Roberge-Weiss periodicity and confinement-deconfinement transition

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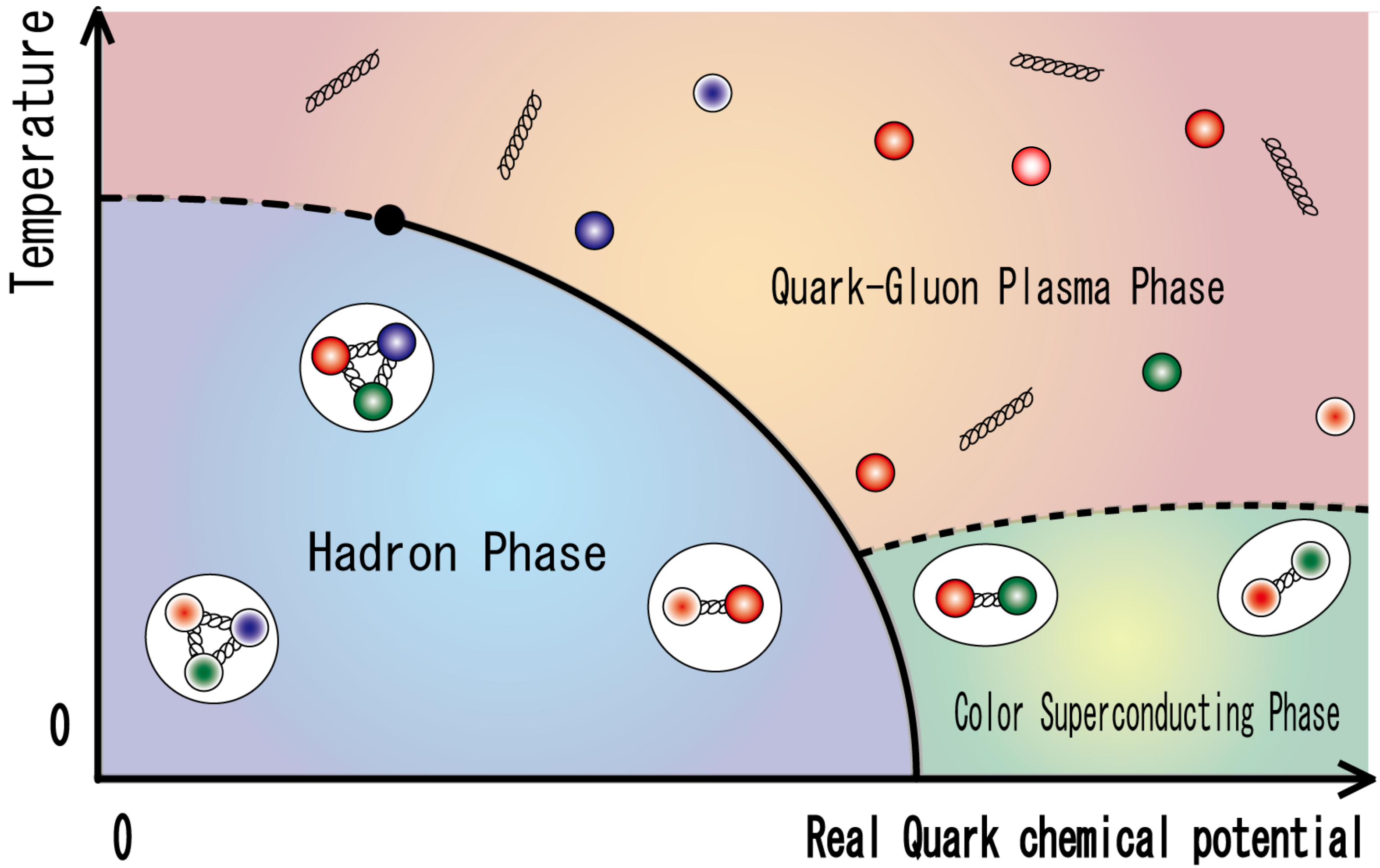
K.K., A. Ohnishi, Phys. Lett. B750 (2015) 282.

K.K., A. Ohnishi, Phys. Rev. D 93 (2016) 116002.

