A gauge invariant Debye mass for the complex heavy-quark potential

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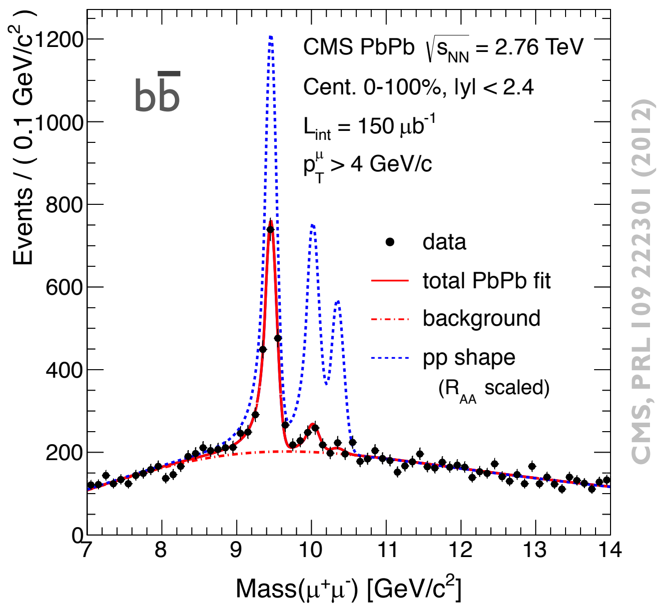
in collaboration with Y. Burnier

References:

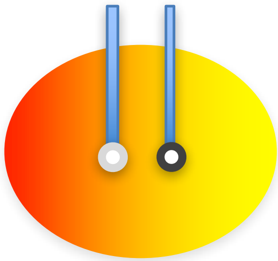
Y. Burnier, A.R.: Phys.Lett. **B753** (2016) 232 (derivation)
Y. Burnier, A.R.: arXiv:1607.04049 (updated SU(3) results)

Motivation: Heavy Quarkonium

- In-medium modification of heavy quark-anti quark bound states



Matsui&Satz's analogy with QED:
Color screening in the QGP
prevents QQ formation



Matsui and Satz, Phys.Lett. B178 (1986) 416

- In QED Debye screening understood: electric gauge bosons acquire T>0 mass

C. Gale and J.I. Kapusta, Finite-Temperature Field Theory

$$\langle \mathbf{E}(\mathbf{x})\mathbf{E}(0)\rangle \sim e^{-m_D|\mathbf{x}|}/|\mathbf{x}|^3$$

$$\lim_{\mathbf{p}\to 0} \Pi_{00}(0,\mathbf{p}) = m_D^2$$

$$\lim_{\mathbf{p}\to 0} \Pi_{ij}(0,\mathbf{p}) = 0$$

- In (lattice) QCD challenging, since **E** and $\Pi_{\mu\nu}$ not gauge invariant

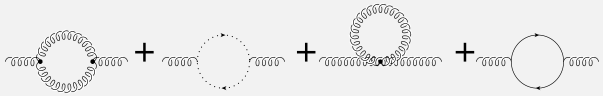
Current approaches to m_D

- Perturbative evaluation in QCD hampered by magnetic sector $O(g^2T)$

$$m_D^{\rm pert} = m_D^{\rm LO} + \frac{Ng^2T}{4\pi}\log\frac{m_D^{\rm LO}}{g^2T} + \kappa_1 g^2T + \kappa_2 g^3 T$$

P. Arnold and L. Yaffe, PRD 52, 7208 (1995)

$$m_D^{\rm LO} = \sqrt{\frac{N_c}{3}+\frac{N_f}{6}}\,gT$$



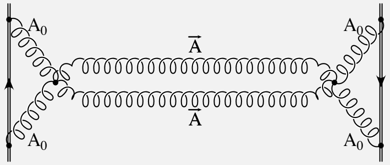
Lattice EFT evaluation (EQCD)

$$L_{\rm eff}[A_0^a, A_i^a] = \frac{1}{4}F_{ij}^aF_{ij}^a + {\rm Tr}[D_i, A_0][D_i, A_0] + m_3{\rm Tr}A_0^3 + \lambda_A({\rm Tr}A_0^2)^2$$

