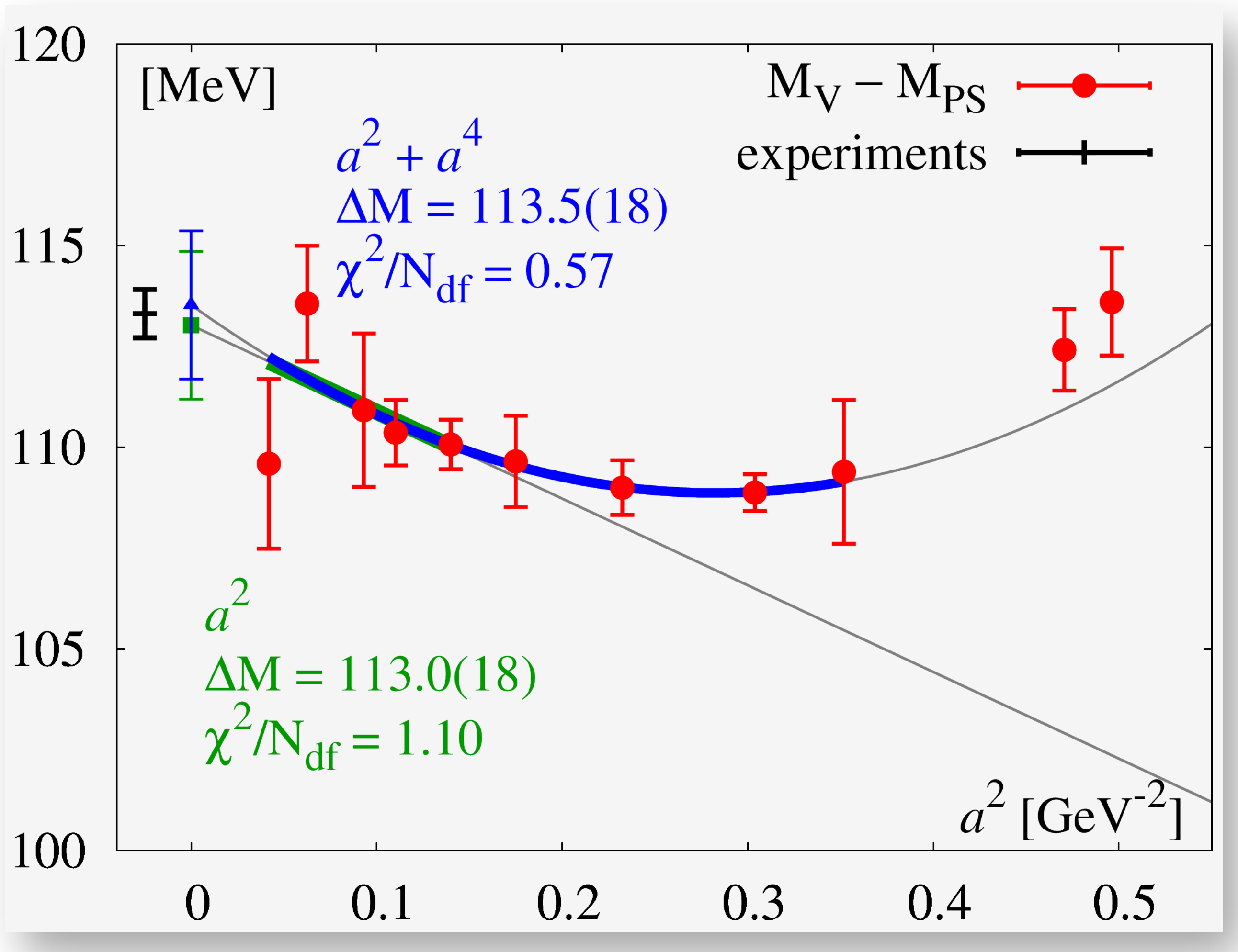
$$M_{\eta_c} = 2.9836(6)\ \mathrm{GeV}$$
$$M_{J/\psi} = 3.09692(1)\ \mathrm{GeV}$$

