Lattice QCD @ nonzero temperature and finite density

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Central China Normal University

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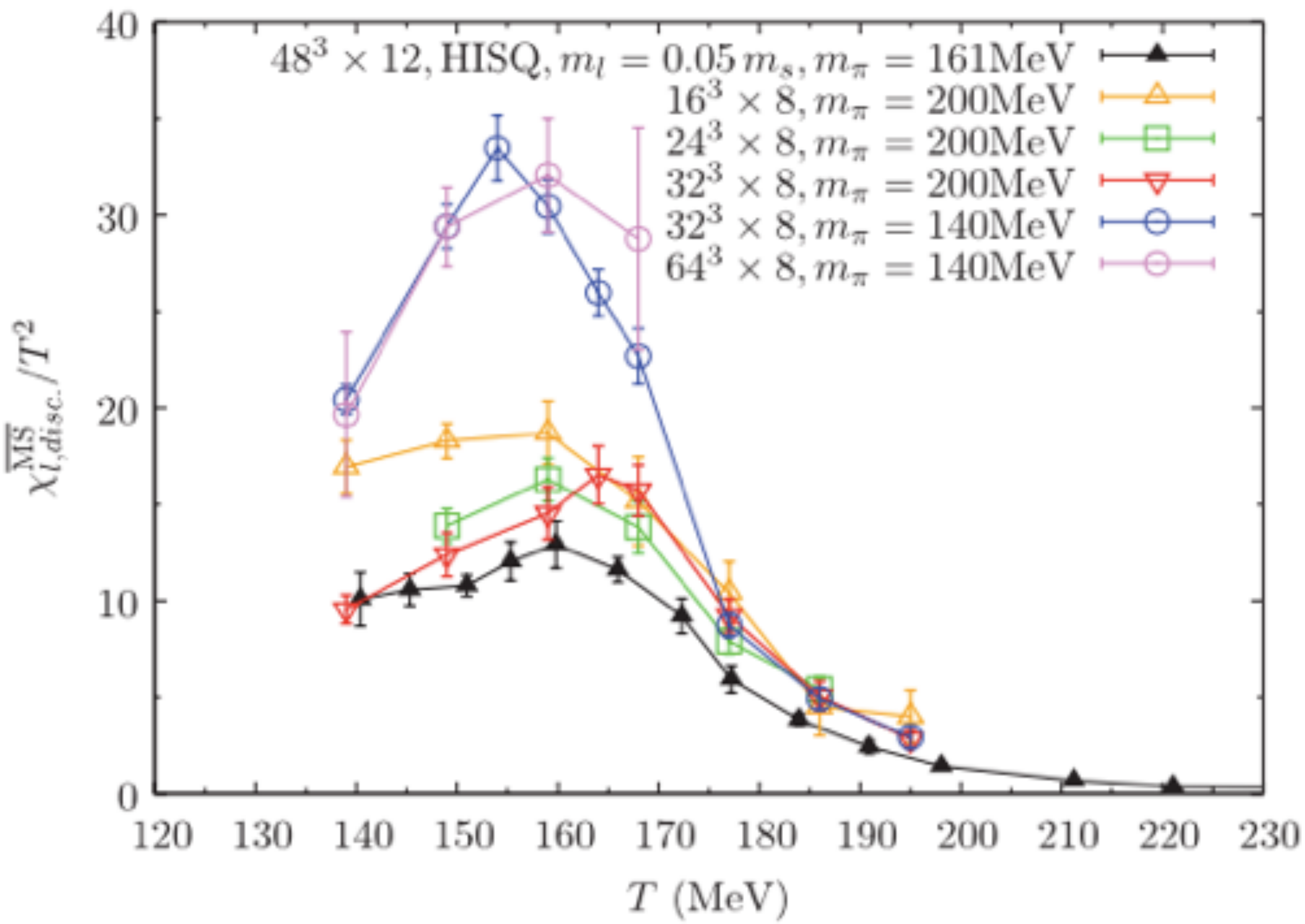
University of Southampton, UK

Outline

- T>0 & μ =0
  - QCD phase structure
  - Properties of QCD medium
- T>0 & $\mu > 0$
  - Equation of State
  - QCD phase structure

Milestone: transition temperature from hadronic phase to QGP phase

Domain wall fermions

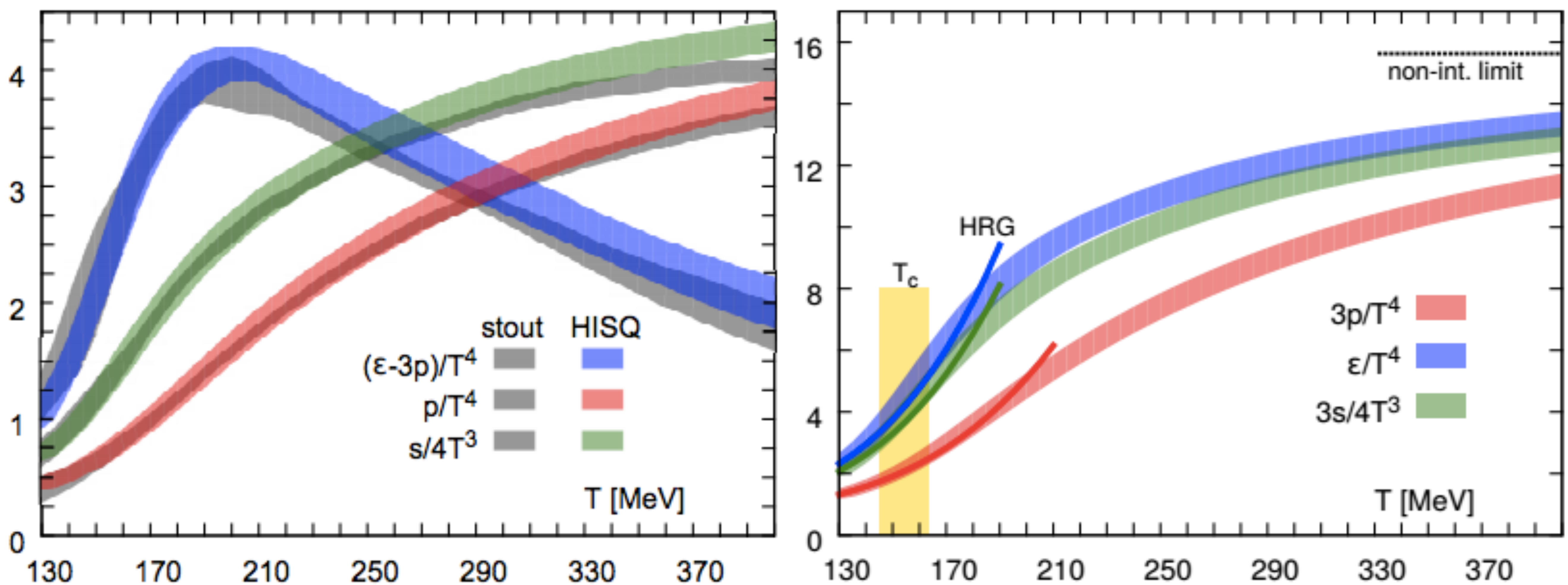


HotQCD: PRL 113 (2014) 082001

- Calculations with Domain wall, HISQ, stout fermions consistently give T_{pc} ~155 MeV
- Not a true phase transition but a crossover

See also the continuum extrapolated results of HISQ, stout & overlap in:
Wuppertal-Budapest: Nature 443(2006)675, JHEP 1009 (2010) 073 , HotQCD: PRD 85 (2012)054503
Borsanyi et al., [WB collaboration], arXiv: 1510.03376, Phys.Lett. B713 (2012) 342

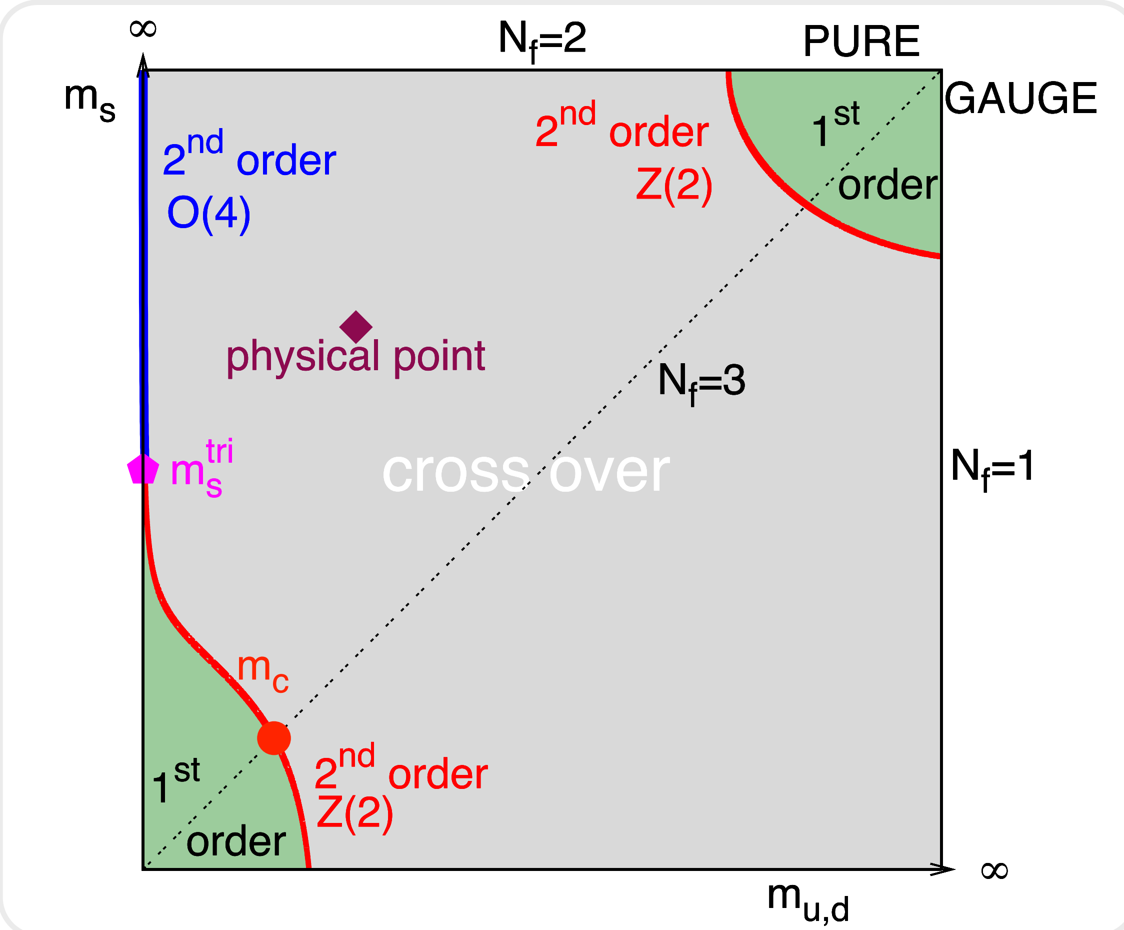
Milestone: QCD Equation of State



HotQCD, PRD 90 (2014) 094503, Wuppertal-Budapest, Phys. Lett. B730 (2014) 99

QCD phase structure in the quark mass plane

columbia plot, PRL 65(1990)2491



HTD, F. Karsch, S. Mukherjee, arXiv:1504.0527

RG arguments:

- m_q=0 or ∞ with N_f=3: a first order phase transition Pisarski & Wilczek, PRD29 (1984) 338
- Critical lines of second order transition
  - N_f=2: O(4) universality class
  - N_f=3: Z(2) universality class

K. Rajagopal & F. Wilczek, NPB 399 (1993) 395

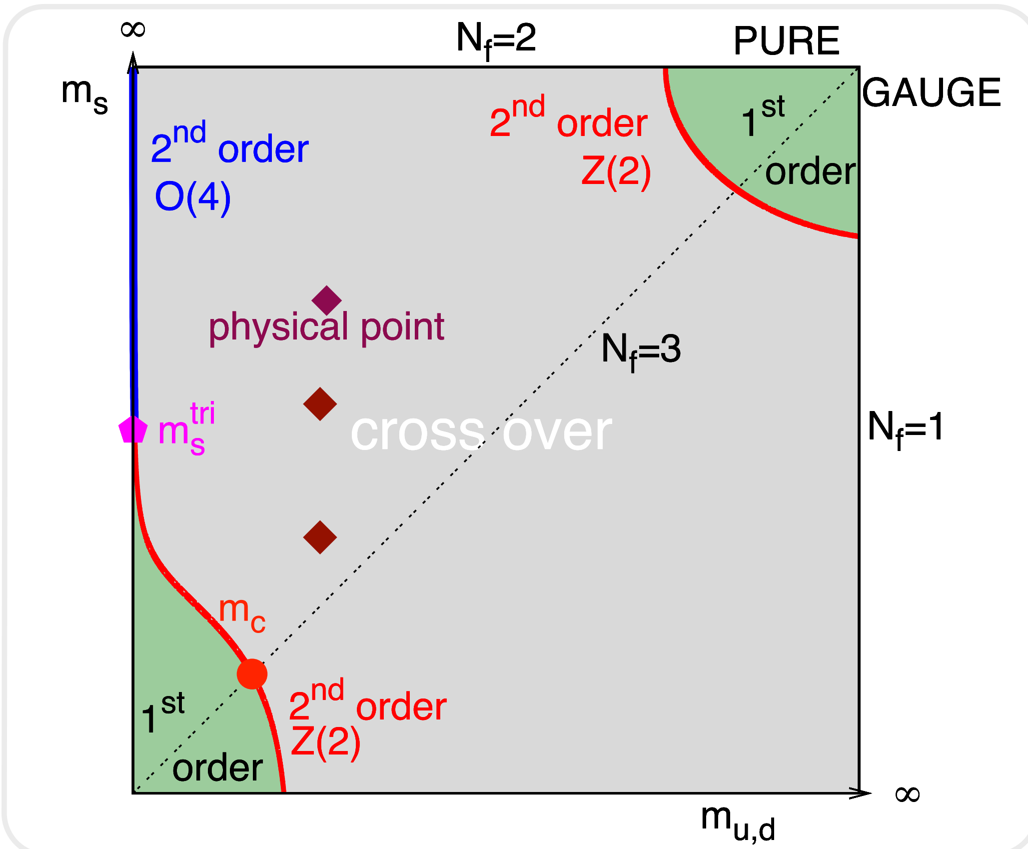
F. Wilczek, Int. J. Mod. Phys. A 7(1992) 3911,6951

Gavin, Gocksch & Pisarski, PRD 49 (1994) 3079

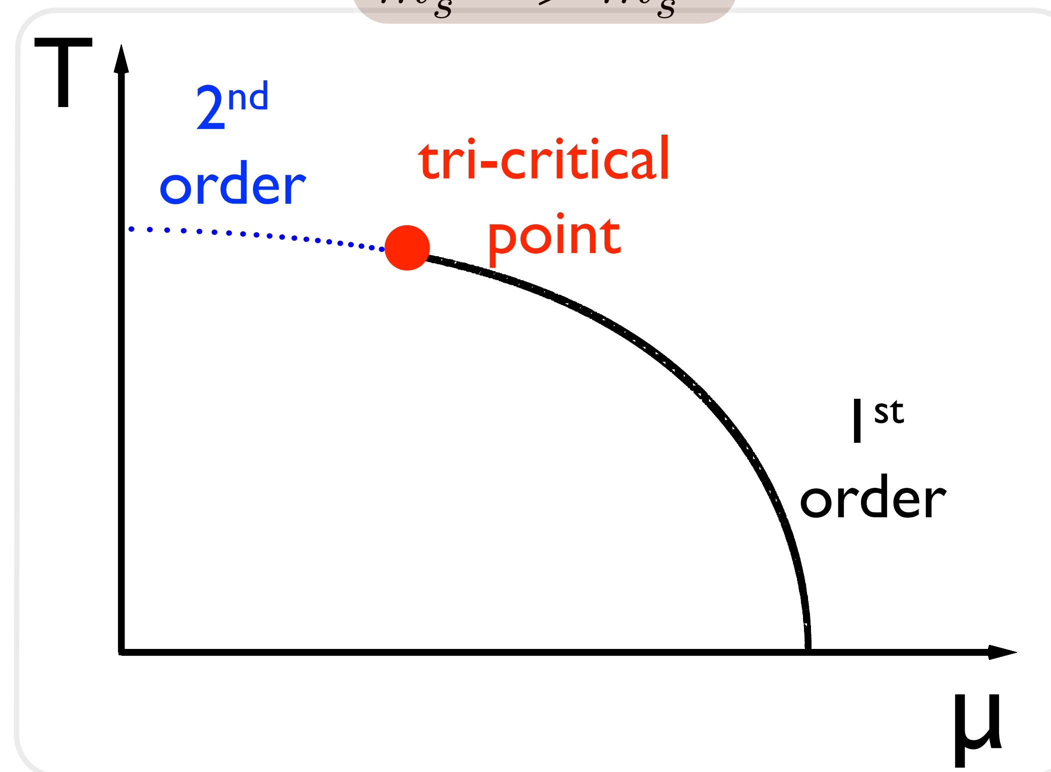
Lattice QCD calculations:

- The value of tri-critical point (m_s^{tri}) ?
- The location of 2^{nd} order Z(2) lines ?
- The influence of criticalities to the physical point ?

scenarios of QCD phase transition at m_l=0



$m_s^{phy} > m_s^{tri}$



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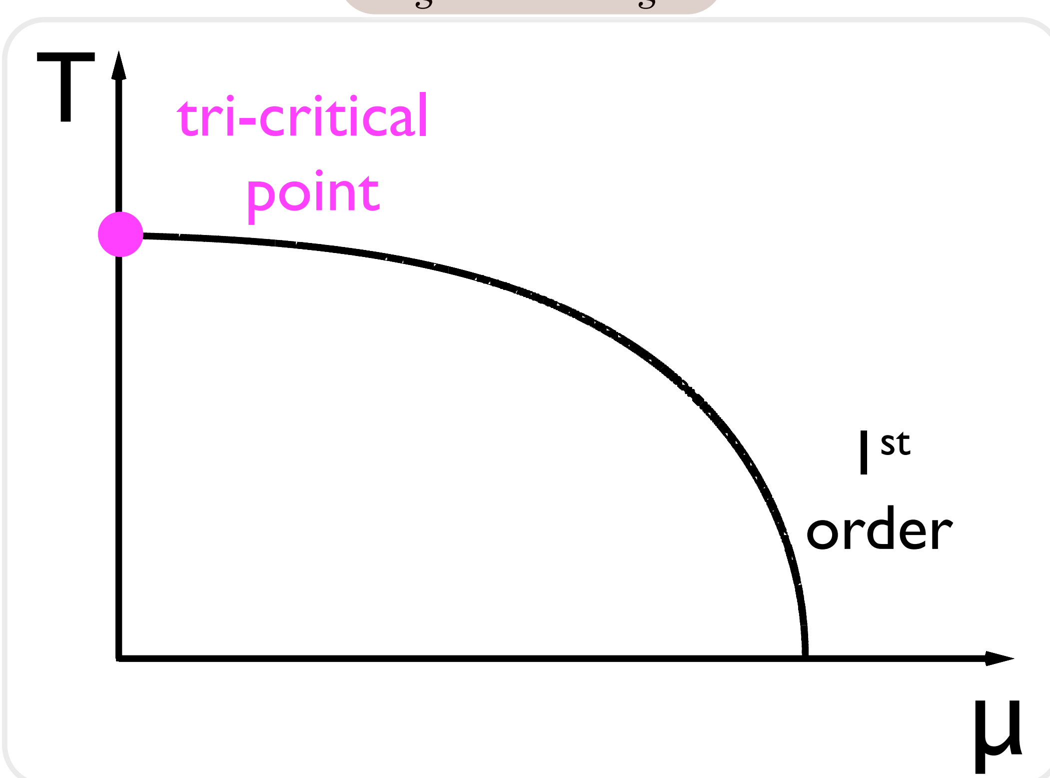

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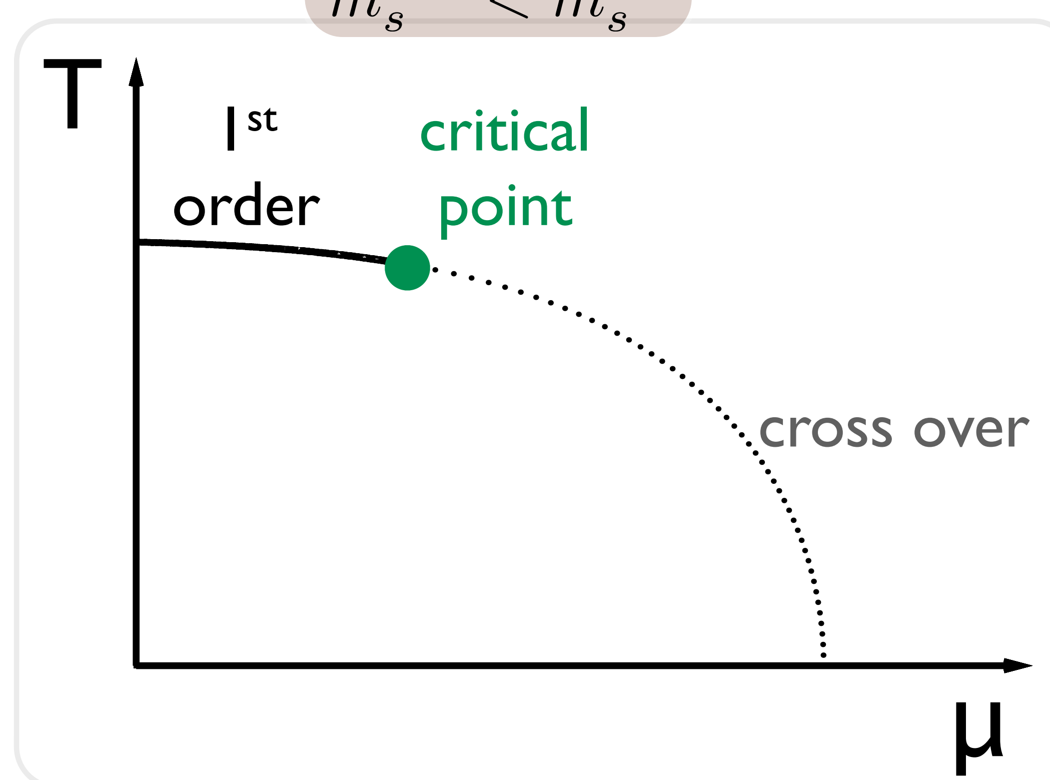
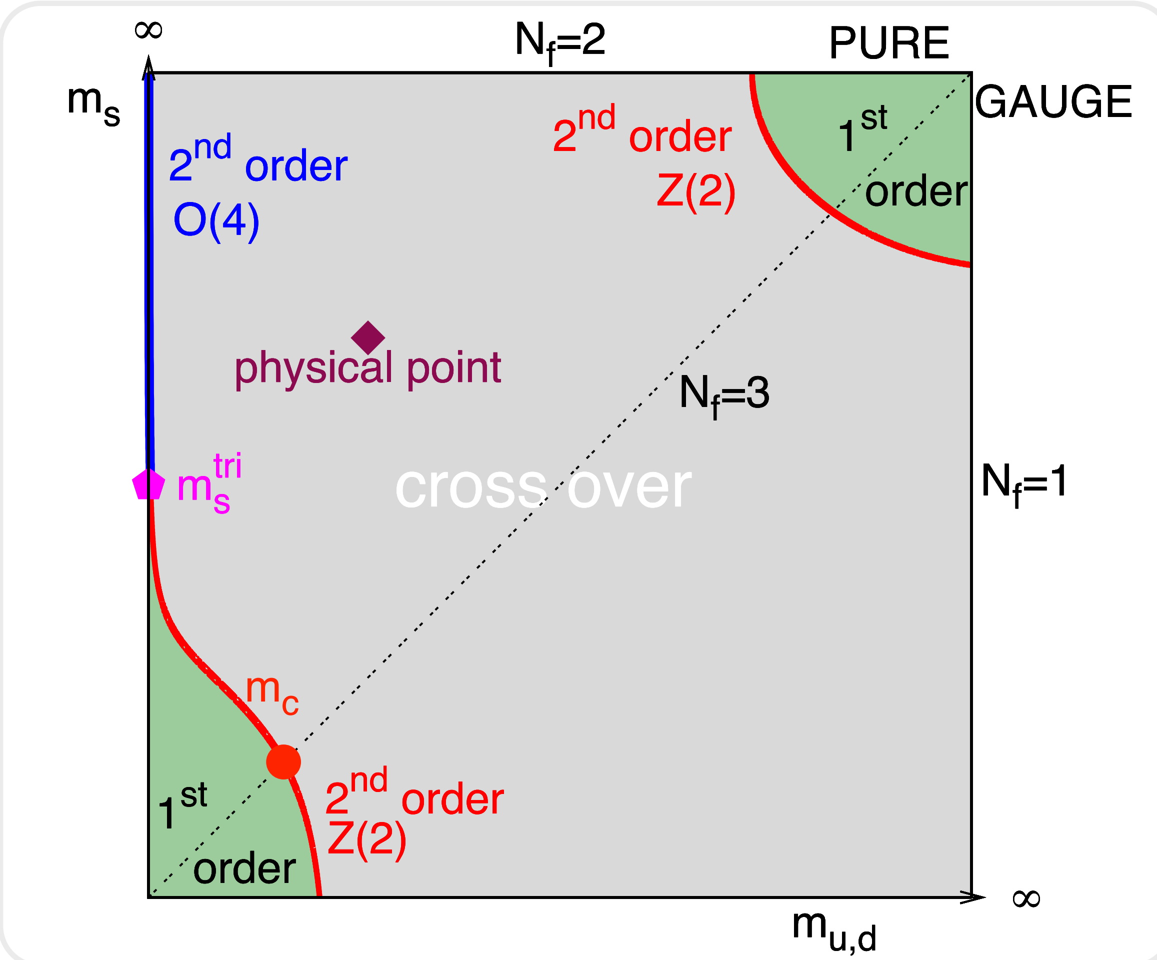
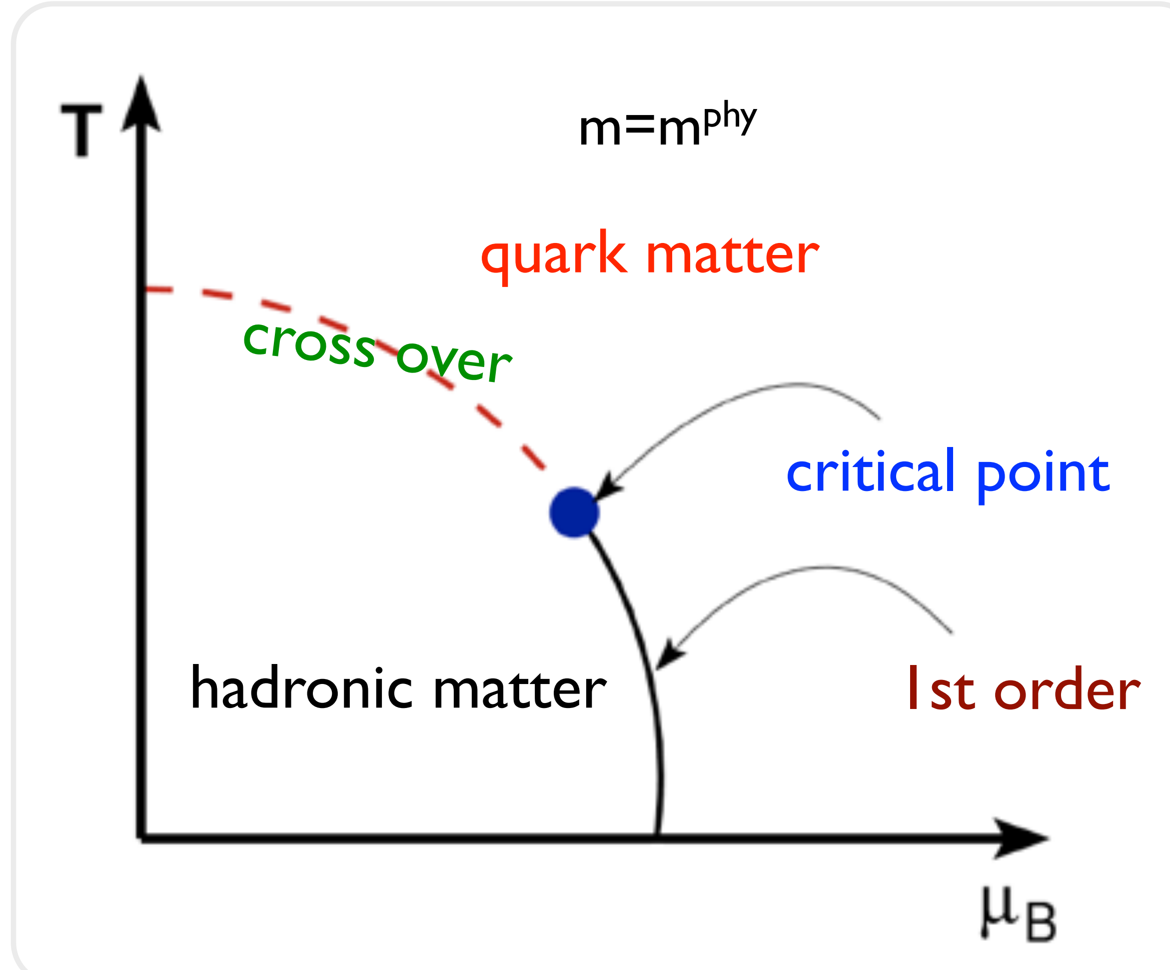
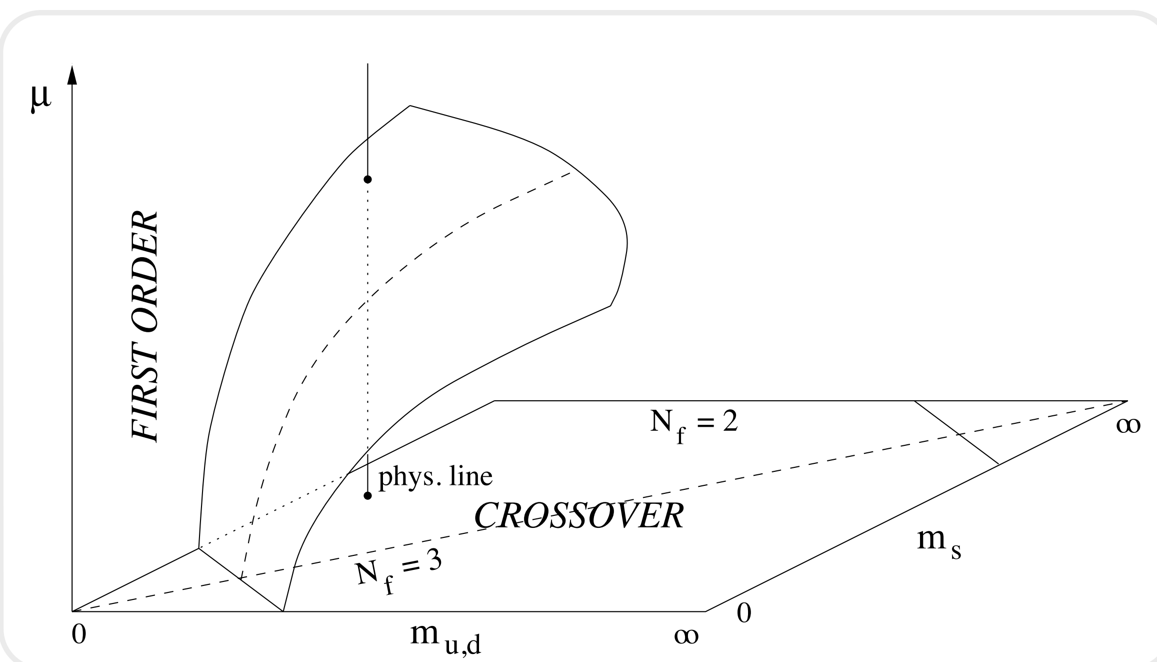