- g_3, m_3 and λ_A matched to QCD perturbatively

K. Kajantie et.al. NPB503 (1997) 357

- Need to disentangle **E** and **B** screening

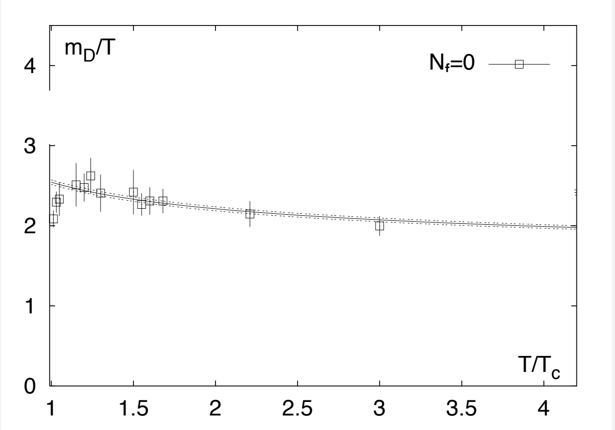


- Measure $\langle h_3h_3\rangle \sim e^{-m_D|x|}$ with $h_i=\epsilon_{ijk}{\rm Tr}[A_0F_{jk}]$
- Large corrections $\kappa_1=2.5(2)$ $\kappa_2=-0.5(2)$

Potential models $F^{(1)}(R)$

Idea of Debye: inspect interactions of test particles in presence of charges

Digal, Kaczmarek, Karsch, Satz, Eur. Phys. J. C 43 (2005) 71 / O. Kaczmarek, F. Zantow, Phys. Rev. D71 (2005) 114510



See also WHOT collaboration PRD 75, 074501 (2007)

- Fit purely Coulombic e^{-m_Dr}/r at large distances
- Much smaller corrections $\kappa_1\approx0.35$ $\kappa_2\approx-0.1$ (SU(3))

Current approaches to m_D

- Perturbative evaluation in QCD hampered by magnetic sector $O(g^2T)$

$$m_D^{pert} = m_D^{LO} + \frac{Ng^2T}{4\pi}\log\frac{m_D^{LO}}{g^2T} + \kappa_1 g^2T + \kappa_2 g^3T$$

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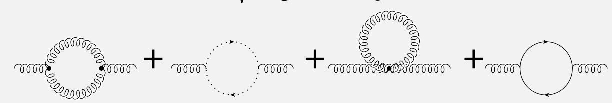

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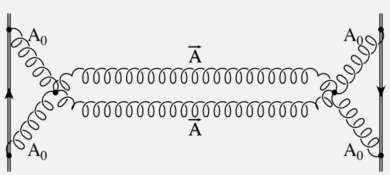

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Potential models $F^{(1)}(R)$

Idea of Debye: inspect interactions of test particles in presence of charges

Our strategy:

1.: instead of a model, use the proper lattice QCD heavy quark potential

2.: elucidate $T>0$ modification of V_{QQ} beyond Coulombic screening

- Fit purely Coulombic e^{-m_Dr}/r at large distances
- Much smaller corrections $\kappa_1 \approx 0.35$ $\kappa_2 \approx -0.1$ (SU(3))

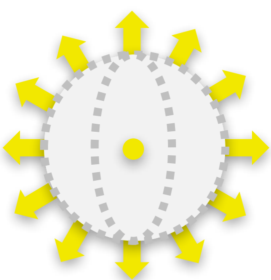
Defining m_D for the proper V_{QQ}

- Challenge: T>0 heavy $Q\bar{Q}$ potential defined using effective field theory is complex

