Towards a Theory of The QCD String

or The Physics of a 750 MeV Boson

Sergei Dubovsky

CCPP (NYU) & Perimeter Institute

w Victor Gorbenko, 1511.01908

earlier work:

w Raphael Flauger, Victor Gorbenko, 1203.1054, 1205.6805, 1301.2325, 1404.0037

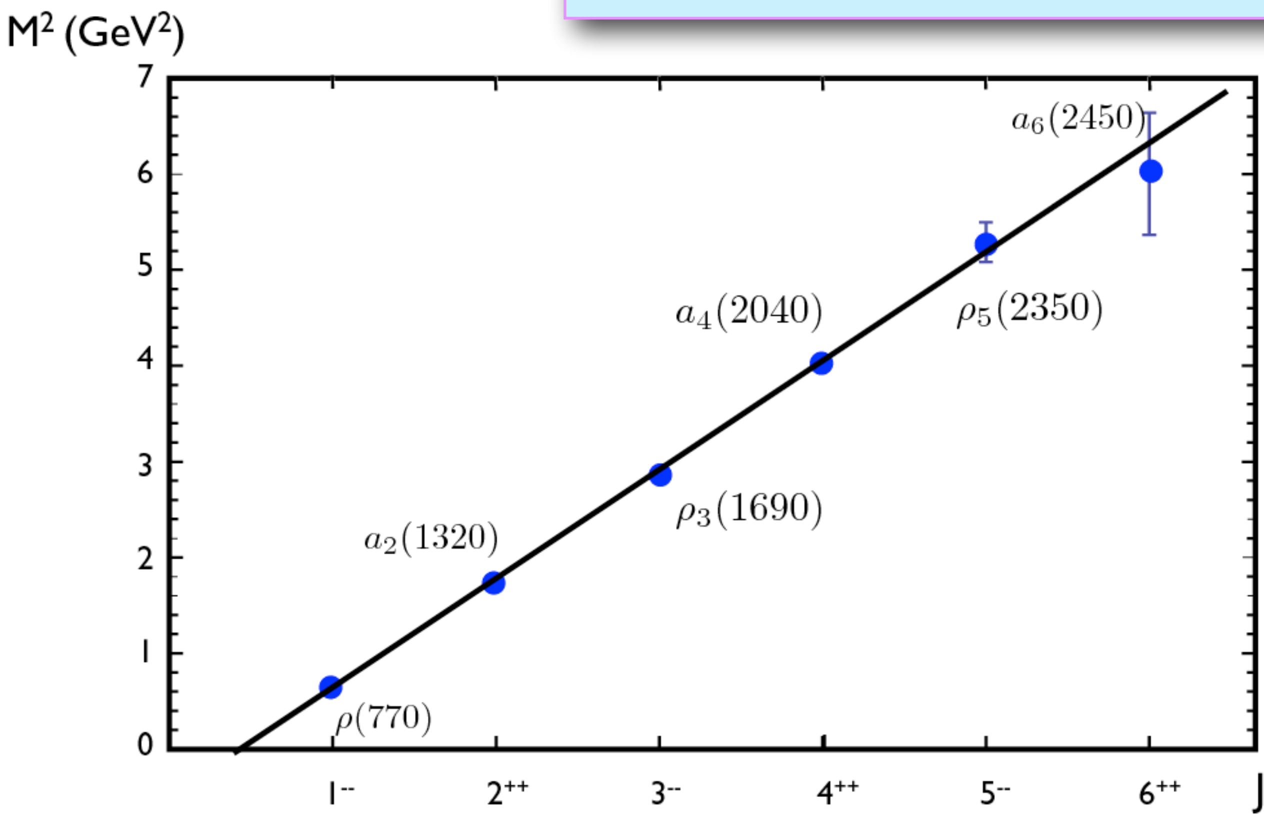
w Patrick Cooper, Victor Gorbenko, Ali Mohsen, Stefano Storace 1411.0703

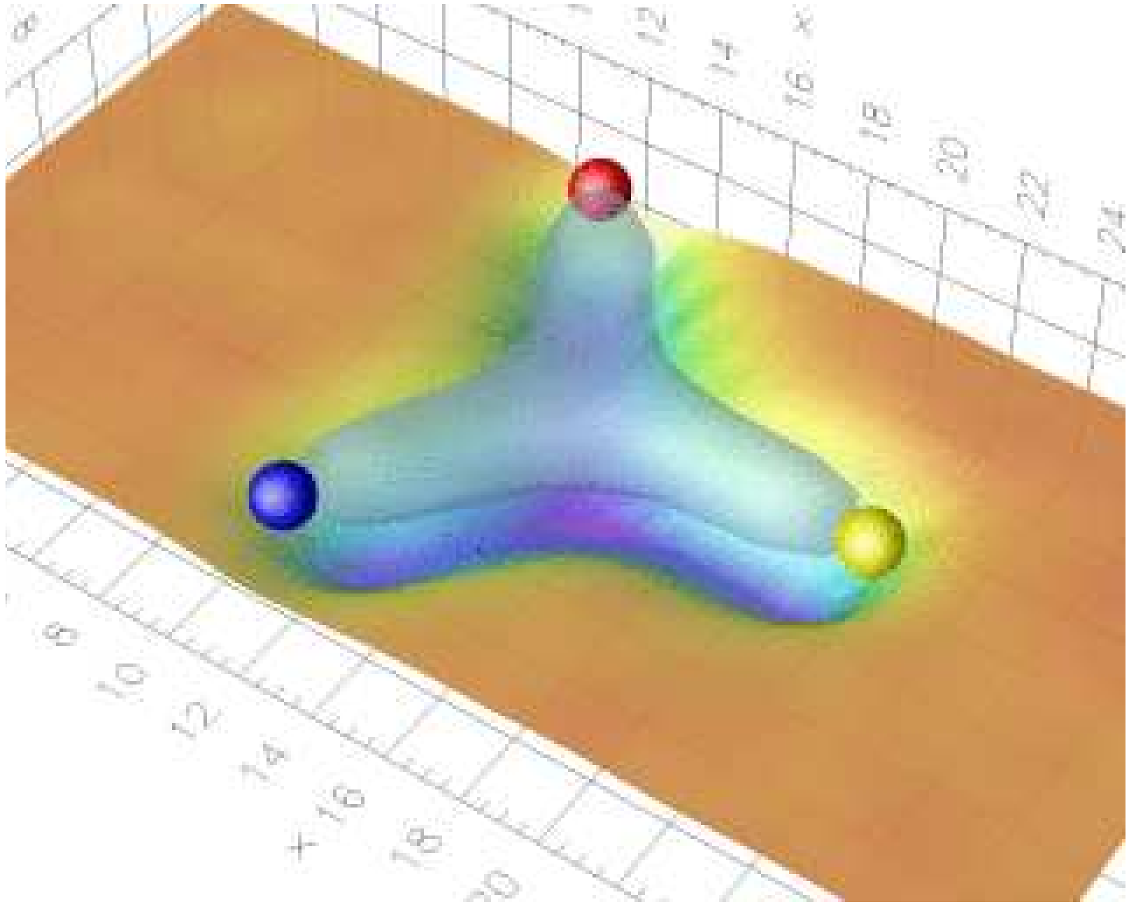
The major underlying question:

$\boxed{\text{What is } SU(\infty) \text{ Yang-Mills?}}$

Old, fascinating and famously hard. Will try to convince you that it may be the right time to attack it.