Schematic QCD phase diagram



In pure gauge theory,
Polyakov-loop becomes the order parameter of deconfinement transition

(Direct relation between deconfinement transition and $\mathbb{Z}_3$ symmetry)

If there are dynamical quarks, Polyakov-loop is **no longer order parameter**

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Usually, phase transitions are induced by the **spontaneous symmetry breaking**, but there are phase transitions **without** the spontaneous symmetry breaking

⬇

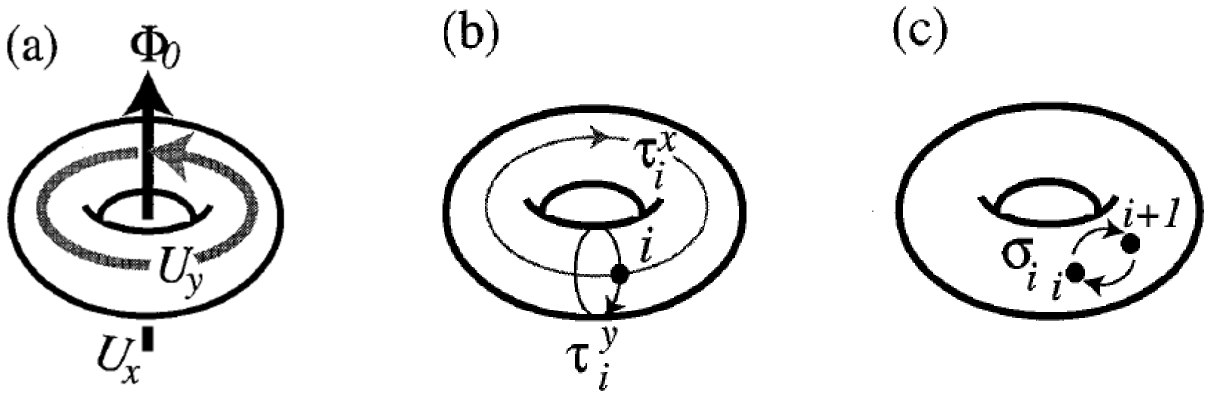
Topological order

X. Wen, Int. J. Mod. Phys. B4 (1990) 239.

In this study,
we investigate deconfinement transition from the topological viewpoint

Three operations

M. Sato, M. Kohmoto and Y.-S. Wu, PRL 97 (2006) 010601.
M. Sato, PRD 77 (2008) 045013.



Braid group

Aharonov-Bohm effect

Integer charge → Commutable

Fractional charge → Non-commutable

Vacuum degeneracy is necessary

T=0

M. Sato, PRD 77 (2008) 045013.

Confined state

Hadrons → No vacuum degeneracy

Deconfined state

Quarks → Vacuum degeneracy

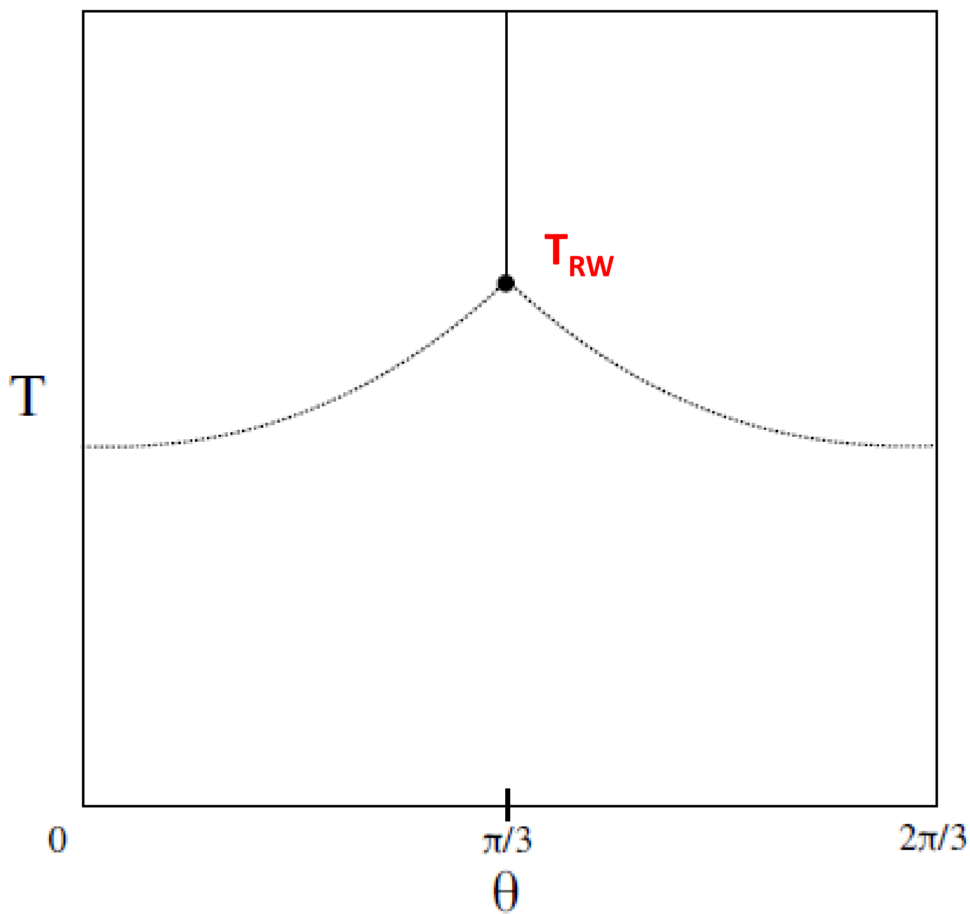
We can clarify the confined and deconfined states from the vacuum degeneracy by **modifying the topology** at zero temperature

