

# Thermalisation properties of various field theories

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# Viscosity of hadronic matter

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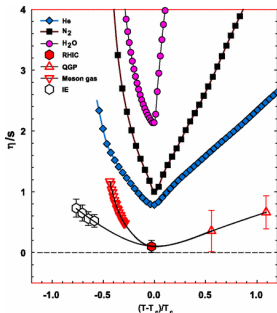
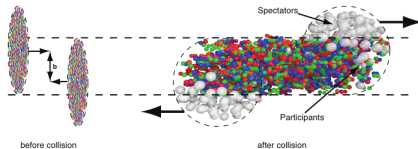
Introduction

Simulation

Basic quantities

histograms

Conclusion




- ▶ observation: nearly ideal liquid
- ▶ relevant quantity:  $\eta/s$   
(damping of hydrodynamic waves)

[R.Lacey et al. (2007),  
PhysRevLett.98.092301]

- ▶ viscosity ( $\eta$ ) in classical field theory
- ▶ small transport coefficient  $\Leftrightarrow$  strongly interacting system  $\rightarrow$  non-perturbative
- ▶ Boltzmann equation, MC (less sensitive for  $\omega \rightarrow 0$ )
- ▶ different approach:
- ▶ test: classical  $\Phi^4$  theory

$$\mathcal{H} = \frac{1}{2}\Pi^2 + \frac{1}{2}(\nabla\Phi)^2 + \frac{m^2}{2}\Phi^2 + \frac{\lambda}{24}\Phi^4 \quad (1)$$

- ▶ "toy model" which may show interesting effects mentioned above
- ▶  Homor, M. M. and Jakovac, A., *Shear viscosity of the  $\Phi^4$  theory from classical simulation*, PhysRevD.92.105011, 2015



- ▶ canonical equations, periodic boundary conditions, leap-frog algorithm
- ▶ initial conditions:  $\{\Pi(t_0 + \frac{\delta t}{2}), \Phi(t_0)\}$
- ▶ uniform random for  $\Pi$
- ▶ Canonical eq. of  $\dot{\Phi}$  (1st part of time step):  
Initial condition  $\rightarrow \Phi(t_0 + \delta t)$
- ▶ Canonical eq. of  $\dot{\Pi}$  (2nd part of time step):  
 $\{\Phi(t_0 + \delta t), \Pi(t_0 + \frac{\delta t}{2})\} \rightarrow \Pi(t_0 + \frac{3\delta t}{2})$
- ▶ input parameters:  $N^3$  lattice size,  $a = 1$  (grid),  $\lambda$  (interaction),  $m^2$  Lagrangian-mass

# Total energy

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Introduction

Simulation

Basic quantities

histograms

Conclusion

$$E = \sum_{i \in U} \frac{1}{2} \Pi_i^2 + \frac{1}{2} (\nabla \Phi)_i^2 + \frac{m^2}{2} \Phi_i^2 + \frac{\lambda}{24} \Phi_i^4, \quad (2)$$

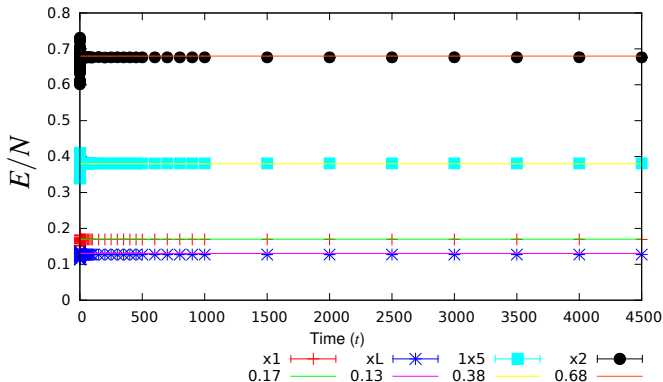


Figure: Time dependence of  $E/N$  where  $N = 50^3$

# Temperature $\langle |\Pi_k|^2 \rangle = 2N^3 T$

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Introduction

Simulation

Basic quantities

histograms

Conclusion

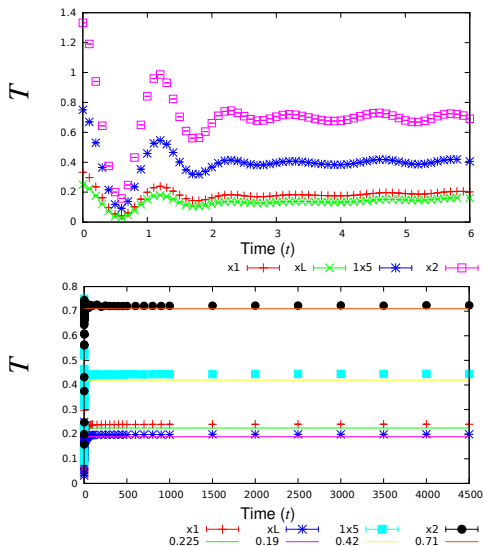


Figure: Time dependence of temperature

# Expectation values of local quantities

- ▶ Local quantity:  $A(\Phi, \Pi)$
- ▶ Real measurement is the time average:

$$\langle A(\Phi, \Pi) \rangle = \frac{1}{t} \int_{t_0}^{t_0+t} dt' A(\Phi(t'), \Pi(t')) \quad (3)$$