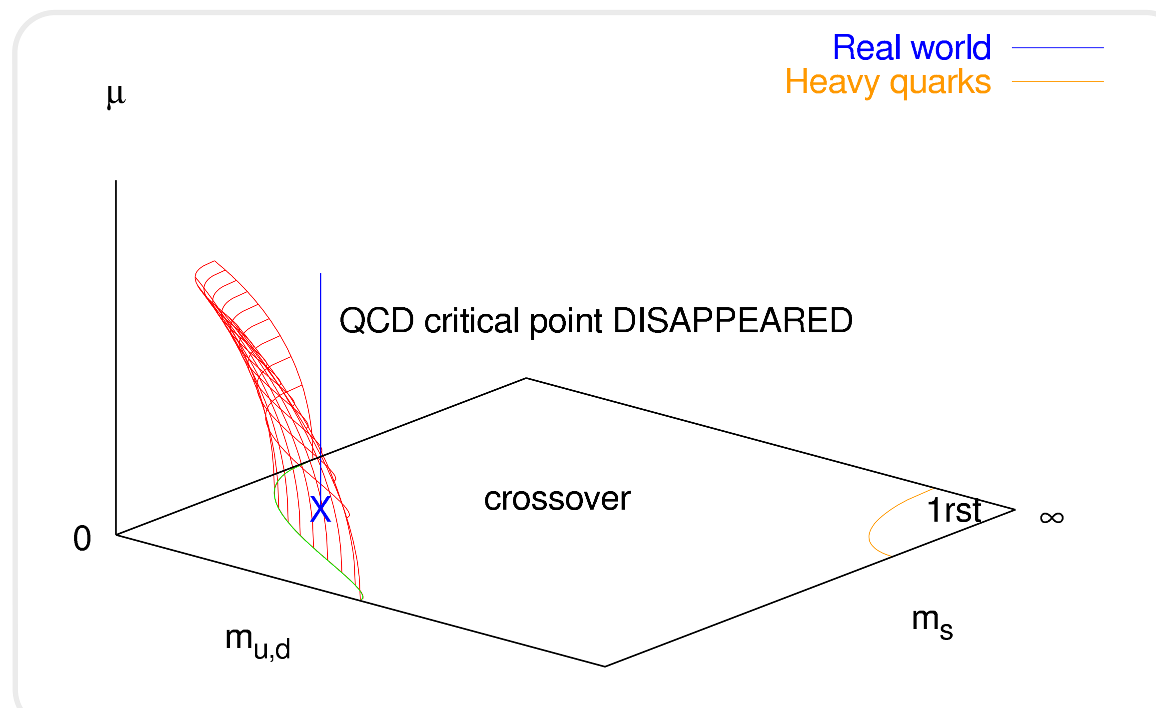
QCD phase transition at the physical point





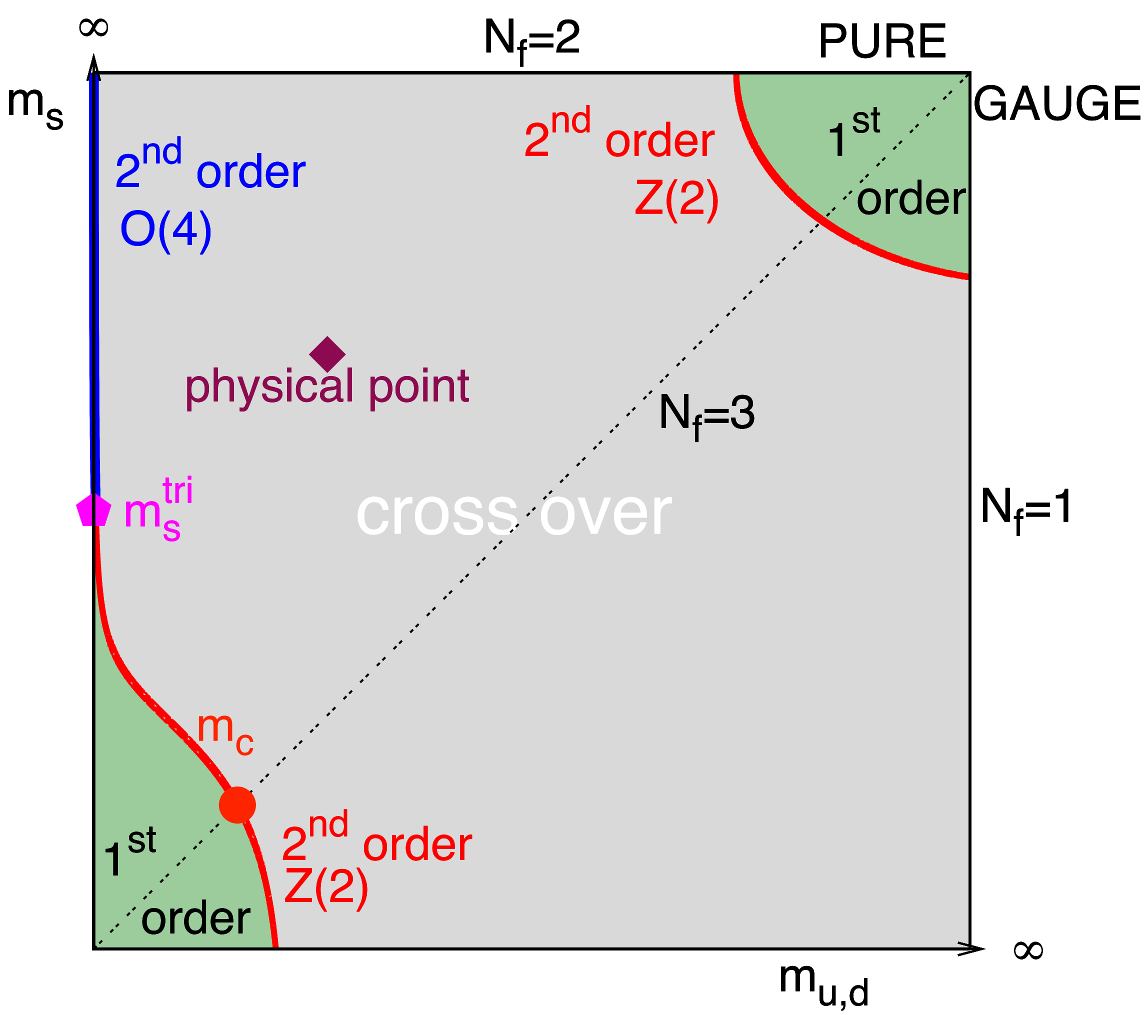


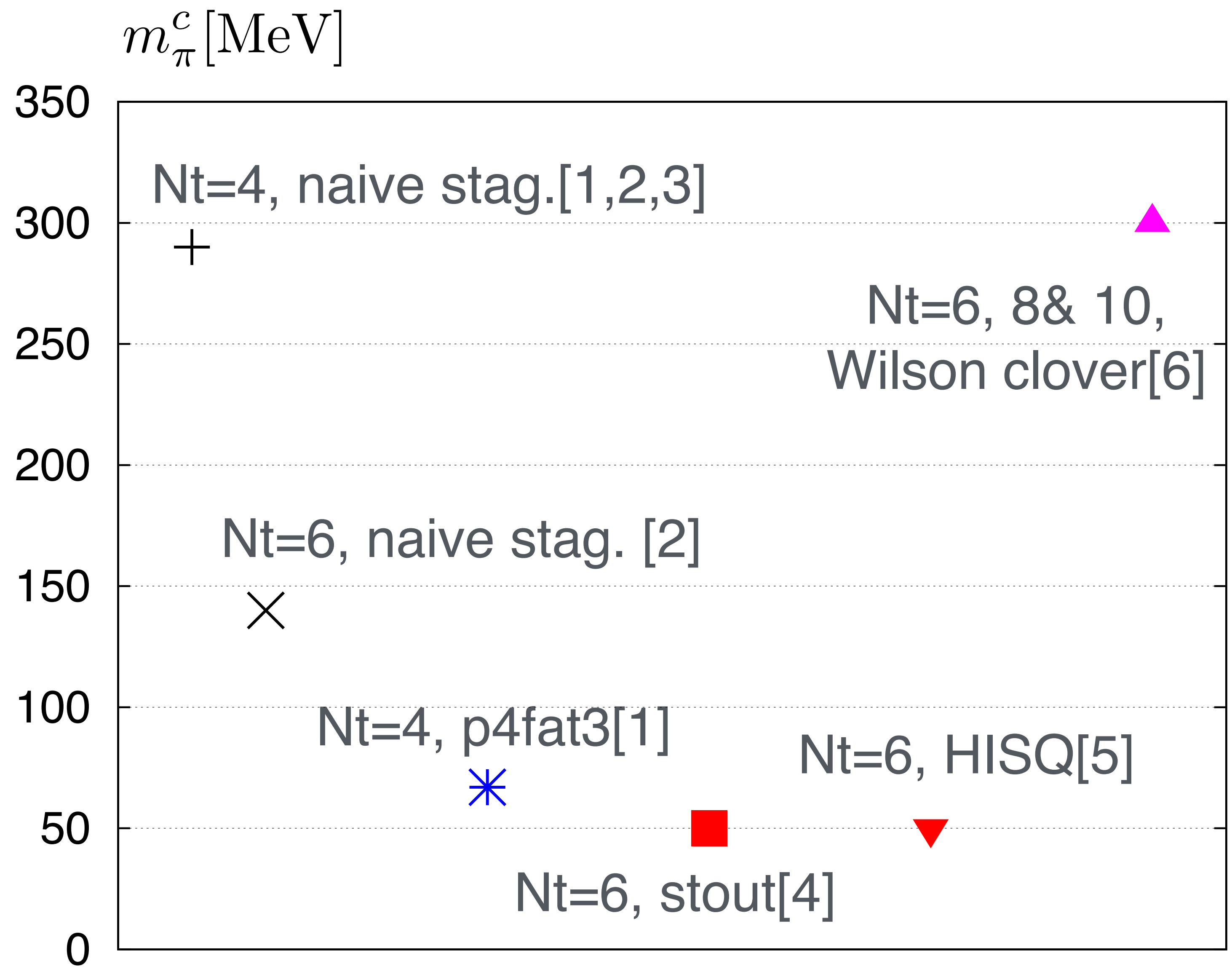
Karsch et al., ’03, Nakamura et al., 15’



de Forcrand & Philipsen, ’07

1st order chiral phase transition region



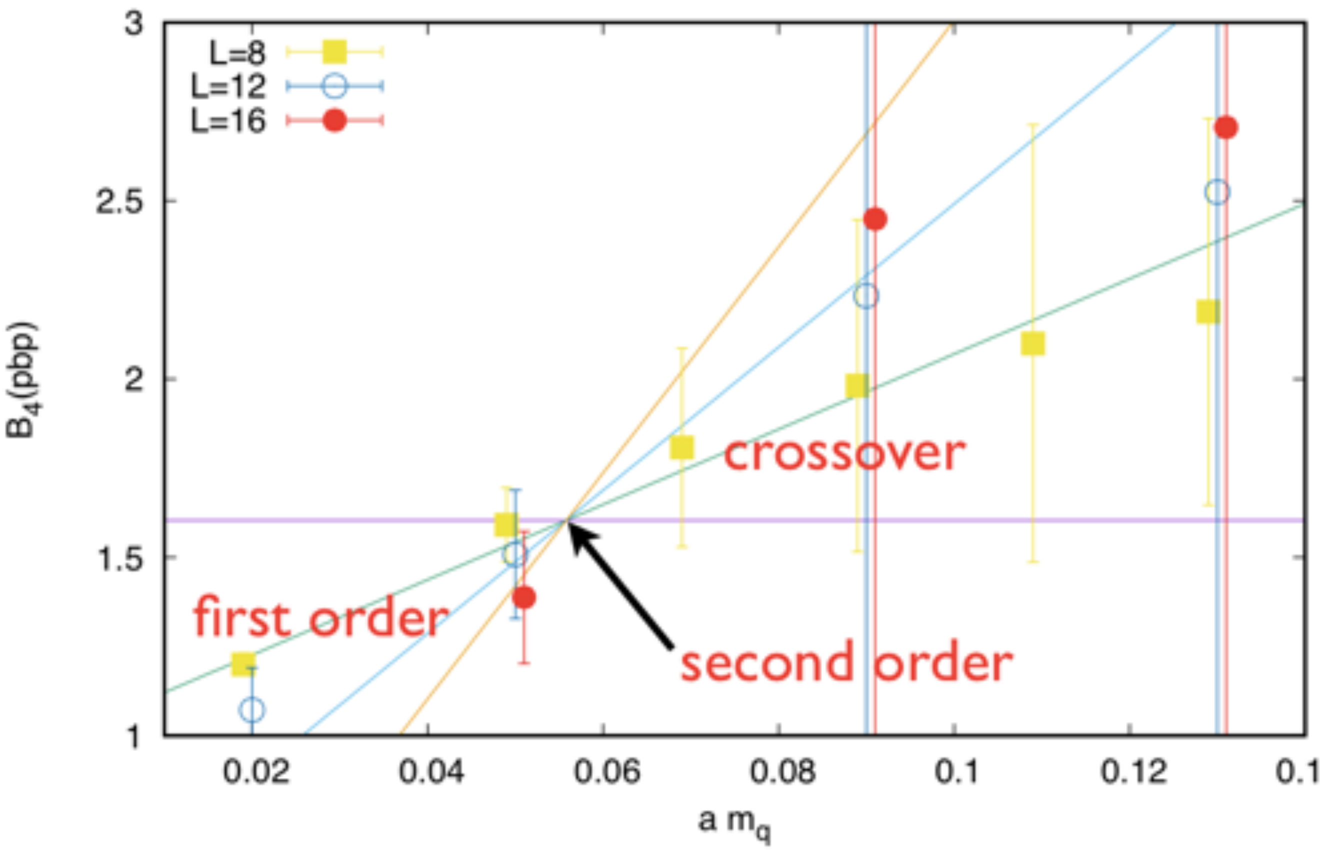


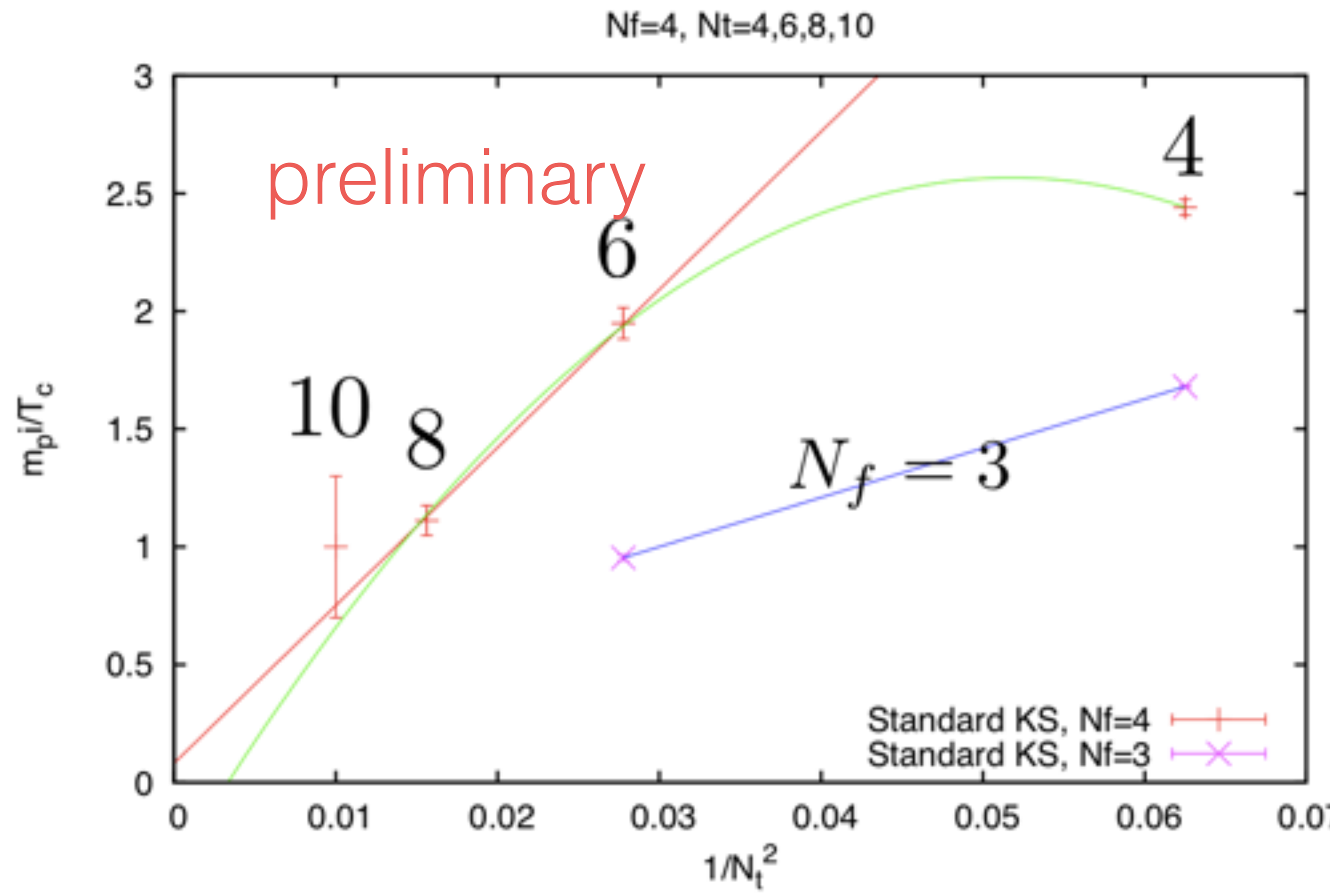
[1]F. Karsch et al., Nucl.Phys.Proc.Suppl. 129 (2004) 614 [2] P. de Forcrand et al, PoS LATTICE2007 (2007) 178
[3]D. Smith & C. Schmidt, Lattice 2011 [4]G. Endrodi et al., PoS LAT2007 (2007) 228
[5] HTD et al., Lattice 15', arXiv: 1511.00553 [6]Y. Nakamura, Lattice 15',PRD92 (2015) no.11, 114511

Rooting issue? Nf=4 staggered QCD

[Philippe De Forcrand, Monday]

Unimproved staggered fermions,
Nt=4, Nf=4





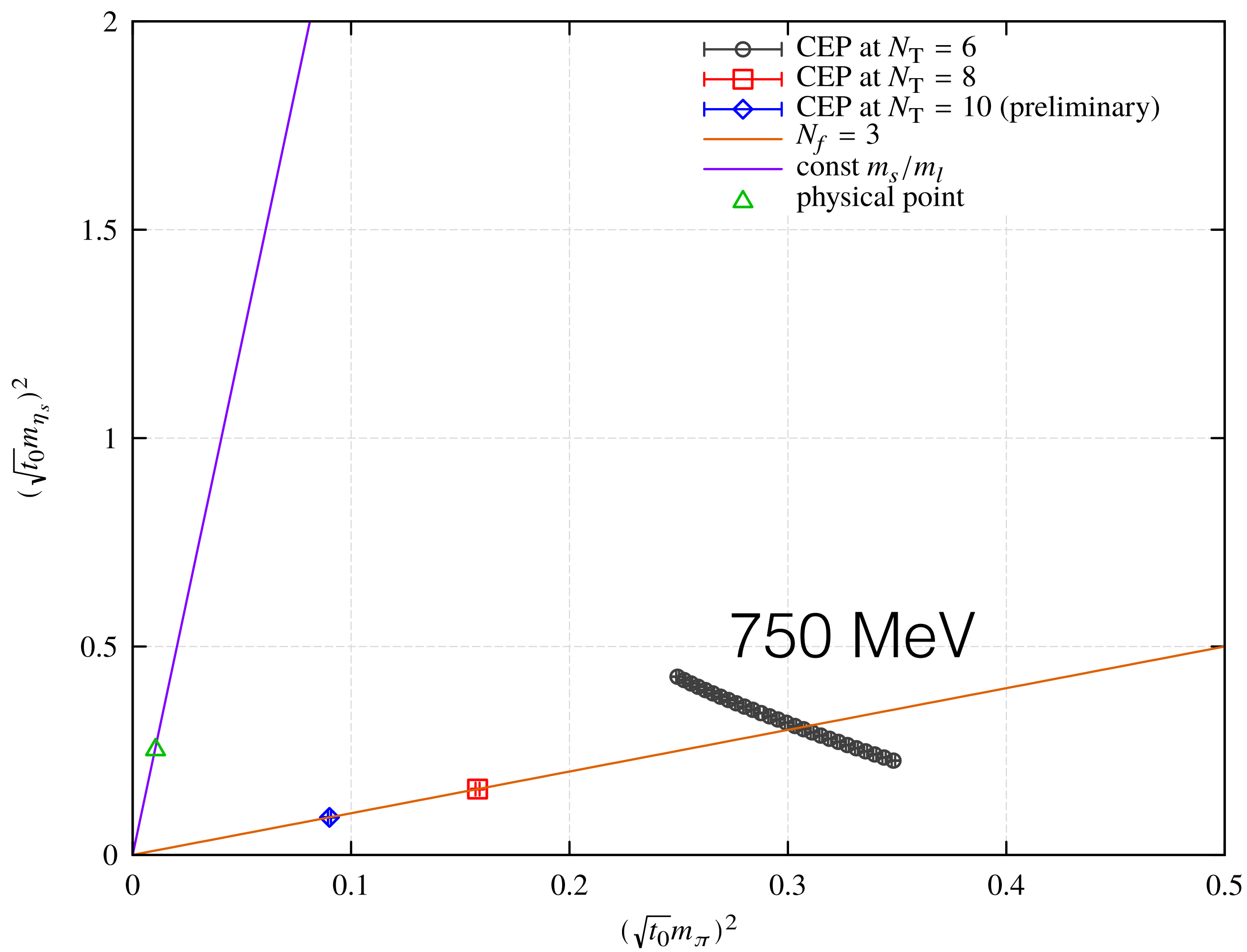
$$B_4(\bar{\psi}\psi) = 1.604$$

Nt↑ m_c ↓

$$N_t = 4 \ \rightarrow \ (am_q)_{\rm crit} = 0.055(2)$$

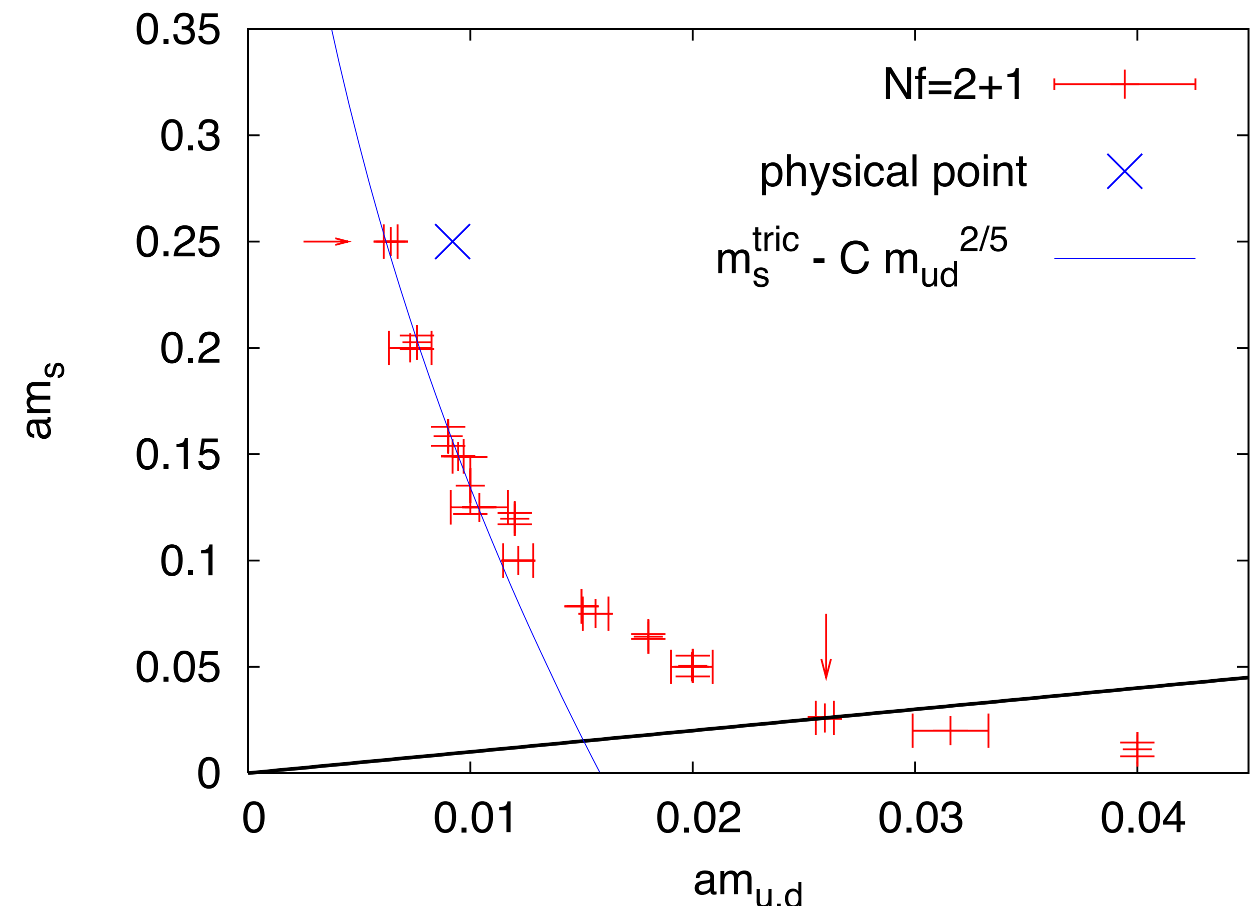
1st order chiral phase transition region