$T = 0 \rightarrow T \neq 0$ thermal excitations ･･･

So …

Response against the imaginary chemical potential

Phase diagram



Symbols
- μ_I : Imaginary chemical pot.
- T : Temperature
- N_C : Nmuber of color

Roberge-Weiss（RW）periodicity

$2\pi/N_c$ periodicity for θ ($\equiv\mu_I/T$)

RW transition

First-order transition at $\theta=(2k-1)\pi/N_c$

RW endpoint

Endpoint of RW transition line

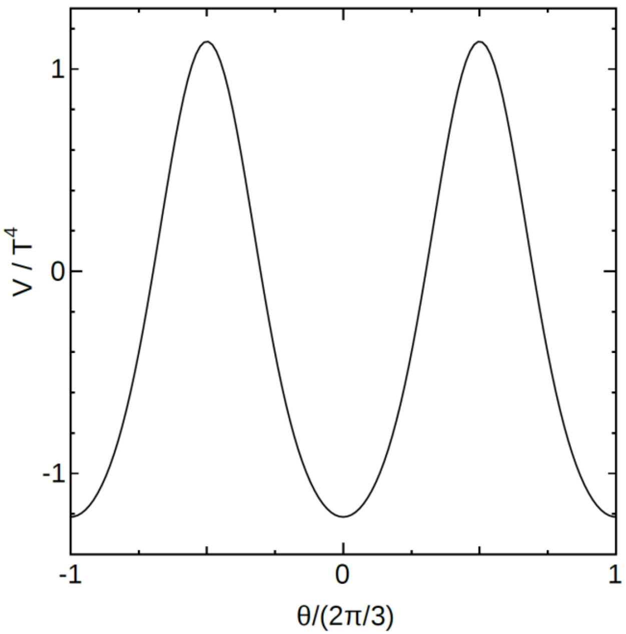
A. Roberge and N. Weiss, Nucl. Phys. B275 (1986) 734

Result 1 : Nontrivial free-energy degeneracy

K.K., A. Ohnishi, Phys. Lett. B750 (2015) 282.

Confined phase

For example,
strong coupling limit



Confined phase No nontrivial free energy degeneracy

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K.K., A. Ohnishi, Phys. Lett. B750 (2015) 282.