- ▶ Inserting  $\delta$  integral  $\langle A(\Phi, \Pi) \rangle =$

$$\int \mathcal{D}\bar{\Phi} \mathcal{D}\bar{\Pi} A(\bar{\Phi}, \bar{\Pi}) \frac{1}{t} \int_{t_0}^{t_0+t} dt' \delta(\bar{\Phi} - \Phi(t')) \delta(\bar{\Pi} - \Pi(t')) \quad (4)$$

- ▶ the second part is a histogram  $f(\bar{\Phi}, \bar{\Pi})$
- ▶ Time-average  $\rightarrow$  Ensemble average

$$\langle A(\Phi, \Pi) \rangle = \int \mathcal{D}\bar{\Phi} \mathcal{D}\bar{\Pi} A(\bar{\Phi}, \bar{\Pi}) f(\bar{\Phi}, \bar{\Pi}) \quad (5)$$

- ▶ e.g. canonical:  $e^{-\beta\mathcal{H}}$



# Energy histogram

Early time energy-distribution function is not Boltzmannian

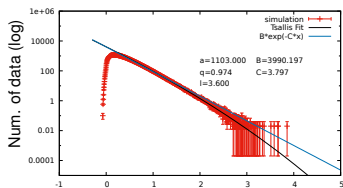
Introduction

Simulation

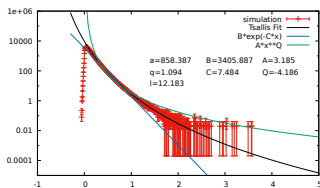
Basic quantities

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Energy-density  
(a) uniform init.



Energy-density  
(b) sech init.

Various fits on logscale energy histogram

Tsallis distribution is an excellent fit!