Laine et al. JHEP 03 (2007) 054; Brambilla et. al. PRD 78 (2008) 014017;
A.R. et. al. PRL108 (2012) 162001, Y. Burnier et.al. PRL 114 (2015) 082001

- How to connect in-medium modification of Re[V] and Im[V] to a Debye mass?

$$V_{Q\bar{Q}}^{T=0} = V_C(r) + V_S(r) = -\frac{\alpha_S}{r} + \sigma r + c$$



$$\vec{\nabla}\left(\frac{\vec{\nabla}V(r)}{r^{a+1}}\right) = -4\pi\, q\, \delta(\vec{r})$$

$$V(r) = a\, q\, r^a \quad \vec{E} = -\vec{\nabla}V(r)$$

V. V. Dixit, Mod. Phys. Lett. A 5, 227 (1990)
see also Digal et. al. Eur.Phys.J. C43 (2005) 71

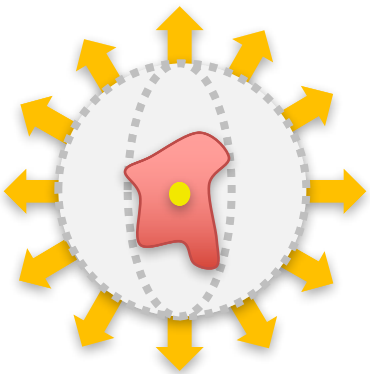
Coulombic: a=-1 q=α_s

$$\vec{\nabla}\left(\vec{\nabla}V_C(r)\right) = -4\pi\, \alpha_S\, \delta(\vec{r})$$

String-like: a=+1 q=σ

$$\vec{\nabla}\left(\frac{\vec{\nabla}V_S(r)}{r^2}\right) = -4\pi\, \sigma\, \delta(\vec{r})$$

Introducing medium effects



In the classical theory of Debye: Boltzmann distr. backgr. charges <ρ>

$$\vec{\nabla}\left(\vec{\nabla}V_C(r)\right) = -4\pi\alpha\left(\delta(\vec{r}) + \langle\rho(\vec{r})\rangle\right)$$

P. Debye, H. Hückel, Phys.Z. 24, 185-206 (1923)

Here instead: Introduce medium via weak coupling HTL permittivity ε

see also: L. Thakur et.al. PRD 89 094020 (2014)

$$p^2 V_C(\vec{p}) = 4\pi\frac{\alpha_s}{\varepsilon(\vec{p}, m_D)} \qquad \varepsilon^{-1}(\vec{p}, m_D) = \frac{p^2}{p^2+m_D^2} - i\pi T\frac{p\, m_D^2}{(p^2+m_D^2)^2}$$

linear response form in which m_D->0 is possible

$$g(x) = 2\int_0^\infty dp \frac{\sin(px)}{px}\frac{p}{p^2+1}$$

$$-\nabla^2 V_C(r) + m_D^2 V_C(r) = \alpha_s\left(4\pi\delta(\vec{r}) - iTm_D^2\, g(m_D r)\right)$$

Y.Burnier, A.R.: PLB753 (2016) 232

solving for Re[V_C] and Im[V_C]: reproduces the HTL result

$V_S(r)$: Gauss Law operator not diagonal in Fourier space: assume validity of linear response

$$-\frac{1}{r^2}\frac{d^2 V_S(r)}{dr^2} + \mu^4 V_S(r) = \sigma\left(4\pi\delta(\vec{r}) - iTm_D^2\, g(m_D r)\right) \qquad \mu^4 = m_D^2\frac{\sigma}{\alpha_S}$$

