Energy-momentum tensor on the lattice: recent developments

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- We have enough motivation to investigate EMT on the lattice.

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- Noether current for the translational invariance $\Leftarrow$ broken on the lattice!
- Generates the Poincaré symmetry and the dilatation
- Its components: energy, momentum, stress densities, and pressure
- Thermodynamics
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Talks on the shear viscosity

- Braguta, "Temperature dependence of shear viscosity in $SU(3)$-gluodynamics," 7/29, 17:50– this afternoon, Building 32 Room 1015.
- Pasztor, "Viscosity of the pure $SU(3)$ gauge theory revisited," 7/29, 18:10– this afternoon, Building 32 Room 1015.

Related investigations in the early days

- K. Fujikawa, “Chiral and conformal anomalies in lattice gauge theory,” Z. Phys. C **25**, 179 (1984).
- A. S. Kronfeld and D. M. Photiadis, “Phenomenology on the lattice: composite operators in lattice gauge theory,” Phys. Rev. D **31**, 2939 (1985).
- G. Martinelli and C. T. Sachrajda, “A lattice calculation of the pion’s form-factor and structure function,” Nucl. Phys. B **306**, 865 (1988).

WT relation on the lattice (Caracciolo et. al. (1990))

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- Would-be translation ($\delta_\xi A_\mu(x) = \xi_\nu(x) F_{\nu\mu}(x)$)

$$\delta_\xi U(x,\mu) = \xi_\nu(x) \hat{F}_{\nu\mu}(x) U(x,\mu)$$

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$$\delta_\xi U(x,\mu) = \xi_\nu(x)\hat{F}_{\nu\mu}(x)U(x,\mu)$$

- Since the lattice action is not translational invariant,

$$\left\langle \left[\hat{\partial}_\mu \hat{T}^{\text{naive}}_{\mu\nu}(x) + a\hat{X}_\nu(x)\right]\hat{\mathcal{O}}(y)\right\rangle = -\left\langle \frac{\delta}{\delta\xi_\nu(x)}\delta_\xi\hat{\mathcal{O}}(y)\right\rangle,$$

where $\hat{T}^{\text{naive}}_{\mu\nu} = \hat{T}^{[1]}_{\mu\nu} + \hat{T}^{[3]}_{\mu\nu}$ and

$$\hat{T}^{[1]}_{\mu\nu} = \frac{1}{g_0^2}(1-\delta_{\mu\nu})\sum_\rho \hat{F}^a_{\mu\rho}\hat{F}^a_{\nu\rho}, \quad \text{(only off-diagonal comps.)}$$

$$\hat{T}^{[3]}_{\mu\nu} = \frac{1}{g_0^2}\delta_{\mu\nu}\left(\sum_\rho \hat{F}^a_{\mu\rho}\hat{F}^a_{\nu\rho} - \frac{1}{4}\sum_{\rho\sigma}\hat{F}^a_{\rho\sigma}\hat{F}^a_{\rho\sigma}\right), \quad \text{(only diagonal comps.)}$$

Operator mixing

- Due to the radiative corrections (under hypercubic symmetry),

$$a\hat{X}_\nu(x) = \left(\frac{Z_T}{Z_\delta} - 1\right)\hat{\partial}_\mu \hat{T}^{[1]}_{\mu\nu}(x) + \left(\frac{Z_T z_t}{Z_\delta} - 1\right)\hat{\partial}_\mu \hat{T}^{[3]}_{\mu\nu}(x)$$
$$+ \frac{Z_T z_s}{Z_\delta}\hat{\partial}_\mu \hat{T}^{[2]}_{\mu\nu}(x) + \frac{1}{Z_\delta} a\hat{R}_\nu(x),$$

  where

$$\hat{T}^{[2]}_{\mu\nu} = \frac{1}{4g_0^2}\delta_{\mu\nu}\sum_{\rho\sigma}\hat{F}^a_{\rho\sigma}\hat{F}^a_{\rho\sigma}, \quad \text{(only diagonal comps.)}$$