$$f(x) = a [1 + (q - 1)\beta x]^{\frac{1}{1-q}} \quad (6)$$

→ consider the time evolution of  $q$

# Not Tsallis?

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Introduction

Simulation

Basic quantities

histograms

Conclusion

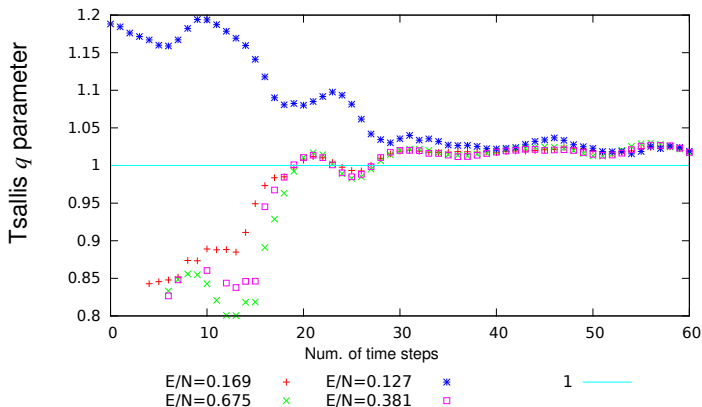


Figure: Time dependence of the Tsallis parameter

# Tsallis?

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field theories

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Introduction

Simulation

Basic quantities

histograms

Conclusion

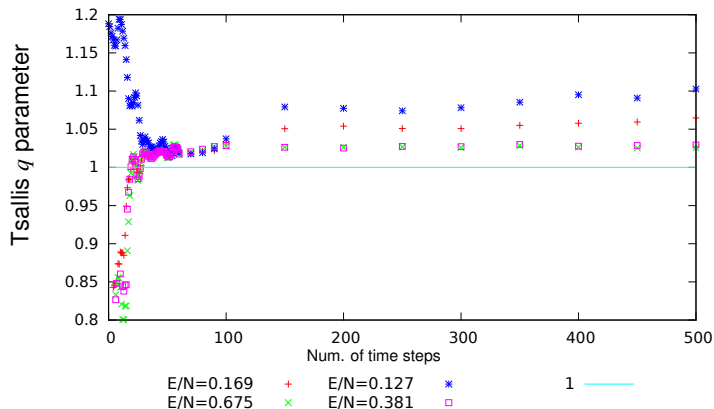


Figure: Time dependence of the Tsallis parameter

# Just pre-thermal?

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Introduction

Simulation

Basic quantities

histograms

Conclusion

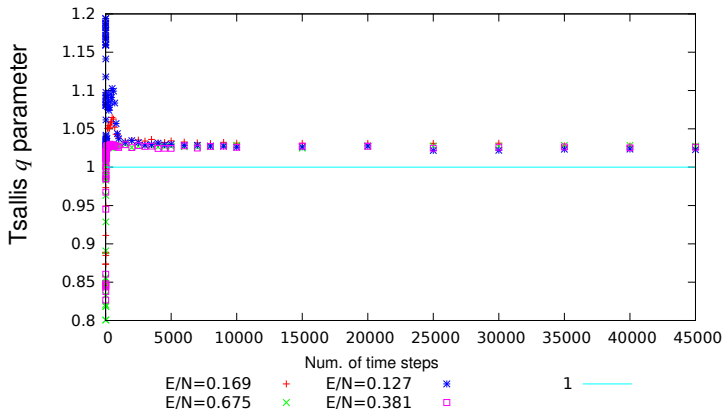


Figure: Time dependence of the Tsallis parameter

$$q \approx 1.026$$

# $\Pi(x)$ histogram

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Introduction

Simulation

Basic quantities

histograms

Conclusion

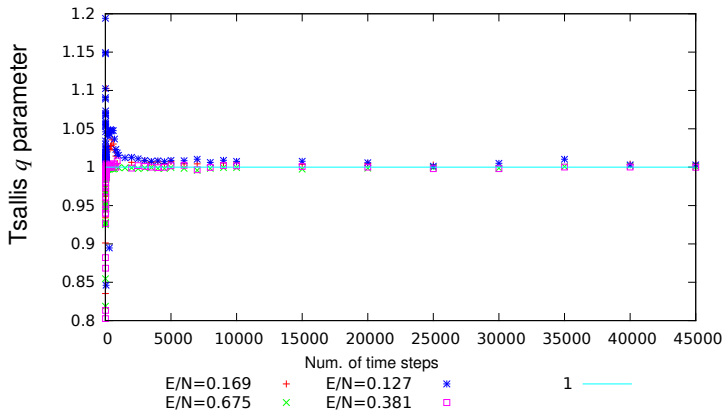


Figure: Time dependence of the Tsallis parameter for  $\Pi$  histogram

$$q \approx 0.999$$

# Lattice size

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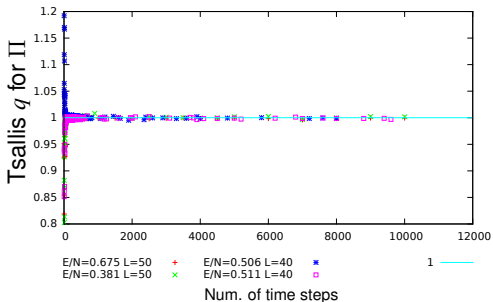
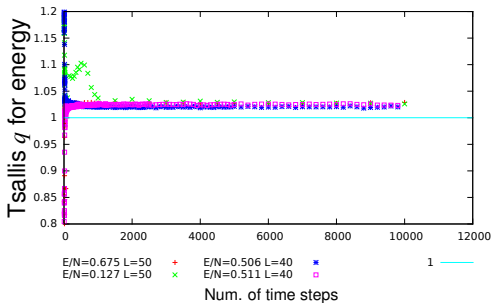
Introduction

Simulation

Basic quantities

histograms

Conclusion





# Interpretation

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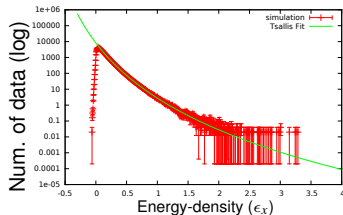
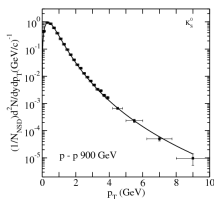
Introduction

Simulation

Basic quantities

histograms

Conclusion



Experiment vs.  $\Phi^4$  simulation

Suggestions:

- ▶ hadrons are created locally
- ▶ creation probability depends on local energy density
- ▶ local energy density distribution can be determined by computer simulations  $\rightarrow$
- ▶ it might be a tool for measuring hadron distribution function



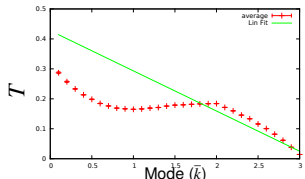
Thank you for your attention!

# Equipartition

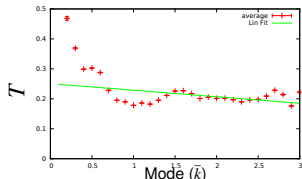
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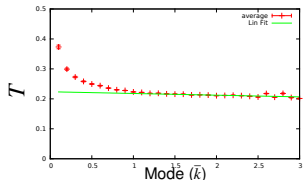
equilibrium  $\rightarrow$  equipartition



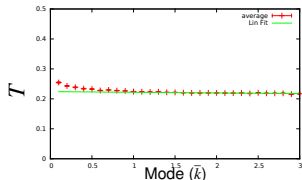
(e)  $t = 2$



(f)  $t = 8$



(g)  $t = 200.1$



(h)  $t = 4500.1$

Figure: Equipartition during time evolution ( $\lambda = 5$ )

Introduction

Simulation

Basic quantities

histograms

Conclusion