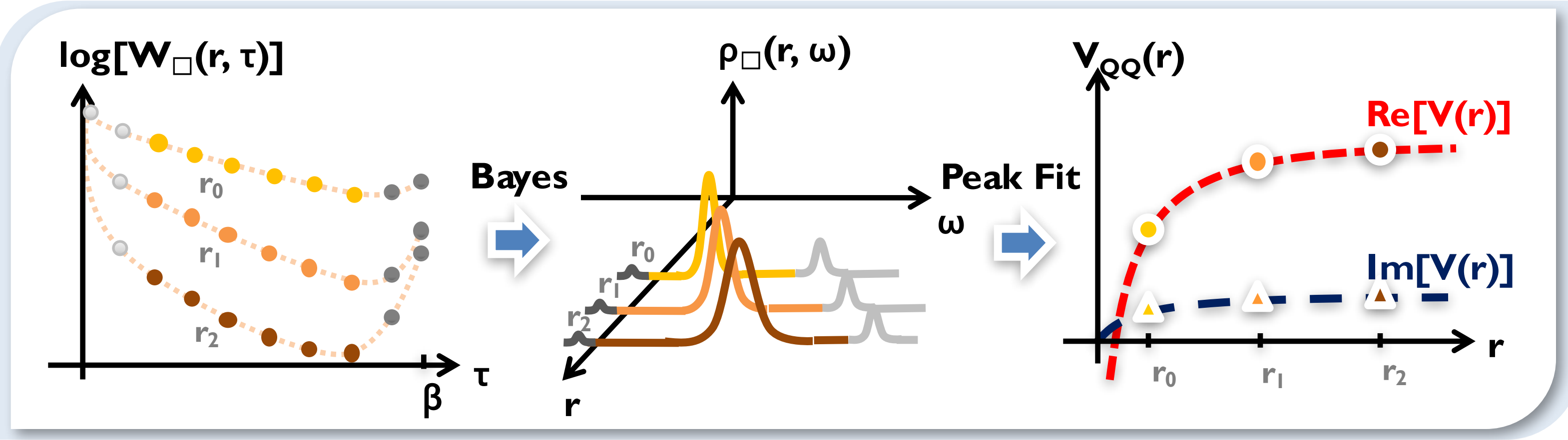
$$\mathrm{Re}V_S(r) = \frac{\Gamma[\frac{1}{4}]}{2^{\frac{3}{4}}\sqrt{\pi}}\frac{\sigma}{\mu}D_{-\frac{1}{2}}(\sqrt{2}\mu r) + \frac{\Gamma[\frac{1}{4}]}{2\Gamma[\frac{3}{4}]}\frac{\sigma}{\mu}$$

Im[V_S] as integral expression using Wronskian

D_ν parabolic cylinder function

Extracting V_{QQ} in SU(3)

- From the Euclidean Wilson loop to the complex heavy quark potential



A.R. MPLA 28, 1330005 (2013)

For technical details see: Y.Burnier, A.R. PRD86 (2012) 051503; PRL111 (2013) 182003

- Quenched lattice QCD: anisotropic lattices with naïve Wilson action $32^3 x N_\tau$
  - Fixed scale approach: $\beta=6.1$ $\xi=a_s/a_\tau=4$ $a_s=0.097$fm

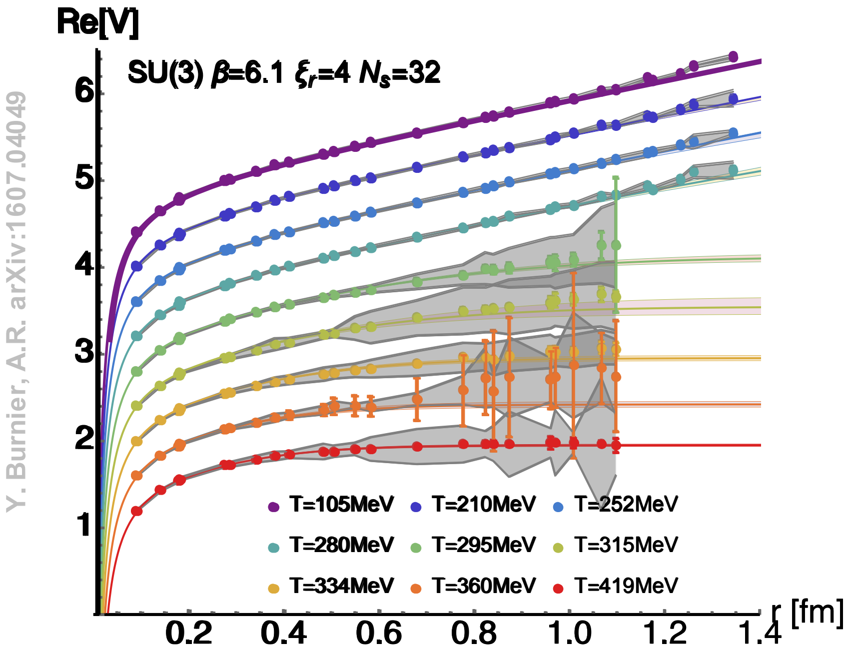
Matsufuru et. al. Phys. Rev. D64, 114503 (2001)