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- Inserting this into

$$\left\langle \left[\hat{\partial}_\mu \hat{T}^{\text{naive}}_{\mu\nu}(x) + a\hat{X}_\nu(x)\right]\hat{\mathcal{O}}(y)\right\rangle = -\left\langle \frac{\delta}{\delta\xi_\nu(x)}\delta_\xi \hat{\mathcal{O}}(y)\right\rangle,$$

Properly normalized EMT

- The identity becomes

$$Z_T \left\langle \hat{\partial}_\mu \left[\hat{T}^{[1]}_{\mu\nu}(x) + z_T \hat{T}^{[3]}_{\mu\nu}(x) + z_s \hat{T}^{[2]}_{\mu\nu}(x) \right] \hat{\mathcal{O}}(y) \right\rangle$$

$$= -\left\langle Z_\delta \frac{\delta}{\delta \xi_\nu(x)} \delta_\xi \hat{\mathcal{O}}(y) + \underbrace{a\hat{R}_\nu(x)\hat{\mathcal{O}}(y)}_{\text{contact term}} \right\rangle$$

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$$\stackrel{a\to 0}{\to} -\delta(x-y) \left\langle \partial_\nu \mathcal{O}(y) \right\rangle + \cdots .$$

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- The properly normalized EMT is given by

$$\hat{T}_{\mu\nu}(x) = Z_T \left[\hat{T}^{[1]}_{\mu\nu}(x) + z_T \hat{T}^{[3]}_{\mu\nu}(x) + z_s \hat{T}^{[2]}_{\mu\nu}(x) \right] - \text{VEV}.$$

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- Determine Z_T, z_t, and z_s.

Various strategies

- Set $y \neq x$ and conservation law determines z_t and z_s. The overall factor Z_T may be fixed by the rest energy $-\hat{T}_{00}$ of a hadronic state (Caracciolo-Curci-Menotti-Pelissetto, Caracciolo-Menotti-Pelissetto)

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- Use gradient flow for the probe operator $\hat{\mathcal{O}}(y)$ (Del Debbio-Patella-Rago, Capponi-Del Debbio-Patella-Rago, Capponi-Rago-Del Debbio-Ehret-Pellegrini)

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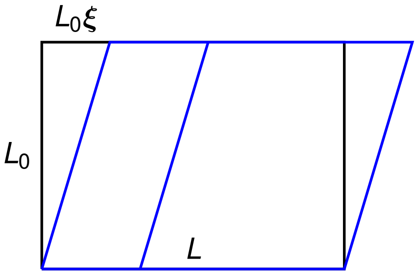
- Use gradient flow for EMT itself; perturbative matching to EMT with dimensional regularization (H.S., Makino-H.S., FlowQCD Collaboration, WHOT QCD Collaboration)

Giusti-Meyer (2011–2013), Robaina-Meyer (2013), Giusti-Pepe (2014–)

- Free energy with a shifted boundary condition

$$f(L_0, \xi) = -\frac{1}{V} \ln \mathrm{Tr}\left[e^{-L_0(H - i\boldsymbol{\xi}\cdot\boldsymbol{P})}\right], \qquad \phi(L_0, \boldsymbol{x}) = \phi(0, \boldsymbol{x} - L_0\boldsymbol{\xi}).$$



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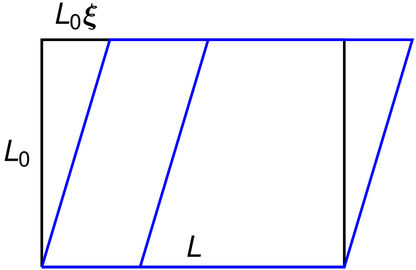

- Derivative wrt ξ_i gives rise to the i-th momentum:

$$\frac{\partial}{\partial\xi_i}f(L_0,\boldsymbol{\xi}) = -\frac{1}{V}iL_0\left\langle P_i\right\rangle_{\boldsymbol{\xi}} = -L_0\left\langle T_{0i}\right\rangle_{\boldsymbol{\xi}}.$$

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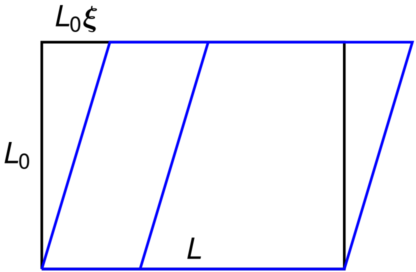

