Long distance contribution to ϵ_K

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1. Introduction
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3. Review of previous talk
4. NLO perturbative matching
5. Results

Introduction

- ϵ_K, with experimental value $2.228(11) \times 10^{-3}$ measures indirect CP violation in $K^0 - \overline{K^0}$ system.
- Standard Model contribution can be separated into short distance and long distance part:
  1. Short distance which is estimated to be dominant contribution.
  2. The long distance part which has been estimated to be few percent, and must be determined using lattice QCD.
- Previous calculation of ϵ_K based on standard model only include the short distance contribution. The error on the results are mostly from CKM matrix. With exclusive V_{cb}, results are $\approx 3\sigma$ away from experiment, while with inclusive of V_{cb}, the resultis consistent with experiment.

Introduction

- ϵ_K is determined by:

$$\epsilon_K = \exp i\phi_\epsilon \sin\phi_\epsilon \left(\frac{\text{Im } M_{0\bar{0}}}{\Delta M_K} + \xi \right)$$

$$\phi_\epsilon = 43.52 \pm 0.005^\circ, \quad \xi = \frac{\text{Im } A_0}{\text{Re } A_0}. \quad M_{0\bar{0}} = \langle \bar{K}^0 | H_W^{\Delta S=2} | K^0 \rangle$$

- The $H_W^{\Delta S=2}$ is given by (the prime means we have used CKM unitarity and do a charm subtraction in the internal quark lines):

$$H_{\text{eff}}^{\Delta S=2} = \frac{G_F^2}{16\pi^2} M_W^2 [\lambda_u^2 \eta_1' S_0'(x_c) + \lambda_t^2 \eta_2' S_0'(x_t) + 2\lambda_u \lambda_t \eta_3' S_0'(x_c, x_t)] Q_{\text{LL}}$$

- We have three terms. λ_u^2 term: real. λ_t^2 term: purely short distance. $\lambda_u \lambda_t$ term: the term needing lattice calculation.

- The relevant part of H_W can be written as (dropping some coefficients):

$$
\begin{aligned}
H_{eff}^{\Delta S=2} &= \frac{G_F^2}{2}\lambda_u\lambda_t\left(\sum_{i=1}^{2}\sum_{j=1}^{6} C_i C_j \sum_{x,y} Q_{ij}(x,y) + \sum_{x} C_7 O_7(x)\right)\\
Q_7 &= (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}\\
Q_{i,j}(x,y) &= \frac{1}{2}T\{2Q_i^{cc}(x)Q_j^{cc}(y) - Q_i^{uc}(x)Q_j^{cu}(y) - Q_i^{cu}(x)Q_j^{uc}(y)\}\ (j=1,2)\\
Q_{i,j}(x,y) &= \frac{1}{2}T\{[(Q_i^{cc}(x) - Q_i^{uu}(x)]\,Q_j(y) + Q_j(x)\,[Q_i^{cc}(y) - Q_i^{uu}(y)]\}\ (j=3,...,6)
\end{aligned}
$$

  Q_j, $j=1,2$ are the current-current operators, and j = 3,...,6 are the QCD penguin operators.

- Results are logarithm divergent when two operators are close to each other. Need a short distance correction and match to perturbative theory.

- We have five types of diagram to evaluate on the lattice.