Confined phase

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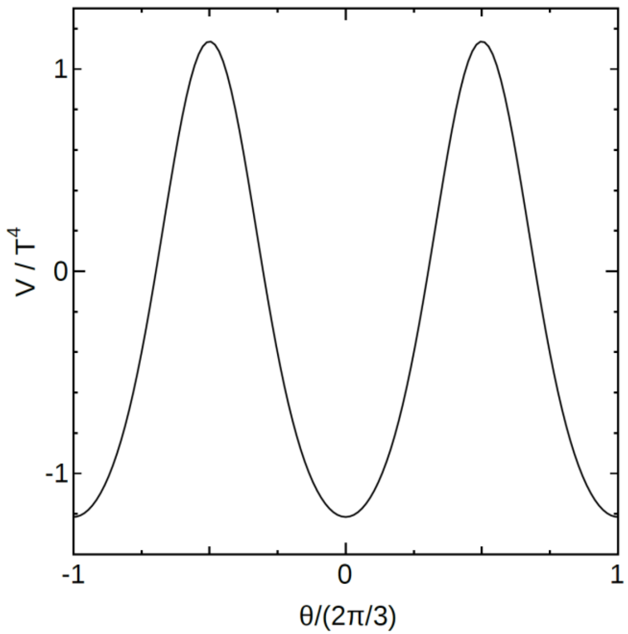
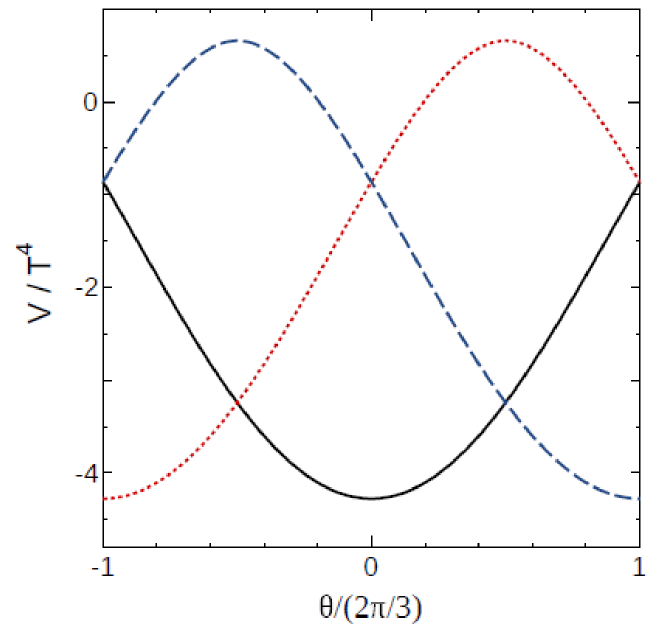


Deconfined phase

For example,
high T limit



Confined phase	No nontrivial free-energy degeneracy
Deconfined phase	Nontrivial free-energy degeneracy

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K.K., A. Ohnishi, Phys. Lett. B750 (2015) 282.

Confined phase

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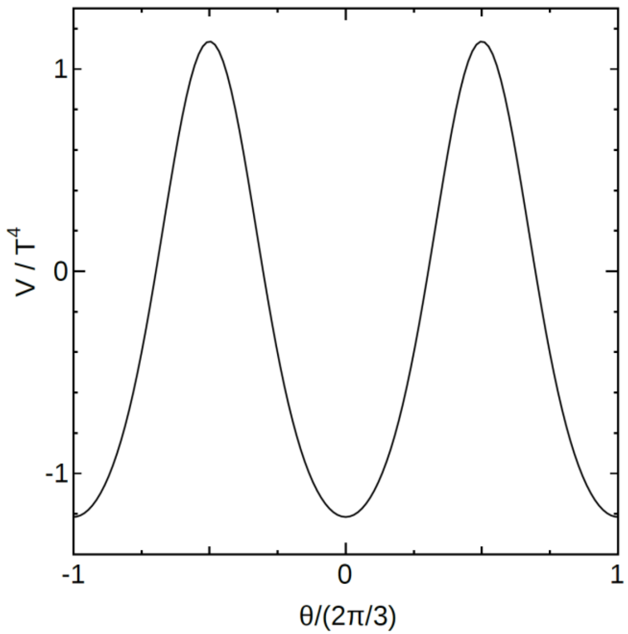