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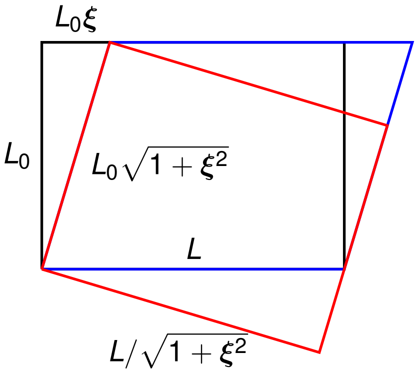
Shifted boundary condition

- The system is however equivalent to an un-shifted one,



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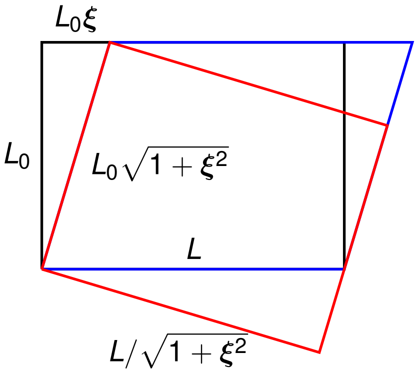

- From the underlying $SO(4)$ covariance, for $L \to \infty$,

$$\langle T_{0i} \rangle_{\xi} = \frac{\xi_i}{1-\xi_i^2} \langle T_{00} - T_{ii} \rangle_{\xi} \,.$$

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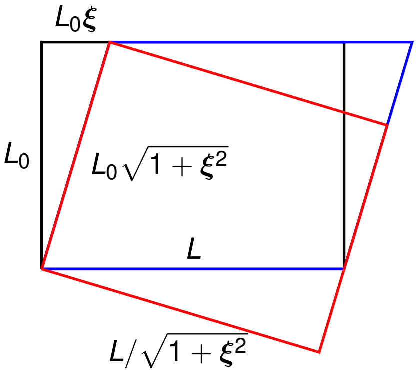

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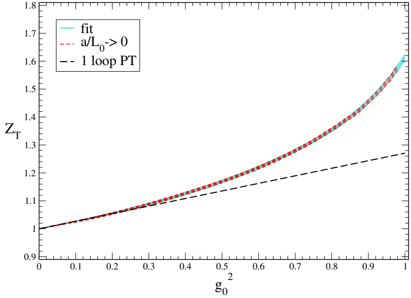
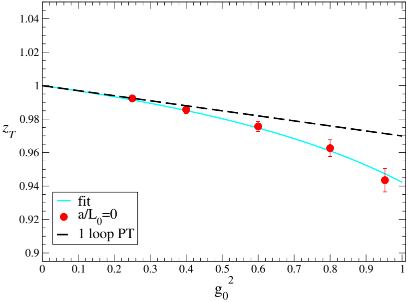
Giusti-Pepe (2015)

- Very accurate determination of $Z_T(g_0^2)$and $z_T(g_0^2)$ (plaquette action, clover $\hat{F}_{\mu\nu}$)





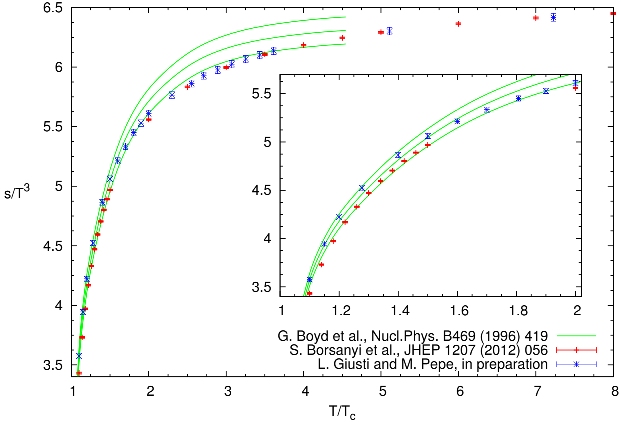
- Fit formulas

$$Z_T(g_0^2) = \frac{1 - 0.4457 g_0^2}{1 - 0.7165 g_0^2} - 0.2543 g_0^4 + 0.4357 g_0^6 - 0.5221 g_0^8,$$