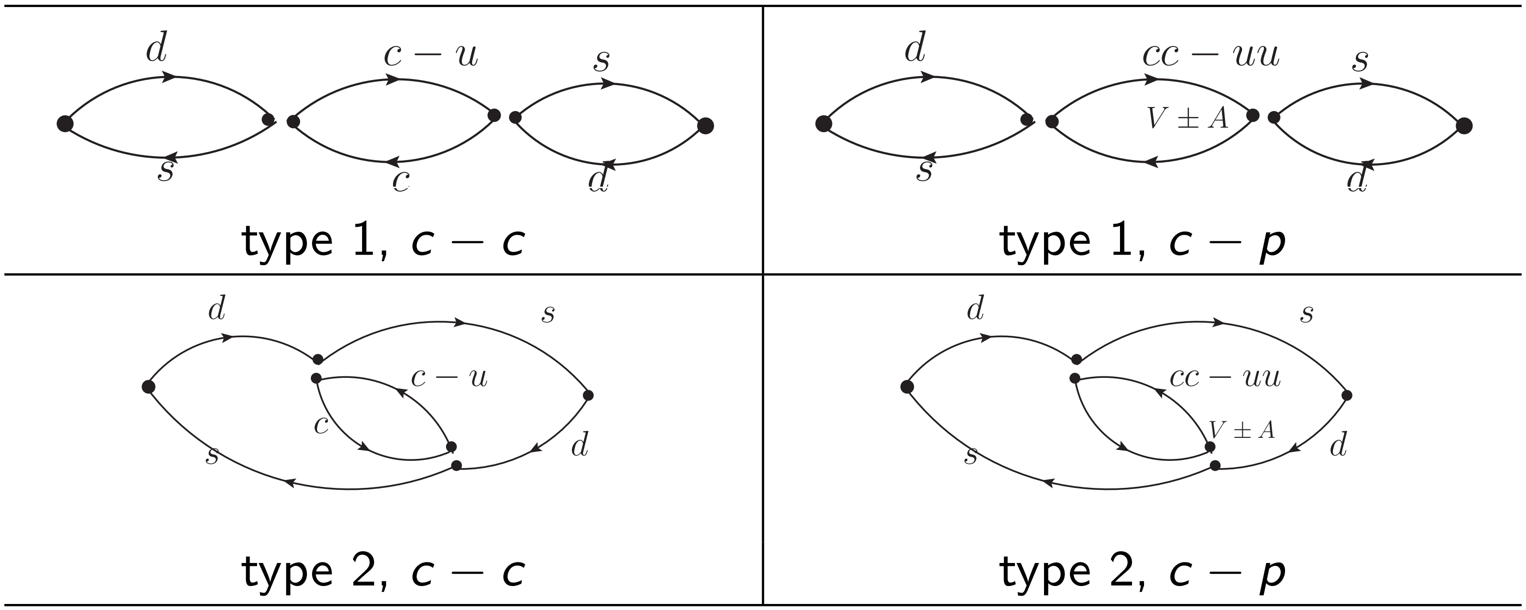


Figure : Type 1 and type 2 four point diagrams. c means current-current operator, p means penguin operator.

Introduction to lattice calculation

- We have five types of diagram to evaluate on the lattice.