QCD is a theory of strings





Bissey et al, hep-lat/0606016

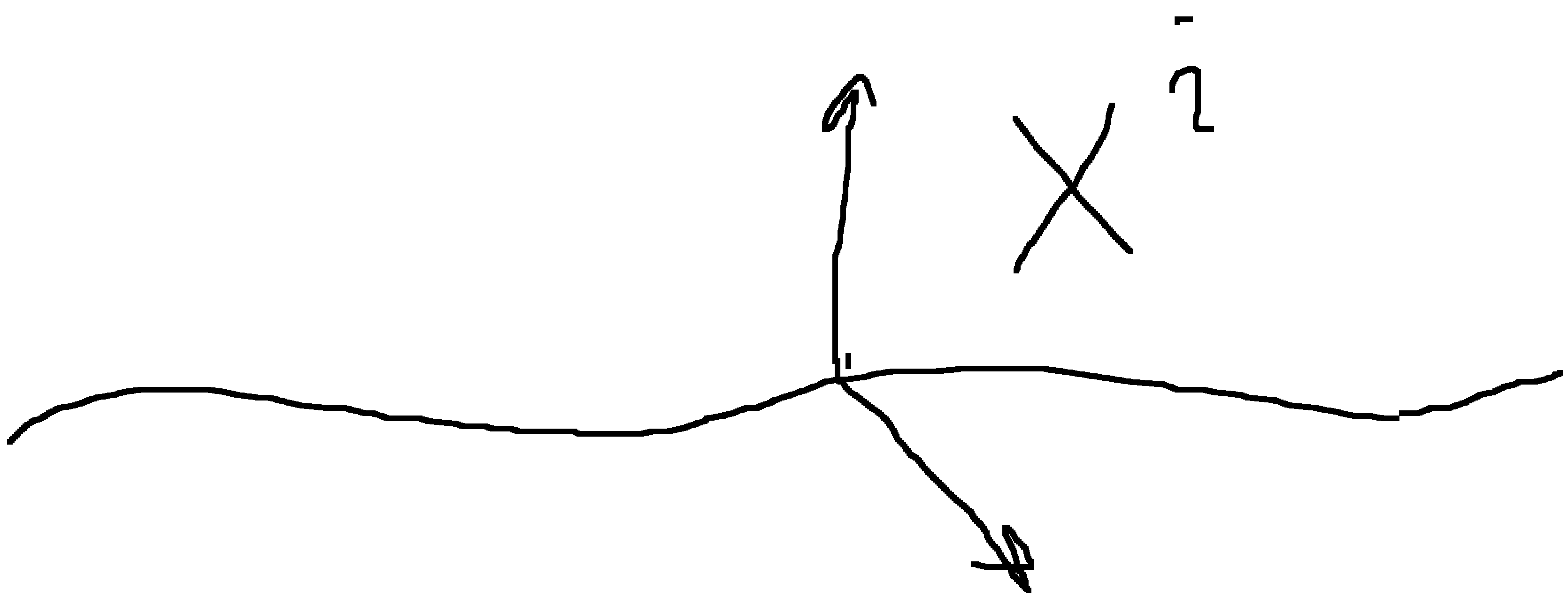
Large N QCD is a theory of free strings

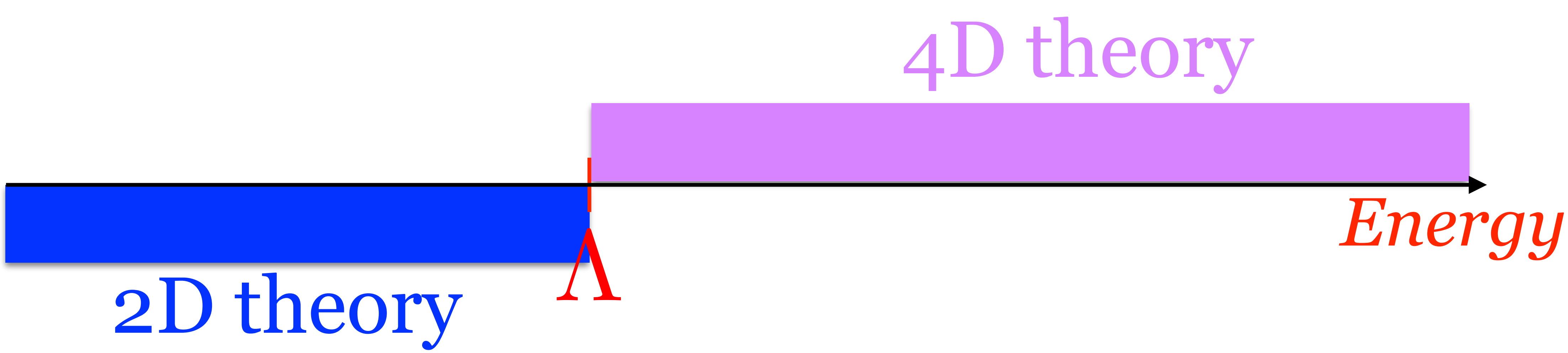
Can we solve this free string theory?