Hyperfine splitting

$$M_{\mathrm{V}} - M_{\mathrm{PS}} = 113.5(18)(7)\ \mathrm{MeV}$$

Experiments:

$$M_{J/\psi} - M_{\eta_c} = 113.3(6)\ \mathrm{MeV}$$



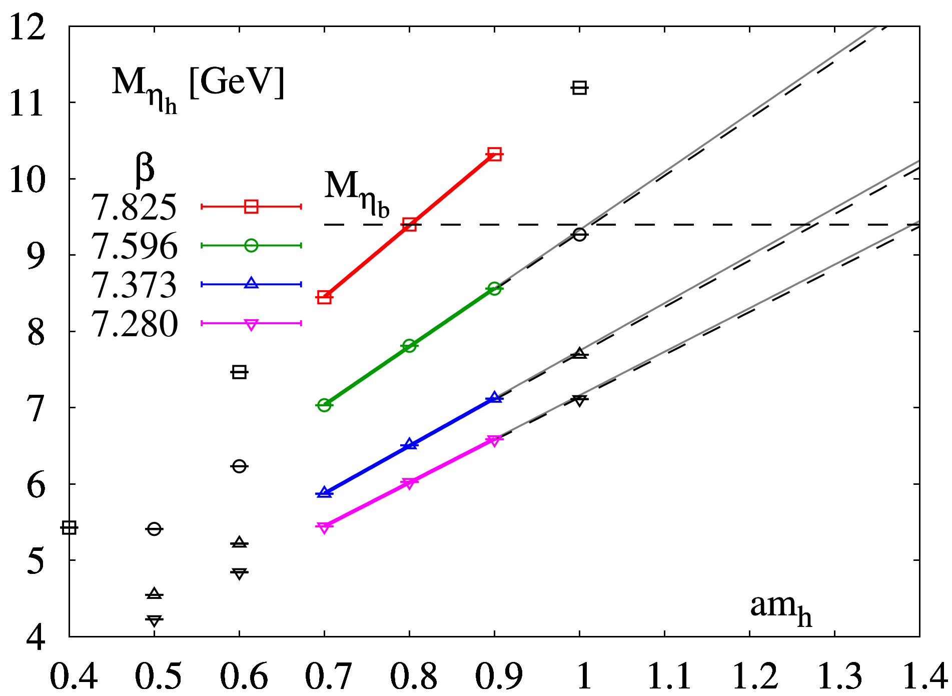
Heavy quark mass

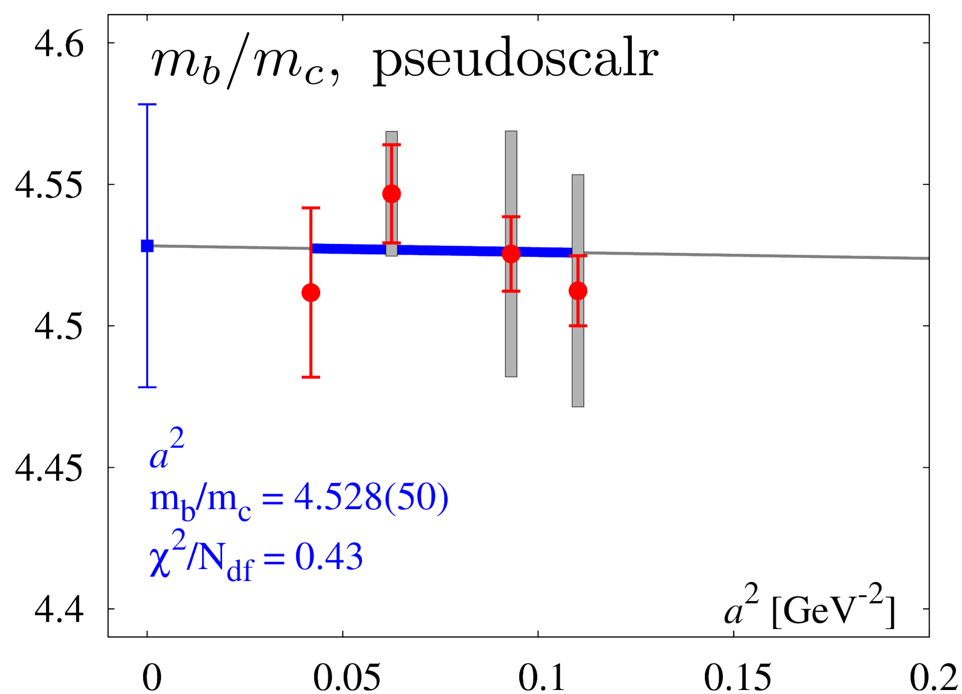
Heavier masses (am_h) up to bottom quarks: difficult $am_b > 1.0 > am_h$