Deconfined phase

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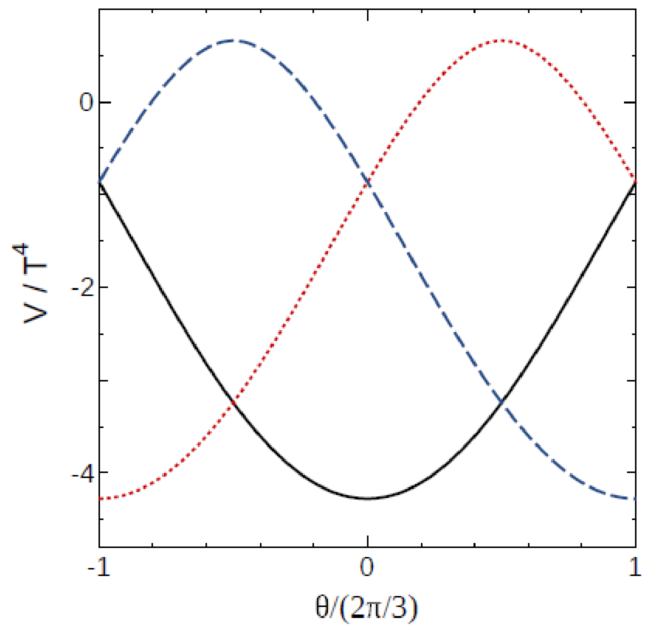
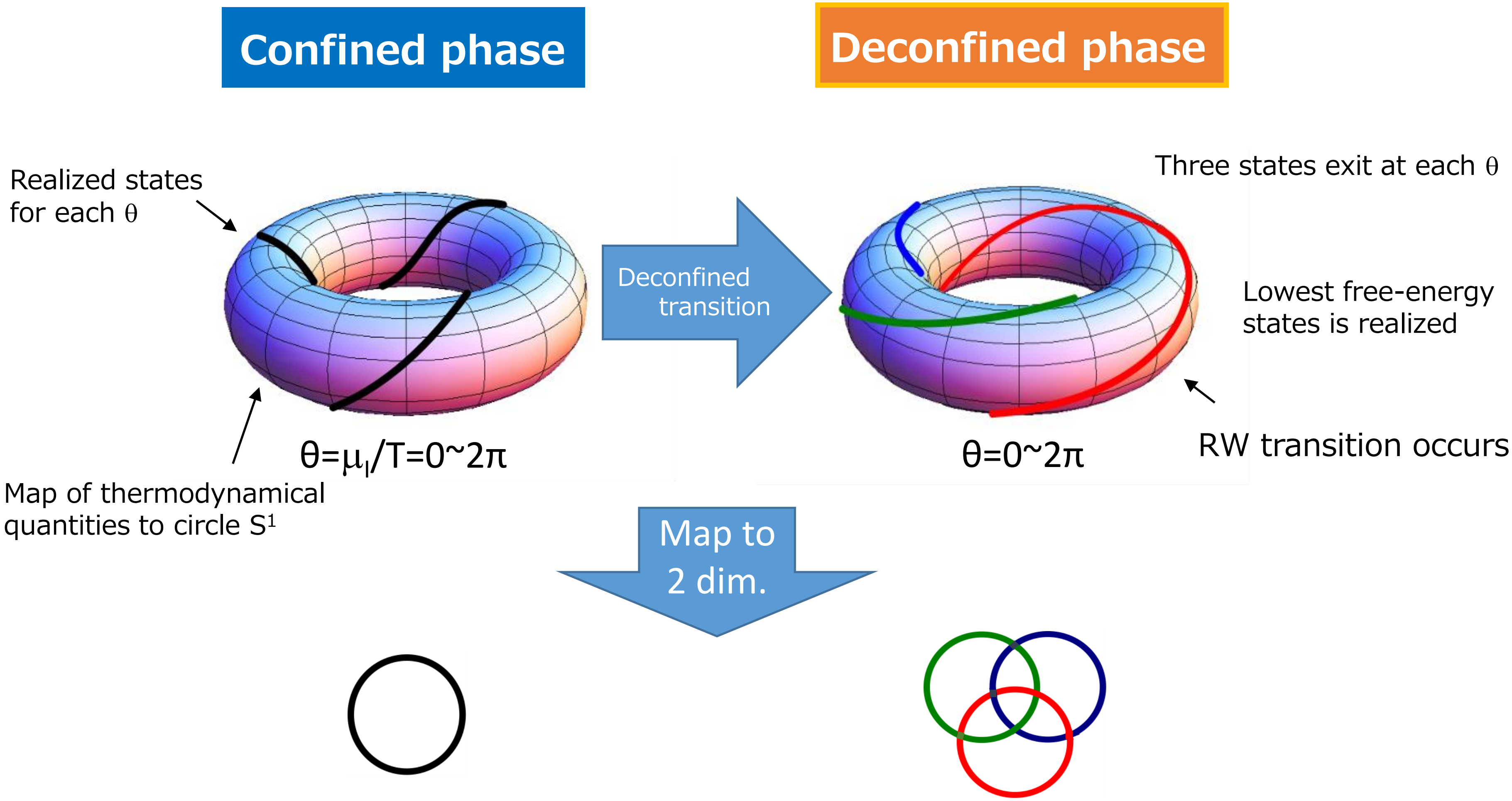


Confined phase	No nontrivial free-energy degeneracy
Deconfined phase	Nontrivial free-energy degeneracy

Infinite quark mas region:

It consist with usual determination → Necessary condition is manifested

Visualization of topological differences



Topological deference between two phases

Result 2 : Quantum order-parameter

K.K., A. Ohnishi, Phys. Rev. D 93 (2016) 116002.