SETUP

✓Confining gauge theory with a gap Λ

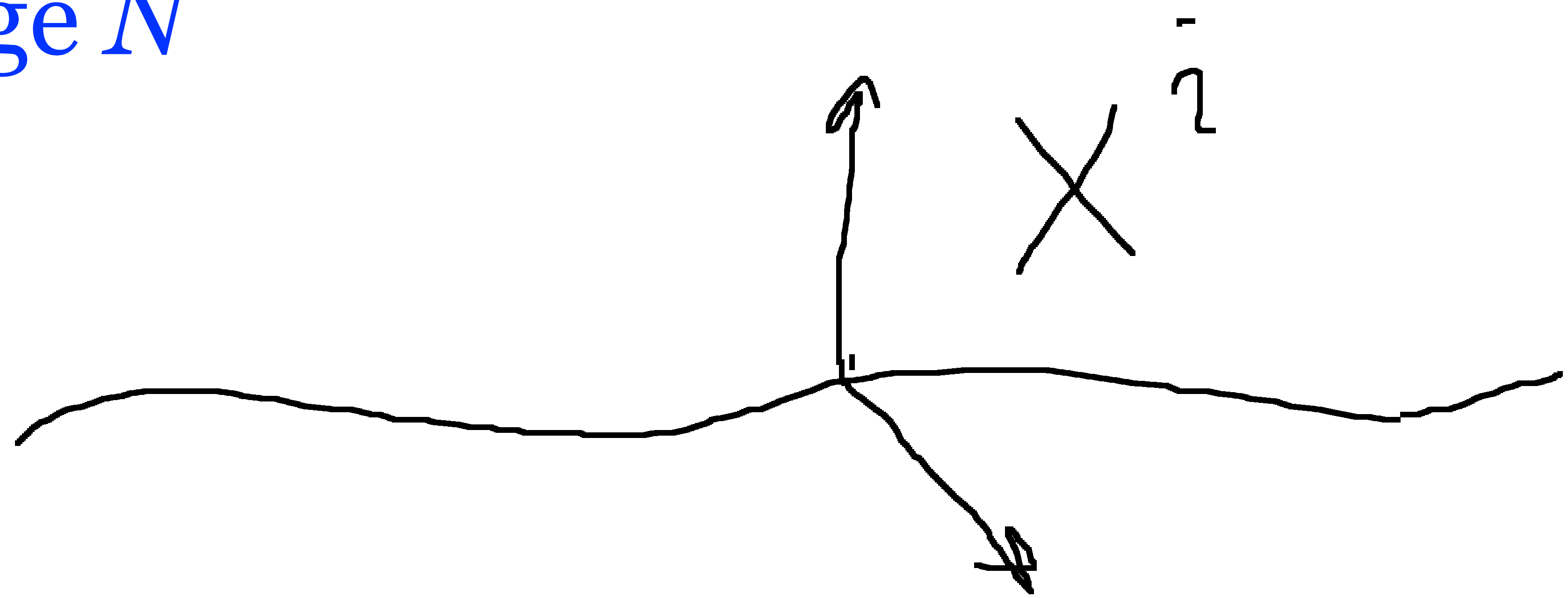
✓Unbroken center symmetry

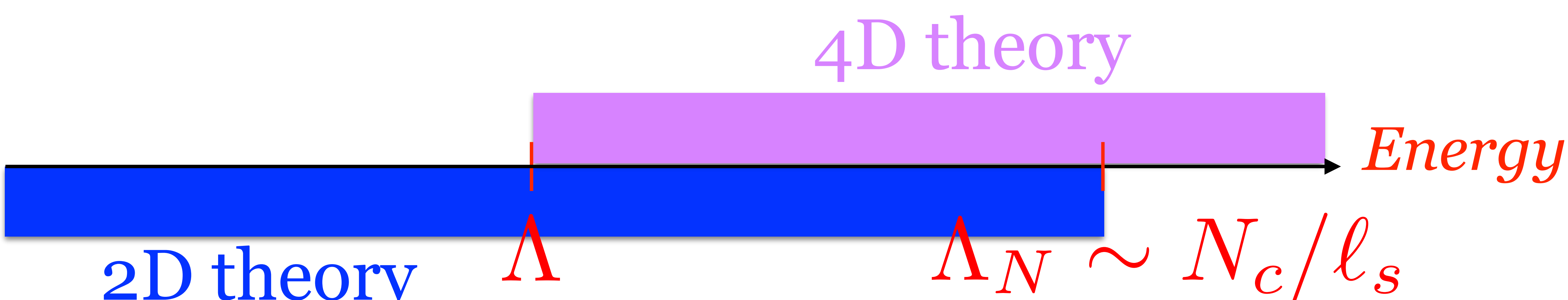




SETUP

- ✓Confining gauge theory with a gap
- ✓Unbroken center symmetry
- ✓Large N

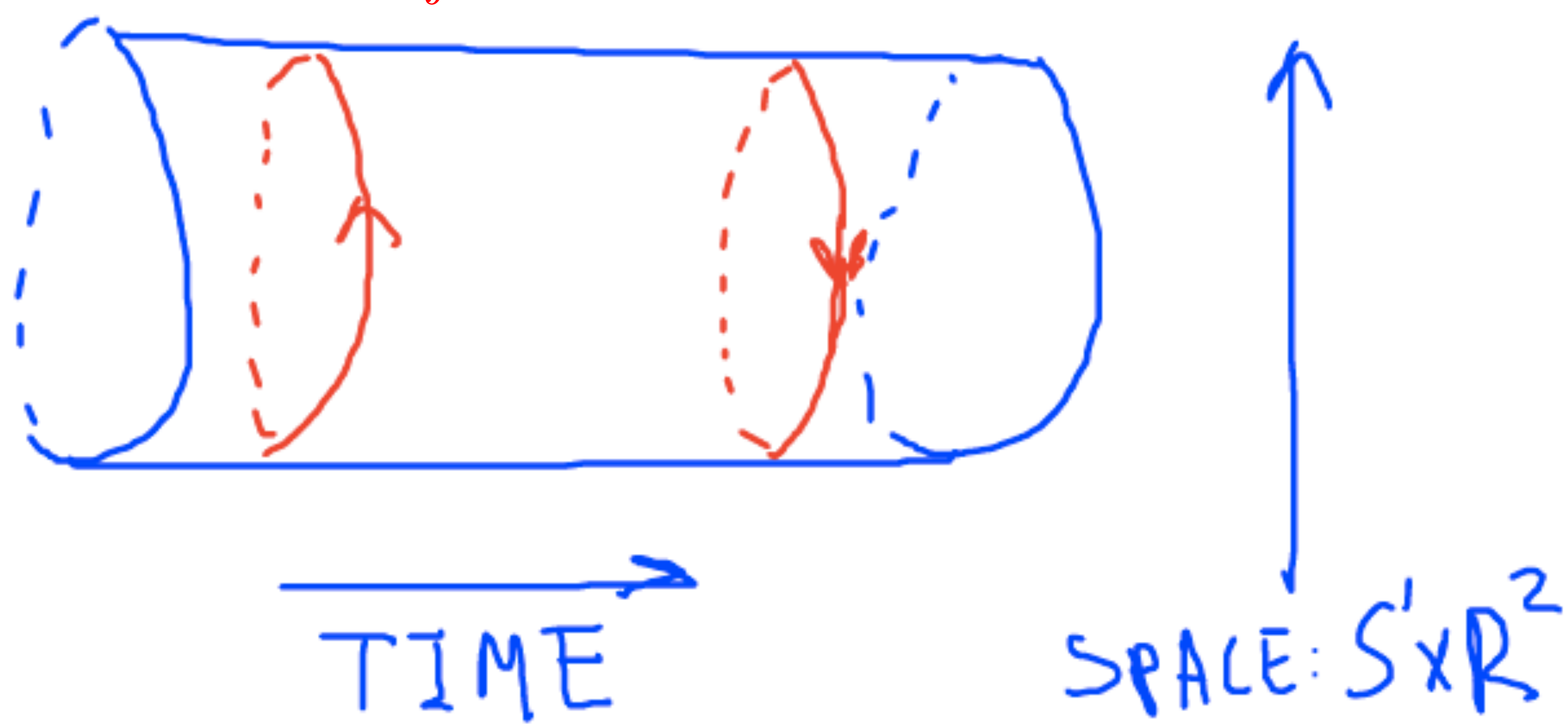




Looks hopeless to solve without experimental data

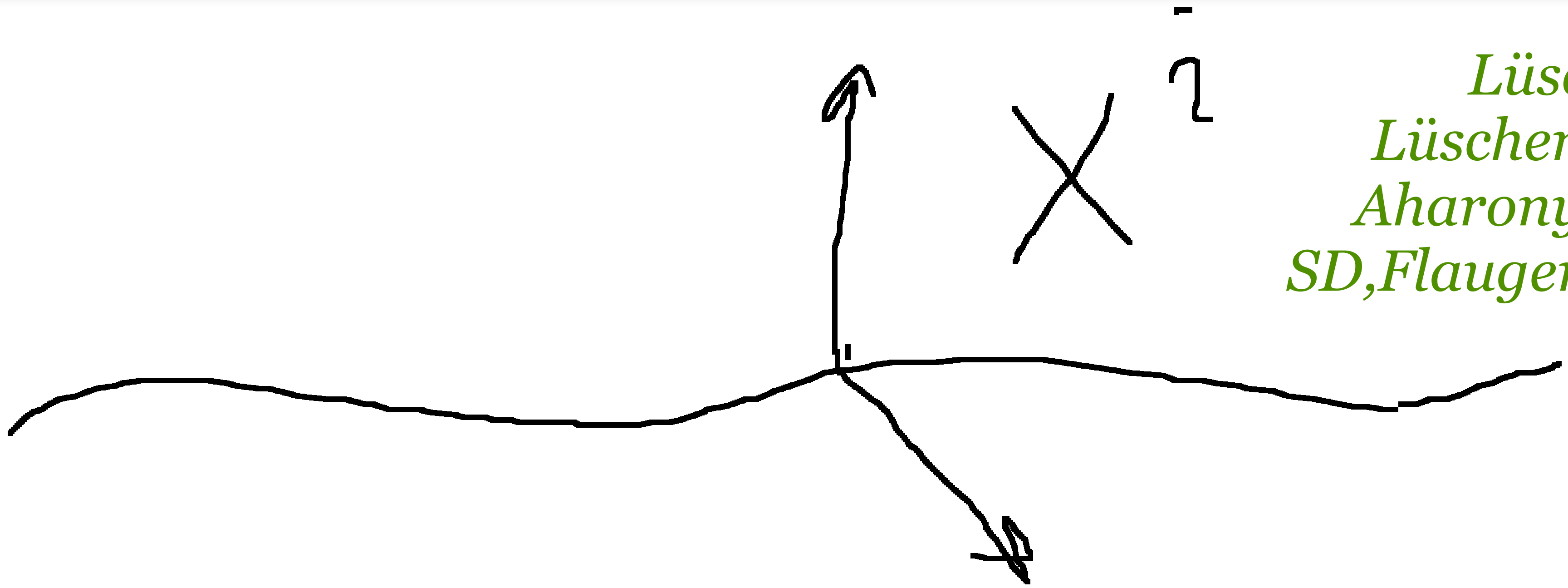
all the data from the papers by Athenodorou, Bringoltz and Teper

$\mathcal{O} = P\exp\{i\oint A\}$



$$\int \mathcal{D}A e^{-S_{YM}}\mathcal{O}(0)\mathcal{O}^\dagger(t) \to e^{-E_{\mathcal{O}}t} + \dots$$

(Long) String as seen by an Effective Field Theorist



Lüscher '81
Lüscher, Weisz '04
Aharony et al '07-11
SD,Flauger, Gorbenko'12

Theory of Goldstone Bosons

$$ISO(1, D-1) \to ISO(1,1) \times SO(D-2)$$

$$\delta_{\epsilon}^{\alpha i} X^j = -\epsilon(\delta^{ij}\sigma^{\alpha} + X^i \partial^{\alpha} X^j)$$

Embedding Coordinates

$$x^\mu = (\sigma^\alpha, \ell_s X^i(\sigma))$$

Induced Metric

$$h_{\alpha\beta} = \partial_\alpha x^\mu \partial_\beta x_\mu$$

$$S_{string} = -\ell_s^{-2} \int d^2\sigma \sqrt{h} +$$ Nambu-Goto

$$\int d^2\sigma \sqrt{h} \left(\frac{1}{\alpha_0} (K^i_{\alpha\beta})^2 + \chi R \right) +$$

rigidity (vanishes on-shell) Euler characteristic

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rigidity (vanishes on-shell) Euler characteristic

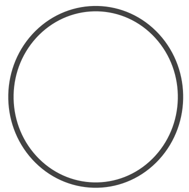
$$C_1 \ell_s^2 \int d^2\sigma \sqrt{h} (K^i_{\alpha\beta})^4 + \dots = C_1 \ell_s^6 \int d^2\sigma (\partial_\alpha \partial_\beta X^i)^4 + \dots$$

higher order non-universal terms start at ℓ_s^6

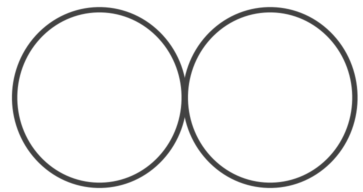
Explains the ground state data

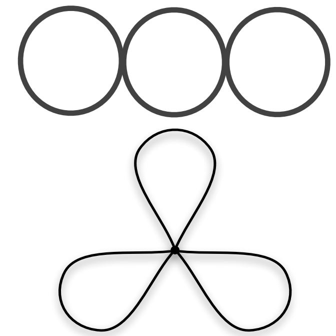
$$E_0(R) = \frac{R}{\ell_s^2} - \frac{(D-2)\pi}{6R} - \frac{(D-2)^2\pi^2\ell_s^2}{72R^3} - \frac{(D-2)^3\pi^3\ell_s^4}{432R^5} + \text{non-universal terms}$$