adjusting : $M_{\eta_b} = 9.398(3)$ GeV, $M_\Upsilon = 9.4603(3)$ GeV

Interpolation: $0.7 \le am_h \le 0.9$ at highest beta

Extrapolation: necessary for other beta -> systematic errors





Moments in Nf=3

$$R_n = \begin{cases} r_4 & (n=4) \\ r_n \cdot \left(m_c^{\rm lat}/m_c(\mu)\right) & (n \ge 6) \end{cases} , \quad r_n = 1 + \sum_{j=1}^{3} r_{nj}(\mu, m_c)\alpha_s^j(\mu)$$

Lattice in continuum

Perturbation

Numerical numbers at $\mu = m_c$ ($N_f = 3$)

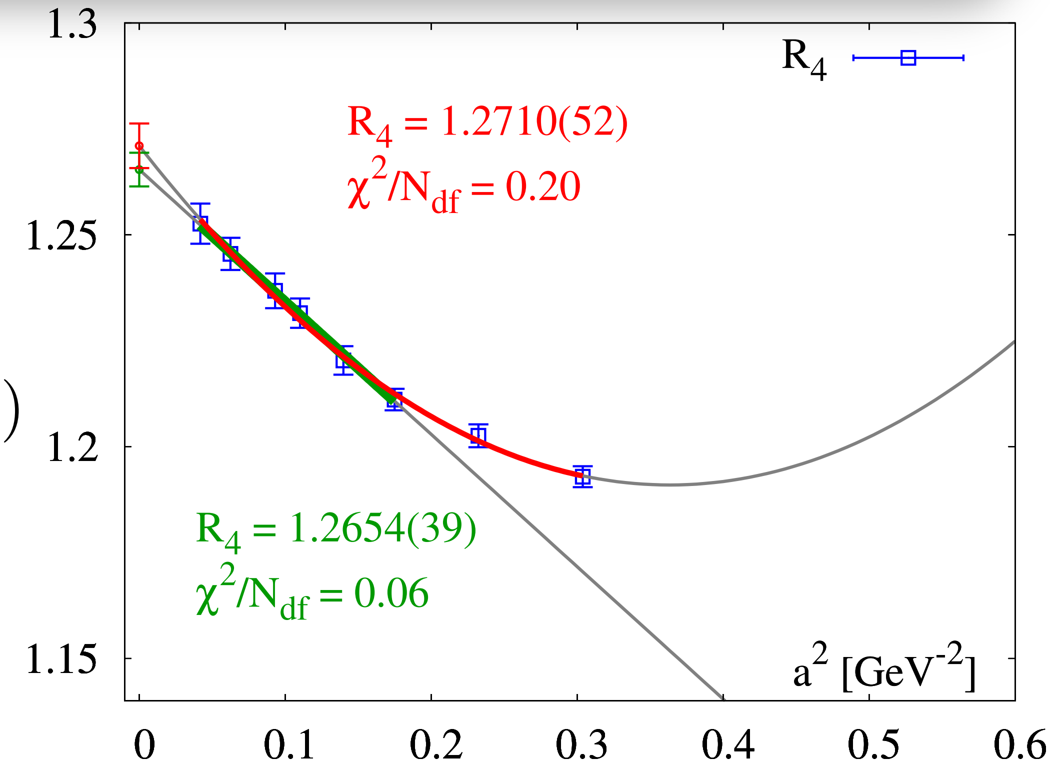
Raw R_4 in our 2+1 flavor calc.

$$R_4 = 1.2654(39)$$

From $R_4 = r_4(\alpha_s; \mu = m_c, n_f = 3)$

$$\alpha_s(m_c) = 0.3638(54)(28) \ : \quad a^2$$

$$\alpha_s(m_c) = 0.3715(72)(31) \ : \quad a^2 + a^4$$



c.f.) 2+1+c-loop flavor with Nf=4 PT

$$\alpha_s(m_c) = 0.3697(54)(15) \ : \quad a^2$$

$$\alpha_s(m_c) = 0.3773(72)(16) \ : \quad a^2 + a^4$$

Comparison of mc(mc)