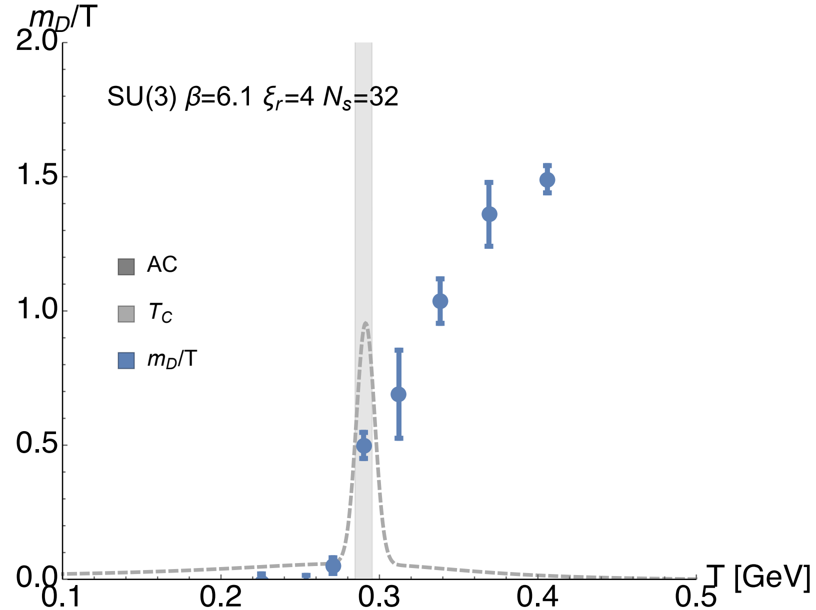
N_τ	20	22	24	26	28	30	32	36	72
T/T_C	1.4	1.27	1.67	1.08	≈1	0.93	0.88	0.78	0.39
N_{meas}	2080	1980	1920	800	1730	900	880	940	950

  - To avoid cusp divergences: use Wilson line correlators in Coulomb Gauge instead of Wilson loops

Re[V_{QQ}] and m_D in SU(3)