$$z_T(g_0^2) = \frac{1 - 0.5090 g_0^2}{1 - 0.4789 g_0^2}.$$

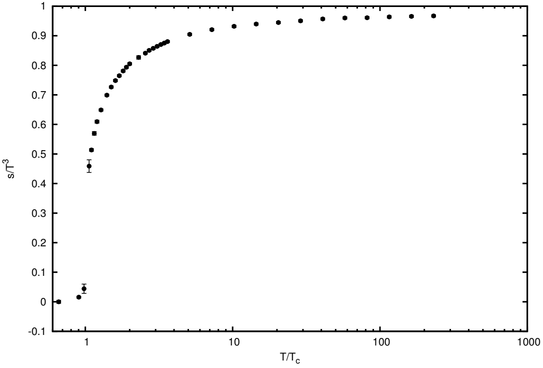
Giusti-Pepe (2015)

- Resulting very accurate entropy density $\sim \langle T_{0i} \rangle_\xi$ to $T \sim 7.5 T_c$



Giusti-Pepe (2016)

- Updated very accurate entropy density $\sim \langle T_{0i} \rangle_\xi$ to $T \sim 250\,T_c$



- Pepe, "Thermodynamics of strongly interacting plasma with high accuracy," 7/29, 16:30– this afternoon, Building 32 Room 1015

Del Debbio-Patella-Rago (2013–)

- The WT relation

$$\left\langle \hat{\partial}_\mu \left[Z_6 \hat{T}^{[1]}_{\mu\nu}(x) + Z_3 \hat{T}^{[3]}_{\mu\nu}(x) + 4 Z_1 \hat{T}^{[2]}_{\mu\nu}(x) \right] \hat{\mathcal{O}}(y) \right\rangle$$

$$= - \left\langle Z_\delta \frac{\delta}{\delta \xi_\nu(x)} \delta_\xi \hat{\mathcal{O}}(y) + \underbrace{a \hat{R}_\nu(x) \hat{\mathcal{O}}(y)}_{\text{contact term}} \right\rangle$$

$$= -\delta_{x,y} \left\langle \hat{\partial}_\nu \hat{\mathcal{O}}(y) \right\rangle + \cdots .$$

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- Use the gradient flow (Narayanan-Neuberger (2006), Lüscher 2009)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x)$$

for the probe operator $\hat{\mathcal{O}}(y)$. For example,

$$\hat{\mathcal{O}}^{[\alpha]}_\nu(t,y) = \hat{\partial}_\rho \hat{T}^{[\alpha]}_{\rho\nu}(y)\Big|_{\text{flowed gauge field}}$$

Del Debbio-Patella-Rago (2013–)

- Then, because of the UV finiteness of the gradient flow (Lüscher-Weisz 2011), no contact term as $a \to 0$,

$$\left\langle \hat{\partial}_\mu \left[Z_6 \, \hat{T}^{[1]}_{\mu\nu}(x) + Z_3 \, \hat{T}^{[3]}_{\mu\nu}(x) + 4 Z_1 \, \hat{T}^{[2]}_{\mu\nu}(x) \right] \hat{\mathcal{O}}^{[\alpha]}_\nu(t,y) \right\rangle$$

$$= -\left\langle Z_\delta \underbrace{\frac{\delta}{\delta \xi_\nu(x)} \delta_\xi \hat{\mathcal{O}}^{[\alpha]}_\nu(t,y)}_{\text{known}} \right\rangle - \left\langle \underbrace{a \hat{R}_\nu(x) \hat{\mathcal{O}}^{[\alpha]}_\nu(t,y)}_{\to 0} \right\rangle$$

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- This can be used to determine the ratios

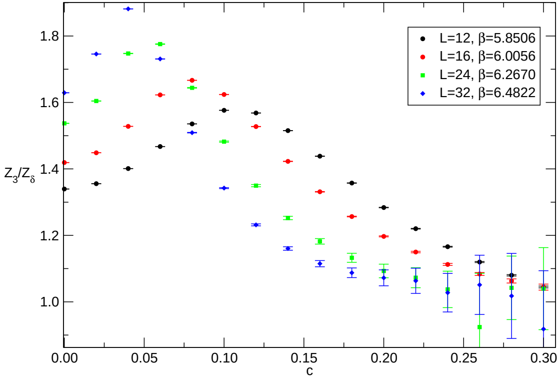
$$\frac{Z_6}{Z_\delta}, \qquad \frac{Z_3}{Z_\delta}, \qquad \frac{Z_1}{Z_\delta},$$

and the multiplicative factor for the translation,

$$Z_\delta.$$

Capponi-Del Debbio-Patella-Rago (2016)