Nt=6, Wilson-Clover



[Yoshifumi Nakamura, Monday]

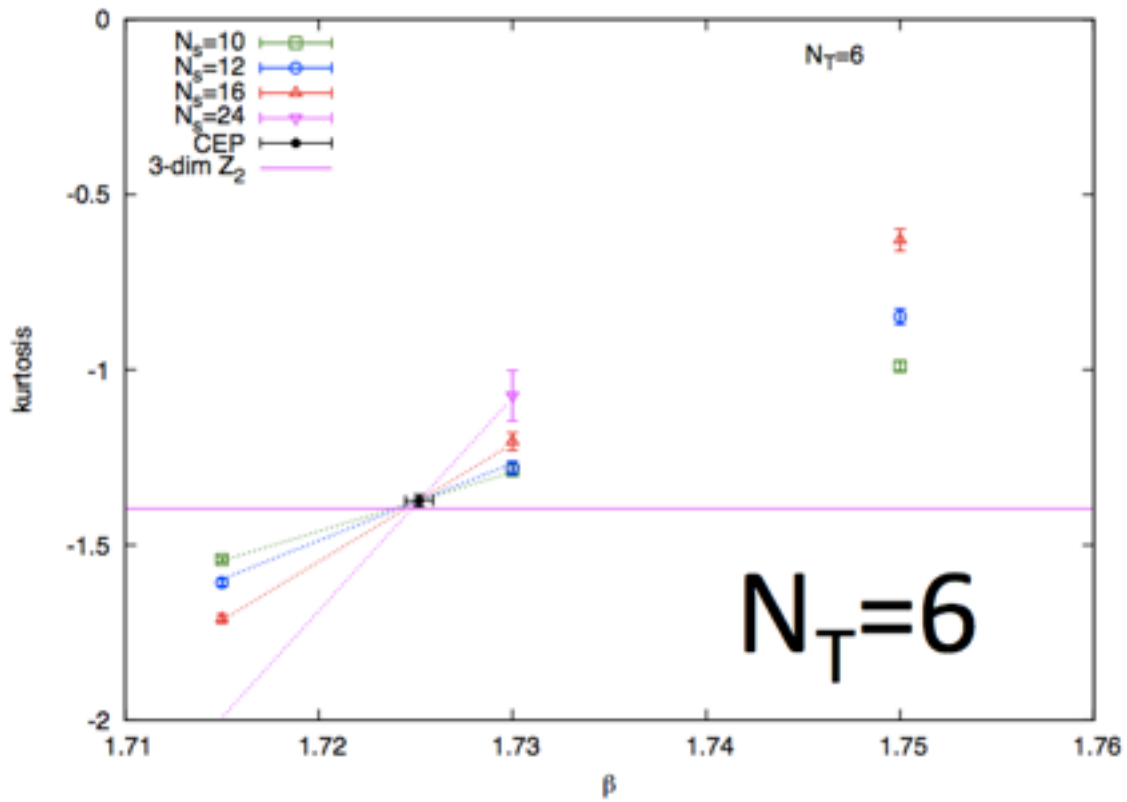
Nt=4, Naive staggered fermions

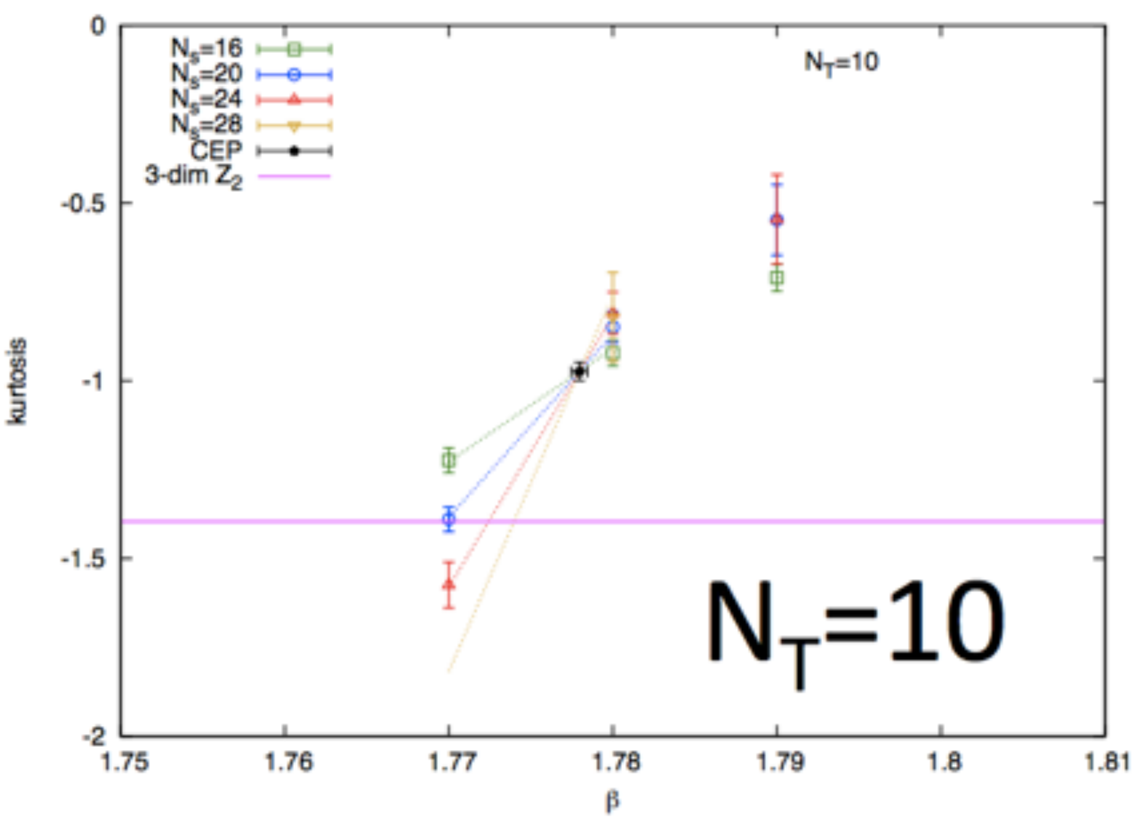


Philippe de Forcrand and Owe Philipsen, JHEP 0701 (2007) 077

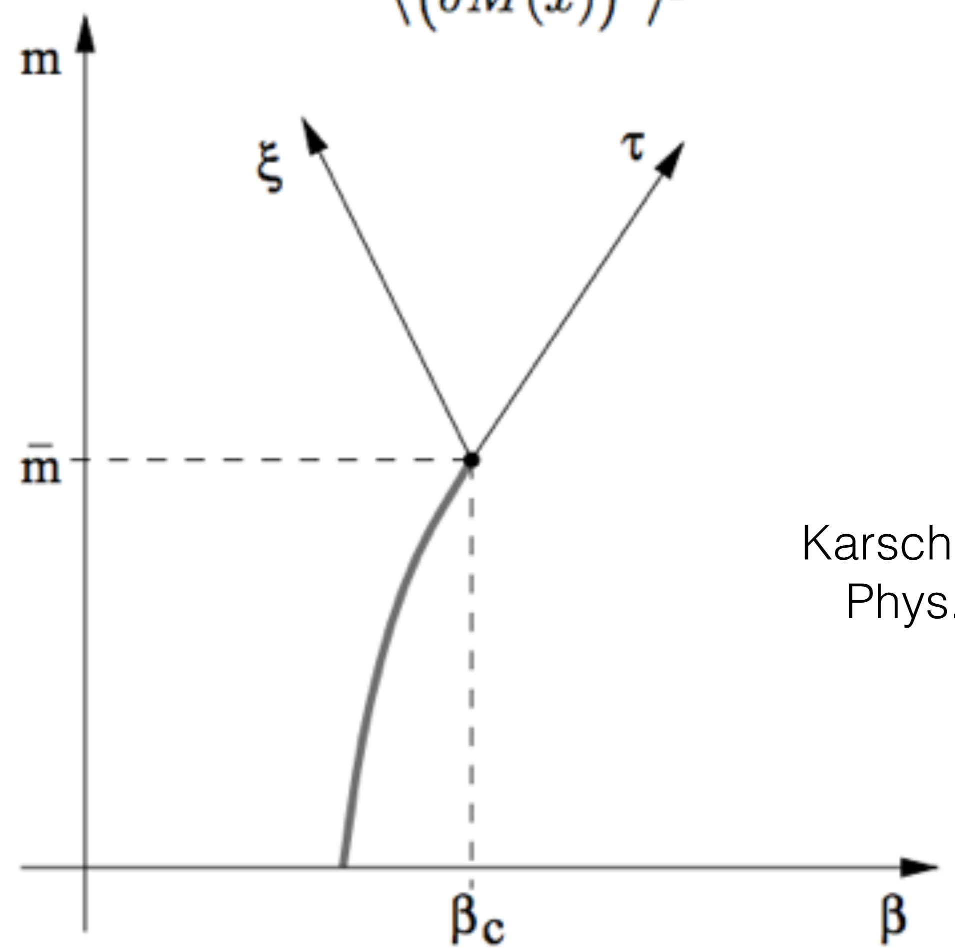
Location of the physical point:
Inconsistency between results from Wilson-clover and Naive staggered fermions on coarse lattices

Proper order parameter of chiral phase transition in Nf=3 QCD [Shinji Takeda, Tuesday]





$$B_4(x) = \frac{\langle(\delta M(x))^4\rangle}{\langle(\delta M(x))^2\rangle^2} \quad , \quad M(x) = \bar\psi\psi + x\, S_G$$



Karsch, Laermann & Schmidt
Phys.Lett. B520 (2001) 41

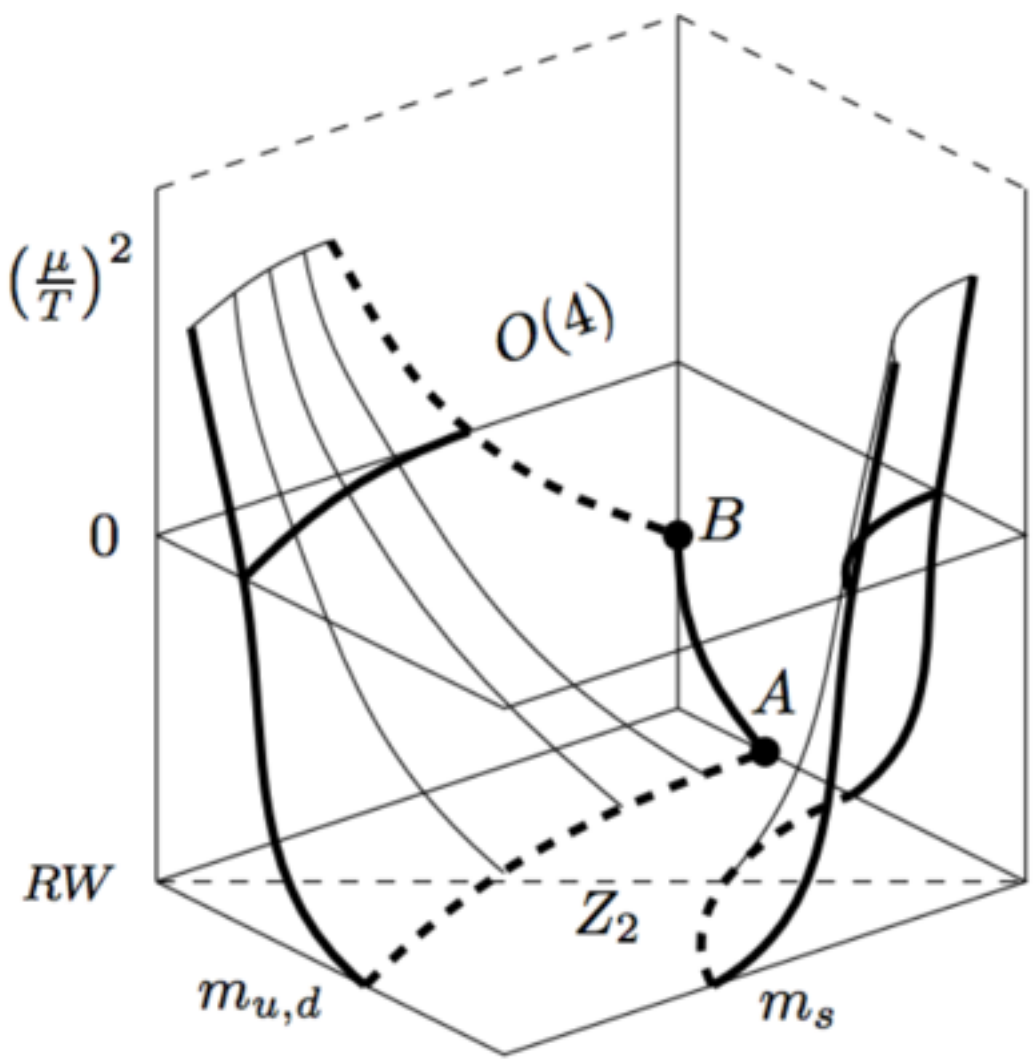
Cross point moves to the Z(2) critical line with:

$$K_{\mathcal{O}} = \left(K_{\rm E} + A N_L^{1/\nu}(\beta - \beta_{\rm CEP})\right)\left(1 + B N_L^{y_t - y_h}\right)$$

new term

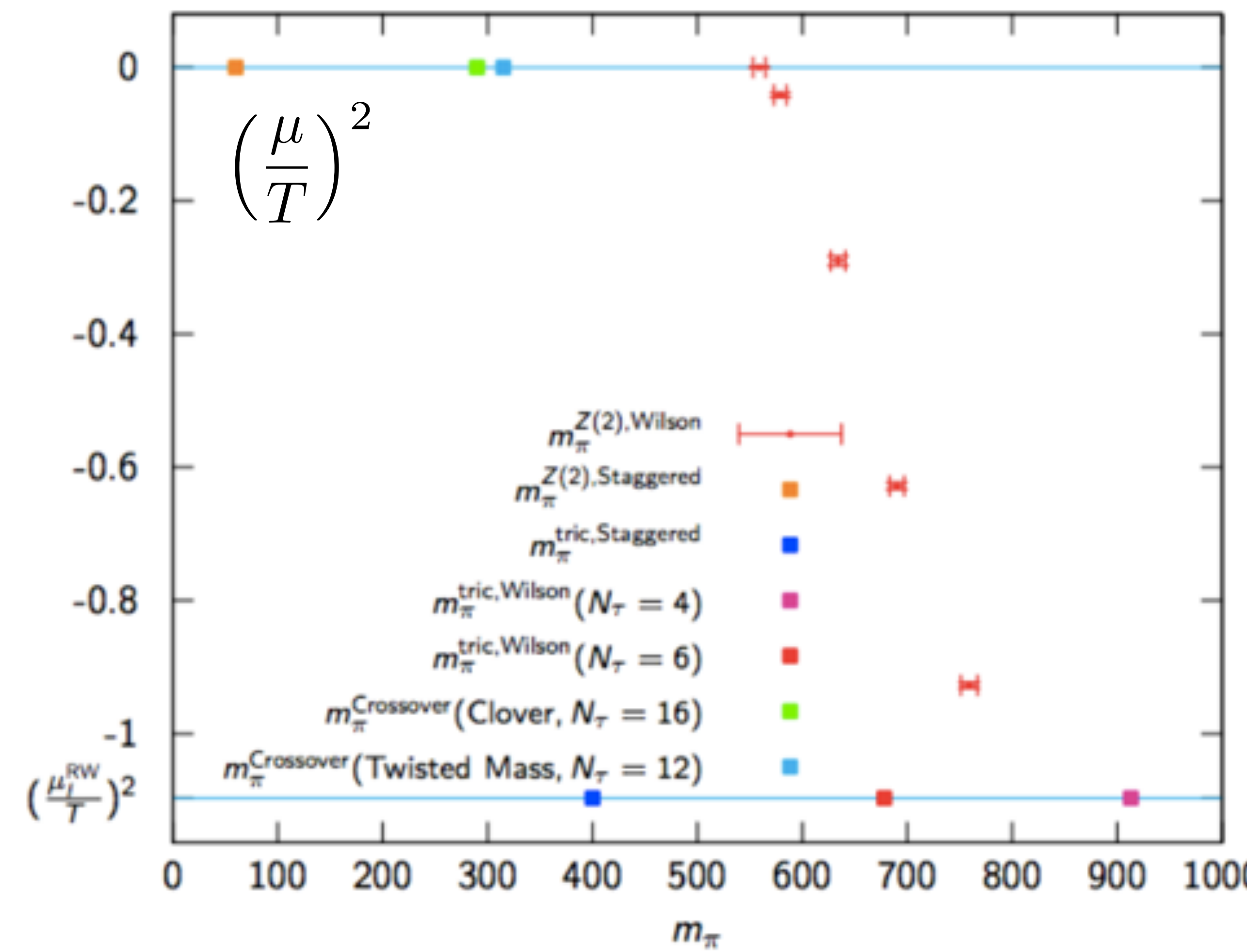
Estimate of critical end point at mu=0: analytical continuation from imaginary mu

[Alessandro Sciarra, Monday]



Bonati et al.,PRD90 7(2014) 074030

unimproved Wilson fermions, Nt=6

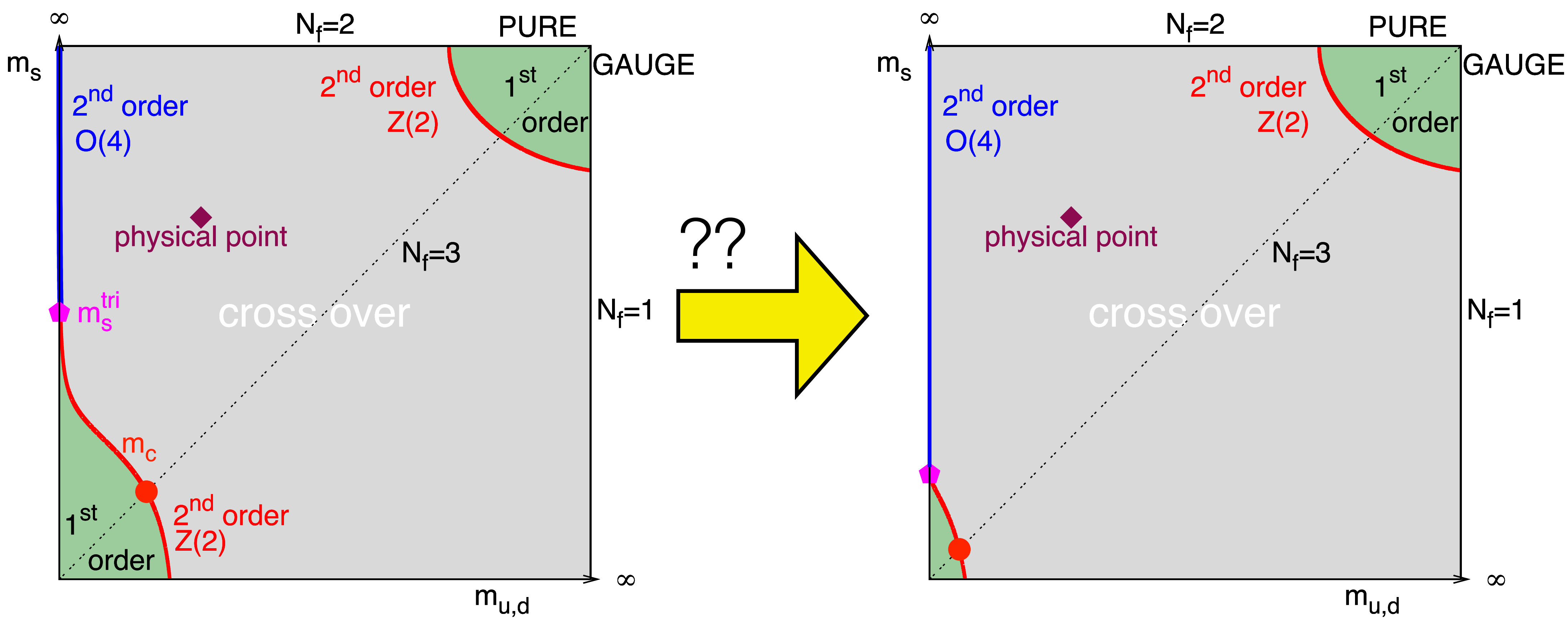


O. Philipsen and C. Pinke, PRD93 11(2016)114507

unimproved Wilson fermions: $m_\pi^c(\mu = 0, N_\tau = 4) \approx 560 \text{ MeV}$

estimate: $m_\pi^c(\mu = 0, N_\tau = 6) \sim 400 \text{ MeV}$

Chiral phase transition region in Nf=3 QCD

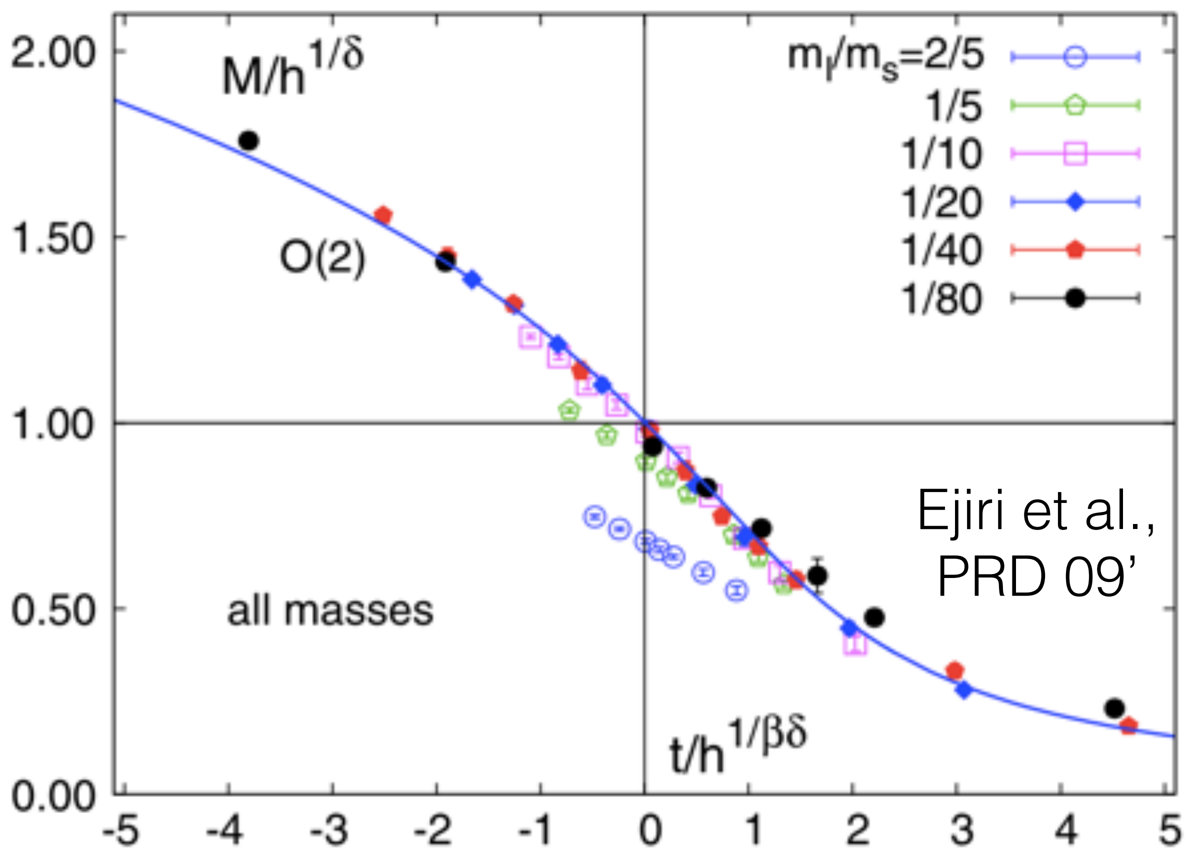


Whether the 1st order chiral phase transition is relevant for the physical point at all?

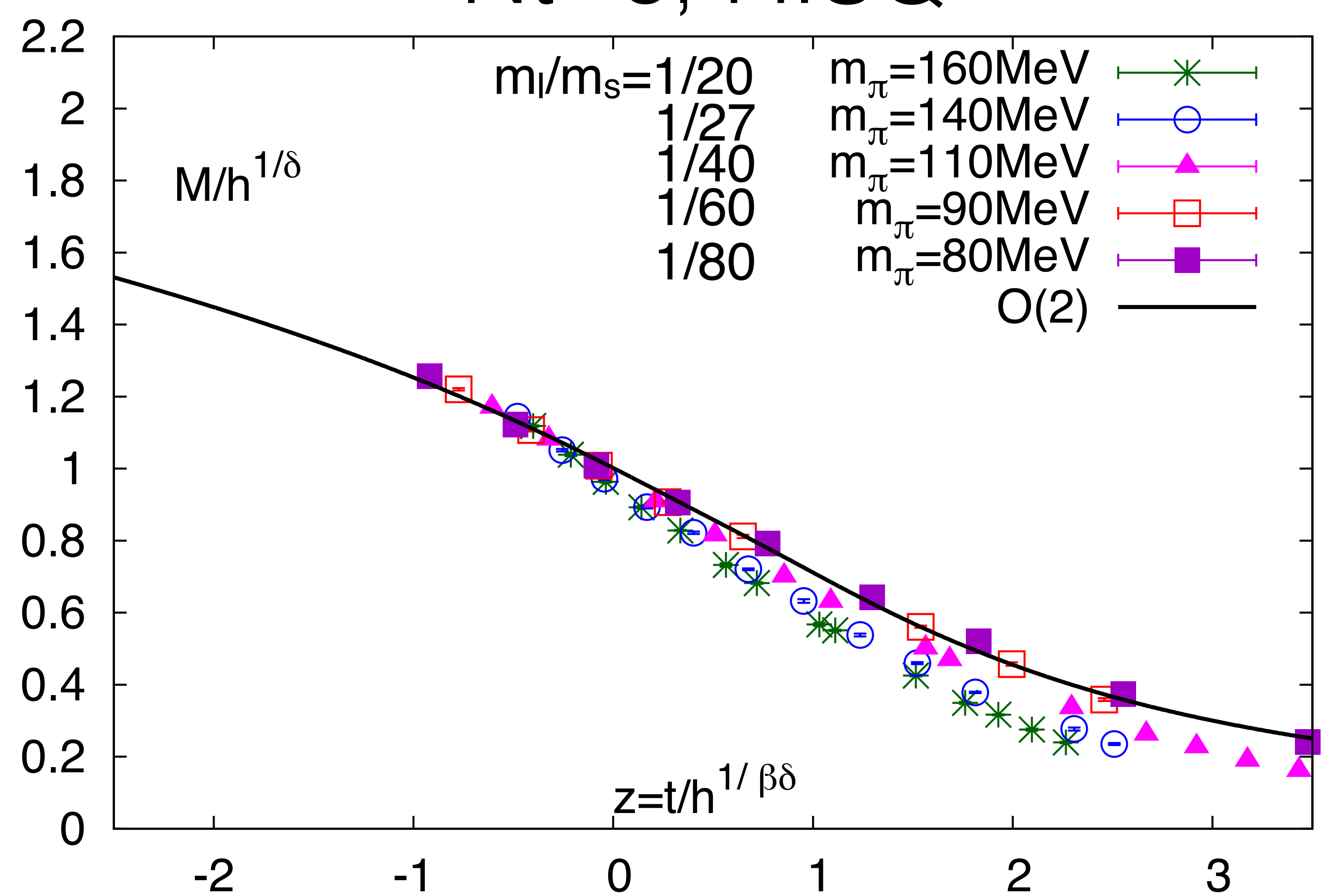
Universal behavior of chiral phase transition in Nf=2+1 QCD

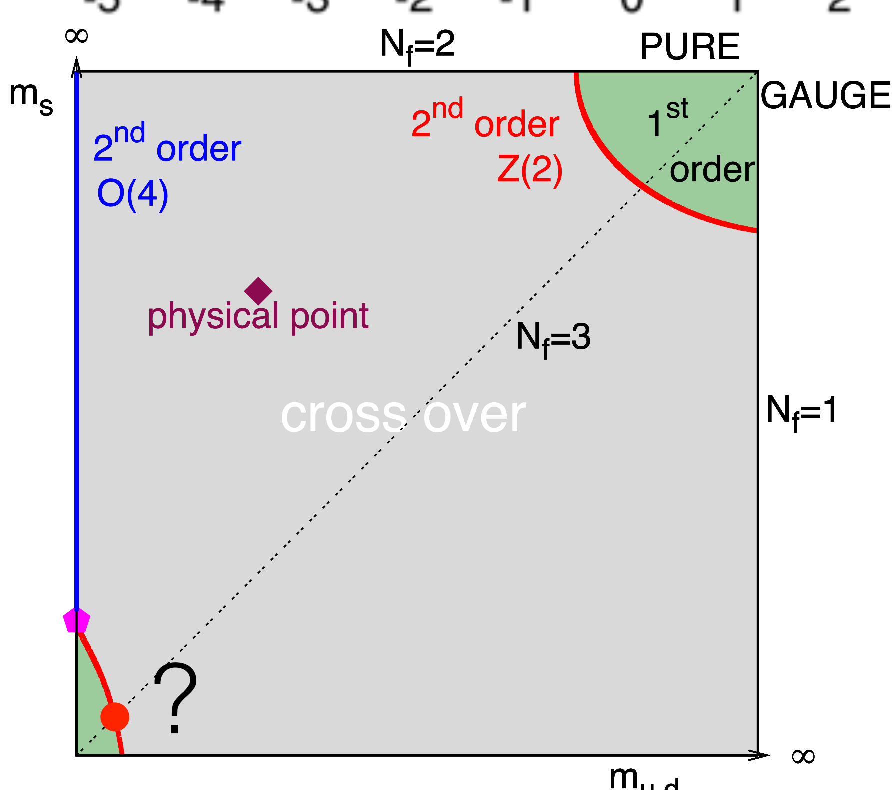
[Sheng-Tai Li, Wednesday]