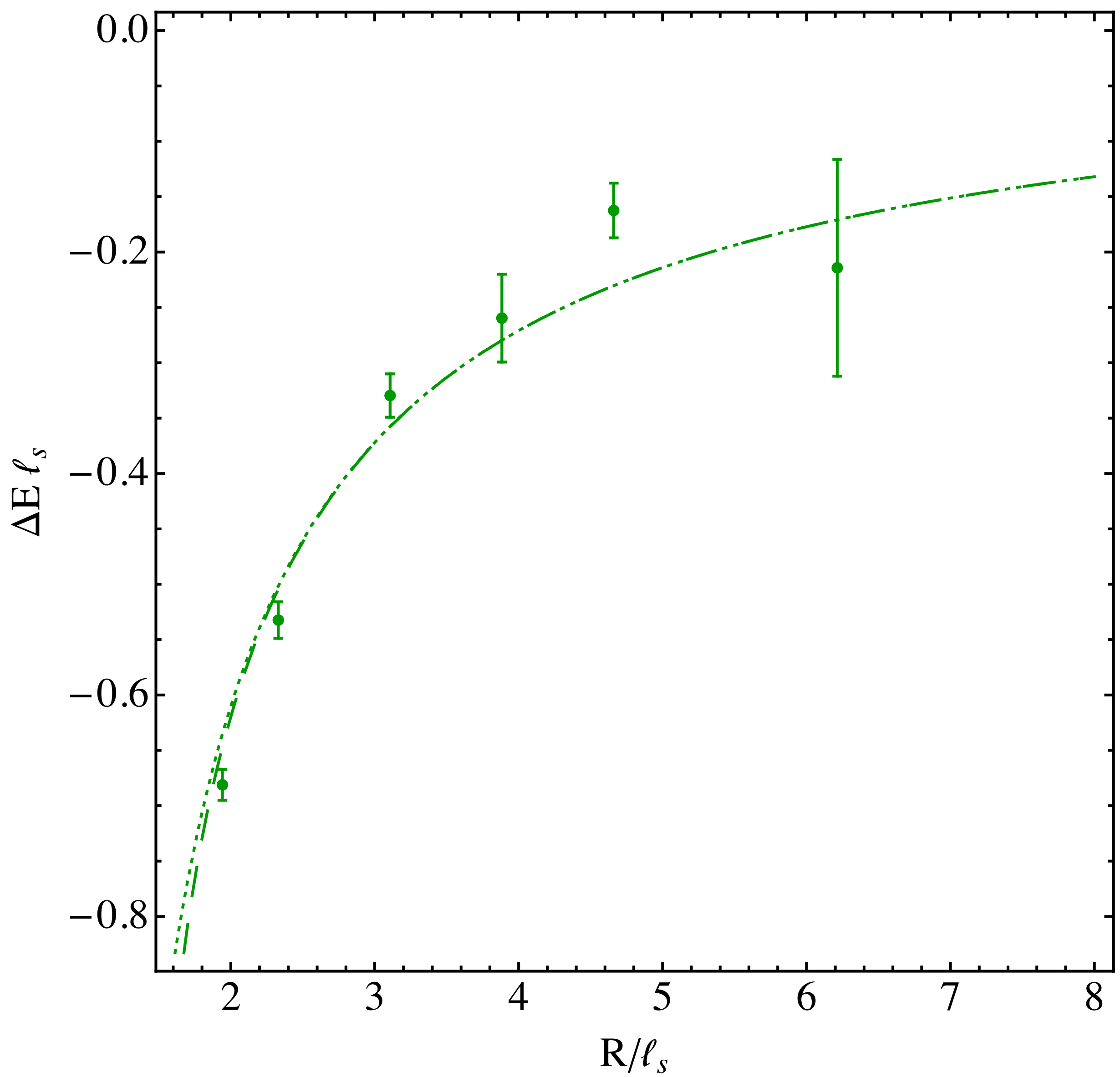
classical



Lüscher term







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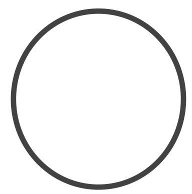

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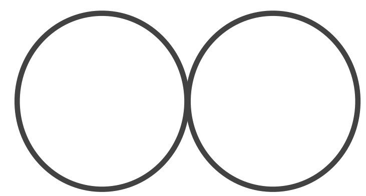
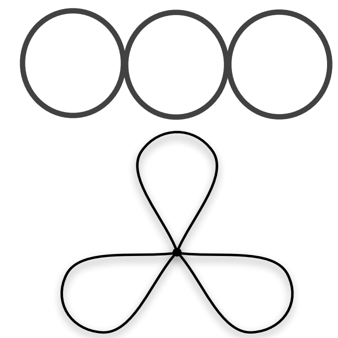
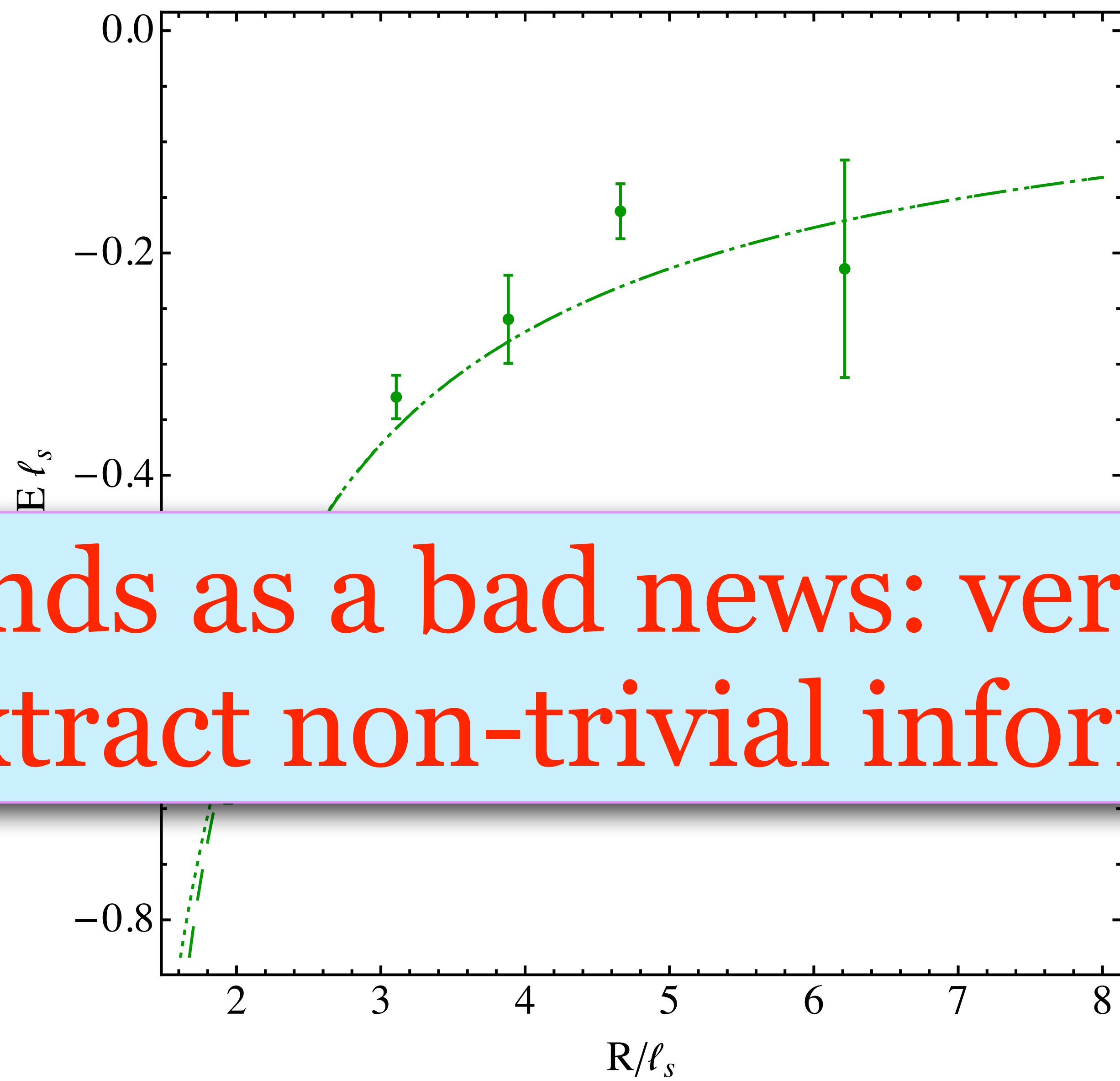