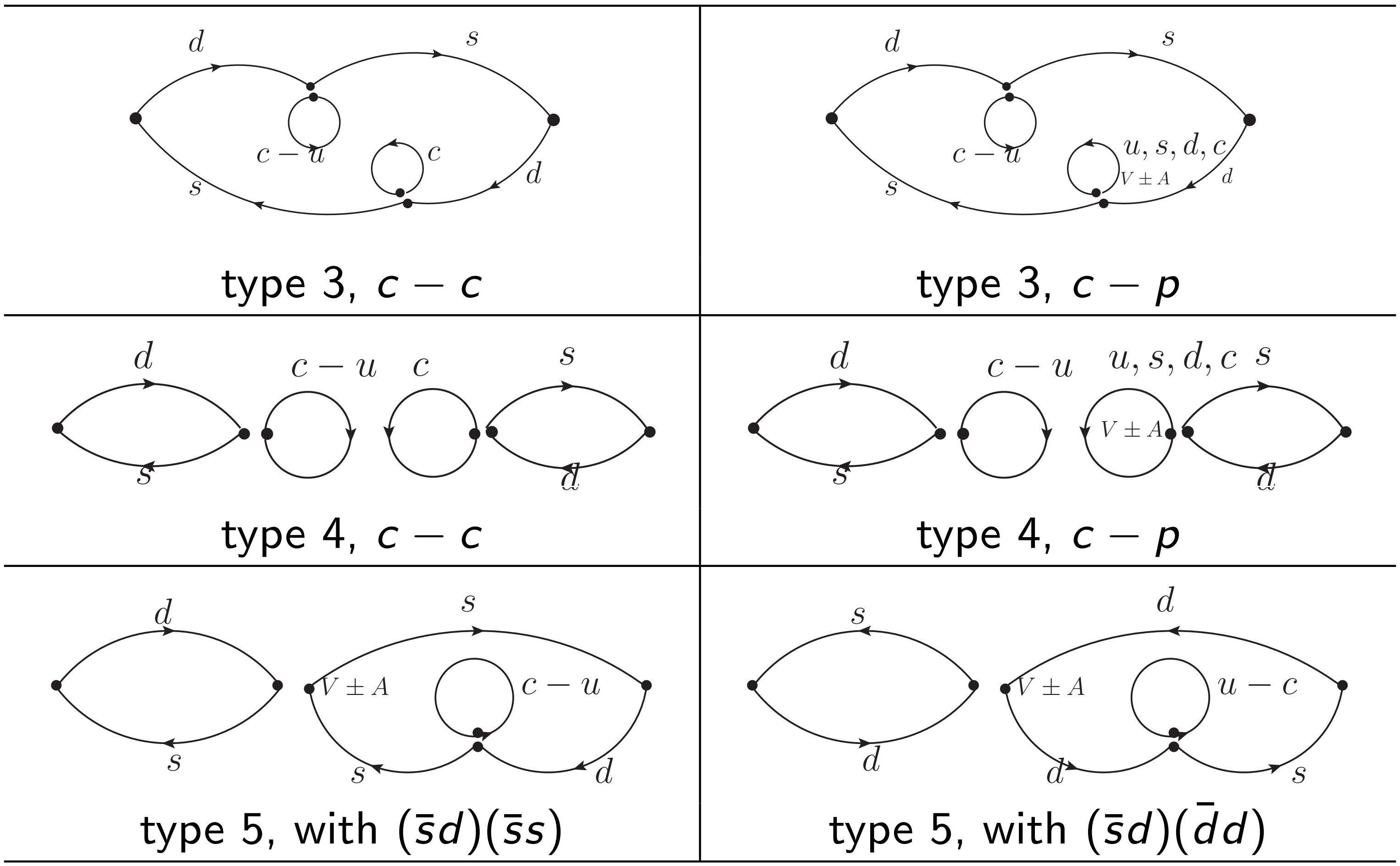


Figure : Type 3, 4 & 5 four point diagrams. c means current-current operator, p means penguin operator. type 5 must have a penguin operator.

Review of previous talk.

- Last year, we presented our preliminary calculation on the same lattice with this work (details follow):
  1. Correct the short distance divergence by performing a LO perturbative matching. This is done by introducing a RI scale μ_{RI} and performing perturbative calculation on the box diagram.
  2. Included type 1 & 2 diagrams in the lattice calculation, leaving out type 3, 4 & 5 diagrams.
- We got the results in the following table. Their dependence on the artificially RI scale μ_{RI} is very small.

μ_{RI} (GeV)	Im $M_{0\bar{0}}^{ut,ld}$	Im $M_{0\bar{0}}^{ut,cont}$	Im $M_{0\bar{0}}^{ut}$
1.54	-0.871(30)	-4.772(56)	-5.642(64)
1.92	-1.065(30)	-4.536(54)	-5.601(62)
2.31	-1.226(31)	-4.350(51)	-5.576(60)

Table : Im $M_{0\bar{0}}^{ut}$ in unit of 10^{-15} MeV.

- With the $\lambda_t\lambda_t$ part added, the final result for ϵ_K is $3.019(45) \times 10^{-3}$, much alrger than experimental value. This is because we only include LO in perturbative calculation, and the NLO correction is quite significant.

NLO PERTURBATIVE MATCHING.

- We define the RI bilocal operator for both lattice and dimensional regularization:

$$[Q_i Q_j]^{RI}(\mu_{RI}) = Z_i^{lat\to RI}(\mu,a) Z_j^{lat\to RI}(\mu,a)[Q_i Q_j]^{lat} - E_{lat}^{i,j}(\mu,a) Z^{lat\to RI}(\mu,a) O_{LL}^{lat}$$

$$[Q_i Q_j]^{RI}(\mu_{RI}) = Z_i^{\overline{MS}\to RI}(\mu,\mu_{RI}) Z_j^{\overline{MS}\to RI}(\mu_{RI})[Q_i Q_j]^{\overline{MS}} - E_{\overline{MS}}^{i,j}(\mu,\mu_{RI}) Z^{\overline{MS}\to RI}(\mu,\mu_{RI}) O_{LL}^{\overline{MS}}$$