Nt=4, p4fat3



Nt=6, HISQ

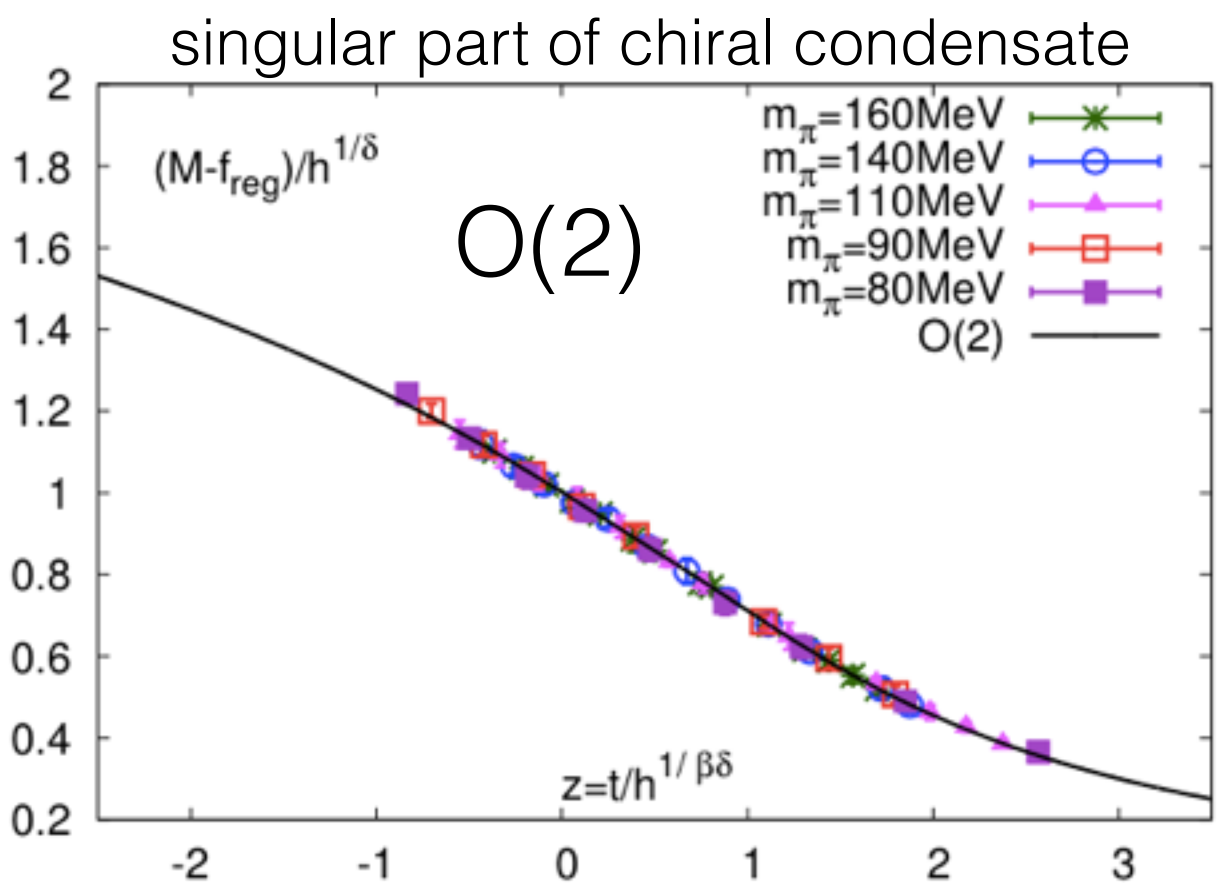


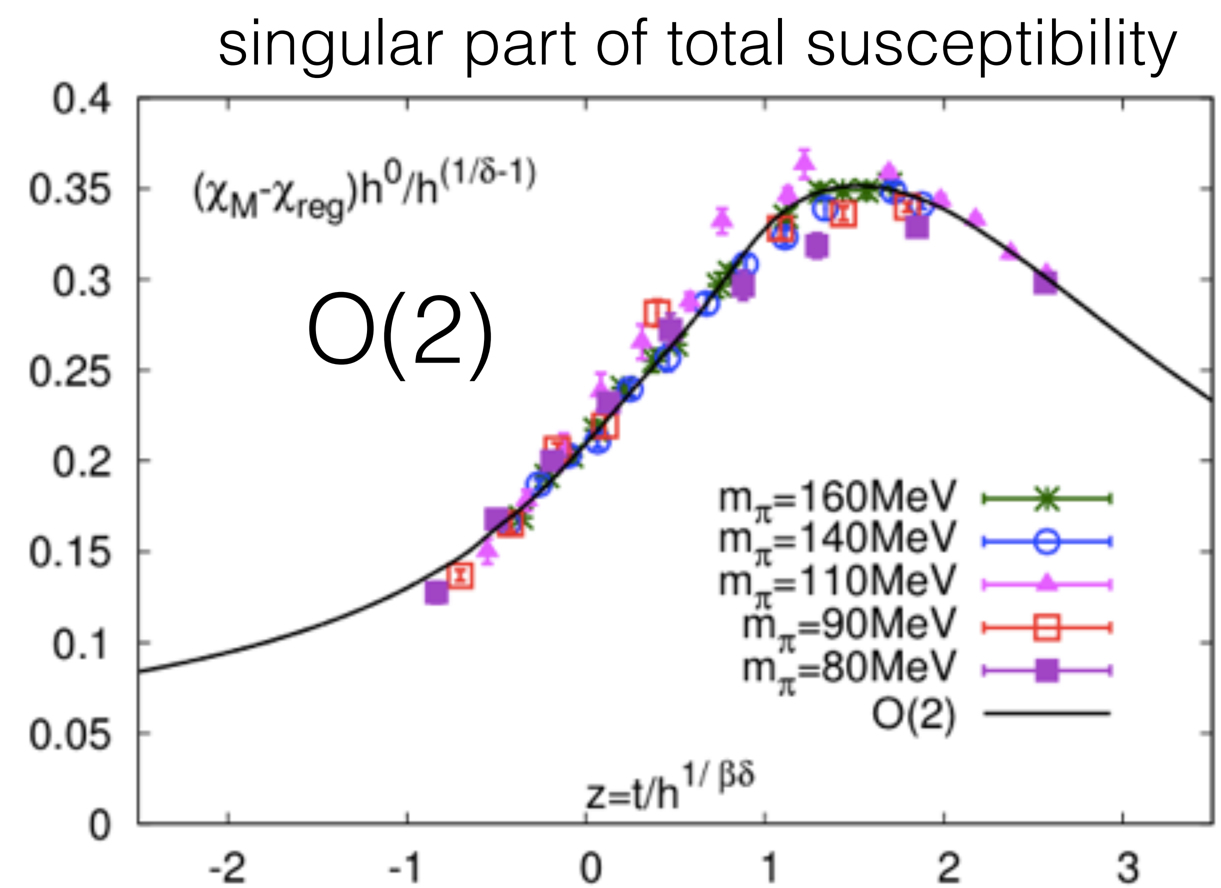


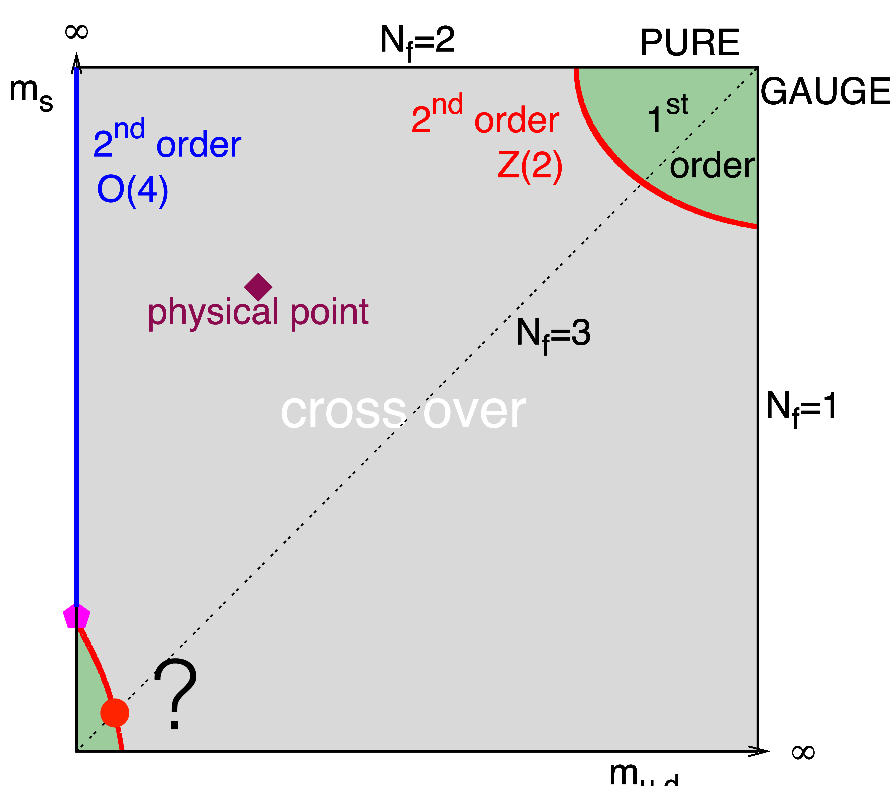
Scaling window becomes smaller from Nt=4 p4fat3 to Nt=6 HISQ results

Universal behavior of chiral phase transition in Nf=2+1 QCD

Nt=6, HISQ [Sheng-Tai Li, Wednesday]





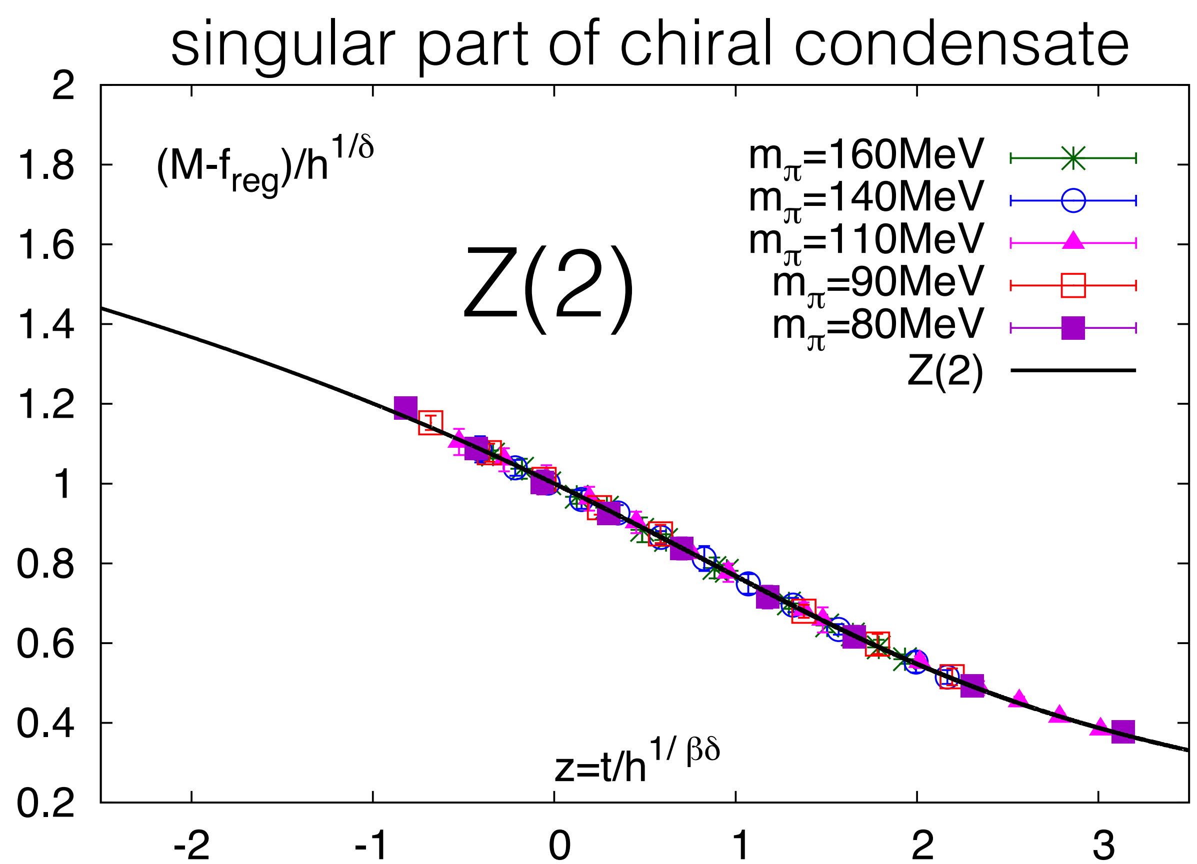


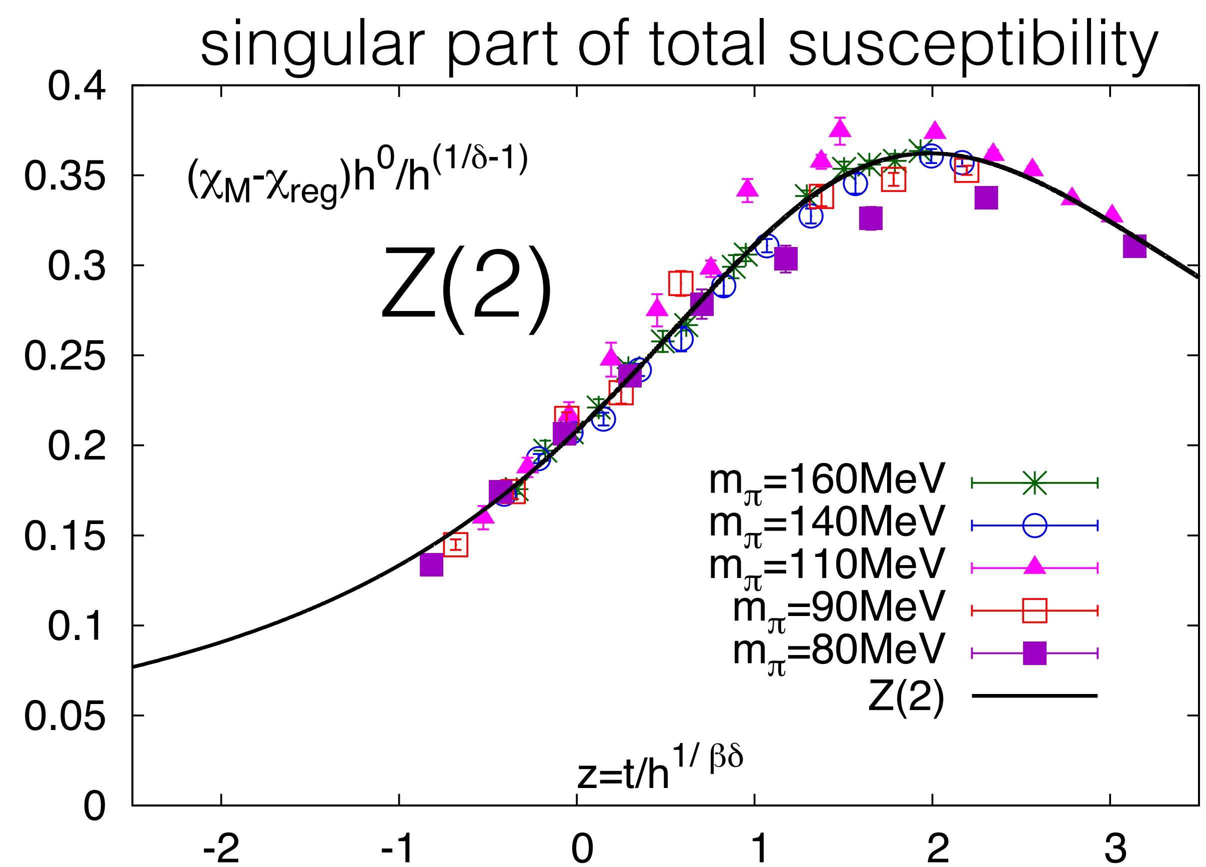
Best evidence of the O(2) scaling

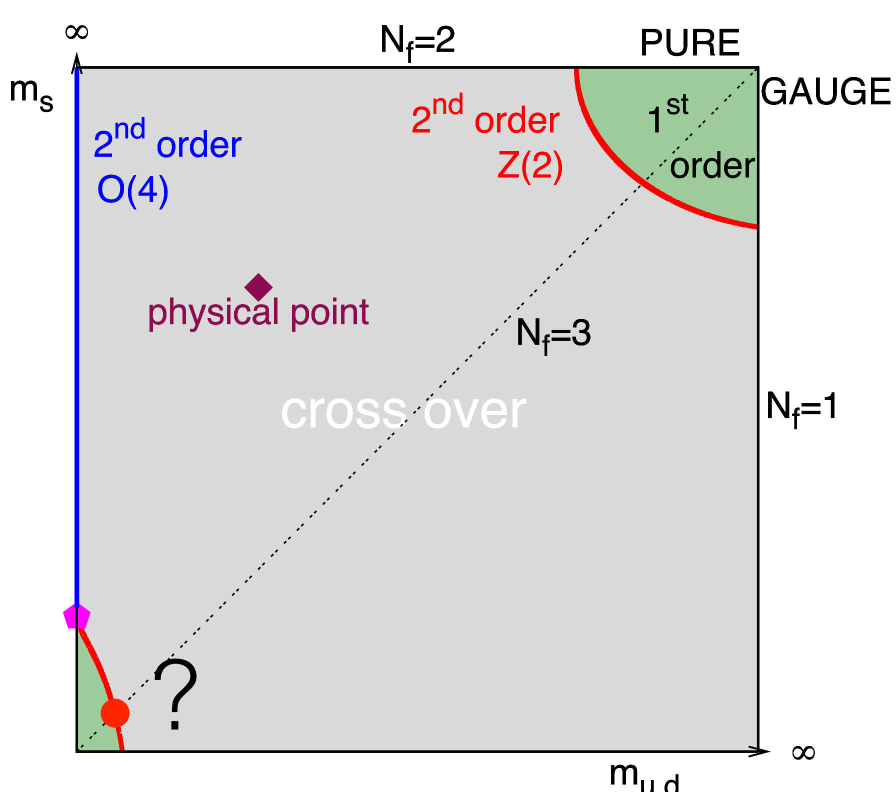
Universal behavior of chiral phase transition in Nf=2+1 QCD

Nt=6, HISQ

[Sheng-Tai Li, Wednesday]







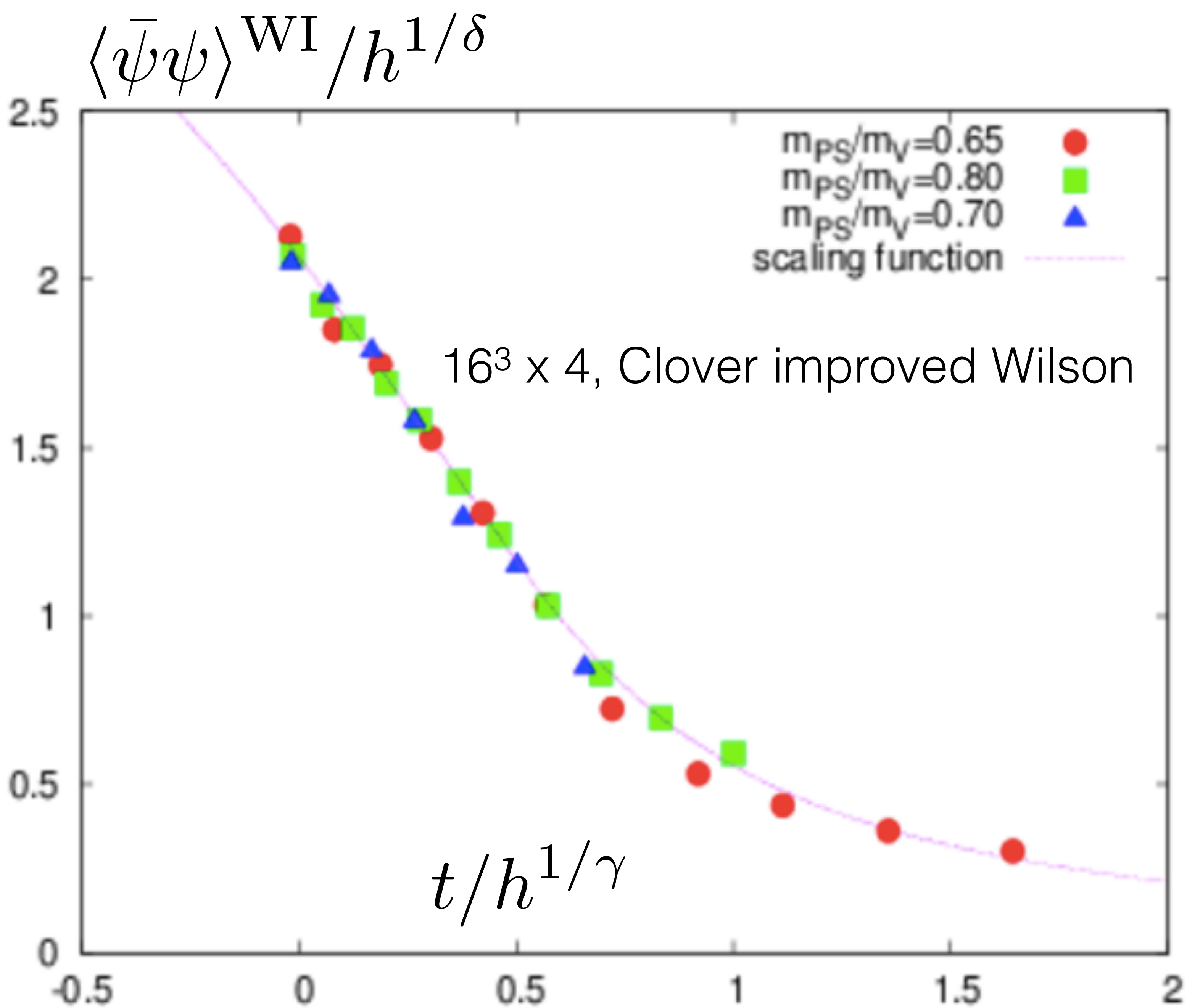
symmetry breaking field: ml - mc

mc ≈ 0 from Z(2) scaling analysis

$$m^s_{phy} > m^s_{tri}$$

O(4) scaling behavior in Nf=2 QCD

[Takashi Umeda, Wednesday]



Chiral condensate defined from Ward identity

Consistent with O(4) scaling

Indication of a 2nd order phase transition in the massless two flavor QCD

fluctuation of Goldstone modes at T<Tc ?

See similar conclusion from the many flavor approach:
Ejiri, Iwami & Yamada, 1511.06126, PRL 110(2013)no. 17, 172001

role of $U_A(1)$ symmetry in Nf=2 QCD

$U_A(1)$ symmetry:

- restored, 1st or 2nd order ($U(2)_L \otimes U(2)_R / U(2)_V$)
- broken, 2nd order (O(4)) phase transition

Pisarski and Wilczek, PRD 29(1984)338

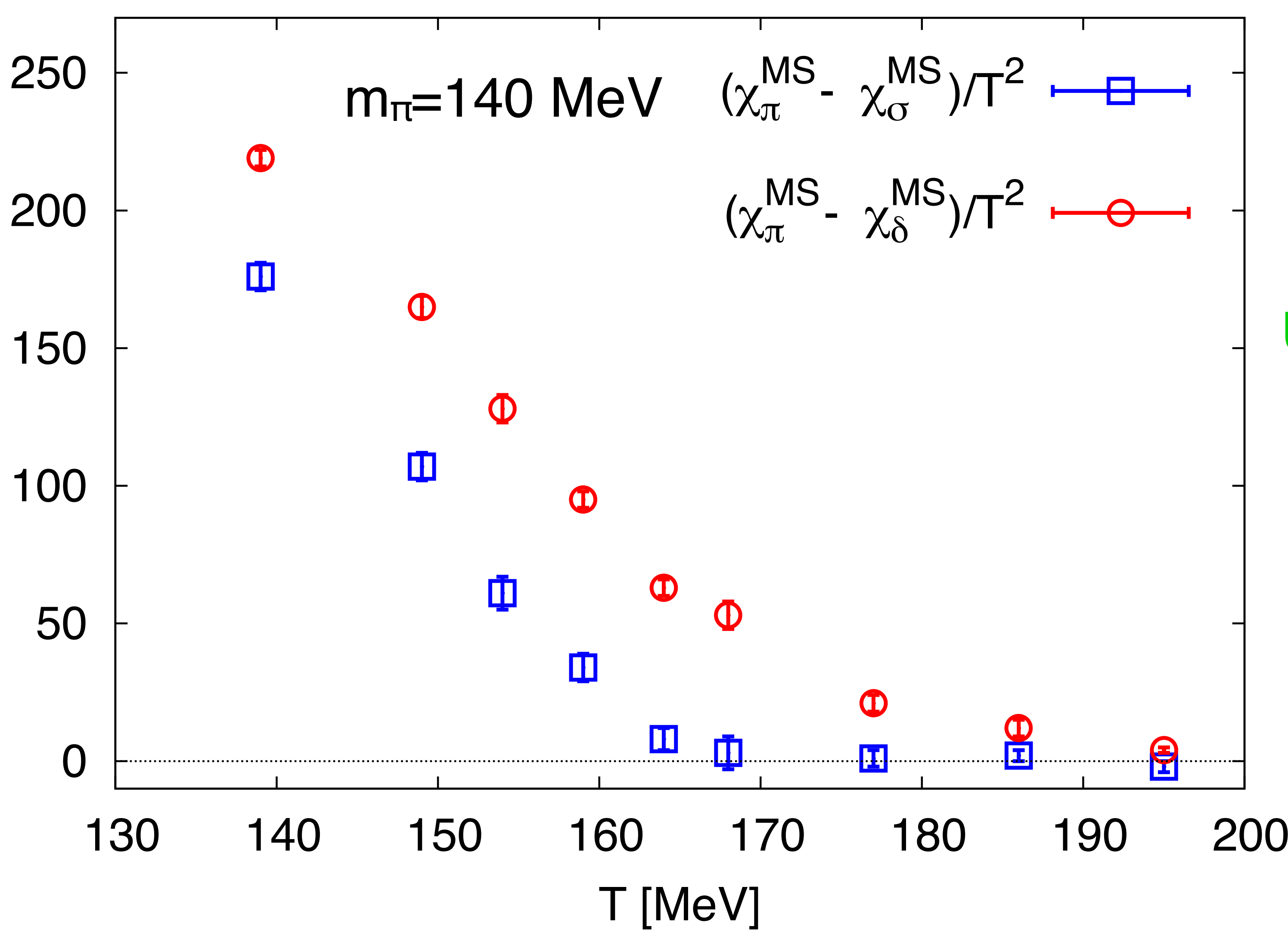
Butti, Pelissetto and Vicar, JHEP 08 (2003)029