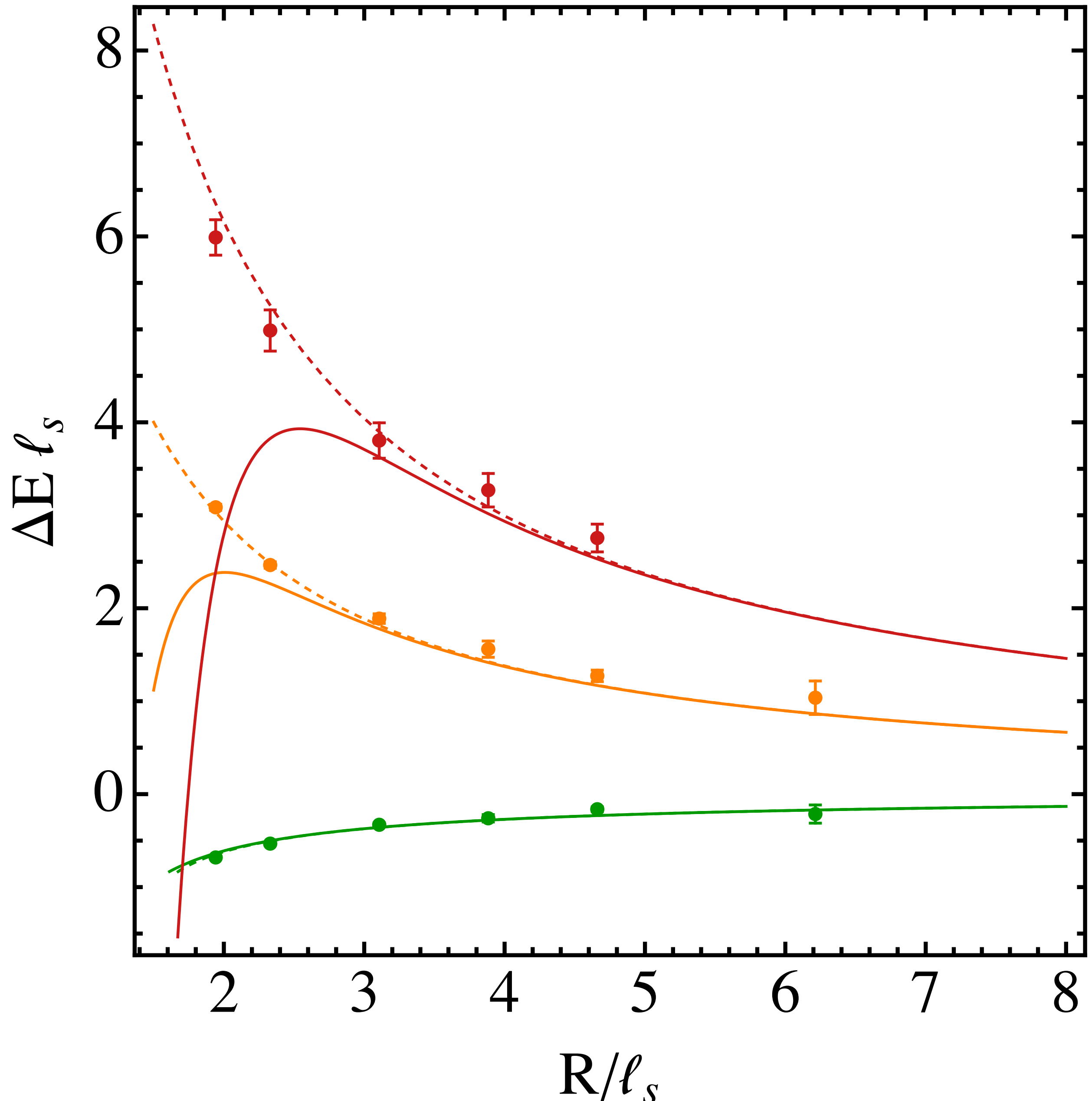




sounds as a bad news: very hard to extract non-trivial information

Excited states are more promising

Left-movers only:



Solid --- universal terms in ℓ_s/R expansion

Dashed --- light cone quantized bosonic string

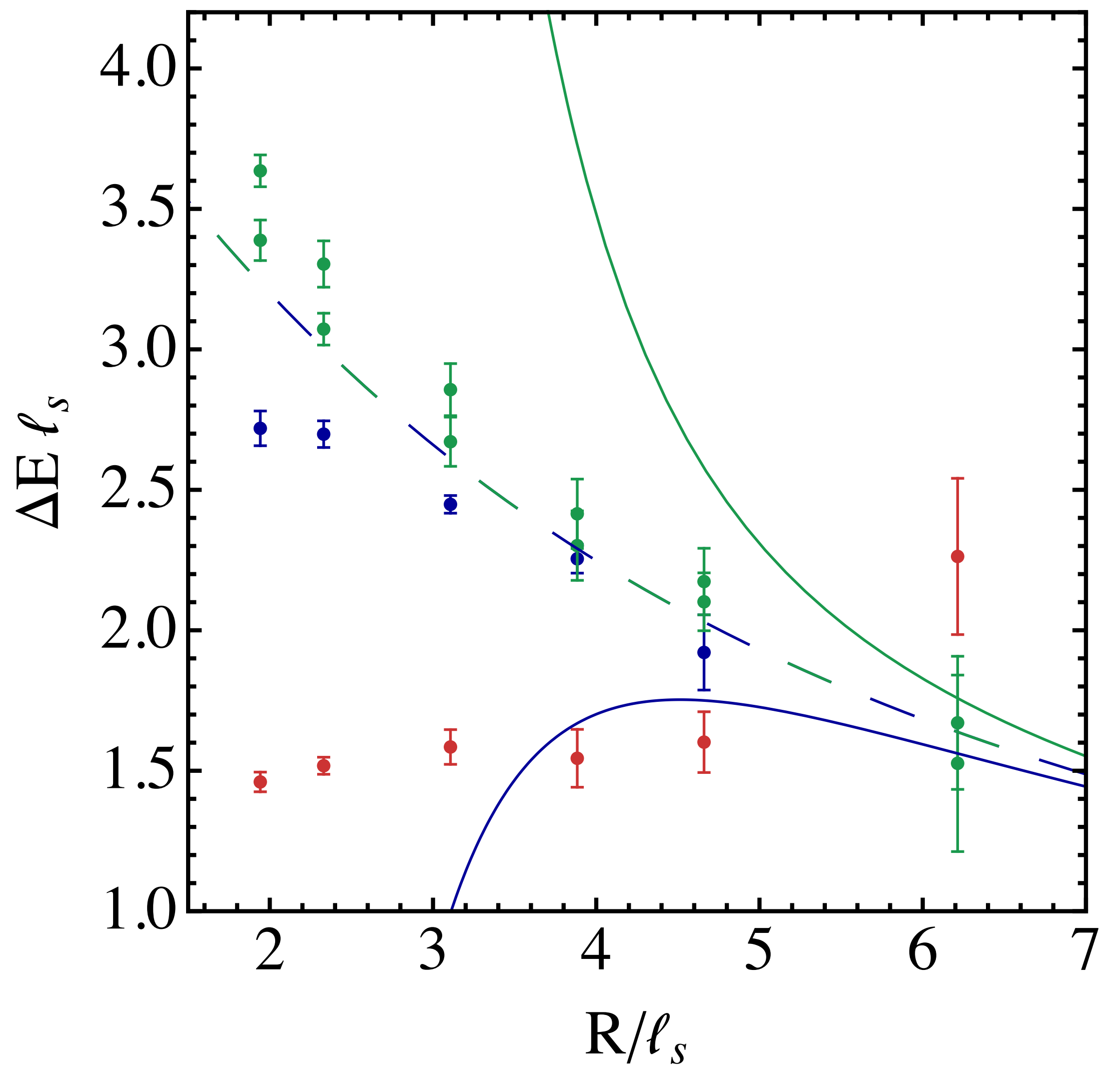
Excited states are more promising

Colliding left- and right-movers:

spin 2

scalar

pseudoscalar



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Dashed --- light cone quantized bosonic string

Goddard,Goldstone,Rebbi,Thorn'73

Light Cone (GGRT) spectrum:

$$E_{LC}(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N+\tilde{N} - \frac{D-2}{12}\right)}$$

ℓ_s/R expansion breaks down for excited states because 2π is a large number!

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for excited states:

$$E = \ell_s^{-1}\mathcal{E}(p_i\ell_s, \ell_s/R)$$

Let's try to disentangle these two expansions

Finite volume spectrum in two steps:

1) Find infinite volume S-matrix

2) Extract finite volume spectrum from the S-matrix

1) is a standard perturbative expansion in $p\ell_s$

2) perturbatively in massive theories (Lüscher)

exactly in integrable 2d theories through TBA

Relativistic string is neither massive nor integrable

But approaches integrable GGRT theory at low energies.