- The RI operators are defined by $\langle Q_i Q_j\rangle^{RI}_{p^2=\mu^2_{RI}} = 0$.
- Finally, we arrive at the following formula. The first two lines are the term we want to evaluate (long distance correction), and the last line is the term that's existing in the conventional ϵ_K calculation (C_j are the Wilson coefficients).

$$H_{eff,ut}^{\Delta S=2} = \sum_{i=1}^{2}\sum_{j=1}^{6}$$

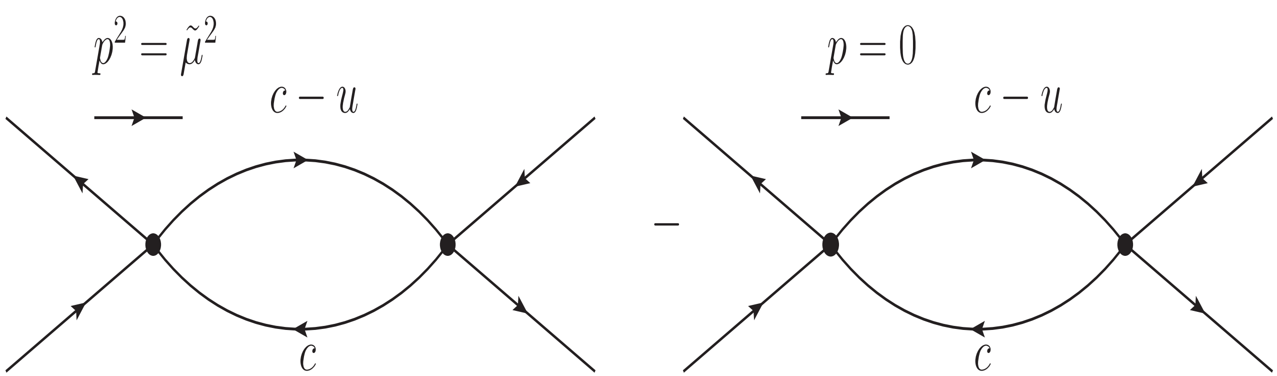
$$\left.\begin{array}{l} C_i^{\overline{MS}}(\mu) C_j^{\overline{MS}}(\mu) Z_i^{lat\to\overline{MS}} Z_j^{lat\to\overline{MS}} (Q_i^{lat} Q_j^{lat} - \tilde{E}_{lat}^{i,j}(\mu_{RI}) O_{LL}^{lat}) \\ + C_i^{\overline{MS}}(\mu) C_j^{\overline{MS}}(\mu) \Delta R_{\overline{MS}}^{i,j}(\mu_{RI}) Z^{lat\to\overline{MS}} O_{LL}^{lat} \end{array}\right\} \text{lattice corrections}$$

$$+ \left[C_i^{\overline{MS}}(\mu) C_j^{\overline{MS}}(\mu) R_{\overline{MS}}^{i,j}(\mu) + C_7^{\overline{MS}}(\mu)\right] Z^{lat\to\overline{MS}} O_{LL}^{lat} \quad \} \text{ conventional operator}$$

NLO perturbative matching.

- To evaluate the $H^{\Delta S=2}_{eff,ut}$ to NLO, which is order $\mathcal{O}(1)$, or order $\mathcal{O}(\alpha_s \ln \mu/M_W)$, we only have to evaluate the $\Delta R^{i,j}_{\overline{MS}}(\mu_{RI})$ to the same order.
- We find $\Delta R^{i,j}_{\overline{MS}}(\mu_{RI})$ by:

$$\Delta R^{i,j}_{\overline{MS}}(\mu_{RI}, m_c) = \tilde{E}^{i,j}_{\overline{MS}}(\mu, \mu_{RI}, m_c) - \tilde{E}^{i,j}_{\overline{MS}}(\mu, 0, m_c)$$
$$\langle Q_i^{\overline{MS}} Q_j^{\overline{MS}} - \tilde{E}^{i,j}_{\overline{MS}}(\mu, \mu_{RI}, m_c) O_{LL}^{\overline{MS}} \rangle_{p^2=\mu^2_{RI}} = 0$$



NLO perturbative matching.