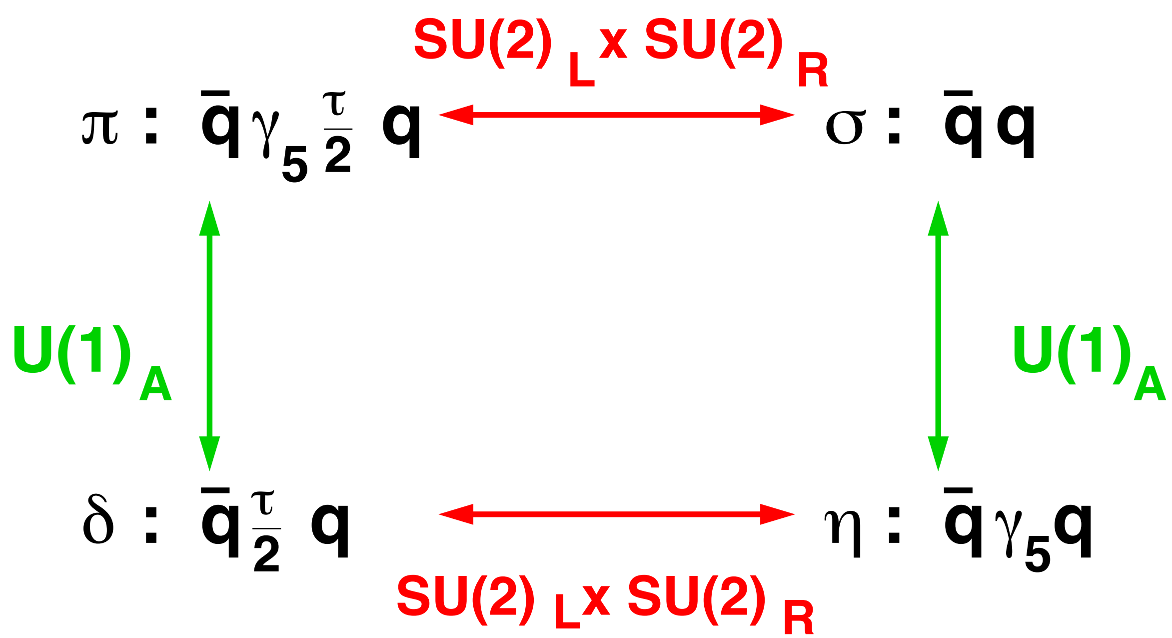
$U_A(1)$ symmetry on the lattice:

- always broken in the Wilson/ Staggered discretization scheme

Fate of chiral symmetries at T=/=0: Nf=2+1 QCD

Domain Wall fermions, 32^3x8, Ls=24,16





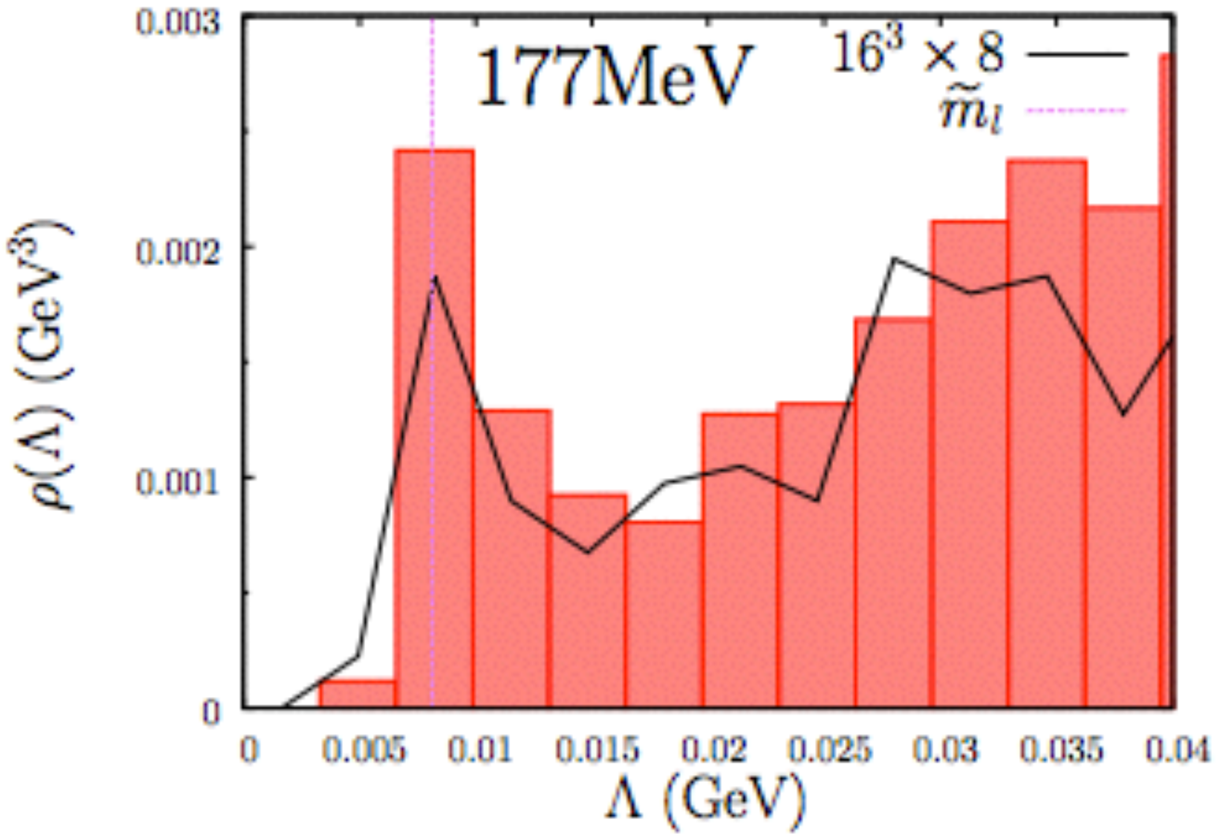
At the physical point, U(1)$_A$ does not restore at $T_{\chi SB}$~170 MeV, remains broken up to 195 MeV ~ 1.16$T_{\chi SB}$

HotQCD&RBC/LLNL, PRL 113 (2014) 082001,PRD 89 (2014) 054514

Underlying mechanism of $U_A(1)$ breaking

Dirac Eigenvalue spectrum:

black lines: 16^3 lattices;
red histograms: 32^3 lattices



Chirality distribution

\# of configurations with N_0 and N_+

32^3x8, T=177 MeV

$N_+ \backslash N_0$	0	1	2	3	4	5
$N_0 = 1$	40	29	-	-	-	-
$N_0 = 2$	11	20	12	-	-	-
$N_0 = 3$	3	11	6	2	-	-
$N_0 = 4$	0	1	2	1	0	-
$N_0 = 5$	0	2	0	0	0	0

N_0: total # of near zero modes

N_+: # of near zero modes with positive chirality

- Density of near zero modes prefers to be independent of V rather than to shrink with 1/sqrt($32^3/16^3$)
- Chirality distribution shows a binomial distribution more than a bimodal one

HotQCD&RBC/LLNL, PRL 113 (2014) 082001,PRD 89 (2014) 054514

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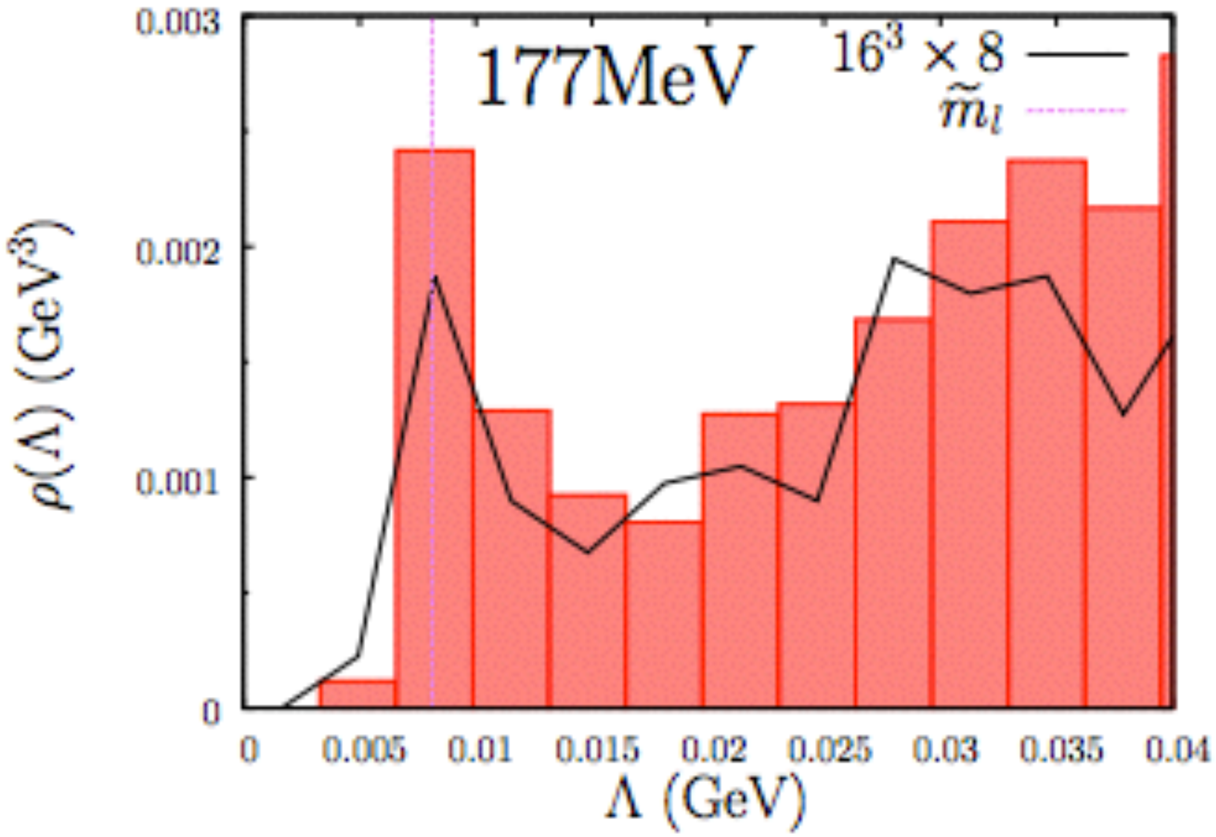

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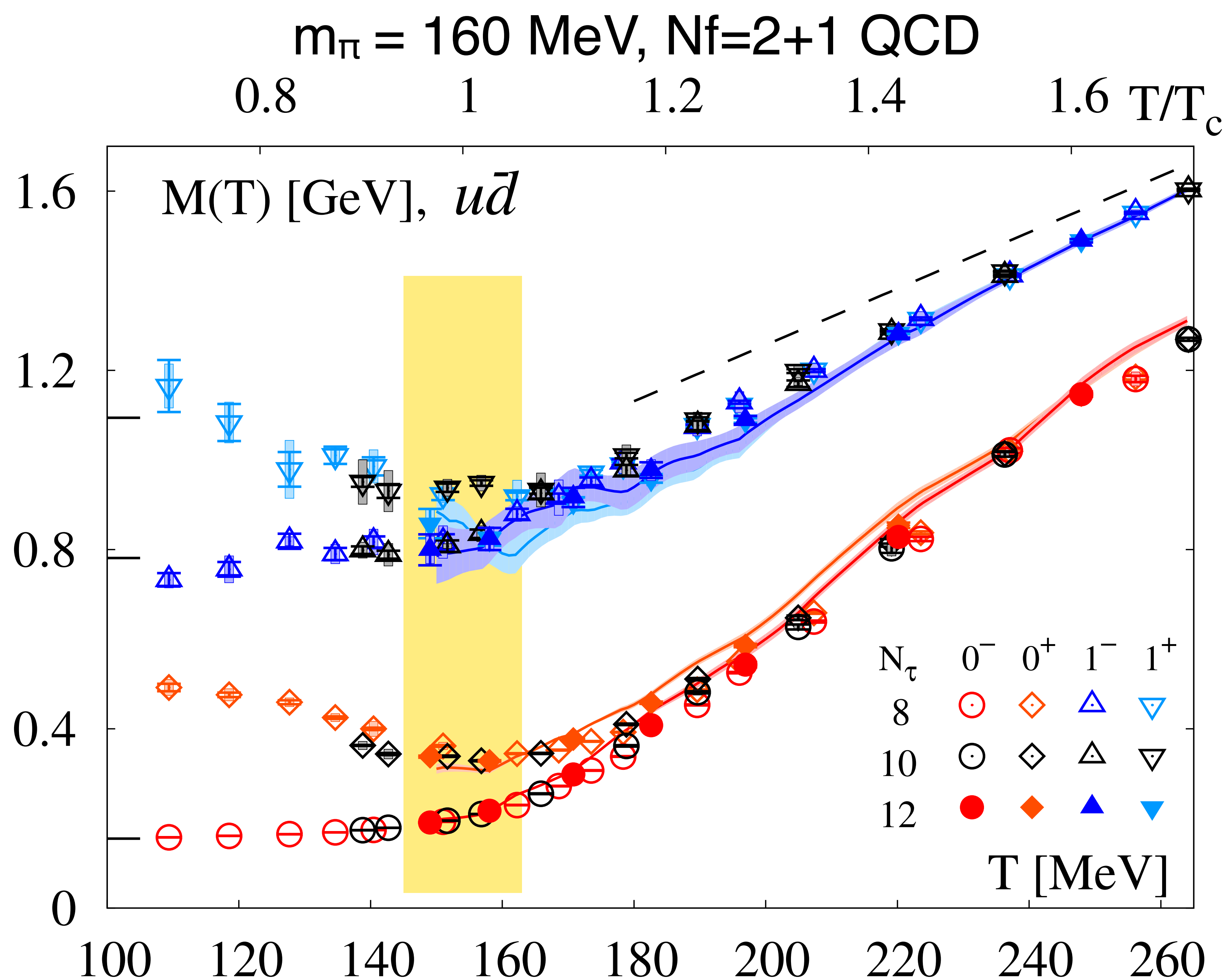
- C
bimodal one

A dilute instanton gas model can describe the non-zero $U_A(1)$ breaking above T_c !

HotQCD&RBC/LLNL, PRL 113 (2014) 082001, PRD 89 (2014) 054514

Fate of chiral symmetries from HISQ calculations

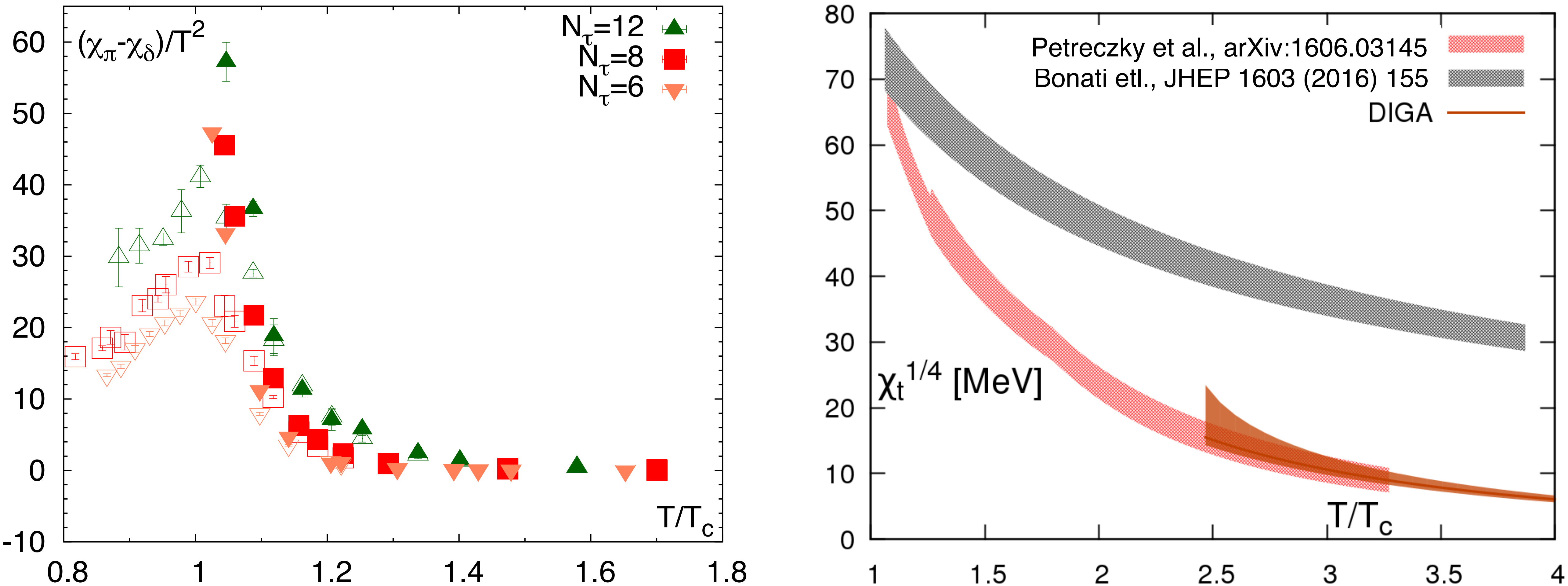


courtesy of Yu Maezewa,
YITP, work in progress

Indication of the breaking of $U_A(1)$ symmetry up to ~1.2 Tc in the continuum limit at m_π = 160 MeV

$U_A(1)$ symmetry from HISQ calculations

m_π = 160 MeV, Nf=2+1 QCD



Petreczky, Schadler & Sharma, arXiv:1606.03145

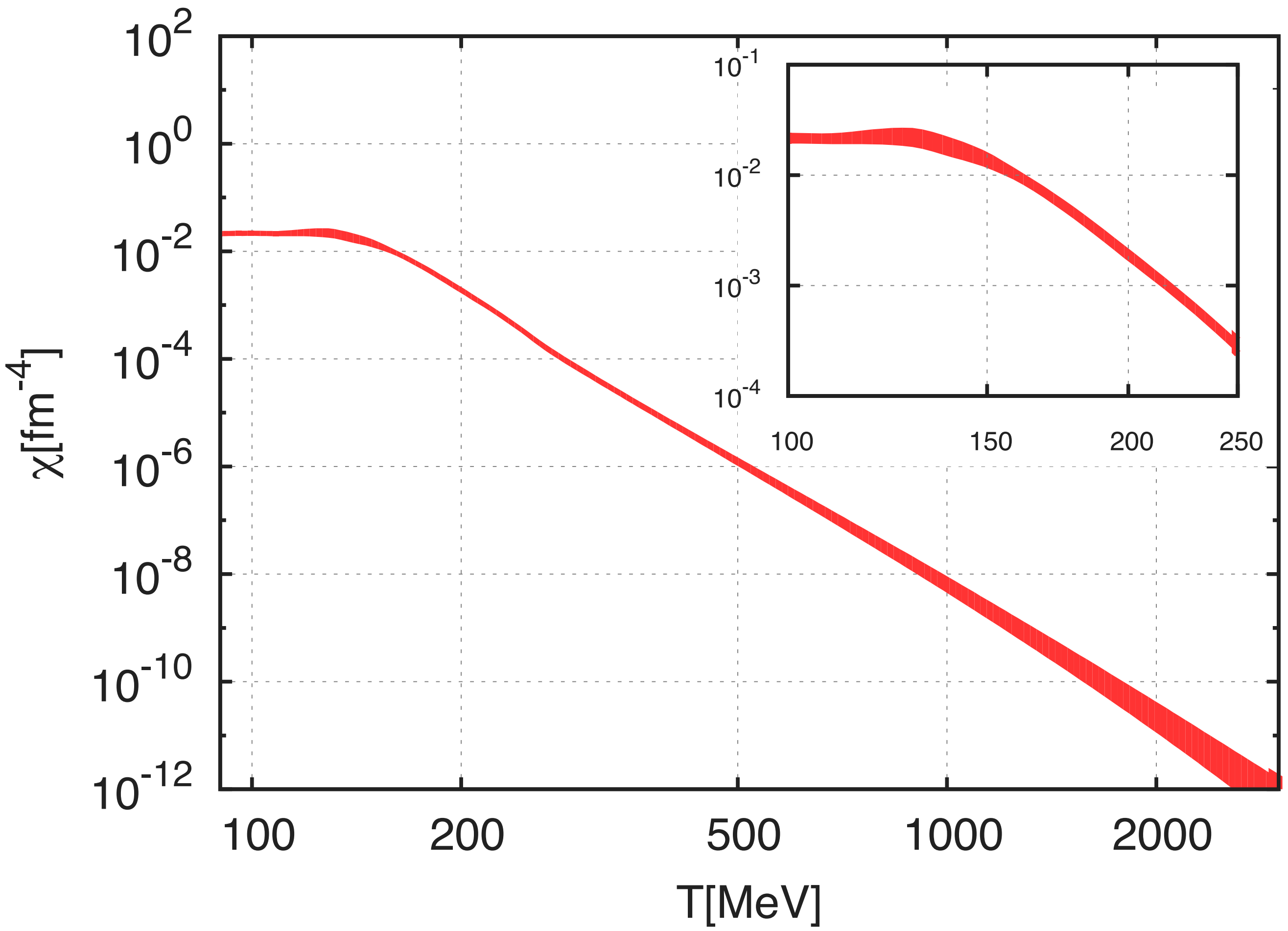
Indication of the breaking of $U_A(1)$ symmetry up to ~1.2 Tc in the continuum limit at m_π = 160 MeV

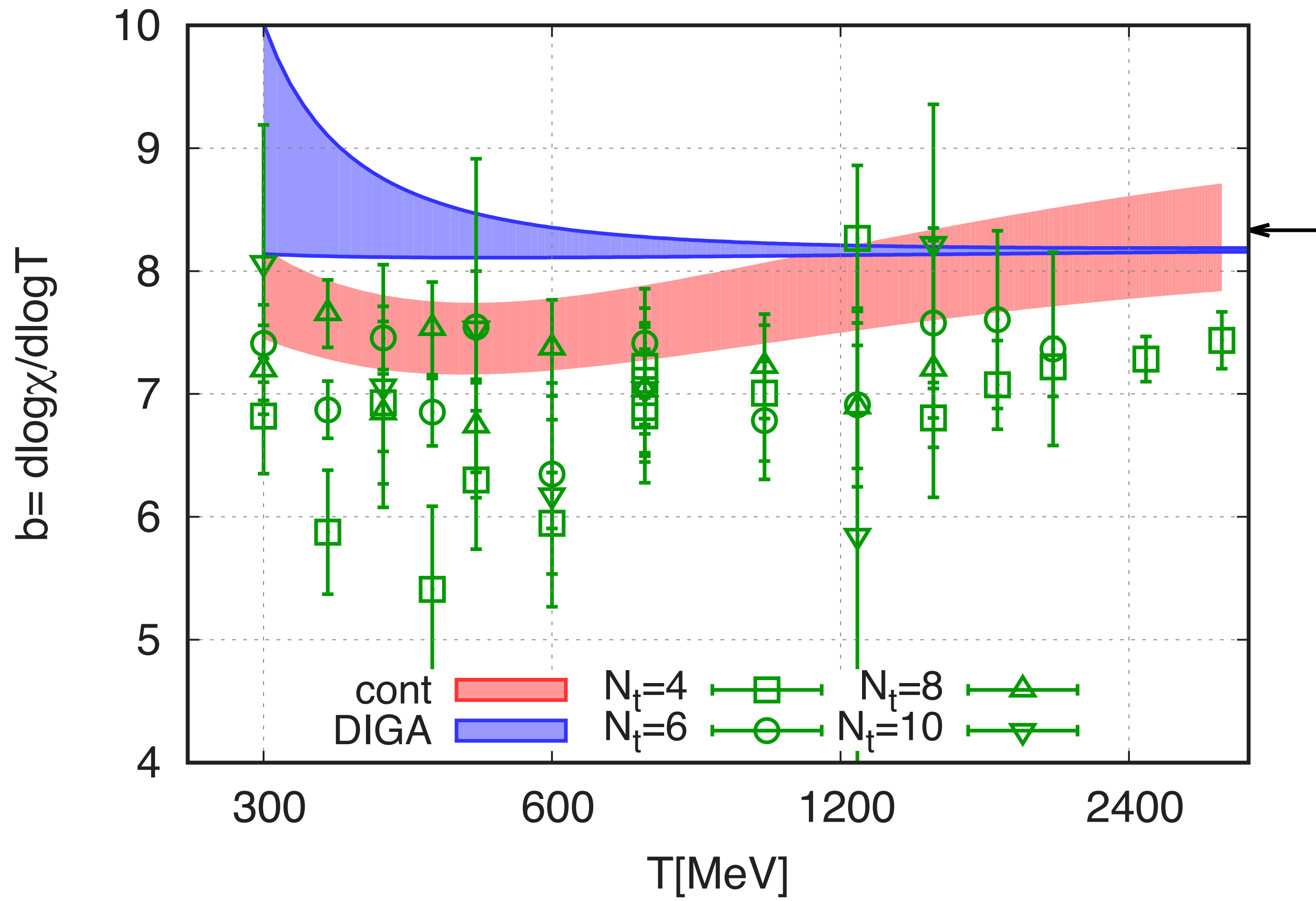
See similar conclusions from measurements of overlap operators on DWF, S. Sharma, lattice 2015, arXiv: 1510.03930

Topological susceptibility up to very high T

[Kalman Szabo , Monday]

4stout, continuum extrapolated



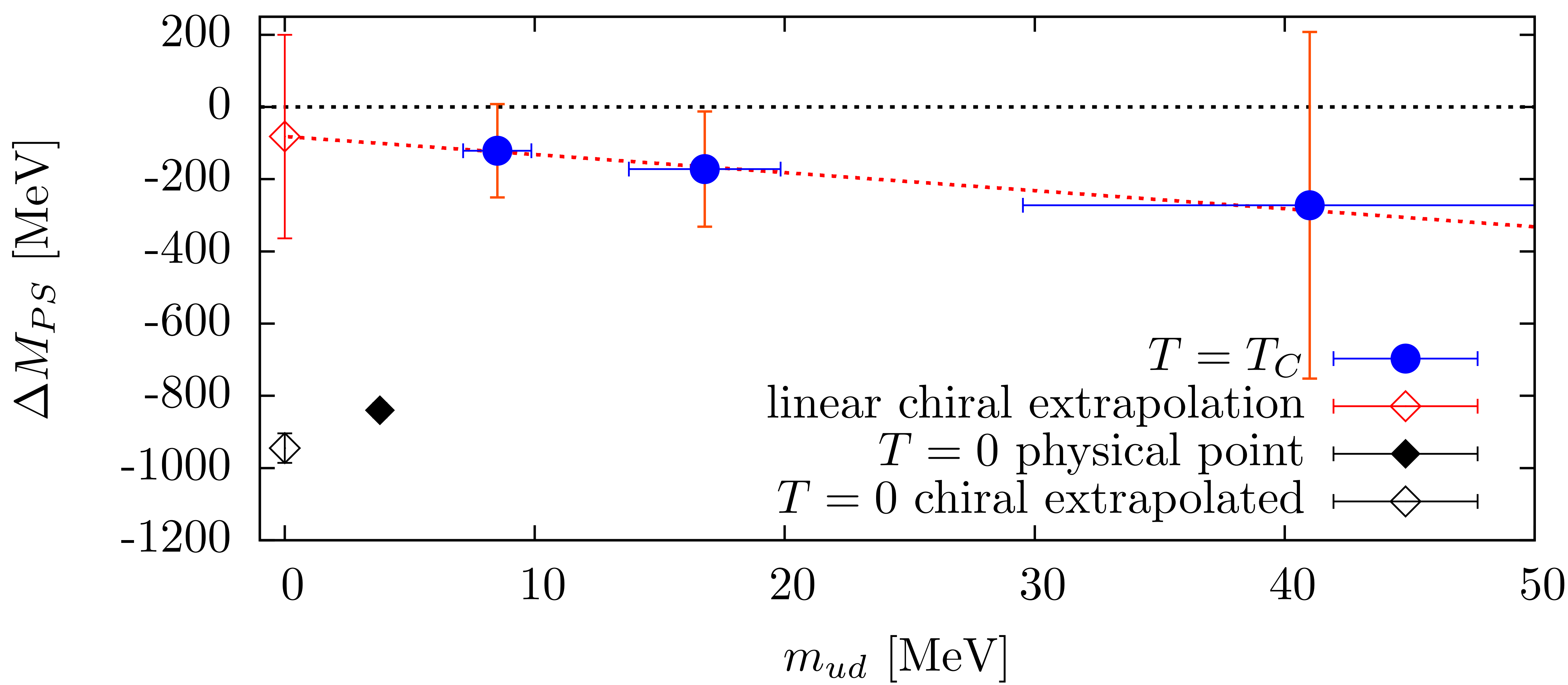


Borsanyi et al., [WB collaboration],1606.07494

The fall-off exponent agrees with Dilute Instanton Gas Approximation (DIGA) /Stefan Boltzmann limit for temperatures above T ~ 1GeV