- At T≈0 Re[V_{QQ}] is well described by the naïve Cornell ansatz

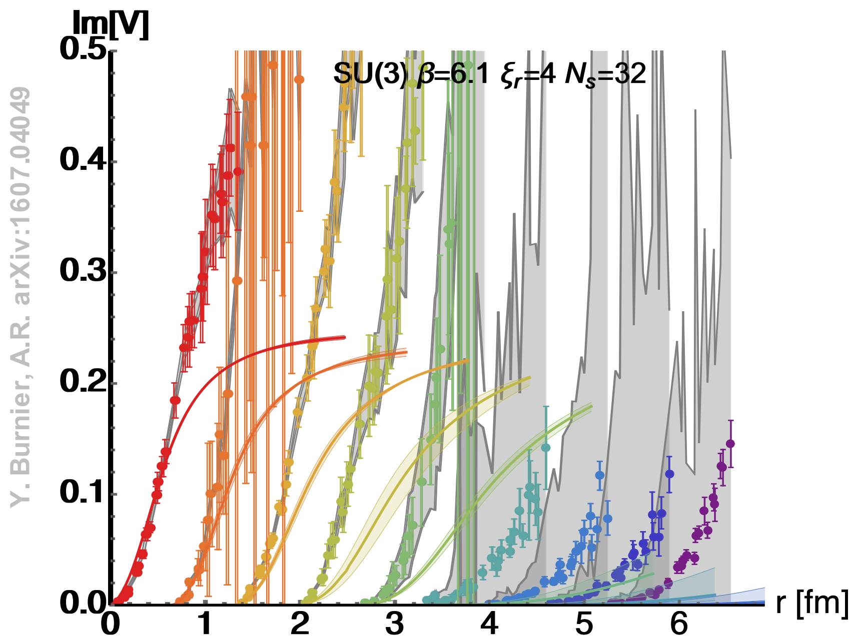


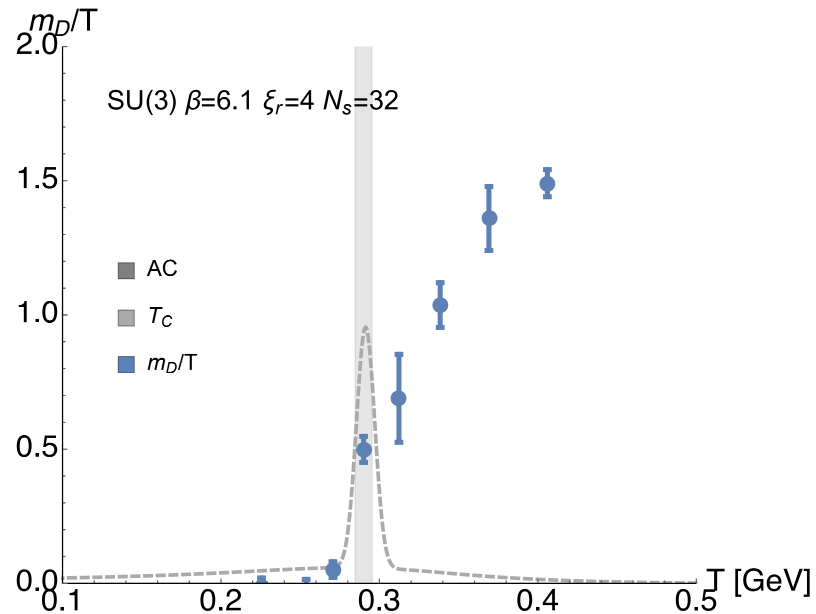


- m_D(T) captures physics of screening: excellent agreement with lattice Re[V_{QQ}]
- T<T_C: m_D compatible with zero, in infinte box expect jump to finite value at T_C
- Once m_D is fixed by Re[V], postdiction of Im[V_{QQ}] possible

Im[V_{QQ}] and m_D in SU(3)