Thermodynamic Bethe Ansatz

Zamolodchikov'91
Dorey, Tateo '96

$$\hat{p}^{(i)}_{k\,L}R+\sum_{j,m}2\delta(\hat{p}^{(i)}_{k\,L},\hat{p}^{(j)}_{m\,R})N^{(j)}_{m\,R}-i\sum_{j=1}^{D-2}\int_0^\infty\frac{dp'}{2\pi}\frac{d\,2\delta(i\hat{p}^{(i)}_{k\,L},p')}{dp'}\ln\left(1-e^{-R\epsilon^j_R(p')}\right)=2\pi n^{(i)}_{k\,L}$$

$$\epsilon^i_L(p)=p+\frac{i}{R}\sum_{j,k}2\delta(p,-i\hat{p}^{(j)}_{k\,R})N^{(j)}_{k\,R}+\frac{1}{2\pi R}\sum_{j=1}^{D-2}\int_0^\infty dp'\,\frac{d\,2\delta(p,p')}{dp'}\ln\left(1-e^{-R\epsilon^j_R(p')}\right)$$

$$E(R)=R+\sum_{j,k}p^{(j)}_{k\,L}+\sum_{j=1}^{D-2}\int_0^\infty\frac{dp'}{2\pi}\ln\left(1-e^{-R\epsilon^j_L(p')}\right)$$

+right-movers

The leading order phase shift reproduces all of the GGRT spectrum

Thermodynamic Bethe Ansatz

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Asymptotic Bethe Ansatz
(~Lüscher's formula)

finite size corrections

$$\hat{p}^{(i)}_{k\,L} R + \sum_{j,m} 2\delta(\hat{p}^{(i)}_{k\,L}, \hat{p}^{(j)}_{m\,R}) N^{(j)}_{m\,R} - i \sum_{j=1}^{D-2} \int_0^\infty \frac{dp'}{2\pi} \frac{d\, 2\delta(i\hat{p}^{(i)}_{k\,L}, p')}{dp'} \ln\left(1 - e^{-R\epsilon^j_R(p')}\right) = 2\pi n^{(i)}_{k\,L}$$

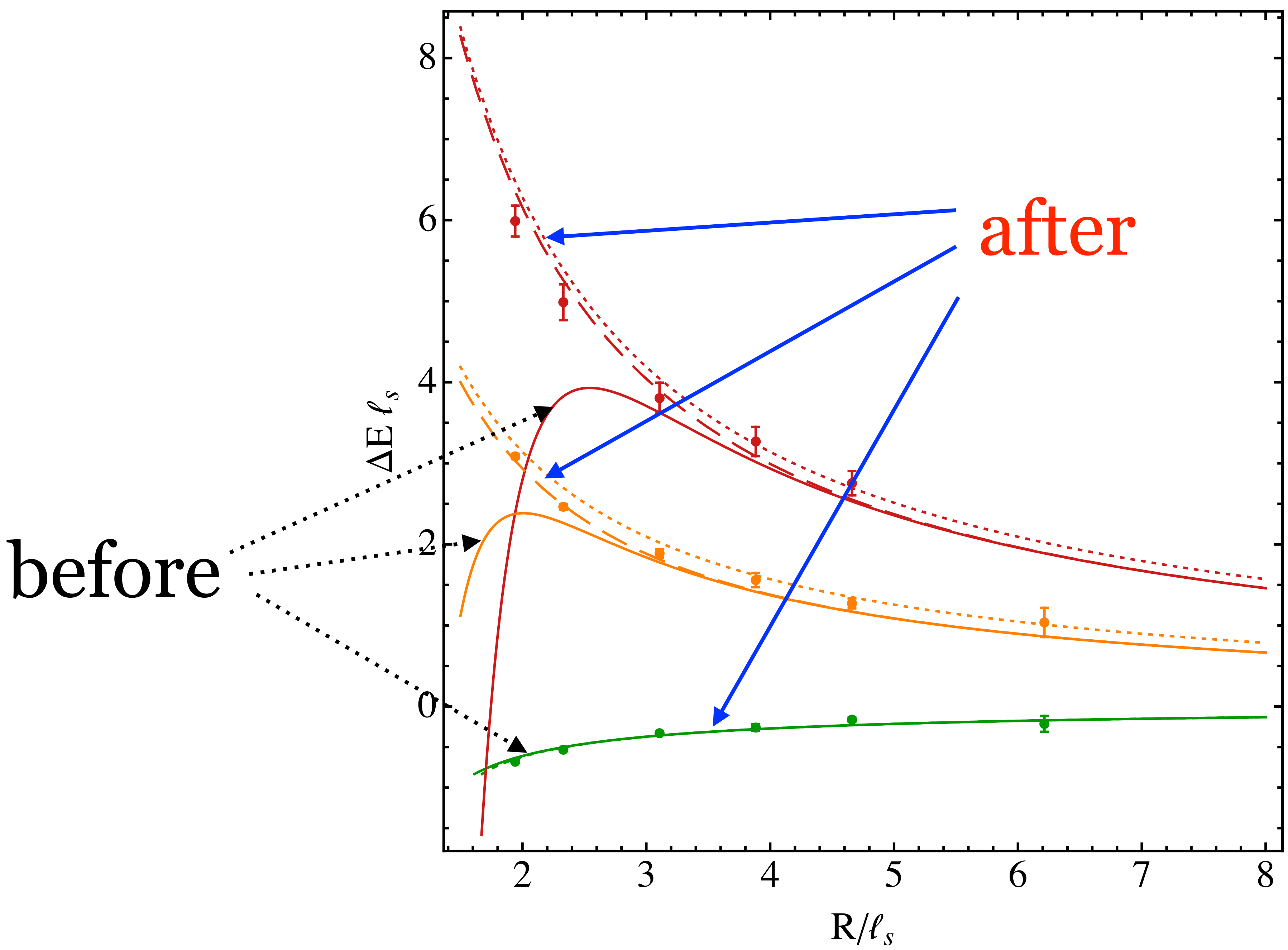
$$\epsilon^i_L(p) = p + \frac{i}{R} \sum_{j,k} 2\delta(p, -i\hat{p}^{(j)}_{k\,R}) N^{(j)}_{k\,R} + \frac{1}{2\pi R} \sum_{j=1}^{D-2} \int_0^\infty dp' \frac{d\, 2\delta(p,p')}{dp'} \ln\left(1 - e^{-R\epsilon^j_R(p')}\right)$$

$$E(R) = R + \sum_{j,k} p^{(j)}_{k\,L} + \sum_{j=1}^{D-2} \int_0^\infty \frac{dp'}{2\pi} \ln\left(1 - e^{-R\epsilon^j_L(p')}\right)$$

+right-movers

The leading order phase shift reproduces all of the GGRT spectrum

Improve your appearance with TBA:

