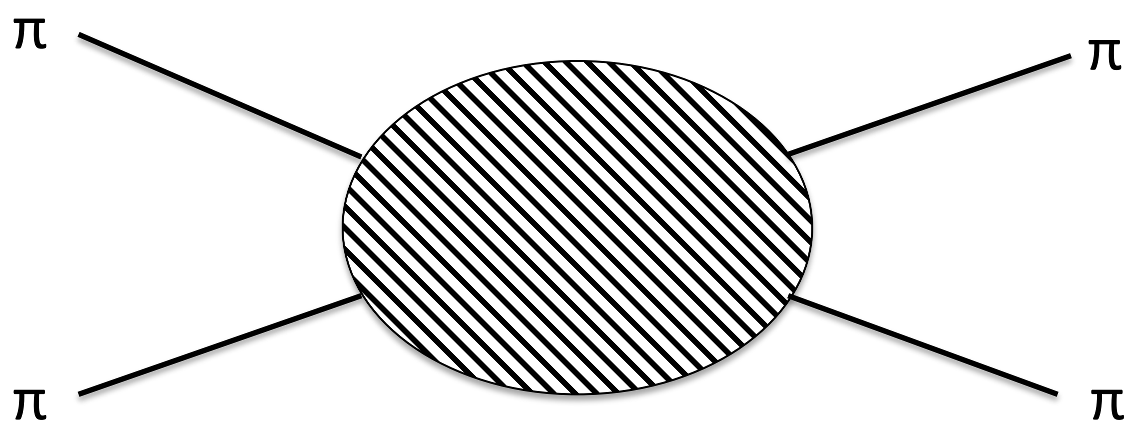
Studying the Low Energy Effective Theory of Eight Flavor SU(3)



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Outline

- Brief review of 8f SU(3) spectrum study
  - Covered by George Fleming in the previous talk
- Candidate low energy effective theories of 8f SU(3)
- Studying the low energy effective theory of 8f SU(3) on the lattice
  - Observables of interest
  - Methods
  - Early results
- Conclusions and Looking forward

8f SU(3) with Staggered Quarks

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \sum_{f=1}^{8} \bar{\psi}_f \left(i\not{D} - m_f \right) \psi_f$$