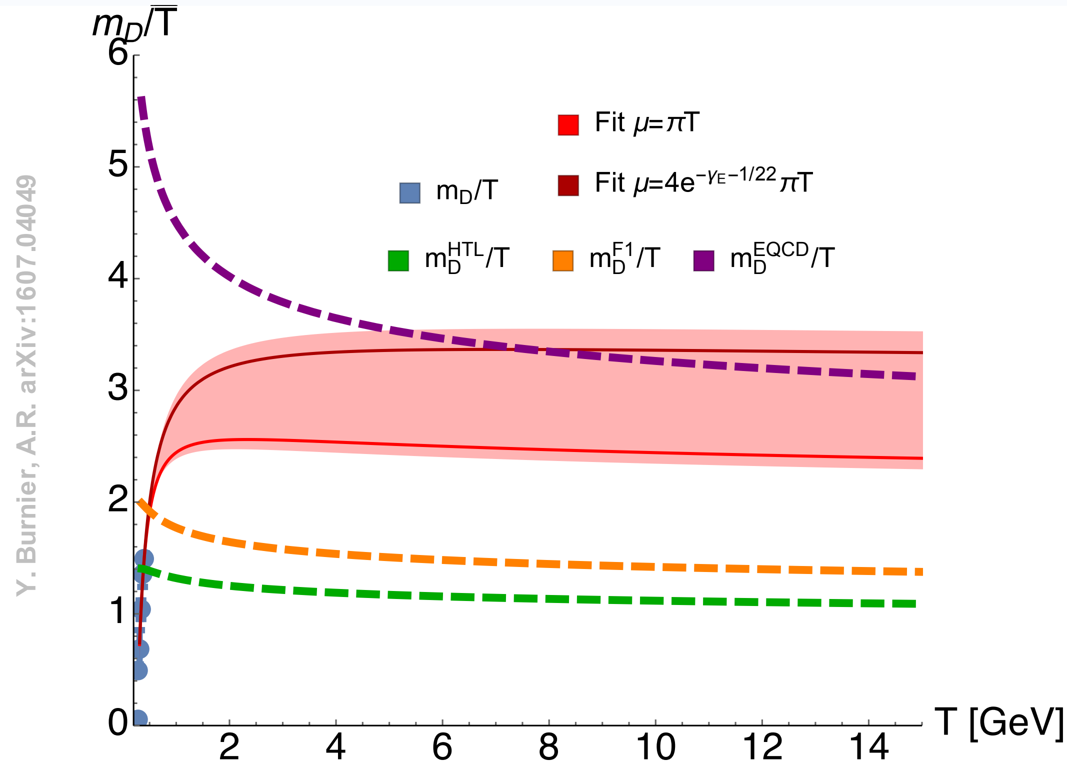
- Im[V_{QQ}] less reliable since based on spectral widths





- $m_D(T)$ not only governs screening but also scattering effects
- $T<T_C$: since m_D compatible with zero also Im[V_{QQ}] vanishes
- Agreement with lattice Im[V_{QQ}] at $T>>T_C$ and small r: need lattices with larger N_τ

The Debye mass parameter



- Interpolate with HTL + non-perturbative corrections ($g(\mu=\pi T)$ 4-loop; fit of $\kappa_1=2.7$ $\kappa_2=-1.5$)

$$m_D = \sqrt{\frac{N_c}{3}}\, g(\mu) T + \frac{N g(\mu)^2 T}{4\pi} \log \frac{\sqrt{\frac{N_c}{3}}}{g(\mu)} + \kappa_1 g(\mu)^2 T + \kappa_2 g(\mu)^3 T$$

P. Arnold and L. Yaffe, PRD 52, 7208 (1995)

Van Ritbergen et.al PLB 400,379 (1997)

  - Evaluation of $<(A_0 F_{ij})^2>$ in EQCD: $\kappa_1=2.5$ $\kappa_2=-0.5$ ($\mu\approx 2\pi T$)   Fit to free energies $\kappa_1=0.3$ $\kappa_2=-0.1$ ($\mu\approx\pi T$)

Summary&Outlook

- Novel definition of m_D from the gauge invariant complex QQ potential at T>0
  - Ingredient 1: Generalized Gauss law together with weak-coupling permittivity ε^{HTL}
    - ➡ Parametrization of Re[V] and Im[V] with m_D as only T dependent parameter
  - Ingredient 2: Proper lattice QCD based values of Re[V] and Im[V]
    - ➡ Validate Gauss-law parametrization by reproducing Re[V] via m_D fit alone
- In SU(3) m_D vanishes in the confined phase, finite above T_C (akin to a phase transition)
- m_D around T_C smaller than LO HTL, for $T>>T_C$ seems to be systematically larger
- Perform continuum limit for SU(3), extract V_{QQ} on realistic HISQ lattices (HotQCD)
- Consequences for phenomenology? (enhanced photon production around T_C...)

Thank you for your attention