- Z_3/Z_δ as a function of $c = \sqrt{8t}/aL$:



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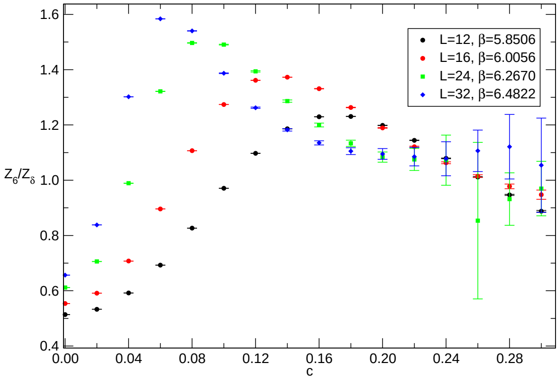

- Also Z_δ; Capponi, 7/27, 9:20– Building B2a Room 2077.

Capponi-Del Debbio-Ehret-Hanada-Jüttner-Pellegrini-Portelli-Rago (2016)

- Application of this strategy in 3D $\lambda\phi^4$-theory.
- Ehret, "Renormalisation of the scalar energy-momentum tensor with the Wilson flow," 7/27, 9:00– Building B2a Room 2077

Universal formula via gradient flow (H.S., Makino-H.S., 2013–)

- Gauge flow, fermion flow (flow time t) (Lüscher 2009–)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x),$$
$$\partial_t \chi(t,x) = \Delta\chi(t,x), \qquad \chi(t=0,x) = \psi(x),$$

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- Composite operators of flowed fields are automatically renormalized ones (Lüscher-Weisz 2011).

Universal formula via gradient flow (H.S., Makino-H.S., 2013–)

- Gauge flow, fermion flow (flow time t) (Lüscher 2009–)

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x),$$
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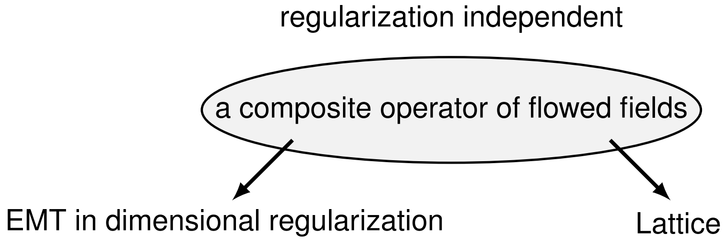
- Strategy



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- For the system containing fermions,

$$
\begin{aligned}
T_{\mu\nu}(x) = \lim_{t\to 0}\Big\{ & c_1(t) G^a_{\mu\rho}(t,x) G^a_{\nu\rho}(t,x) \\
& + \left[c_2(t) - \frac{1}{4} c_1(t)\right] \delta_{\mu\nu} G^a_{\rho\sigma}(t,x) G^a_{\rho\sigma}(t,x) \\
& + c_3(t) \mathring{\bar{\chi}}(t,x) \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right) \mathring{\chi}(t,x) \\
& + [c_4(t) - 2c_3(t)]\, \delta_{\mu\nu} \mathring{\bar{\chi}}(t,x) \overleftrightarrow{\not{D}} \mathring{\chi}(t,x) \\
& + c_5(t) m \mathring{\bar{\chi}}(t,x) \mathring{\chi}(t,x) - \text{VEV} \Big\},
\end{aligned}
$$

and

Universal formula for EMT

- the coefficients can be determined a priori (asymptotic freedom for $t \to 0$):

$$c_1(t) = \frac{1}{\bar{g}^2} - b_0 \ln \pi - \frac{1}{(4\pi)^2} \left[\frac{7}{3} C_2(G) - \frac{3}{2} T(R) N_f \right],$$