Definition

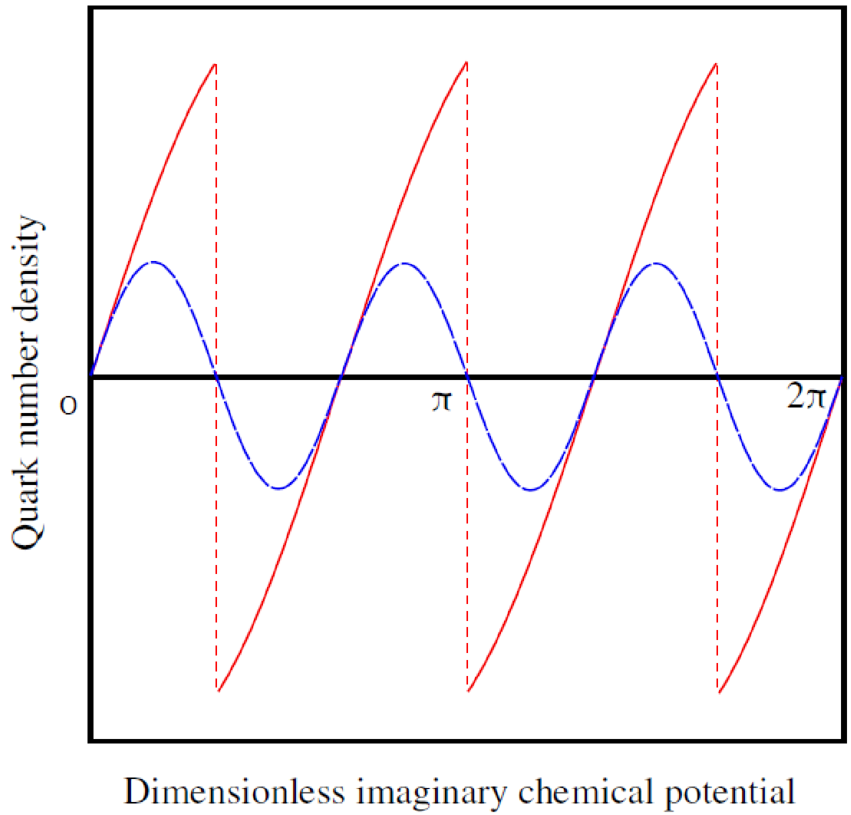
Quark number susceptibility

$$\Psi = \left[\oint_0^{2\pi} \left\{ \mathrm{Im} \left(\frac{d\tilde{n}_q}{d\theta} \Big|_T \right) \right\} d\theta \right]$$

Dimensionless quark number density

$$\tilde{n}_q \equiv C n_q$$

$C = T^3$



It is similar with **Uhlmann phase**

O. Viyuela, A. Rivas, M. Martin-Delgado,
Phys. Rev. Lett. 112 (2014) 130401
and references therein.

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K.K., A. Ohnishi, Phys. Rev. D 93 (2016) 116002.