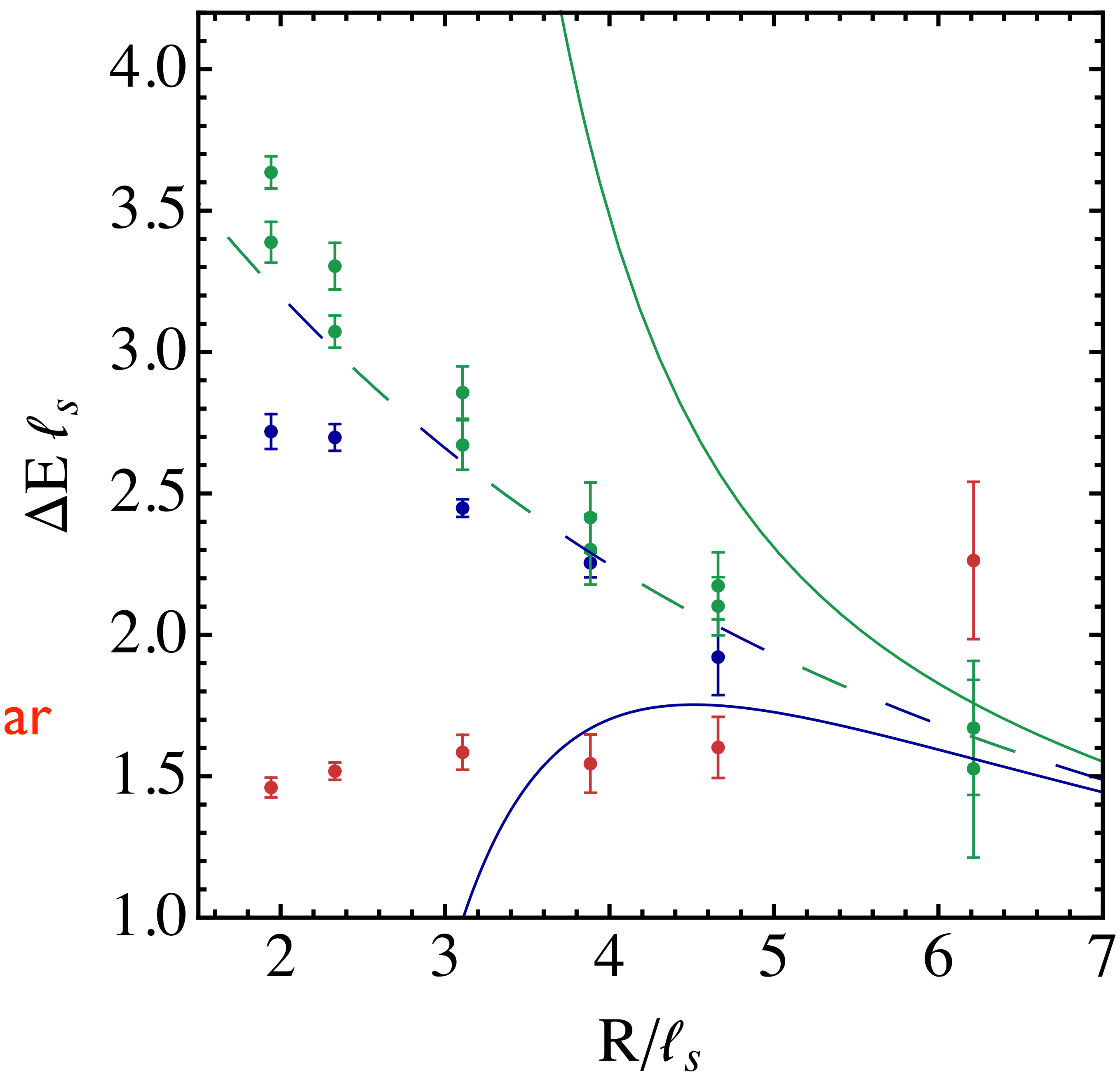


Colliding left- and right-movers

spin 2

scalar

pseudoscalar



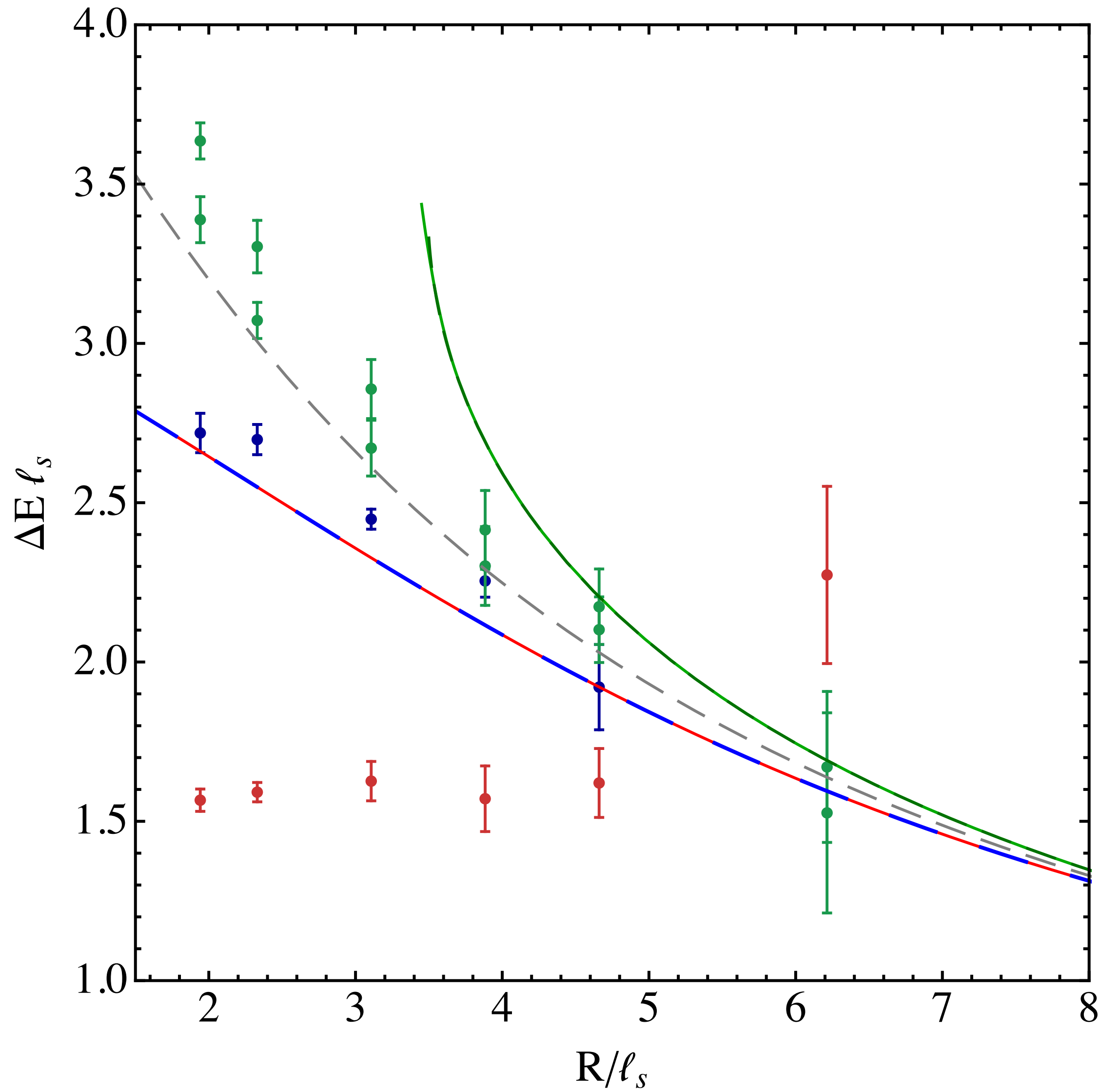
before

Colliding left- and right-movers

spin 2

scalar

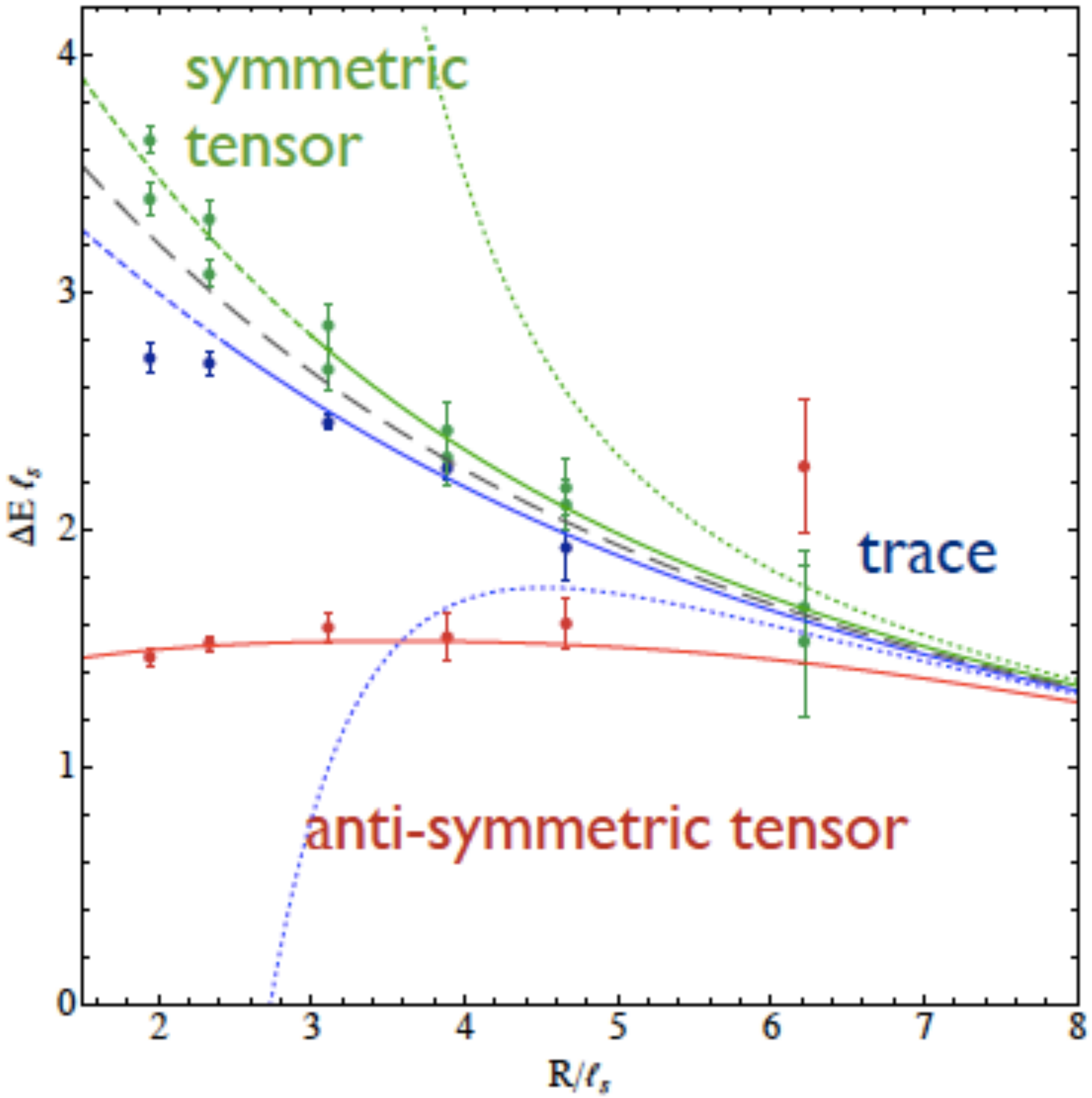
pseudoscalar



after

Red points:
A new massive state appearing as a resonance in the antisymmetric channel!

$$S = \int d^2\sigma \frac{1}{2}\partial_\alpha \phi \partial^\alpha \phi - \frac{1}{2}m^2\phi^2 + Q\phi\epsilon^{\alpha\beta}\epsilon_{ij}K^i_{\alpha\gamma}K^{j\gamma}_\beta$$



~750 MeV

$$m\ell_s \approx 1.85^{+0.02}_{-0.03} \qquad Q \approx 0.382 \pm 0.004$$

SD, Flauger, Gorbenko, 1301.2325

Alternative and Equivalent View on TBA:

Use finite volume spectrum to reconstruct S-matrix

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GGRT spectrum

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corresponds to an integrable theory with