$$c_2(t) = \frac{1}{8} \frac{1}{(4\pi)^2} \left[\frac{11}{3} C_2(G) + \frac{11}{3} T(R) N_f \right],$$

$$c_3(t) = \frac{1}{4} \left\{ 1 + \frac{\bar{g}^2}{(4\pi)^2} C_2(R) \left[\frac{3}{2} + \ln(432) \right] \right\},$$

$$c_4(t) = \frac{1}{8} d_0 \bar{g}^2,$$

$$c_5(t) = -\frac{\bar{m}}{m} \left\{ 1 + \frac{\bar{g}^2}{(4\pi)^2} C_2(R) \left[3 \ln \pi + \frac{7}{2} + \ln(432) \right] \right\}.$$

where

$$\bar{g} = \bar{g}(1/\sqrt{8t}), \qquad \bar{m} = \bar{m}(1/\sqrt{8t}),$$

are running parameters in the MS scheme.

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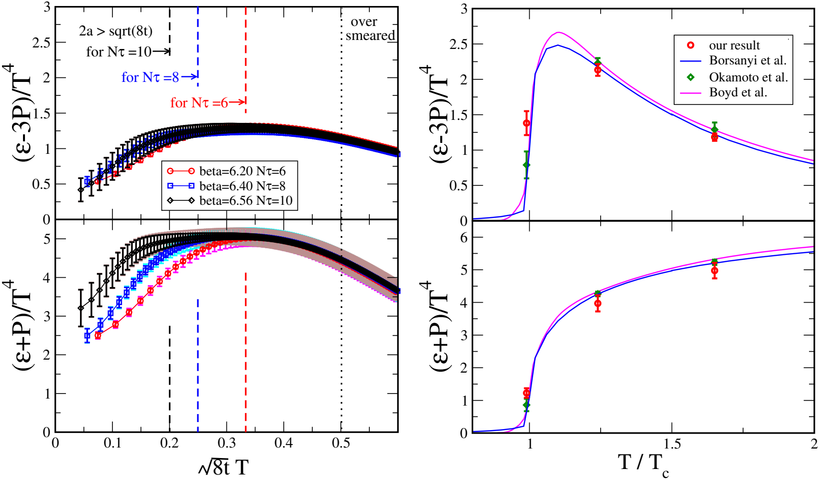
- Extrapolation to $t \to 0$ is a possible source of the systematic error.

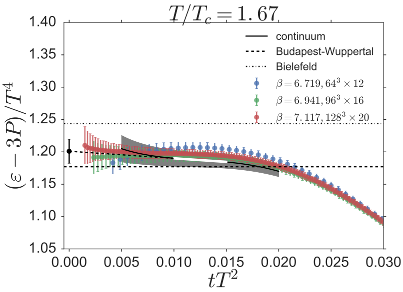
FlowQCD Collaboration (Asakawa-Hatsuda-Itou-Kitazawa-H.S.) (2013)

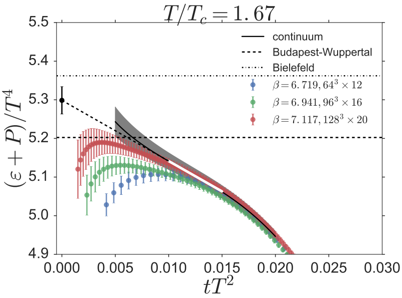
- Pure $SU(3)$. $a = 0.041$–0.11 fm, $N_s = 32$, $N_\tau = 6$–10, ~ 300 configs.



FlowQCD Collaboration (Asakawa-Hatsuda-Iritani-Kitazawa-H.S.) (preliminary)

- Pure $SU(3)$. $a = 0.013$–0.061 fm, $N_s = 64$–128, $N_\tau = 12$–24, ~ 1000–2000 configs.