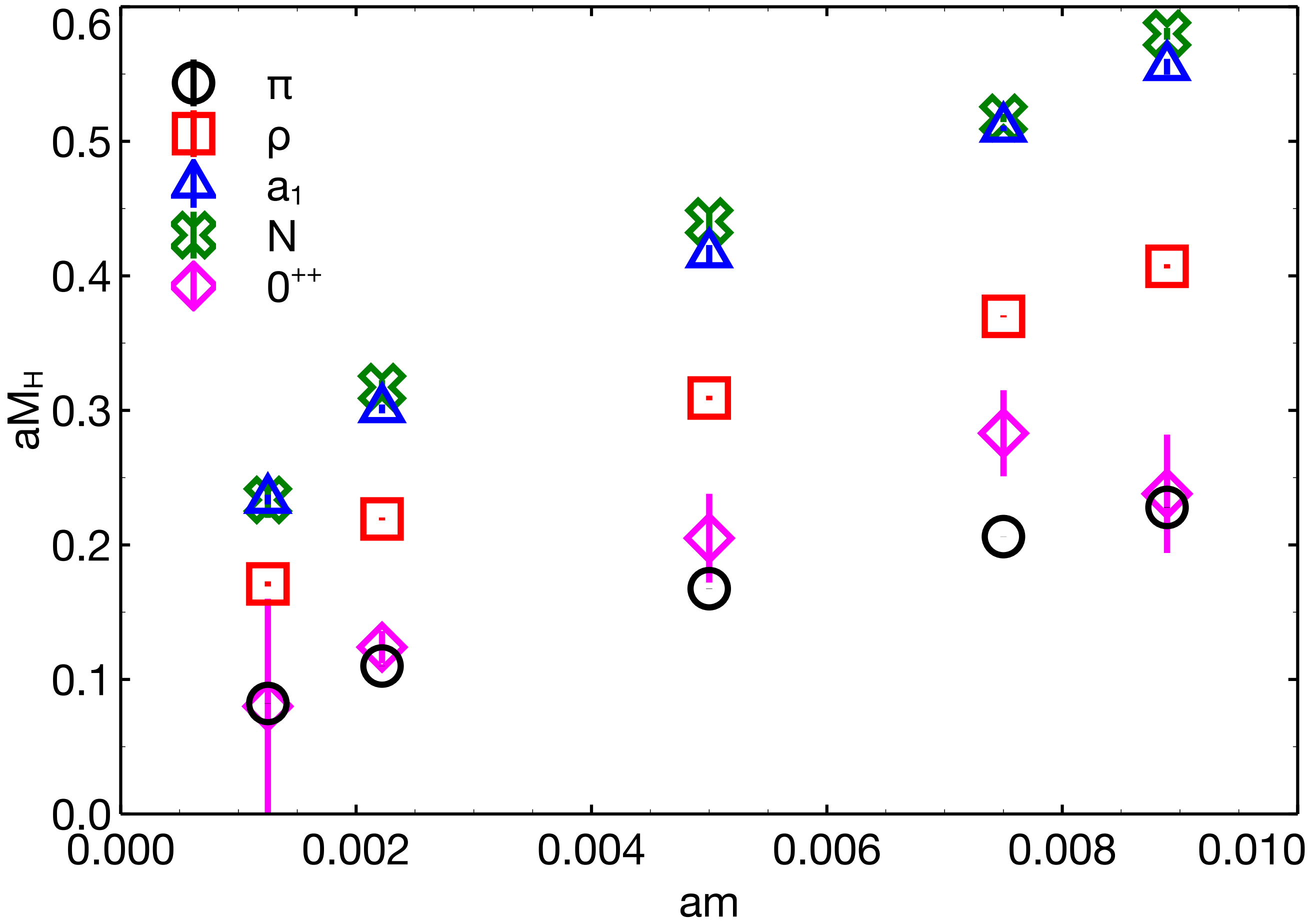
- 8 flavors in the fundamental representation
- Work under conventional assumption that model is chirally broken[1][2][3][4][5]
- Study the mass degenerate theory
  - One could consider splitting masses to connect to:
    - Dilatonic Higgs scenario
    - PNGB Higgs scenario
- We study the sector in isolation,
  - One may consider assigning standard model couplings
  - Want to understand the novel dynamics of the gauge sector

8f SU(3) on the lattice

- Staggered quarks
  - Relatively inexpensive
  - Exact $U(1)_V$ x $U(1)_A$ lattice symmetry of staggered action
  - Exact $SU(2)_f$ "isospin" flavor symmetry
- Lattices generated via nHYP smeared staggered quark action and Fundamental adjoint plaquette gauge action, $\beta = 4.8$, $\beta_a = -\beta/4$
- Currently using the same lattices as used in the 8f spectrum analysis (previous talk)
  - Early study performed on subset of coarse lattices
  - Finer lattices already exist from the spectrum calculation
  - We are also generating new ensembles more tailored to the specific task of studying scattering and form factors

Volume	$m_q a$	MDTU
24^3x48	0.00889	25k
32^3x64	0.0075	25k
32^3x64	0.005	22k

Light 0^{++} in 8f SU(3)



Appelquist,, et al (LSD Collaboration). arXiv:1601.04027 (2016).

[6]

- Low lying spectrum: pions and a sigma
- **What is the low energy effective theory?**

Candidate Low Energy Effective Theories

- XPT makes definite prediction for low energy observables
  - scattering observables
    - Scattering length, effective range
    - I=2,1,0
- At leading order, linear sigma model gives same prediction as XPT
- At NLO linear sigma and XPT differ.
  - Undetermined low energy constants in XPT
- Some alternative low energy effective theories
  - Dilatonic Effective Theory of Goldberger, Grinstein, and Skiba (arxiv 0708.1463) [12] ,
  - Dilatonic Effective Theory of Golterman and Shamir (arxiv *1603.04575*) [15]
  - XPT + scalar theory, e.g. Soto et al [13][14]
    - Additional undetermined low energy constants