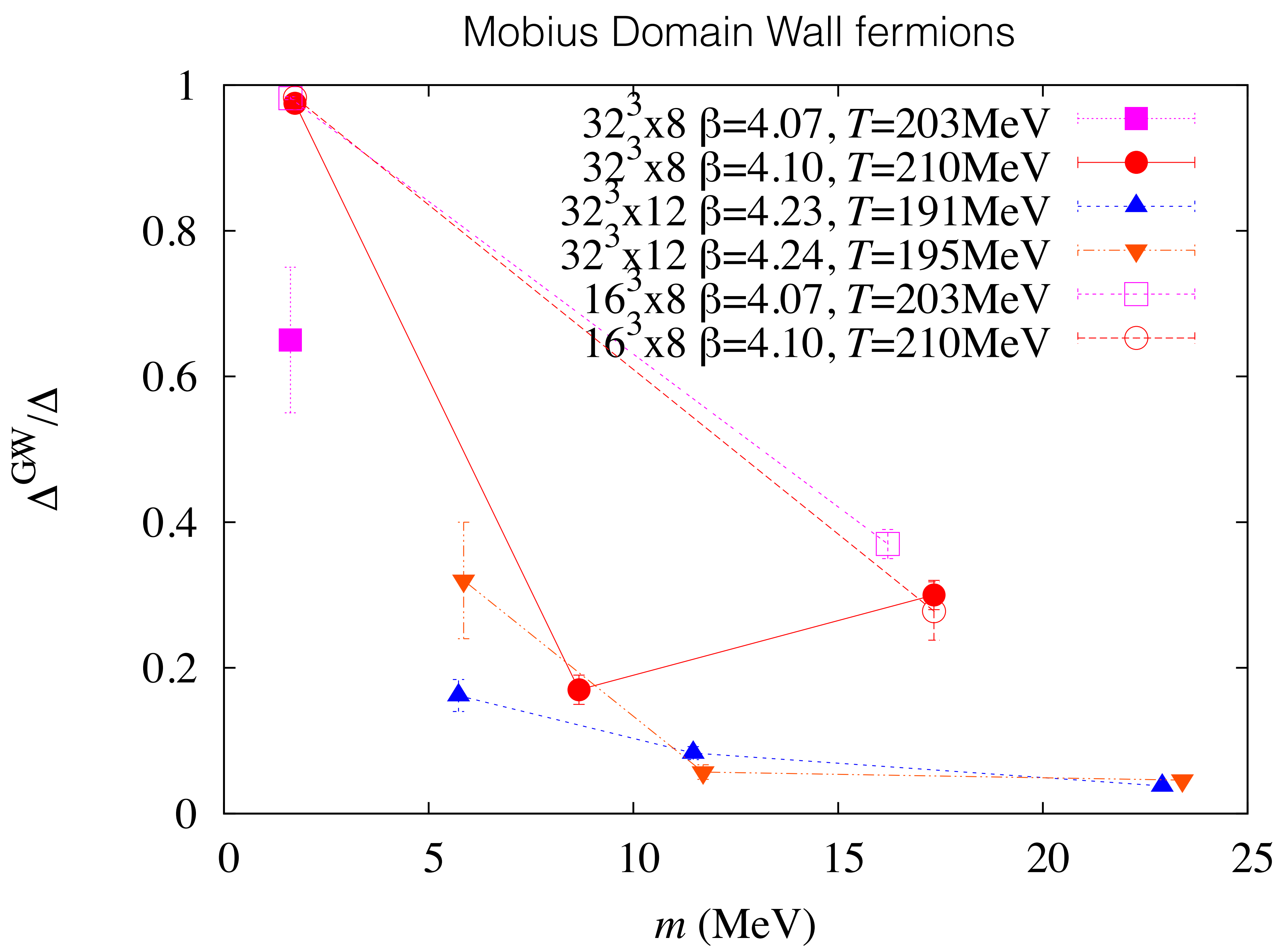
Marching to the chiral limit…

clover-improved Wilson fermions on Nt=16 lattices, Nf=2



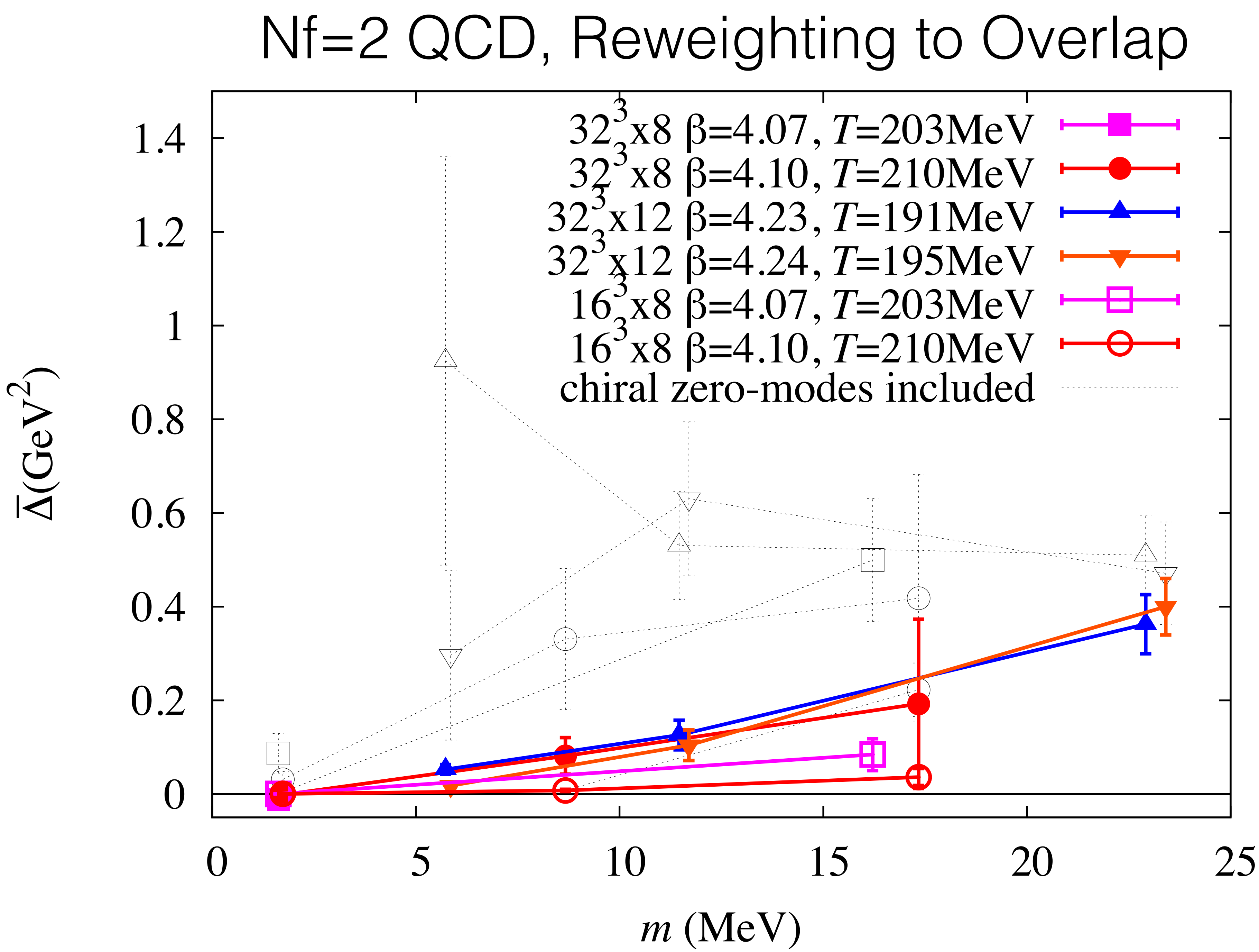
courtesy of Bastian Brandt, Univ. of Frankfurt,
updated results of 1310.8326 (lattice 2013), work in progress

Violation of Ginsparg-Wilson relation



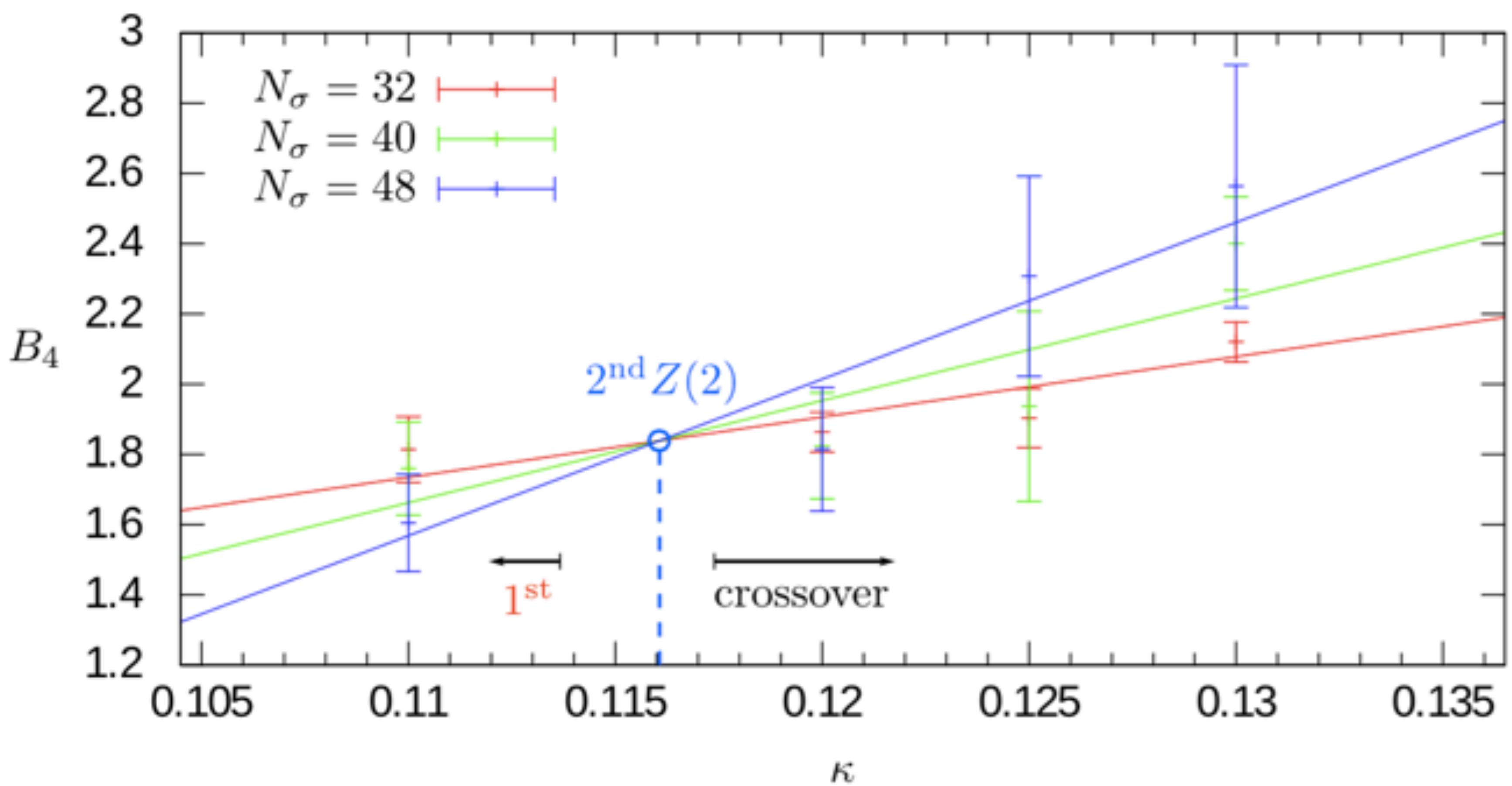
Courtesy of Guido Cossu, University of Edinburgh,
JLQCD, G. Cossu et al., PRD93 (2016) no.3, 034507,1511.05691, A. Tomiya, 1412.7306

Marching to the chiral limit…



Courtesy of Guido Cossu, University of Edinburgh,
JLQCD, G. Cossu et al., PRD93 (2016) no.3, 034507,1511.05691, A. Tomiya, 1412.7306

Columbia plot in the heavy quark mass region



[Christopher Czaban, Monday]

Nt=8, unimproved Wilson fermions

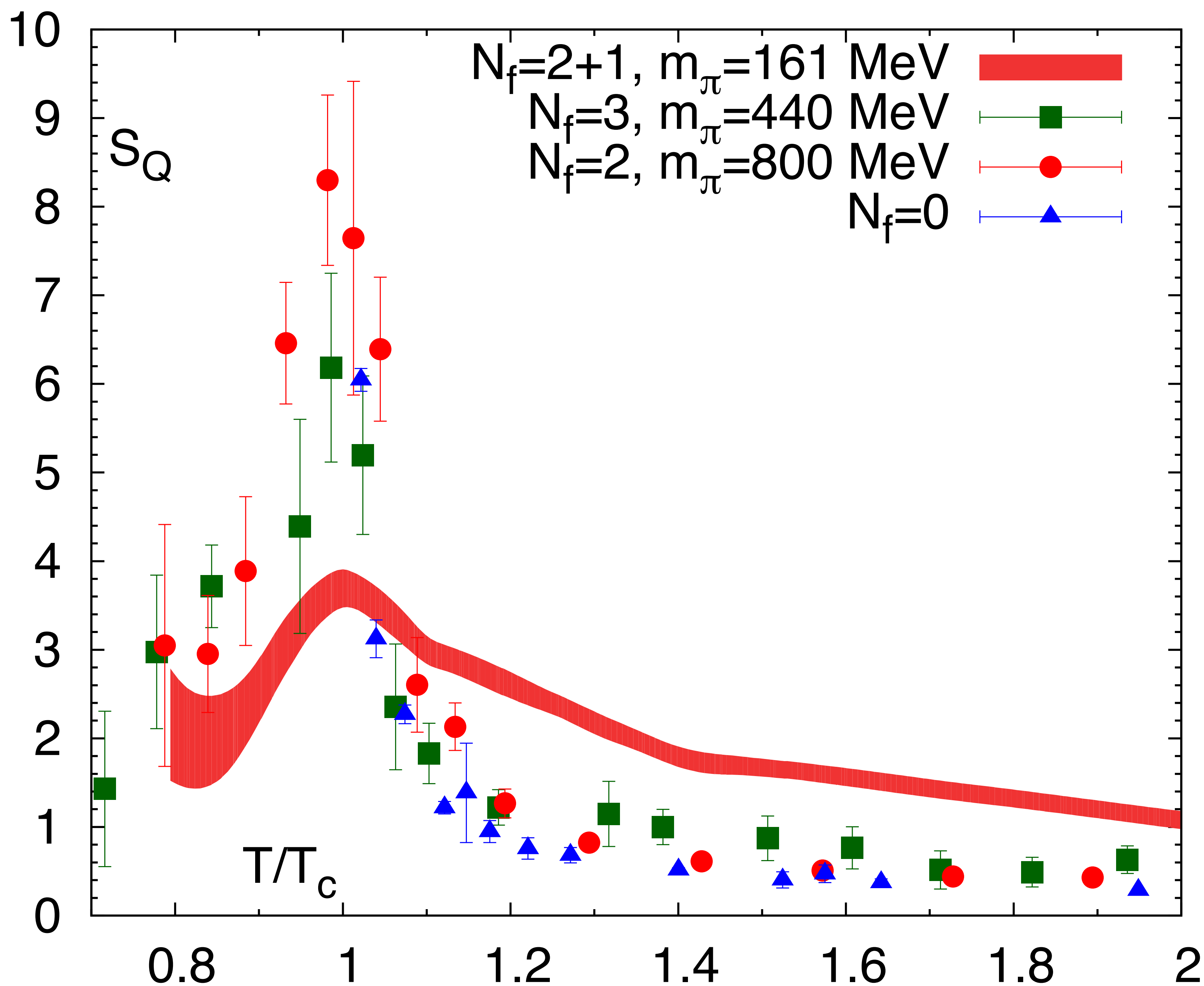
$$B_4^{\text{fit}}(\kappa_c, \infty) = 1.8387(984)$$

B_4=1.604, 2nd order of Z(2)

κ	β_c	a [fm]	am_π	m_π [MeV]	T_c [MeV]
0.1100	6.0303	0.0895(5)	2.1310(6)	4690(28)	275(2)
0.1300	5.9491	0.0947(6)	1.3964(5)	2904(17)	260(2)

C.f. WHOT collaboration results on 24^3x4 lattices using standard Wilson fermions, PRD84 (2011) 054502, Erratum: PRD85 (2012) 079902

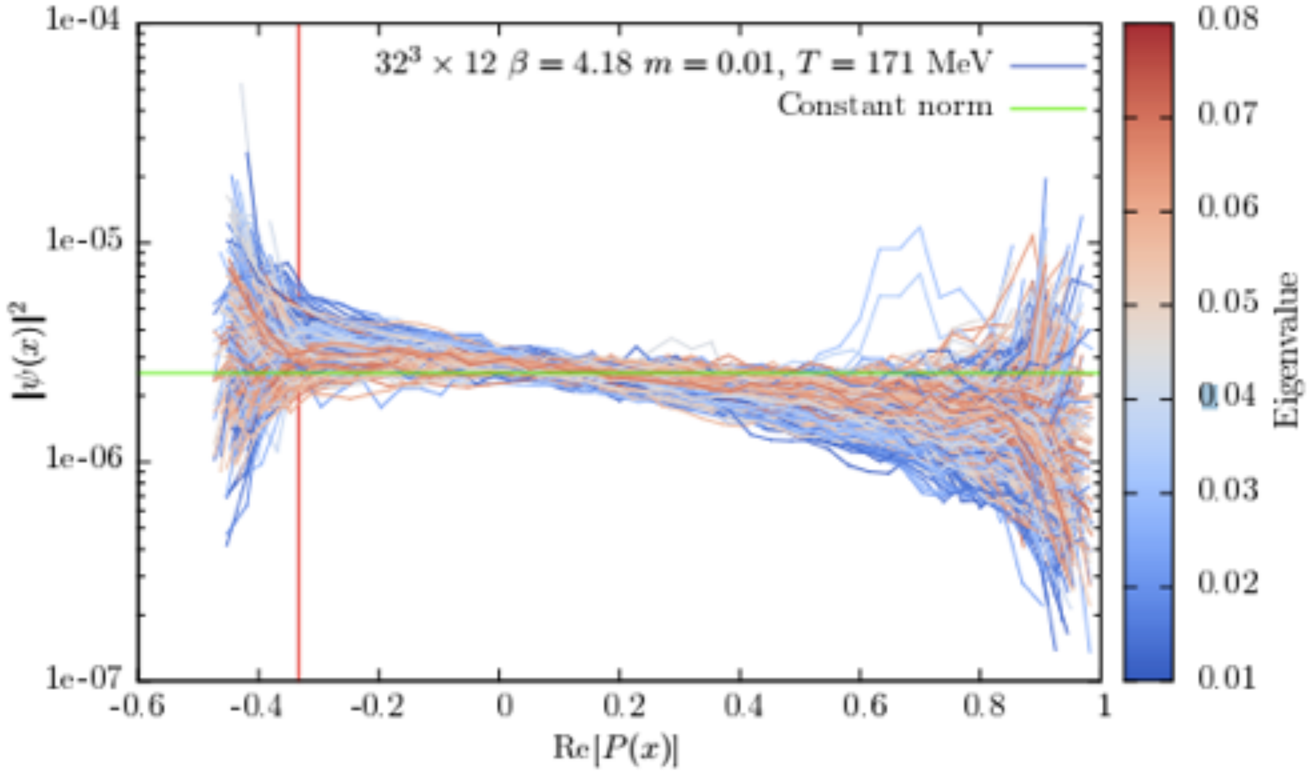
Possible connections between deconfinement & chiral aspects of the cross over

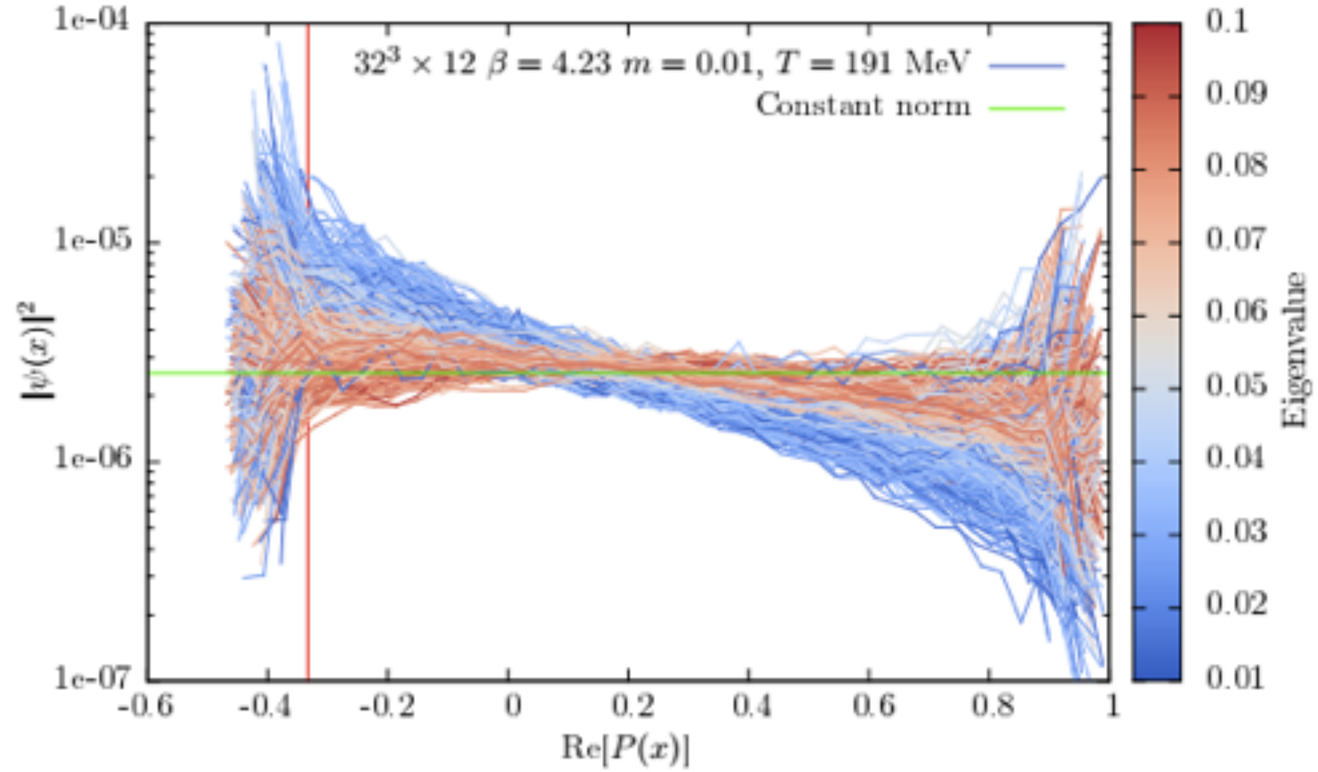


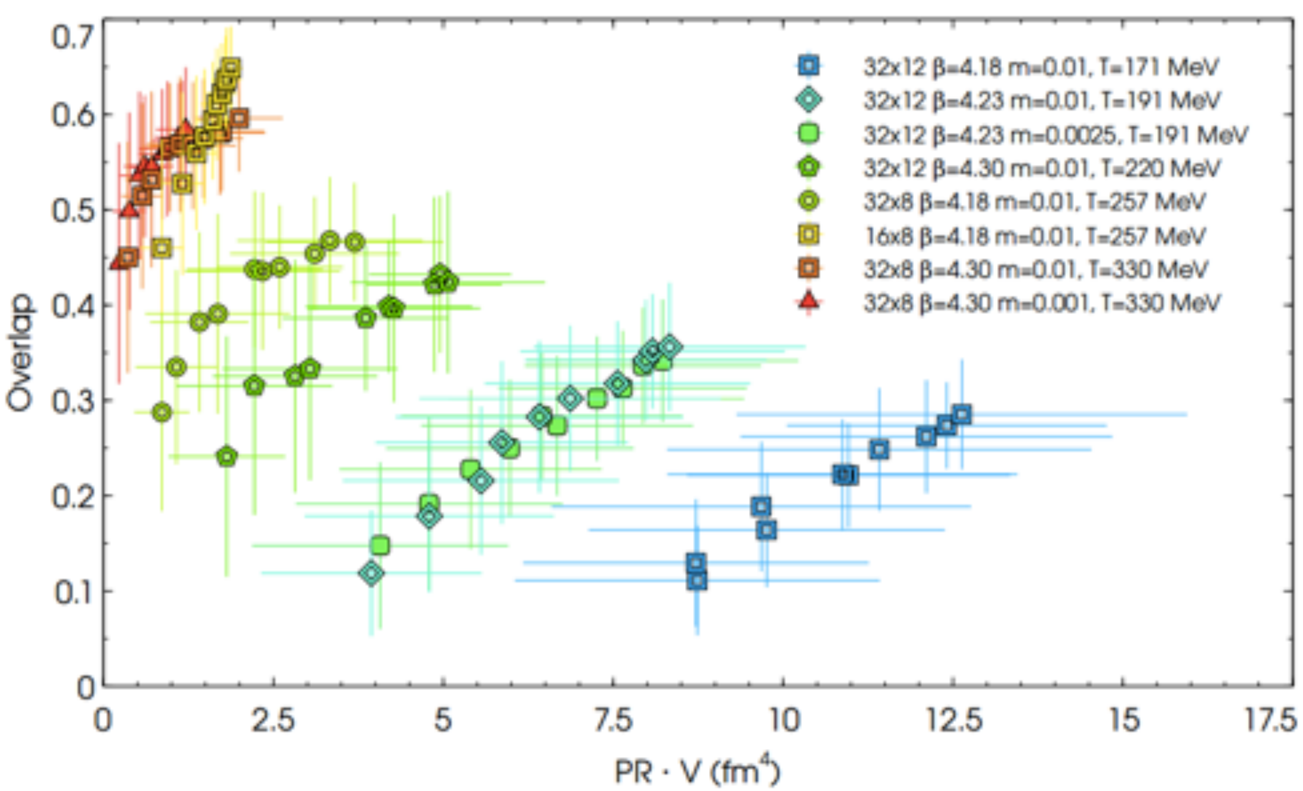
A. Bazavov et al., PRD93 (2016) no.11, 114502

Causes of transitions? Analogy to Anderson Localization

[Guido Cossu@Friday]







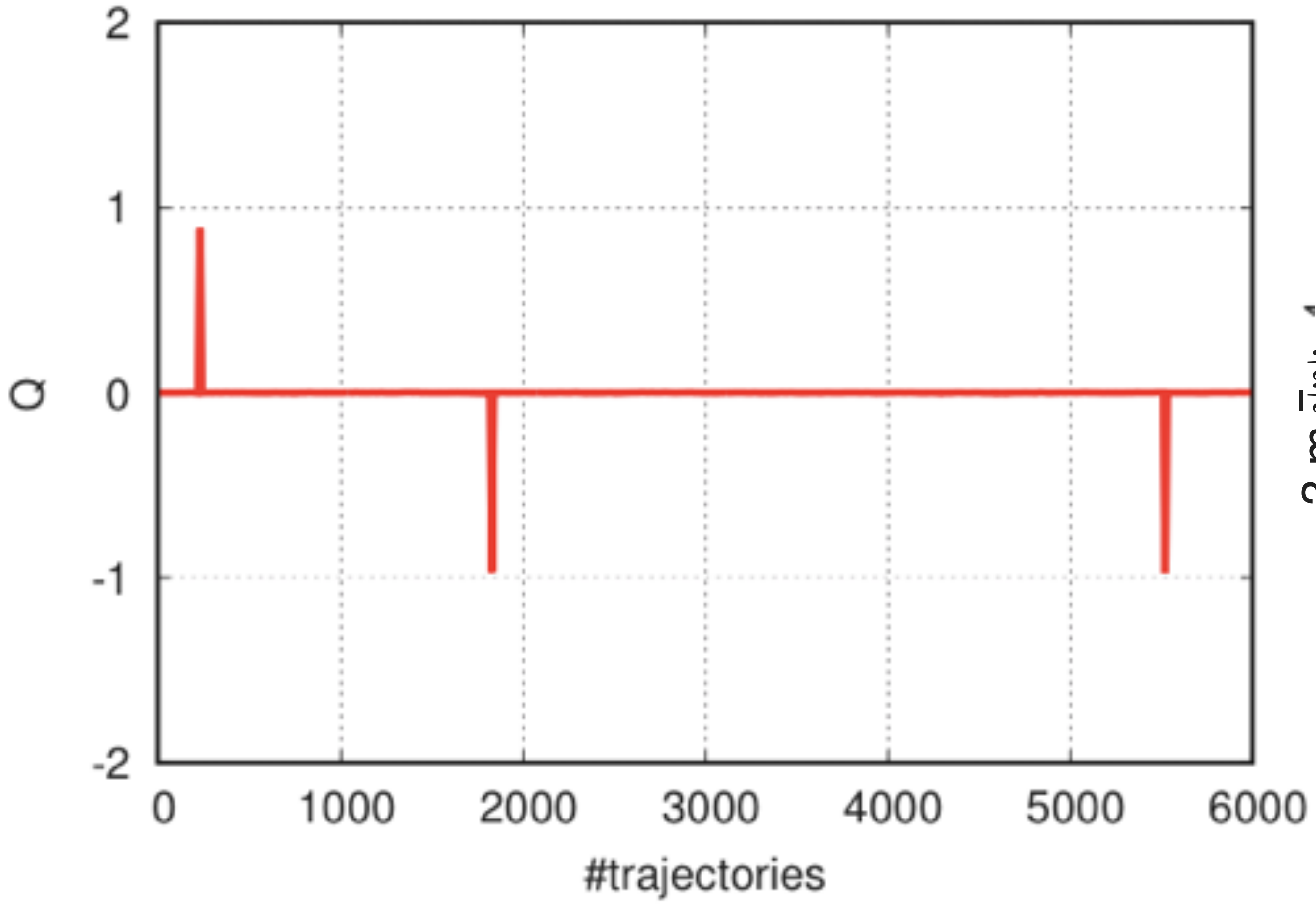
- Near zero modes are correlated with Polyakov loop
- Conjecture: localization of low modes causes the restoration of chiral symmetry