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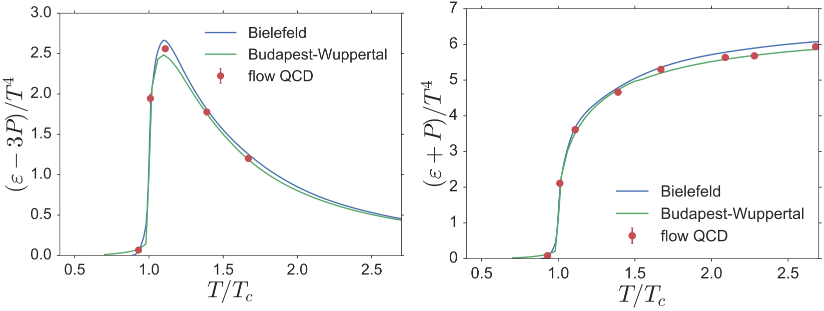

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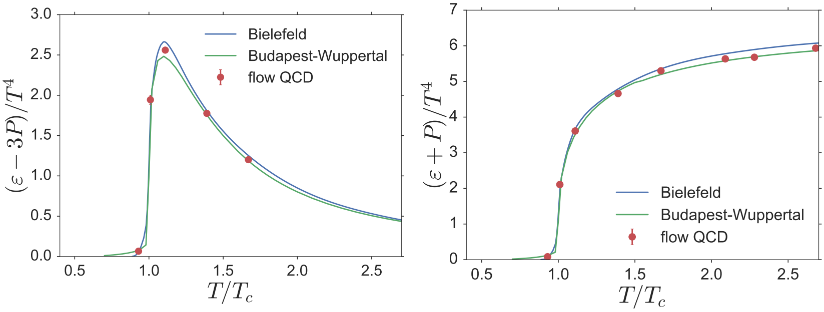
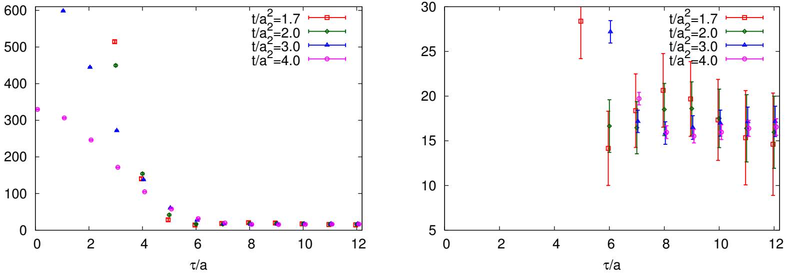


- Works very well!

FlowQCD Collaboration (Asakawa-Hatsuda-Iritani-Itou-Kitazawa) (preliminary)

- Pure $SU(3)$. $a = 0.017\,\text{fm}$, $N_s = 96$, $T = 1.66\,T_c$, ~ 50000 configs.

$$\frac{1}{T^5}\int d^3x\ \langle \delta T_{00}(x,\tau)\delta T_{00}(0)\rangle$$



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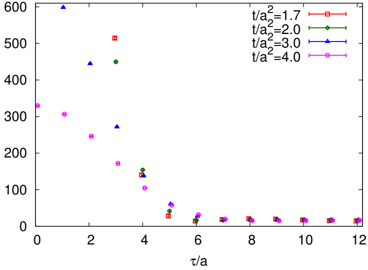
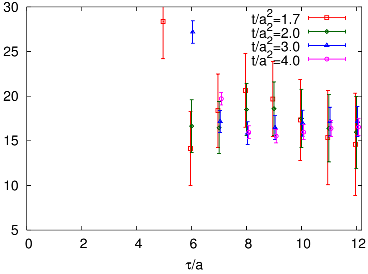


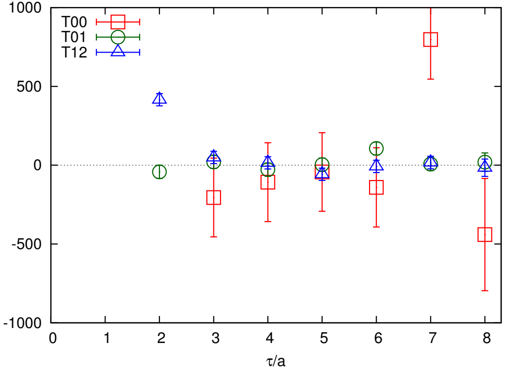


- Very clean signal. Indication of the energy conservation!