- In the obove equation, $\Delta R^{i,j}_{\overline{MS}}(\mu_{RI})$ is finite (no ultra-violet divergence). We found (results are preliminary):

$$
\begin{aligned}
\Delta R^{i,j}_{\overline{MS}}(\mu_{RI})^{(c-u,c),LL} &= 4m_c^2 \left\{ \frac{1}{32\pi^2} \int_0^1 \mathrm{dx} \ln \frac{m_c^2}{x(1-x)\mu_{RI}^2 + m_c^2} \right. \\
&\quad \left. + \frac{-(\mu_{RI}^2 + m_c^2)}{32\pi^2 m_c^2} \int_0^1 \mathrm{dx} \ln \frac{x(1-x)\mu_{RI}^2 + m_c^2}{x\mu_{RI}^2 + m_c^2} \right\} \tau_{i,j} \\
\Delta R^{i,j}_{\overline{MS}}(\mu_{RI})^{(cc-uu),LL} &= \frac{-m^2}{8\pi^2} \left\{ -\frac{p^2}{m^2} \int_0^1 \mathrm{dx} \ln \frac{x(1-x)p^2 + xm^2}{x(1-x)p^2} \right. \\
&\quad \left. + \int_0^1 \mathrm{dx} \ln \frac{x(1-x)p^2 + m^2}{x(1-x)p^2 + xm^2} - 1 \right\} \tau_{i,j} \\
\Delta R^{i,j}_{\overline{MS}}(\mu_{RI})^{(cc-uu),LR} &= 8m_c^2 \times \frac{1}{16\pi^2} \int_0^1 \mathrm{dx} \ln \frac{m^2}{x(1-x)p^2 + m^2} \tau_{i,j}
\end{aligned}
$$

Lattice calculation details.

- We work on a $24^3 \times 64$ lattice, with $1/a = 1.78 GeV$. The $m_\pi = 329$ MeV, $m_K = 575$ MeV, and the input charm mass is 941 MeV.
- Two wall sources are used for the kaons, and we use random volume source propagator to evaluate the self loop propagators in type 3/4/5 diagrams.
- Lanczos algorithm is used to accelerate the light quark inversion with 300 eigenvectors.
- The code runs on a half rack of BGQ, and takes 3 hours per configuration.
- We use non-perturbative method to remove the short distance divergence in the lattice calculation, which is by adding a local operator $O_{LL} = (\bar{s}d)_{V-A}(\bar{s}d)_{V-A}$ with coefficient $\tilde{E}^{i,j}_{lat}$ found by:

$$\langle Q_i^{lat} Q_j^{lat} - \tilde{E}^{i,j}_{lat}(\mu_{RI}) O_{LL}^{lat} \rangle_{p^2=\mu_{RI}^2} = 0$$

Results with type 1/2 diagrams.

- What we calculate on the lattice is the 'long distance correction', and the final ϵ_K is found by adding our result to the conventional short distance calculation.

$$
\begin{aligned}
H^{\Delta S=2}_{eff,ut,ld\ corr} &= C_i^{\overline{MS}}(\mu) C_j^{\overline{MS}}(\mu) Z_i^{lat\to\overline{MS}} Z_j^{lat\to\overline{MS}} (Q_i^{lat} Q_j^{lat} - \tilde{E}^{i,j}_{lat}(\mu_{RI}) O_{LL}^{lat}) \\
&\quad + C_i^{\overline{MS}}(\mu) C_j^{\overline{MS}}(\mu) \Delta R^{i,j}_{\overline{MS}}(\mu_{RI}) Z^{lat\to\overline{MS}} O_{LL}^{lat}
\end{aligned}
$$

- We call the first term long distance lattice result, and we call the second term the correction term, which is used to match to the conventional short distance perturbation calculation.
- The result only inlcuding type 1/2 diagrams is given by:

Results with type 1/2 diagrams.

μ_{RI}	Im $M_{0,\bar{0}}^{ut,RI}$ from lat	Im $M_{0,\bar{0}}^{ut,RI\to\overline{MS}}$ from PT	Im $M_{0,\bar{0}}^{ut,ld\ corr}$ the sum	contribution to $\lvert\epsilon_K\rvert$
1.54	-0.871(30)	0.1890	-0.682(30)	0.1384×10^{-3}
1.92	-1.065(30)	0.3343	-0.731(30)	0.1483×10^{-3}
2.11	-1.151(31)	0.4250	-0.726(31)	0.1473×10^{-3}
2.31	-1.226(31)	0.5335	-0.693(31)	0.1405×10^{-3}
2.56	-1.302(30)	0.6879	-0.614(30)	0.1246×10^{-3}

Table : Im $M_{0,\bar{0}}$ at different scale μ_{RI} (unit 10^{-15}MeV), and there contribution to $\lvert\epsilon_K\rvert$. We have fixed $\mu = 2.15$ GeV, which is the energy scale we find the Wilson coefficients.