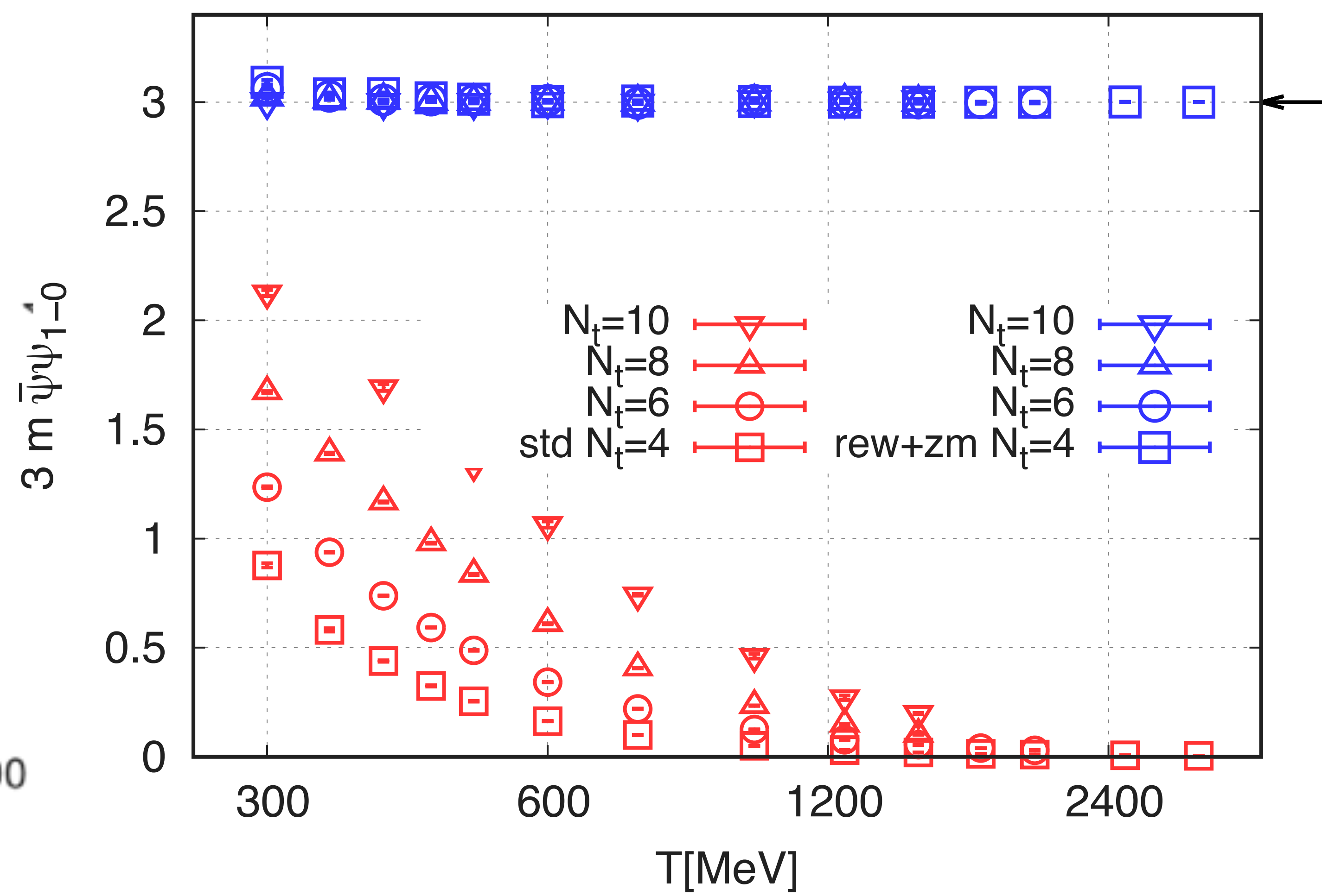
G. Cossu and S. Hashimoto, JHEP 1606 (2016) 056

Similar topics [Matteo Giordano, Friday]

QCD thermodynamics at very high temperature

[Kalman Szabo , Monday]





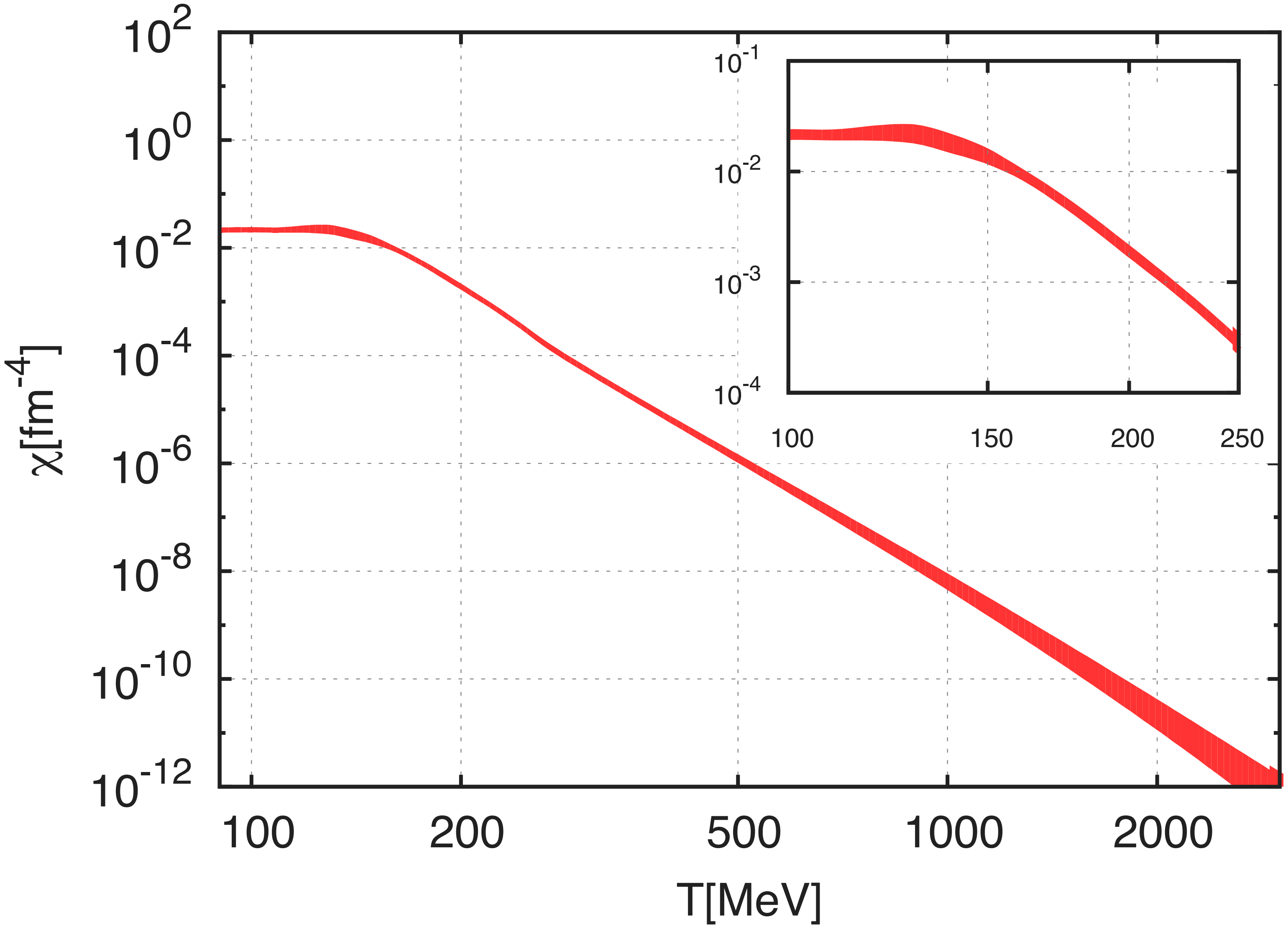
Borsanyi et al., [WB collaboration],1606.07494

Fixed Q Integration approach

For the method see also Frison et al., 1606.07175

Topological susceptibility up to very high T

[Kalman Szabo , Monday]



Borsanyi et al., [WB collaboration],1606.07494

$$m_A^2 = \chi / f_A^2,$$

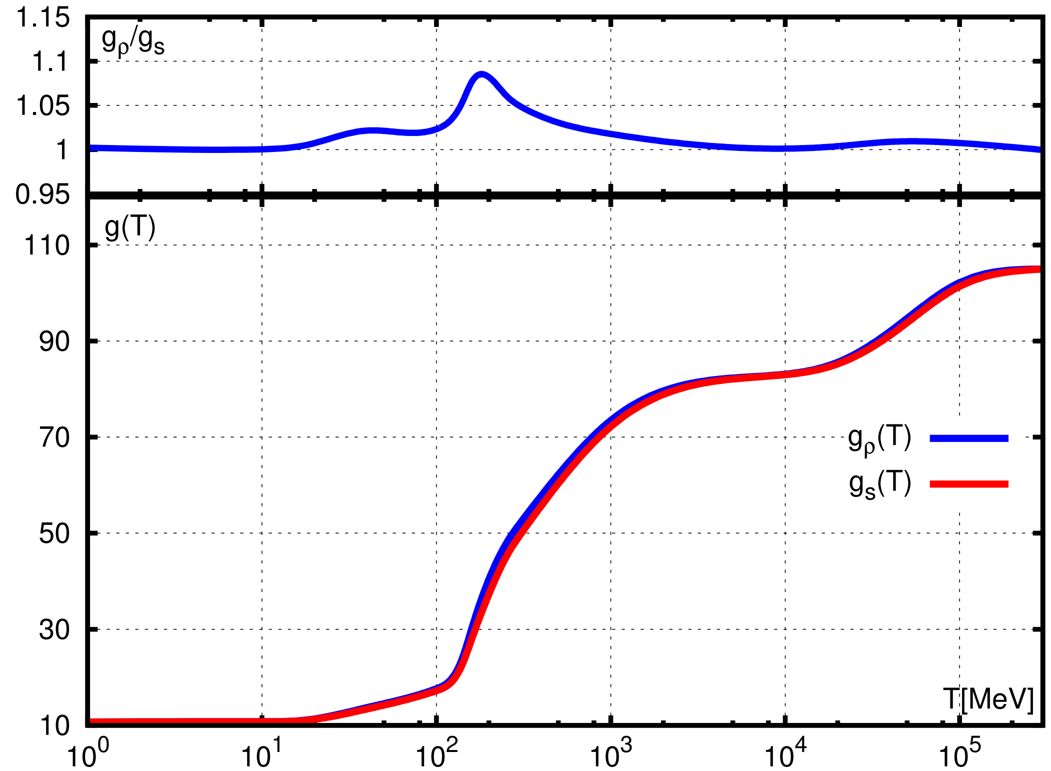
An axion mass of 50(4)μeV

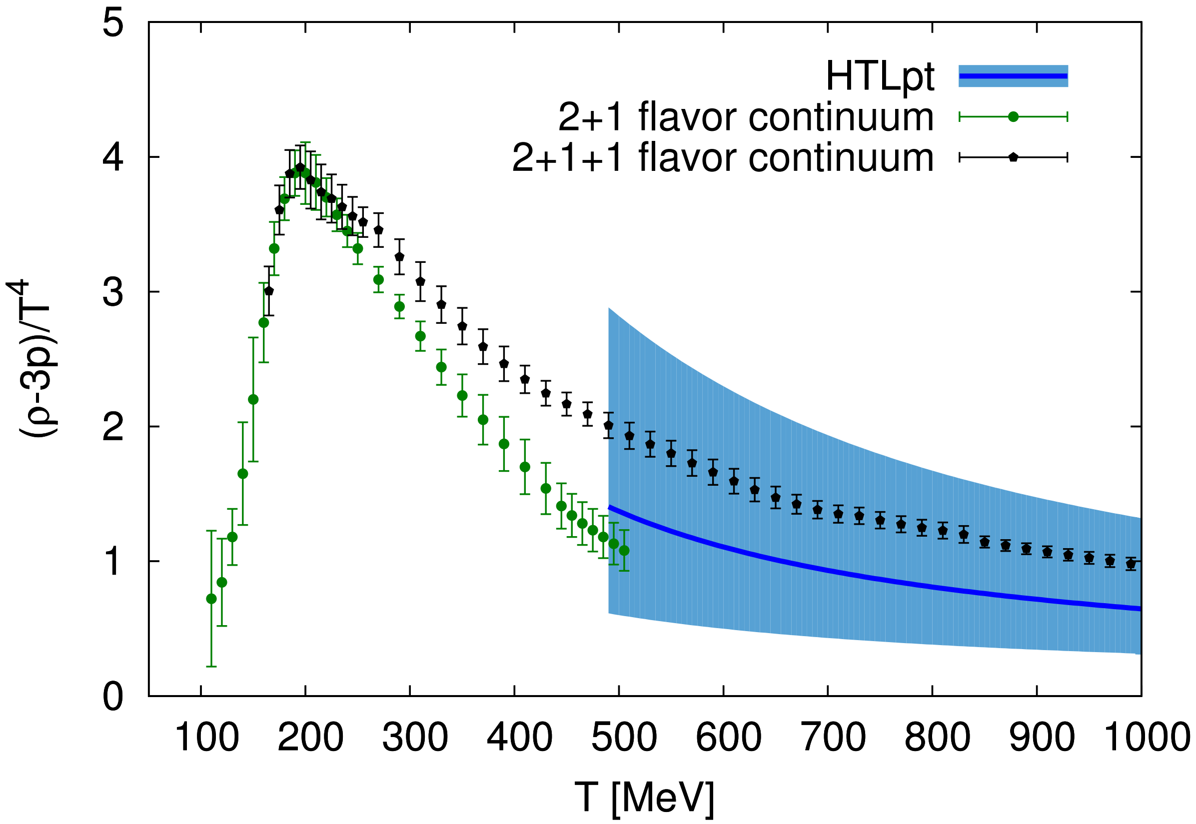
Relevance for Dark Matter [Enrico Rinaldi, Saturday]

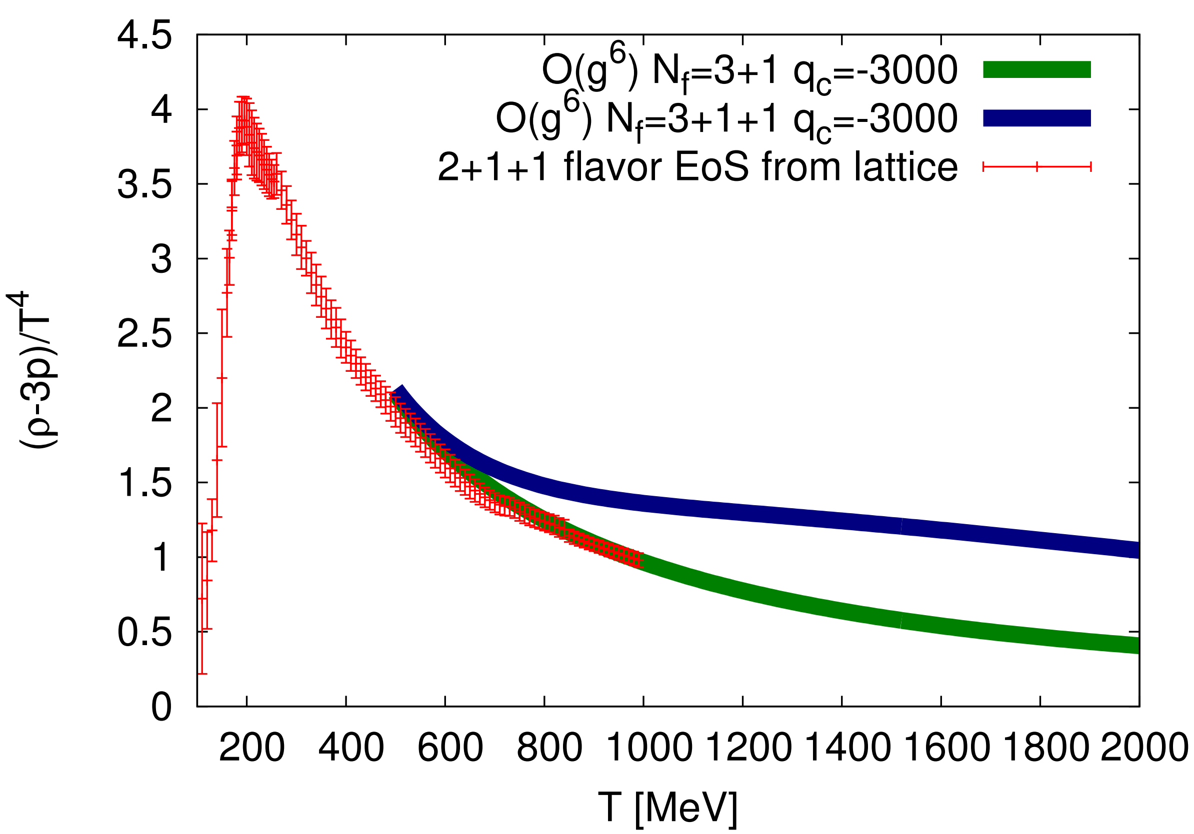
See also Petreczky, Schadler & Sharma, arXiv:1606.03145, Bonati et al., JHEP 1603 (2016) 155

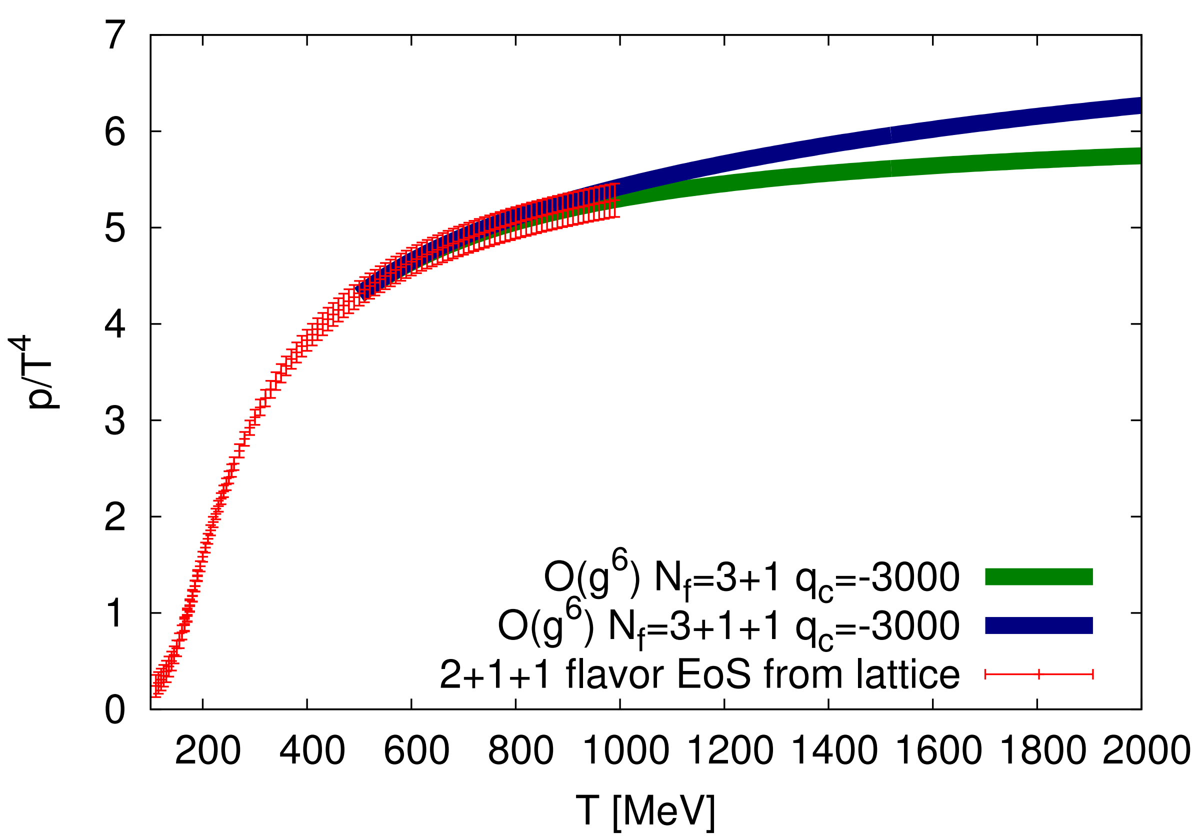
Equation of State up to very high T

[Szabolcs Borsanyi, Monday]

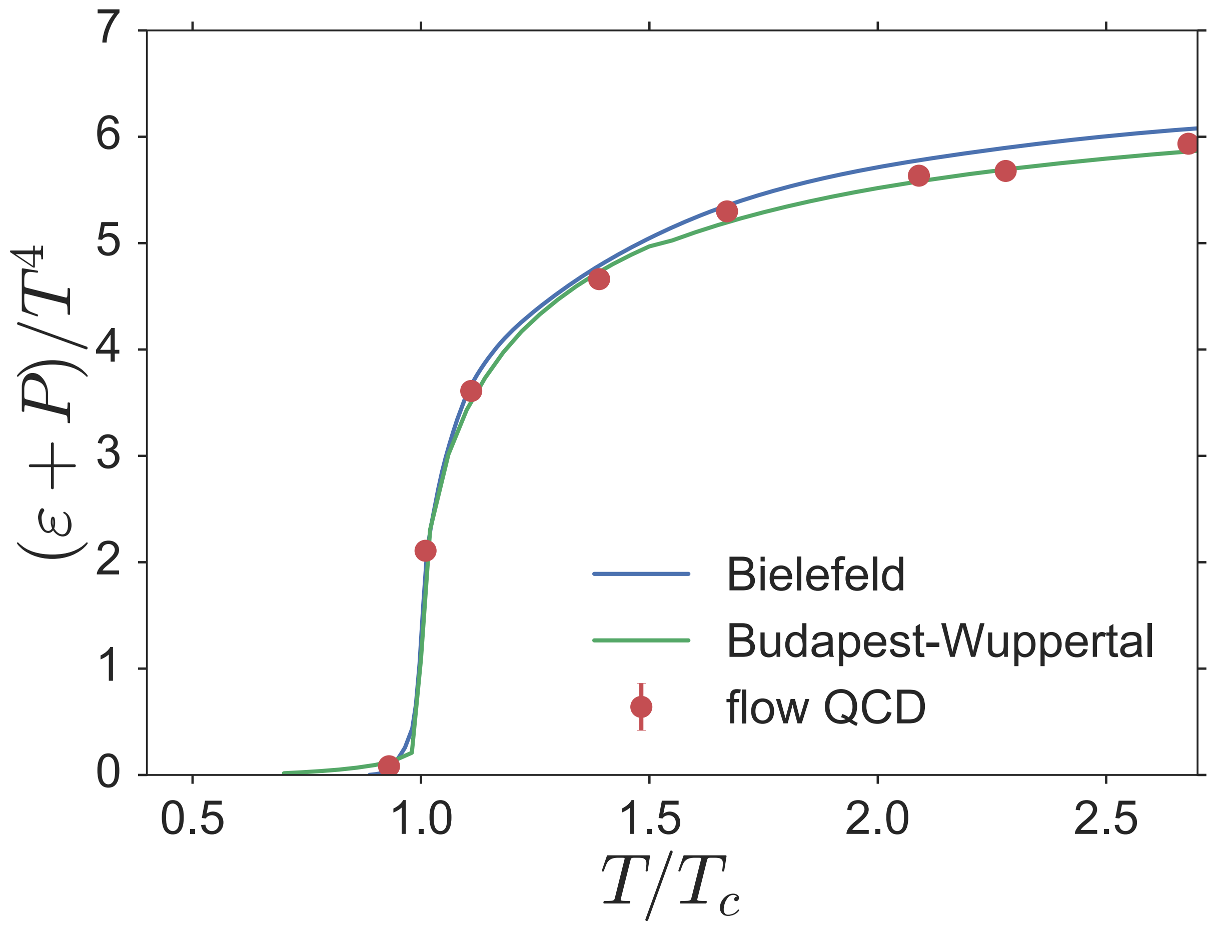


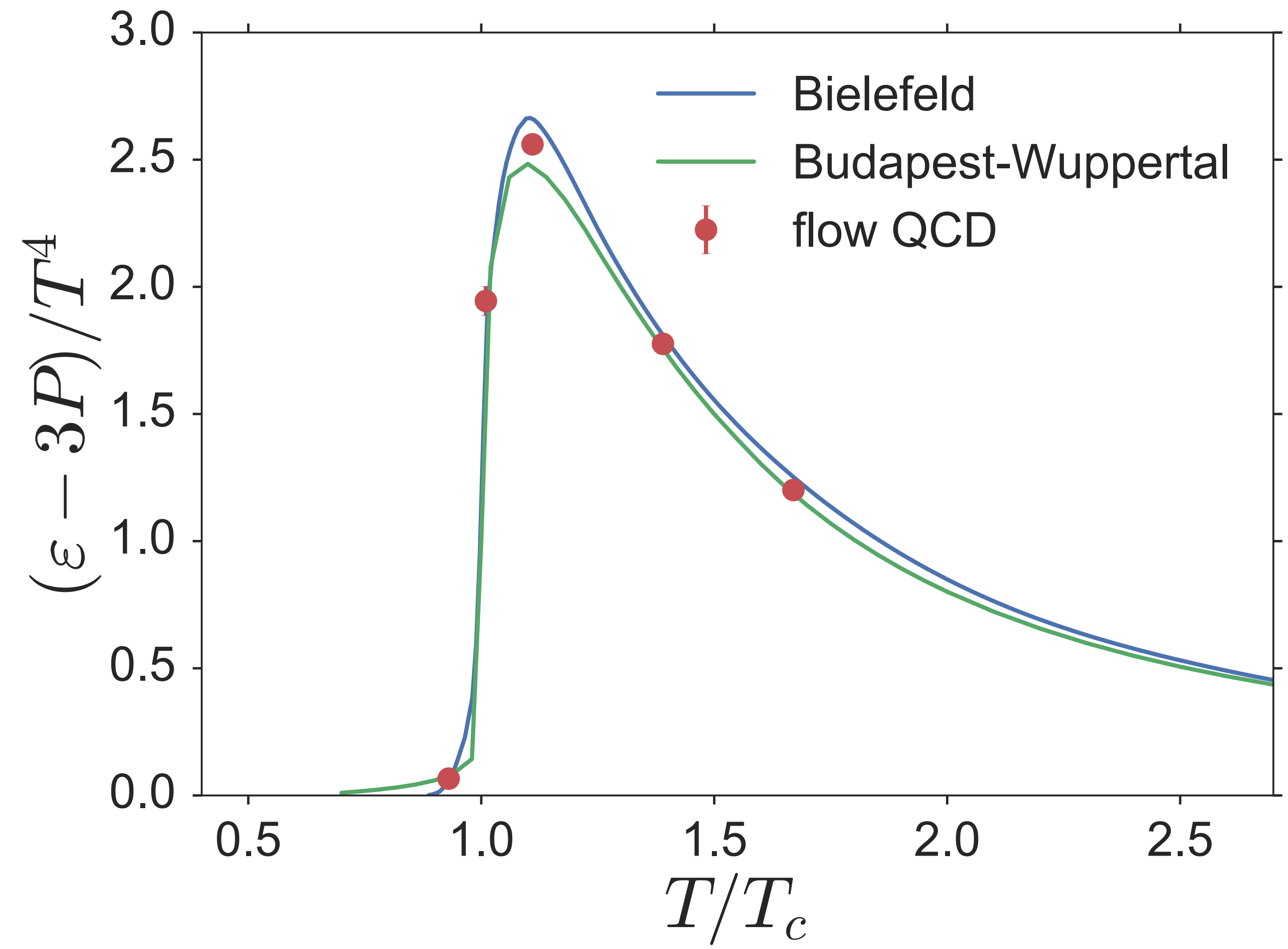






SU(3) thermodynamics from Gradient flow





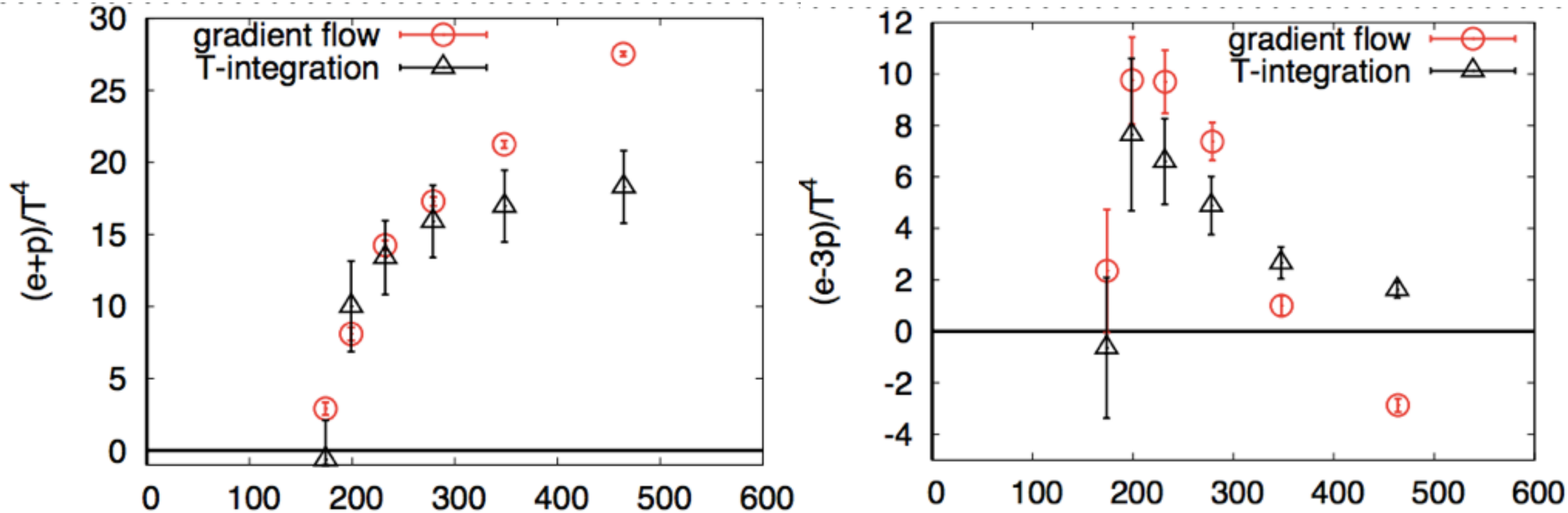
Numerical analysis is performed on Nt=12 - 24 lattices.