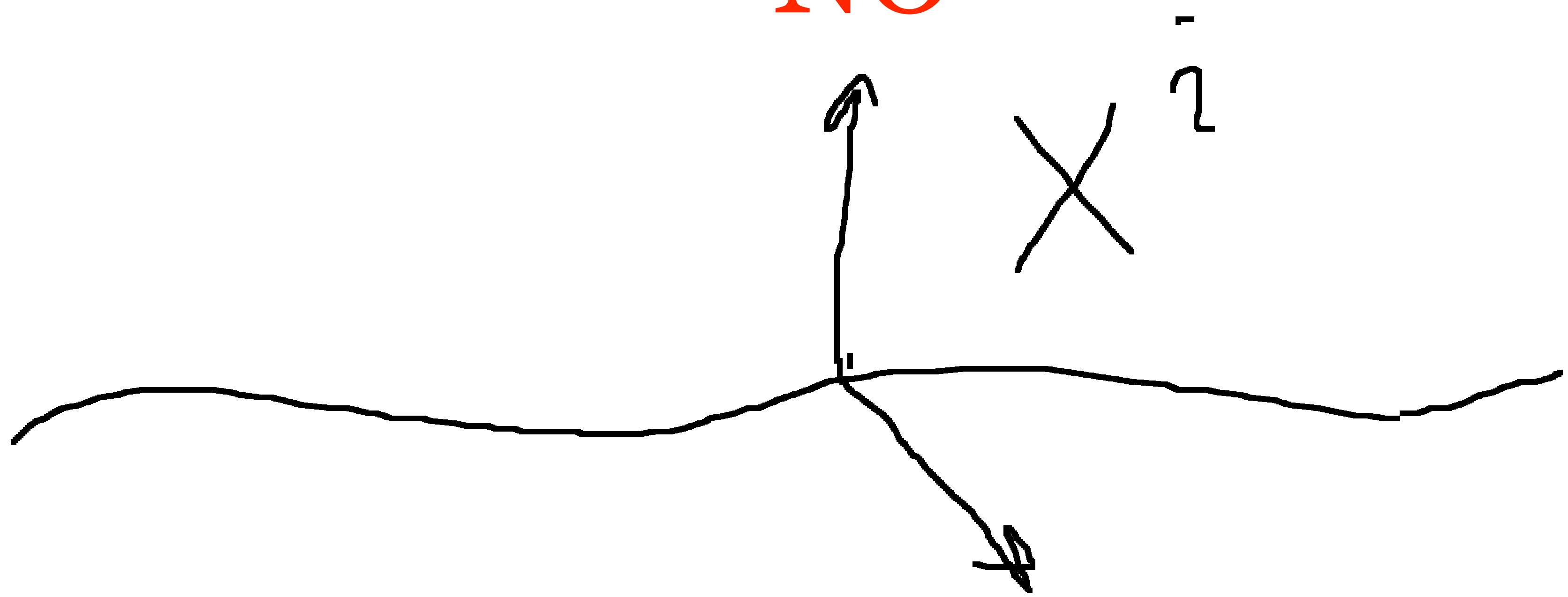
$$\boxed{e^{2i\delta(s)} = e^{is\ell^2/4}}$$

✶Time delay proportional to the collision energy

✶*Scale survives all the way to the UV !*

Could QCD string be integrable for pure glue?

NO



$$S_{string} = -\ell_s^{-2} \int d^2\sigma \sqrt{-det(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i)} + \mathcal{O}(\ell_s^2)$$

✳Integrable at tree level

✳*Universal* one-loop particle production if $D \neq 26, 3$

All these one-loop amplitudes can be explicitly calculated

A simple option to restore integrability:

$$\boxed{S_{string} = -\ell_s^{-2} \int d^2\sigma \sqrt{-det(\eta_{\alpha\beta} + \partial_\alpha X^i \partial_\beta X^i + \partial_\alpha \phi \partial_\beta \phi)} + Q \int d^2\sigma \phi R[X] + \ldots}$$

$$Q = \sqrt{\frac{25 - D}{48\pi}}$$

$$\boxed{e^{2i\delta(s)} = e^{is\ell^2/4}}$$

This is also known as a linear dilaton background

Another simple option to restore integrability:

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SD, Gorbenko, 1511.01908

$$Q = \sqrt{\frac{25 - D}{48\pi}} = \sqrt{\frac{7}{16\pi}} \approx 0.373176\ldots$$

$$\boxed{e^{2i\delta(s)} = e^{is\ell^2/4}}$$

Compare to

$$Q_{lattice} \approx 0.382 \pm 0.004$$

???

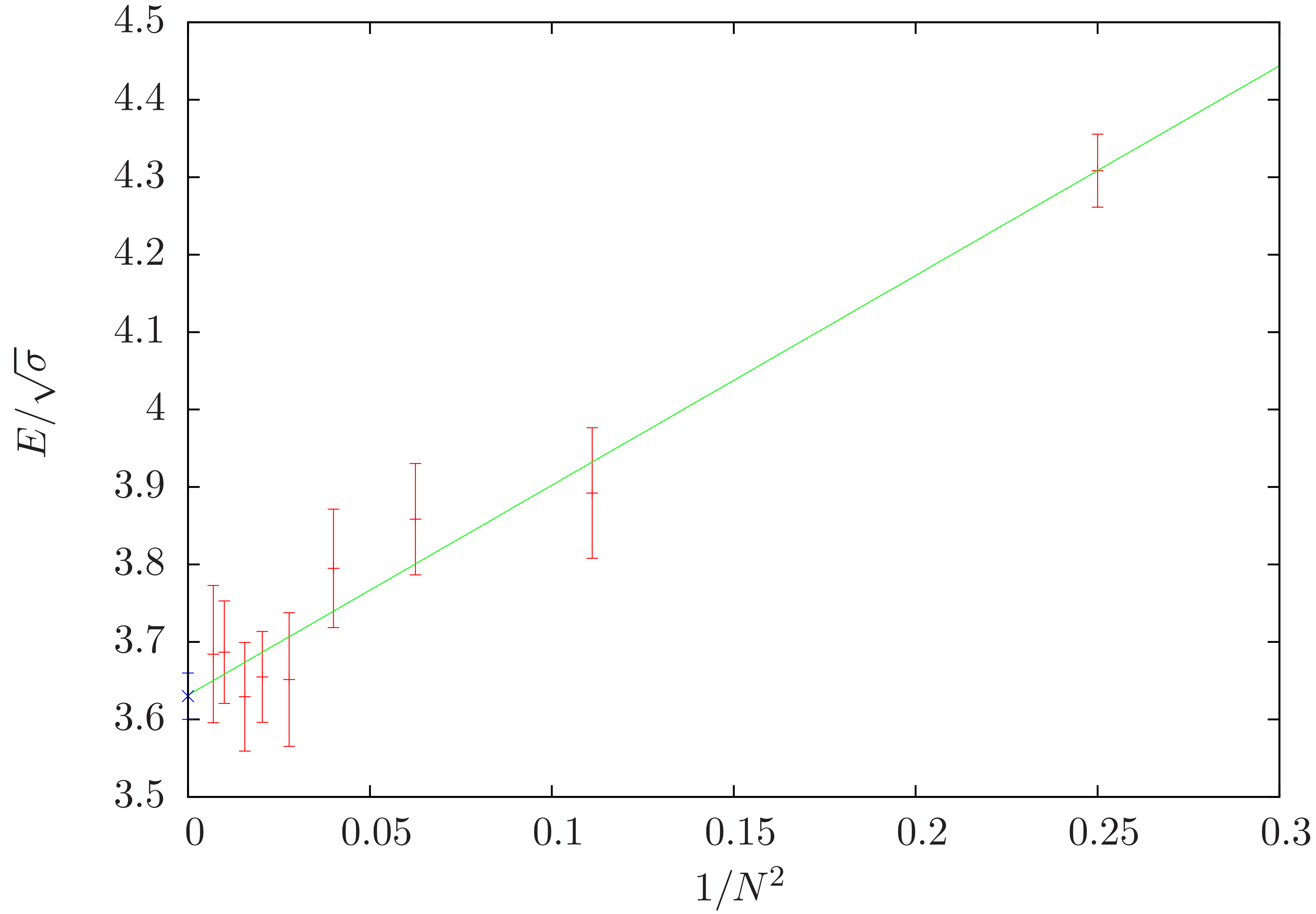
What this could mean?

✳Numerology

✳In the planar limit axion becomes massless and the planar QCD string is integrable

✳ This is the UV asymptotics of the planar QCD string

Athenodorou, Teper, to appear



the second option is excluded

To conclude:

$\boxed{\text{(Strawman) proposal for the structure of the QCD string in D=3,4:}}$

✶Matter content:

Goldstones+massive antisymmetric O(D-2) tensor

✶Integrable UV asymptotics with

$$\boxed{e^{2i\delta(s)} = e^{is\ell^2/4}}$$

✶Future checks: confront with lattice data for winding strings and glueball spectra