Results with all diagrams.

- The type 1&2 diagram contribution to Im $M_{0\bar{0}}$ is in the following table. The total contribution to ϵ_K is $2.16(4) \times 10^{-4}$.

$Q_1 Q_1$	$Q_1 Q_2$	$Q_1 Q_3$	$Q_1 Q_4$	$Q_1 Q_5$	$Q_1 Q_6$
	$Q_2 Q_2$	$Q_2 Q_3$	$Q_2 Q_4$	$Q_2 Q_5$	$Q_2 Q_6$
0.4072(58)	-0.4610(097)	-0.0849(43)	-0.0017(07)	0.0337(24)	-0.1049(037)
	1.6395(261)	-0.0024(11)	-0.1733(65)	0.0197(27)	-0.2068(165)

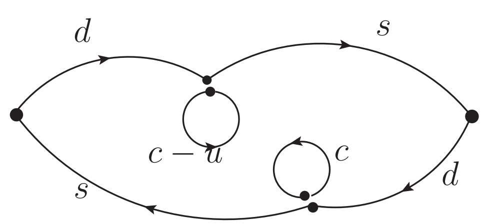
Table : contribution to Im $M_{0,\bar{0}}$ from type 1/2 diagrams, with all the relevant Wilson coefficient multiplied. We used $\mu_{RI} = 1.92$ GeV.

Results with all diagrams.

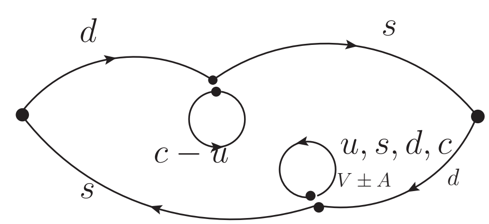
- The type 3 diagram contribution to Im $M_{0\bar{0}}$ is in the following table. The total contribution to ϵ_K is $3.67(63) \times 10^{-4}$.

$Q_1 Q_1$	$Q_1 Q_2$	$Q_1 Q_3$	$Q_1 Q_4$	$Q_1 Q_5$	$Q_1 Q_6$
	$Q_2 Q_2$	$Q_2 Q_3$	$Q_2 Q_4$	$Q_2 Q_5$	$Q_2 Q_6$
-0.0011(43)	0.0780(0377)	0.0045(14)	-0.0138(050)	-0.0379(121)	0.3238(1042)
	0.3347(1066)	0.0166(50)	-0.0605(167)	-0.1512(387)	1.3177(3263)

Table : contribution to Im $M_{0,\bar{0}}$ from type 3 diagrams, with all the relevant Wilson coefficient miltiplied.



type 3, $c - c$



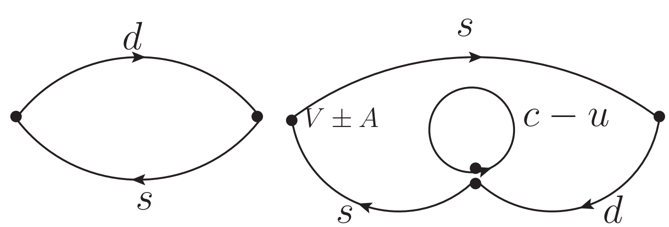
type 3, $c - p$

Results with all diagrams.

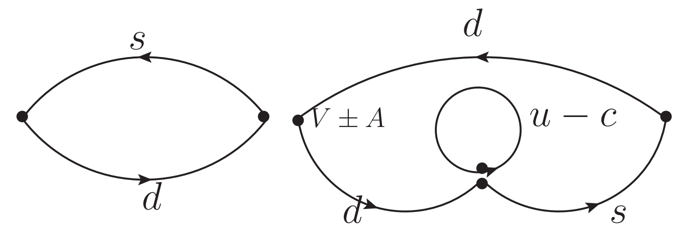
- The type 5 diagram contribution to Im $M_{0\bar{0}}$ is in the following table. The total contribution to ϵ_K is $2.95(63) \times 10^{-4}$.