Probing the Low Energy Effective Theory

- Interesting physical observables computable on the lattice beyond the spectrum
- I=2 ππ scattering
  - Corresponds to longitudinal W^+W^+ scattering in composite Higgs scenario
  - Scattering length and effective range are comparable to EFTs
  - In QCD, domainated by two pion exchange
  - In 8f SU(3), t-channel exchange of light sigma may contribute significantly
- I=0,1 scattering
  - Give further insight into the scalar,vector states of the theory
  - I=0 problem different from QCD because sigma is light
    - Coupled channel problem
- Form factor computations give a non-perturbative probe of an interaction vertex in the EFT.
  - We're chiefly interested in ππV, ππσ, σσσ
  - Initial results for vector and scalar form factors of pion will be in proceedings
  - Certain EFTs dictate relationships between form factors
- EFTs predict dominant tree level contributions to 2->2 scattering amplitudes
- Compute form factors (3 point functions) and 2->2 scattering amplitudes (4 point functions) on the lattice
- **We report on initial I=2 scattering results here.**

I=2 Pi Pi Scattering

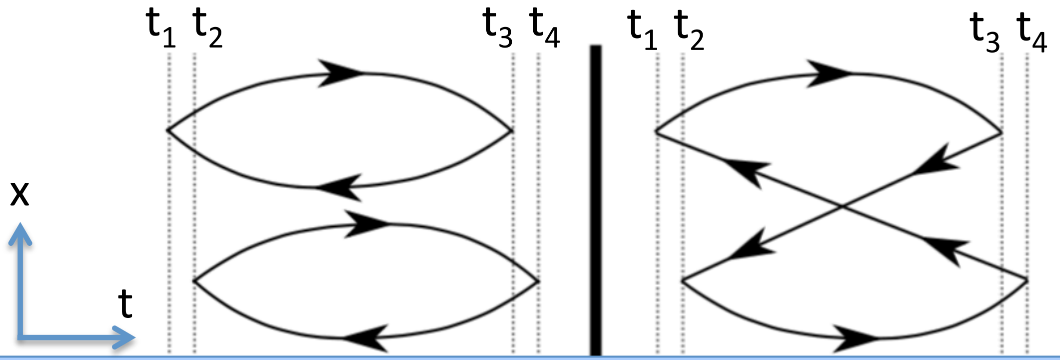
- Two types of contractions, "direct" and "crossed"

$$\langle \pi^+(t_1)^\dagger \pi^+(t_2)^\dagger \pi^+(t_3)\pi^+(t_4)\rangle = C_D(13;24) + C_D(14;23) - C_C(1324) - C_C(1423)$$

- Where

$$C_D(ik;jl) = \mathrm{Tr}(G^\dagger_{ik} G_{ik}) \mathrm{Tr}_C(G^\dagger_{jl} G_{jl})$$
$$C_C(ijkl) = \mathrm{Tr}(G_{ik} G^\dagger_{jk} G_{jl} G^\dagger_{il})$$

- Assume $t_1, t_2 \ll t_3, t_4$.
  - Can pull off energy of two pion state by using usual technique of large time separated correlator
  - Choose $t_2 = t_1 + 1$ and $t_4 = t_3 + 1$
    - Prevents projection onto unwanted states due to Fierz Identity



Extracting Scattering Observables Using Luscher's Method

- The momentum transfer is computed from the two pion energy

$$k^2 = \frac{1}{4}E_{\pi\pi}^2 - m_\pi^2$$

- One may then extract the scattering phase shift

$$k\cot\delta_0(k) = \frac{2\pi}{L}\pi^{-3/2}Z_{00}\left(1, \left(\frac{kL}{2\pi}\right)^2\right)$$