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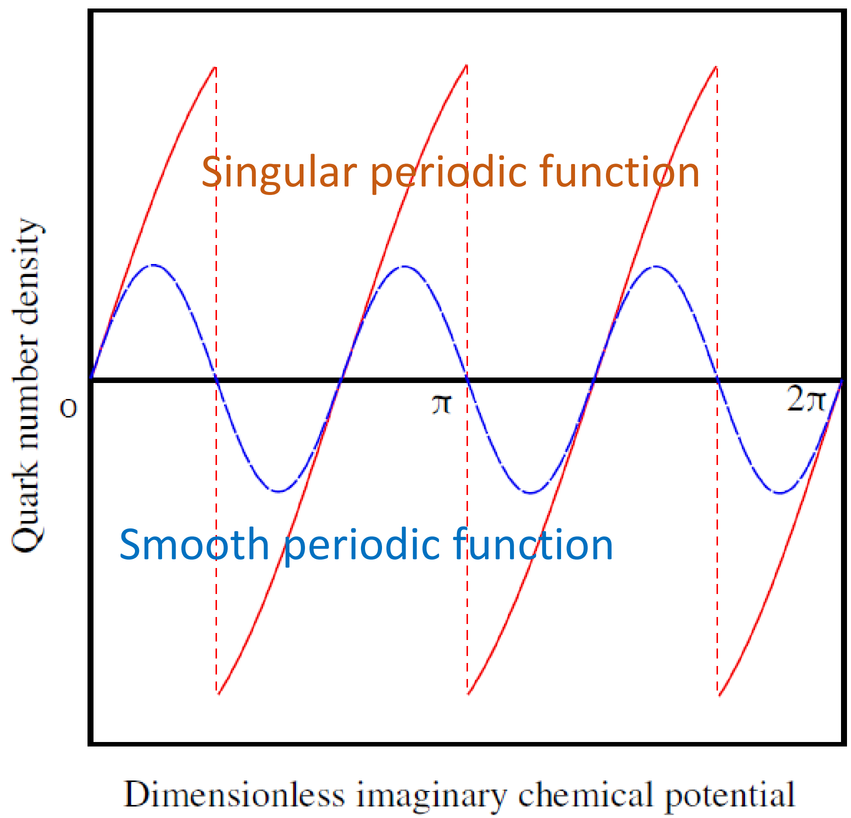

Quark number density :

High T Singular periodic function

→ Deconfined phase

Low T Smooth periodic function

→ Confined phase

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K.K., A. Ohnishi, Phys. Rev. D 93 (2016) 116002.

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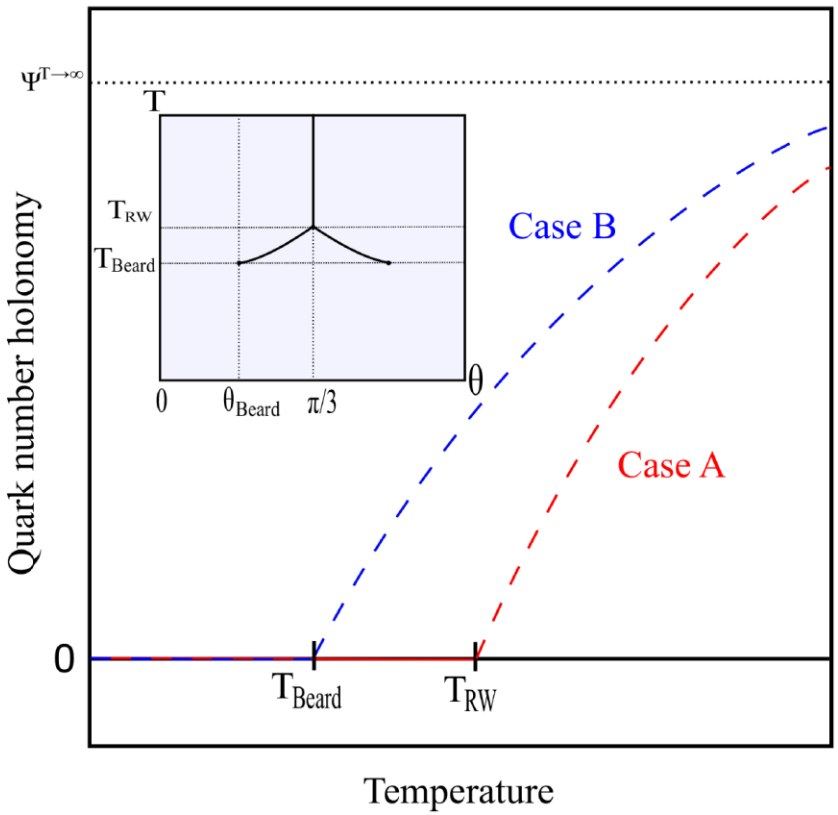

Two scenarios

In both cases,
there are no qualitative difference

Starting point where quark number holonomy has non-zero value is different

We investigate the deconfinement transition from topological viewpoints

To discuss the deconfinement transition at finite T, we here use the nontrivial free energy degeneracy

Confined phase	No nontrivial free-energy degeneracy
Deconfined phase	Non trivial free-energy degeneracy

New order-parameter

$$\Psi = \left[\oint_0^{2\pi} \left\{ \mathrm{Im}\left(\frac{d\tilde{n}_q}{d\theta}\Big|_T \right) \right\} d\theta \right]$$