$Q_1 Q_1$	$Q_1 Q_2$	$Q_1 Q_3$	$Q_1 Q_4$	$Q_1 Q_5$	$Q_1 Q_6$
	$Q_2 Q_2$	$Q_2 Q_3$	$Q_2 Q_4$	$Q_2 Q_5$	$Q_2 Q_6$
0	0	-0.0062(07)	0.0118(13)	-0.0087(129)	-0.4144(1260)
0	0	-0.0261(29)	0.0492(51)	0.1440(462)	-1.2042(4208)

Table : contribution to Im $M_{0,\bar{0}}$ from type 5 diagrams, with all the relevant Wilson coefficient miltiplied.



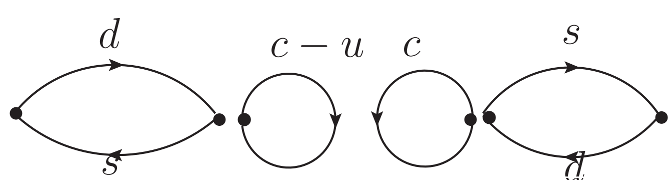
type 5, with $(\bar{s}d)(\bar{s}s)$



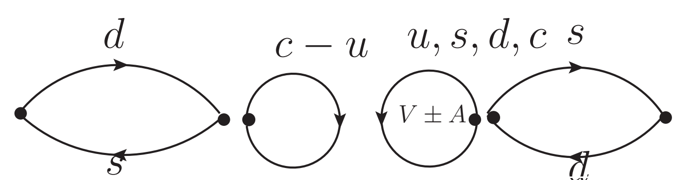
type 5, with $(\bar{s}d)(\bar{d}d)$

Results with all diagrams.

- The type 4 diagram has very large error, due to the fact that the Q_5, Q_6 has very strong coupling to vacuum, and we are doing a not well-correlated vacuum subtraction.
- We are re-running some measurements to perform better vacuum subtraction so we can have better accuracy.



type 4, $c - c$



type 4, $c - p$

Results with all diagrams.

- The following table is the contribution to Im $M_{0\bar{0}}$ when we include all 5 types of diagrams.

$Q_1 Q_1$	$Q_1 Q_2$	$Q_1 Q_3$	$Q_1 Q_4$	$Q_1 Q_5$	$Q_1 Q_6$
	$Q_2 Q_2$	$Q_2 Q_3$	$Q_2 Q_4$	$Q_2 Q_5$	$Q_2 Q_6$
0.664(42)	-1.977(576)	-0.125(20)	0.179(081)	0.923(182)	-4.216(1472)
0(0)	2.487(2311)	0.040(71)	-0.129(340)	0.852(759)	-7.683(6506)

Table : contribution to Im $M_{0,\bar{0}}$ from type 5 diagrams, with all the relevant Wilson coefficient miltiplied.

- The contribution to ϵ_K is

$$\epsilon_K^{ut,ld} - 1.8(12) \times 10^{-3}.$$

  This is a very large number because of the large error on type 4 diagram (with a Q_5 or Q_6 operator).

- Although the result above has very large error, but it show us that when we include type 4 diagrams, it may cancel the contribution of other 4 types of diagrams (final answer changes sign).

Conclusion.

- We are now able to do NLO perturbative matching and produce reasonable result for the long distance correction of ϵ_K.
- With the perturbative matching done, our type 1/2 diagram contribution to ϵ_K is $1.48(4) \times 10^{-4}$, type 3 diagram contribution is $3.67(63) \times 10^{-4}$, type 5 contribution is $2.95(63) \times 10^{-4}$.
- Without type 4 diagram, our long distance correction to ϵ_K is $8.1(9) \times 10^{-4}$. This is about 30% of the total experimental ϵ_K. But we are expecting that type 4 diagram will cancel some of this result when done correctly.
- We are currently re-running the type 4 measurements using a more precise vacuum subtraction method. This should gives us much better error for the type 4 diagrams and allow us to resolve the final long distance correction to ϵ_K.
- Comparison with experiment is for orientation only since we are using non-physical kinematics with $m_\pi = 329$MeV, $m_c = 941$ MeV on a $1/a = 1.78$ GeV lattice.