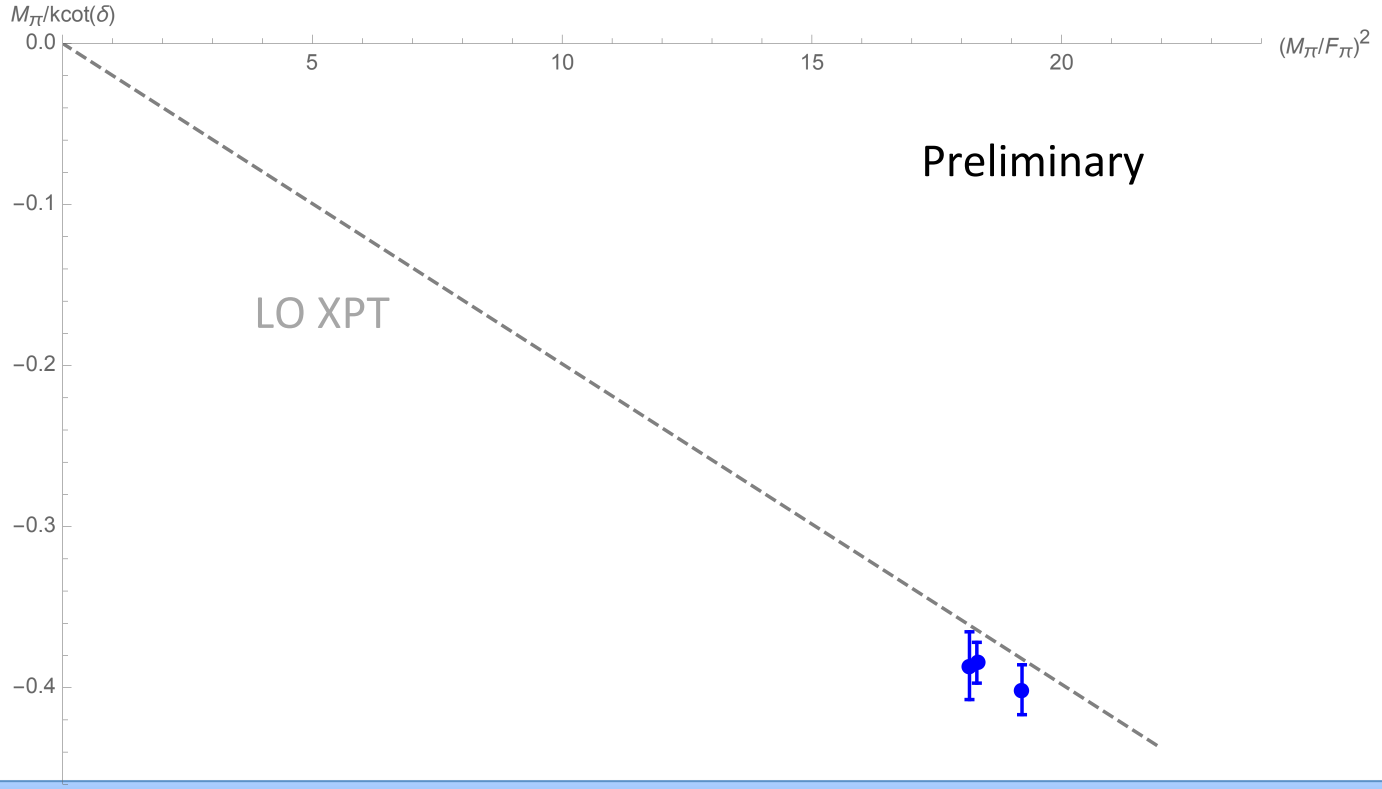
$$Z_{00}(1;q^2) = \frac{1}{\sqrt{4\pi}}\sum_{n\in\mathbb{Z}^3}\frac{1}{n^2-q^2}$$

- Scattering phase shift has a small k effective range expansion

$$k\cot(\delta_0(k)) = \frac{1}{a} + \frac{rM_\pi^2}{2}\left(\frac{k^2}{M_\pi^2}\right) + O\left(\left(\frac{k^2}{M_\pi^2}\right)^2\right)$$

Result for scattering phase shift on 24^3 and 32^3 ensembles

- $M_\pi/k\cot(\delta) \sim= M_\pi a$
- Additional ensembles and moving frames will provide many more data points
- Dotted line is LO XPT prediction.
  - Rough agreement, some tension
  - Well outside radius of convergence of XPT



Conclusions and Looking Forward

- We have measured the I=2 scattering phase shift on coarse ensembles
  - Rough agreement with LO XPT or linear sigma model
- Many interesting results to look forward to.
  - Measurement of I=2 on finer lattices
  - Extraction of scattering length and effective range
  - Moving frames for more values of k^2
  - Heading towards I=0,1 scattering
    - Need larger basis of interpolating operators
    - And dilution techniques for disconnected diagrams
  - Vector form factor of pion
    - Will give deeper insight into VMD and the KSRF relationships
  - Scalar form factor of scalar
    - Direct test of GGS dilatonic theory vs linear sigma model

Thank you for your attention!