Reference: A naive EMT without flow (Kitazawa)

- Pure $SU(3)$. $a = 0.019\,\mathrm{fm}$, $N_s = 64$, $T = 2.2T_c$, ~ 50000 configs.

$$\frac{1}{T^5}\int d^3x\ \langle \delta T_{\mu\nu}(x,\tau)\delta T_{\mu\nu}(0)\rangle$$



- No useful signal. . .

WHOT-QCD Collaboration (Taniguchi-Ejiri-Iwami-Kanaya-Kitazawa-H.S.-Umeda-Wakabayashi) (preliminary)

- $N_f = 2 + 1$ QCD. $a = 0.070\,\mathrm{fm}$, $m_\pi/m_\rho \simeq 0.63$, $m_{\eta_{ss}}/m_\phi \simeq 0.74$, $N_s = 32$, $N_\tau = 4$–56, $\sim$ 100–1000 configs.

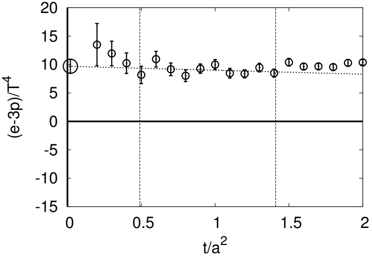
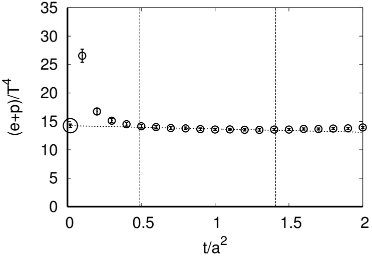


Figure: $(e - 3p)/T^4$, $T = 232\,\mathrm{MeV}$



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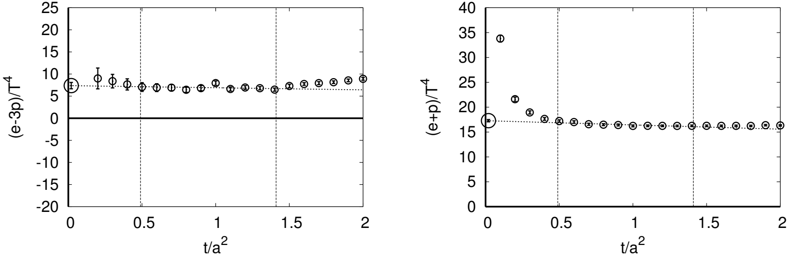

Figure: $(e - 3p)/T^4$, $T = 279\,\mathrm{MeV}$ Figure: $(e + p)/T^4$, $T = 279\,\mathrm{MeV}$

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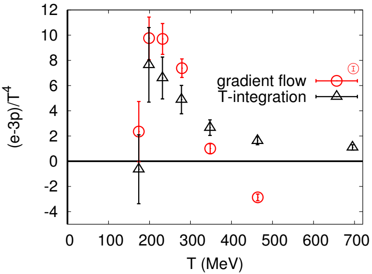
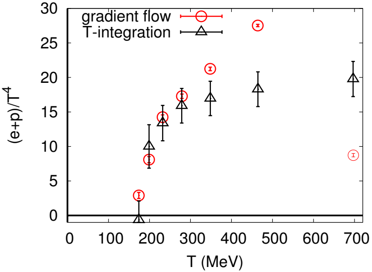


Figure: Black: T. Umeda et al. [WHOT-QCD Collaboration] (2012)



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Related presentations

- Kanaya, "Equation of state in $(2+1)$-flavor QCD with gradient flow," 7/27, 11:30– Building 32 Room 1015
- Ejiri, "Determination of latent heat at the finite temperature phase transition of $SU(3)$ gauge theory," 7/29, 16:50– this afternoon, Building 32 Room 1015

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Summary and prospects

- I hope I convinced you that...
- there have been rapid developments recently for this old, but important problem in lattice field theory, with encouraging results.
- So far, tests/applications of new ideas are limited mostly to bulk thermodynamics (and a few on viscosities)...
- Remembering possible vast applications, spin/momentum structure, (quasi-)conformal field theory, large anomalous dimension, gravity, ... (mainly related to correlation functions), we expect much to be explored.

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- Bulk thermodynamics shows encouraging results.
- As a by-product, correlation functions are typically quite clean!
- Still, we need to understand/reduce the systematic error associated with the $t \to 0$ extrapolation.