References

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BACKUP SLIDES

I=2 Scattering Observables Data Table

Out[205]=

	f8l24t48b48m00889	f8l24t48b48m0075	f8l32t64b48m0075
$L^3 xT$	24^3x48	24^3x48	32^3x64
am_q	0.00889	0.0075	0.0075
M_π	0.22764(16)	0.20965(27)	0.206194(62)
F_π	0.051941(62)	0.047205(92)	0.048197(31)
$(M_\pi/F_\pi)^2$	19.207(50)	19.726(99)	18.303(26)
$E_{I=2}$	0.46396(37)	0.43002(64)	0.41649(17)
k^2	0.001995(95)	0.00228(14)	0.000850(33)
kcot(δ}	-0.567(22)	-0.510(25)	-0.536(17)
M_π/kcot(δ)	-0.401(15)	-0.411(20)	-0.385(13)

XPT expression for scattering length at NLO

$$M_\pi a_{I=2} = -\frac{M^2}{16\pi F^2}\left(1+\frac{M^2}{16\pi F^2}\left(b^r_{I=2}(\mu) - \frac{2(N_f-1)}{N_f^2} + A(N_f)\log\left(\frac{M^2}{\mu^2}\right)\right)\right)$$

$$A(N_f) = \frac{(2-N_f+2N_f^2+N_f^3)}{N_f^2}$$

$$b^r_{I=2}(\mu) = -256\pi^2\left(\left(N_f-2\right)\right)\left(L^r_4(\mu)-L^r_6(\mu)\right)+L^r_0(\mu)+2L^r_1(\mu)+L^r_3(\mu)\right)$$

[7][8][9][10][11]

Construction of I=2 4-point amplitude on lattice

$$\pi^+(x) = \bar{\chi}_2(x)\epsilon(x)\chi_1(x)$$

$$\langle \pi^+(t_1)^\dagger \pi^+(t_2)^\dagger \pi^+(t_3)\pi^+(t_4)\rangle$$
$$= \sum_{\vec{x_1}\vec{x_2}\vec{x_3}\vec{x_4}} \epsilon(x_1)\epsilon(x_2)\epsilon(x_3)\epsilon(x_4)\langle \bar{\chi}_1(x_1)\chi_2(x_1)\bar{\chi}_1(x_2)\chi_2(x_2)\bar{\chi}_2(x_3)\chi_1(x_3)\bar{\chi}_2(x_4)\chi_1(x_4)\rangle$$

$$\langle \bar{\chi}_1(x_1)\chi_2(x_1)\bar{\chi}_1(x_2)\chi_2(x_2)\bar{\chi}_2(x_3)\chi_1(x_3)\bar{\chi}_2(x_4)\chi_1(x_4)\rangle$$
$$= \langle \bar{\chi}_1(x_1)\chi_2(x_1)\bar{\chi}_1(x_2)\chi_2(x_2)\bar{\chi}_2(x_3)\chi_1(x_3)\bar{\chi}_2(x_4)\chi_1(x_4)\rangle$$
$$+ \langle \bar{\chi}_1(x_1)\chi_2(x_1)\bar{\chi}_1(x_2)\chi_2(x_2)\bar{\chi}_2(x_3)\chi_1(x_3)\bar{\chi}_2(x_4)\chi_1(x_4)\rangle$$
$$+ \langle \bar{\chi}_1(x_1)\chi_2(x_1)\bar{\chi}_1(x_2)\chi_2(x_2)\bar{\chi}_2(x_3)\chi_1(x_3)\bar{\chi}_2(x_4)\chi_1(x_4)\rangle$$
$$+ \langle \bar{\chi}_1(x_1)\chi_2(x_1)\bar{\chi}_1(x_2)\chi_2(x_2)\bar{\chi}_2(x_3)\chi_1(x_3)\bar{\chi}_2(x_4)\chi_1(x_4)\rangle$$