Relatively low cost compared to the standard method

Courtesy of Masakiyo Kitazawa, Osaka Univ., work in progress

EoS of full QCD from Gradient flow

[Kazuyuki KANAYA ,Wednesday]



Results agree with T-integration method at T<=300 MeV

Larger Nt-s at high temperature are needed

See [Yusuke Taniguchi, Friday] on topological susceptibility from Gradient Flow

See [Saumen Datta, Friday],arXiv:1512.04892 on the deconfinement transition from GF

Properties of QGP through spectral functions

Spectral functions: in-medium heavy hadron properties (**[Seyong Kim, Today's plenary]**), transport properties, dissociation T of hadron, electromagnetic properties of QGP

$$G_H(\tau,\vec{p},T) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\rho_H(\omega,\vec{p},T)\,\frac{\cosh(\omega(\tau-1/2T))}{\sinh(\omega/2T)}$$

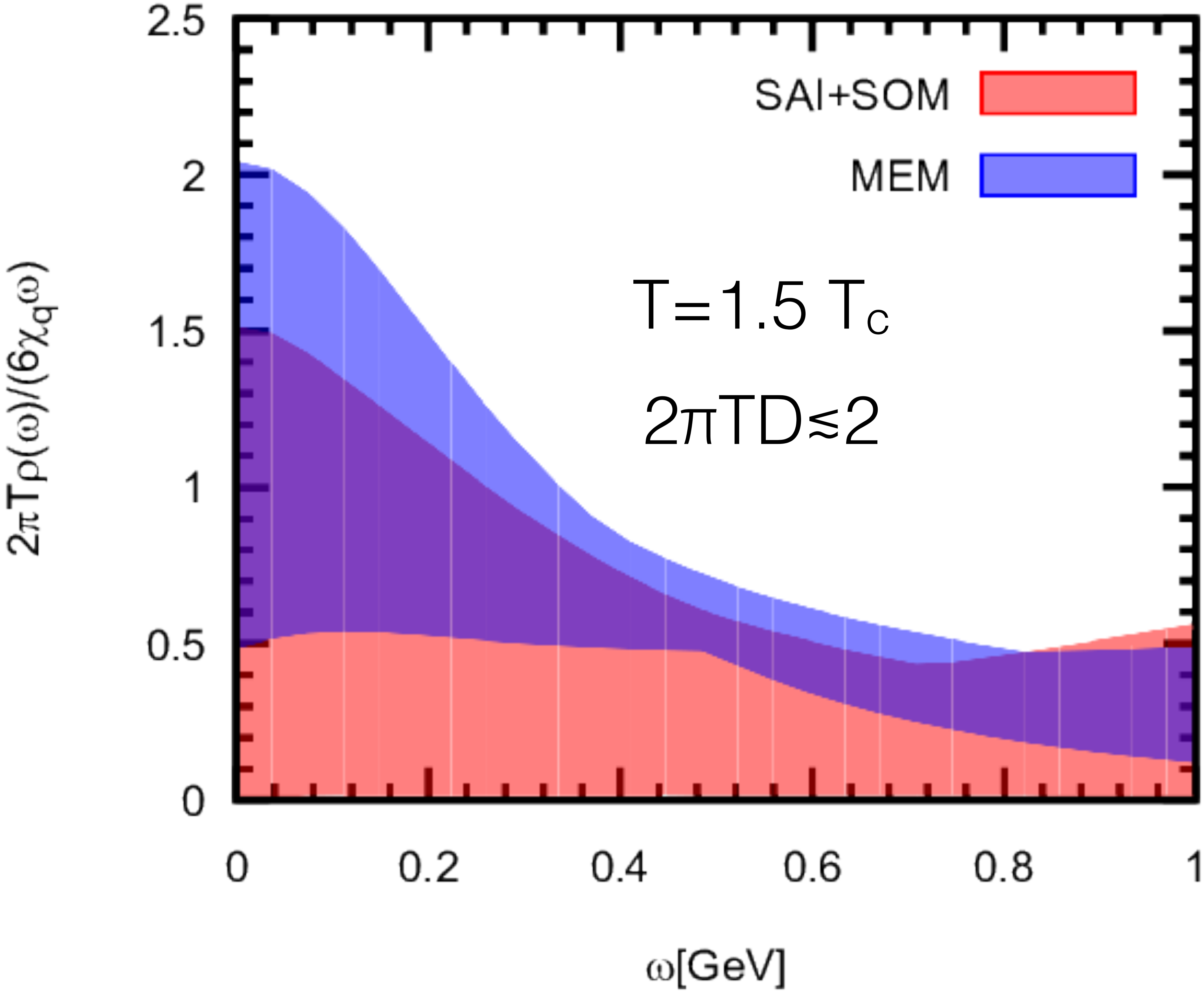
Methods to solve the ill-posed problem:

- Maximum Entropy Method (MEM): Based on Bayesian theorem using Shannon-Jaynes Entropy M. Asakawa, T. Hatsuda & Y. Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459
- Improved Maximum Entropy Method: Similar with MEM but with a different Entropy term Y. Burnier & A. Rothkopf, PRL. 111(2013)182003
- Backus-Gilbert Method [Daniel Robaina, Tuesday]
- Stochastic Analytical Interference (SAI) & Stochastic optimization method (SOM) [Hai-Tao Shu, Thursday]

Charm quark diffusion coefficient

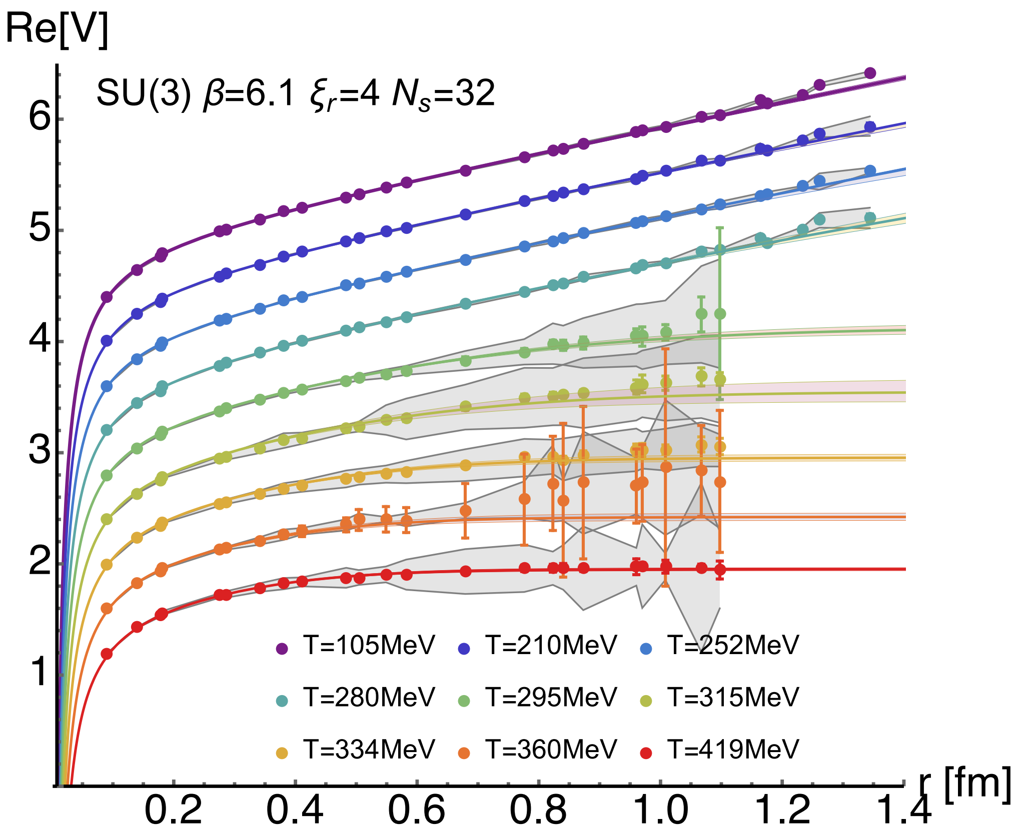
[Hiroshi Ohno, Thursday]

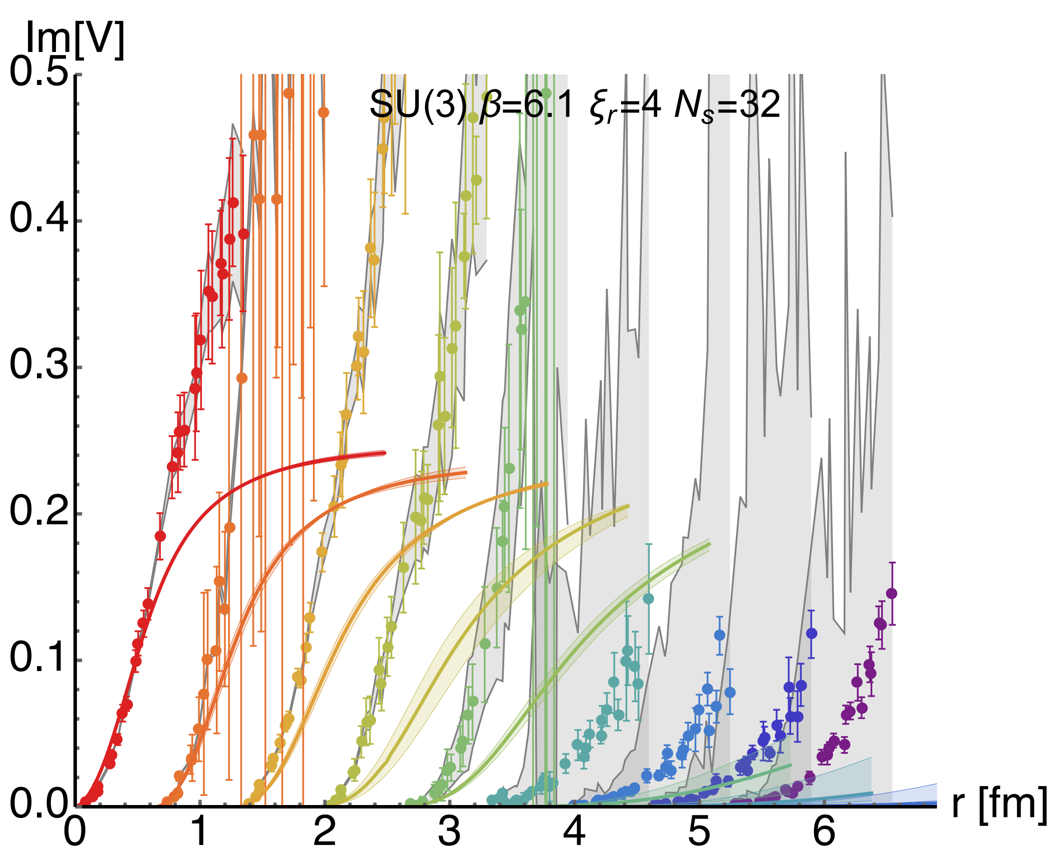
192^3x48, quenched QCD

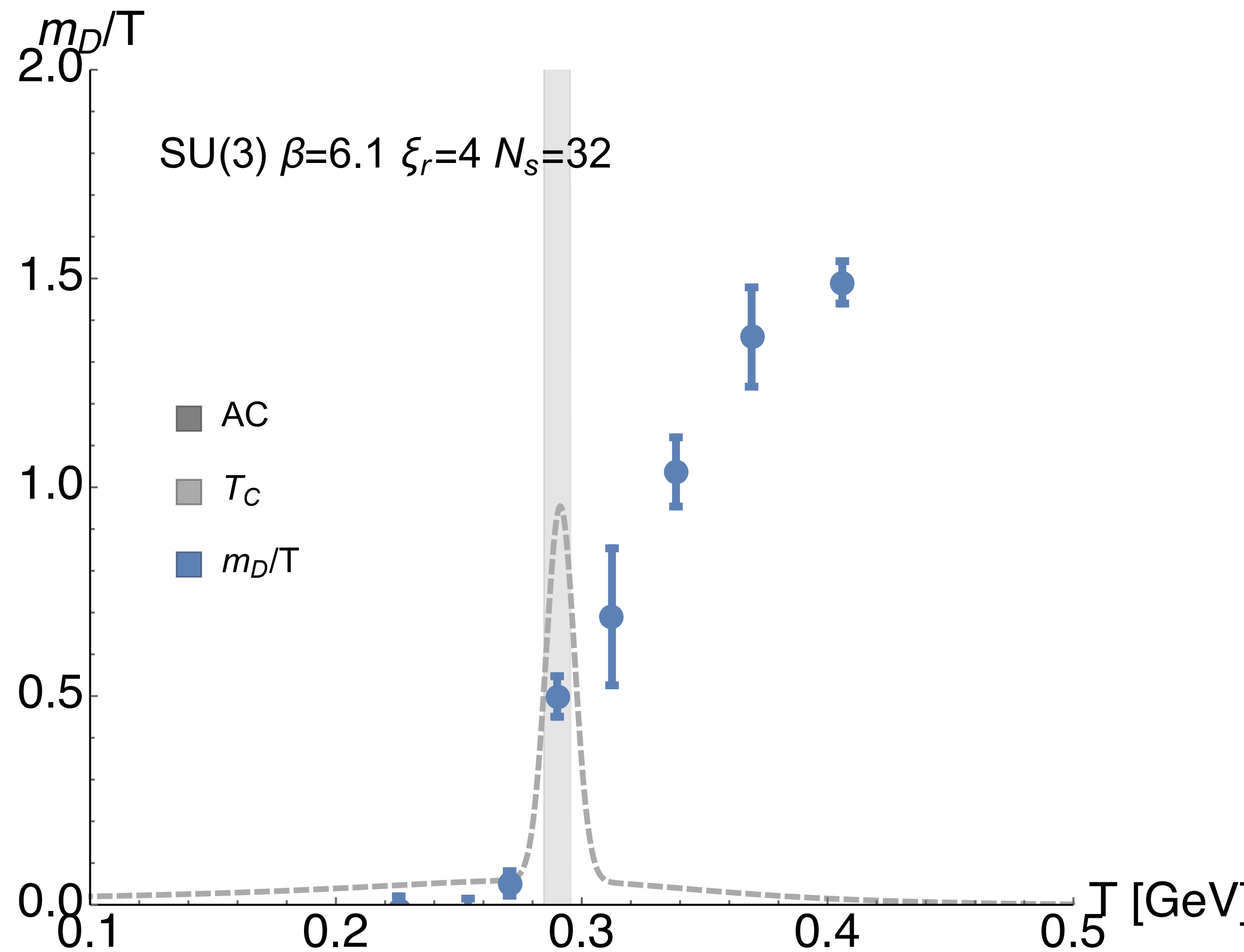


Debye mass for a complex heavy quark potential

[A. Rothkopf, Thursday]



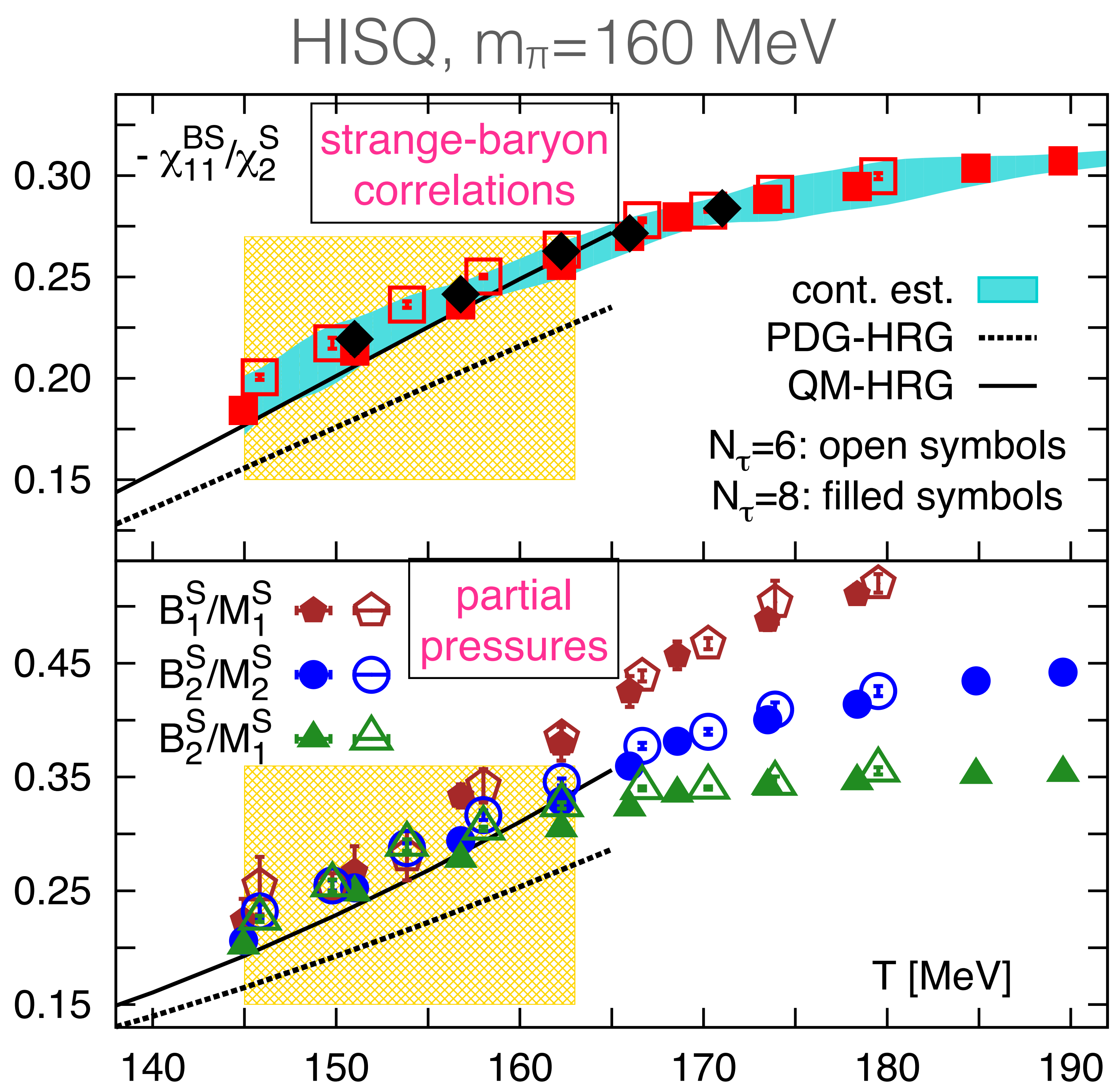




m_D includes both screening & scattering effects

Y. Burnier & A. Rothkopf. Phys.Lett. B753 (2016) 232, arXiv.1607.04049

Indirect evidence of experimentally not yet observed strange states hinted from QCD thermodynamics



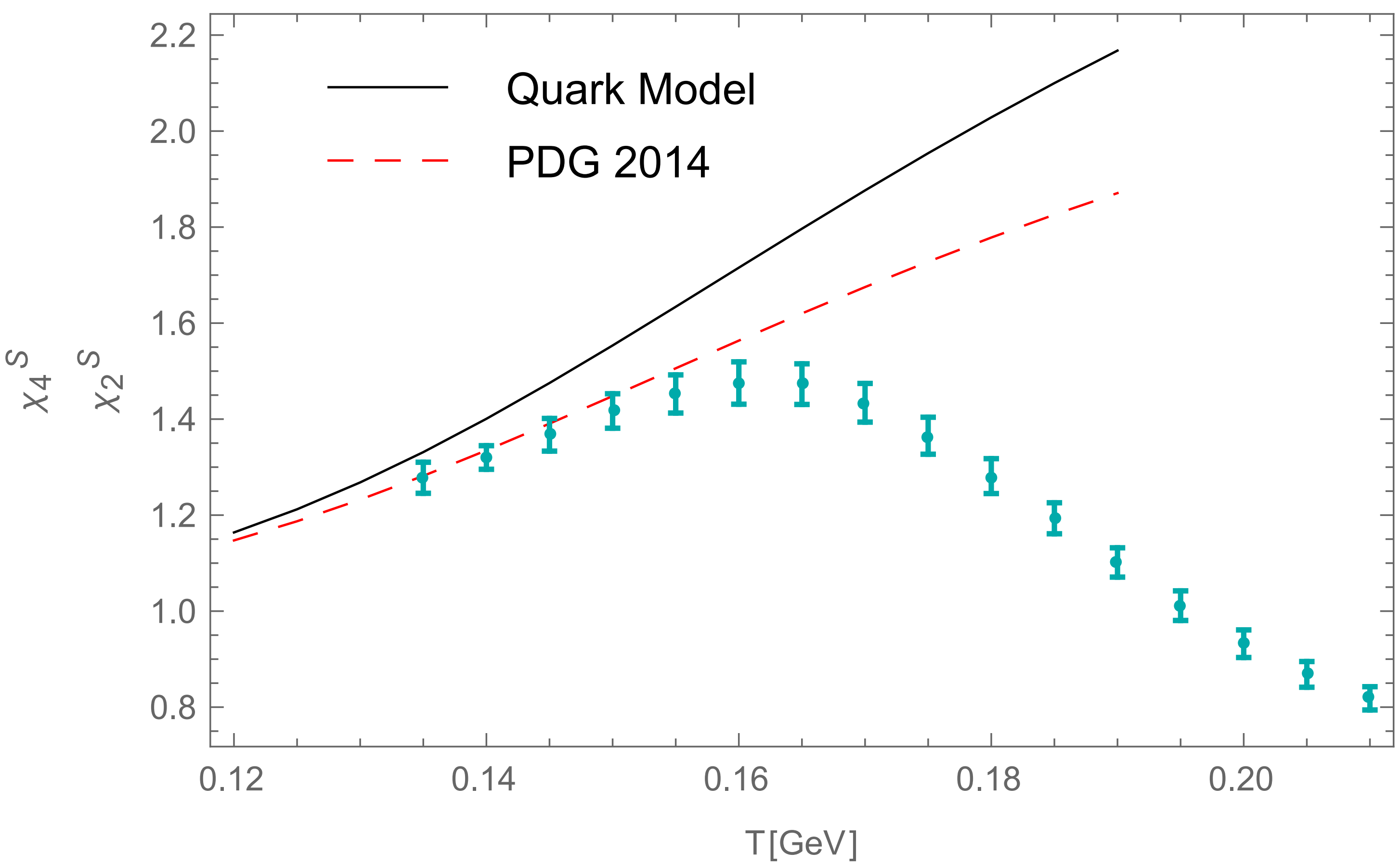
PDG-HRG: Hadron Resonance Gas model calculations with spectrum from PDG

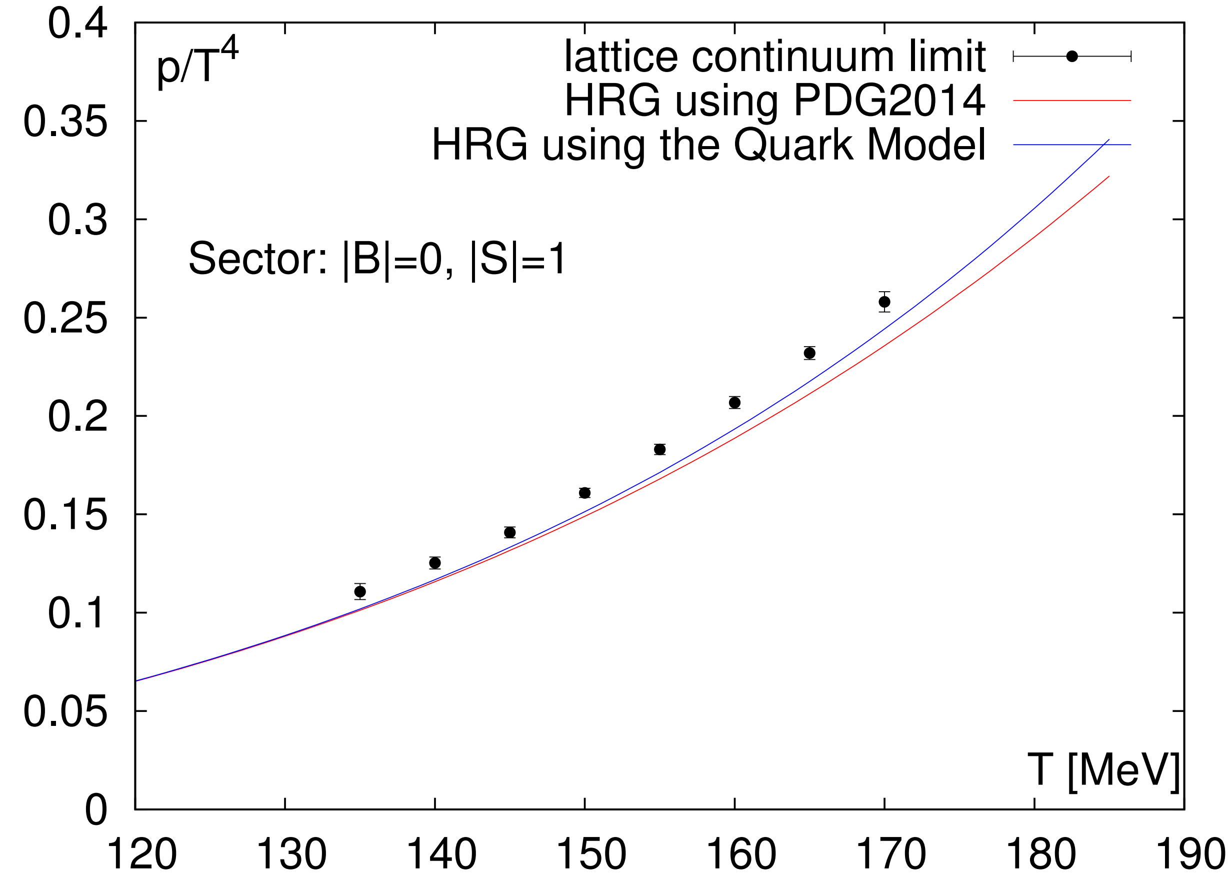
QM-HRG: Similar as PDG-HRG but with spectrum from Quark Model

A. Bazavov et al.[BNL-Bielefeld-CCNU], Phys. Rev. Lett. 113 (2014)072001

Strange mesons in PDG & Quark Model

[Szabolcs Borsanyi, Monday]





Relative abundance in strange baryons to strange mesons are well described by QM-HRG

Some single-strange mesons are missing in QM

$T>0$ & $\mu > 0$

Fluctuations of conserved charges

Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507

Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Taylor expansion coefficients at μ=0

$$\chi_{ijk}^{BQS} \equiv \chi_{ijk}^{BQS}(T) = \frac{1}{VT^3} \frac{\partial P(T, \hat{\mu})/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\Big|_{\hat{\mu}=0}$$

- Thermodynamic relations

$$\frac{\epsilon - 3p}{T^4} = T \frac{\partial P/T^4}{\partial T} = \sum_{i,j,k=0}^{\infty} \frac{T\, \mathrm{d}\chi_{ijk}^{BQS}/\mathrm{d}T}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

Pressure of hadron resonance gas (HRG)

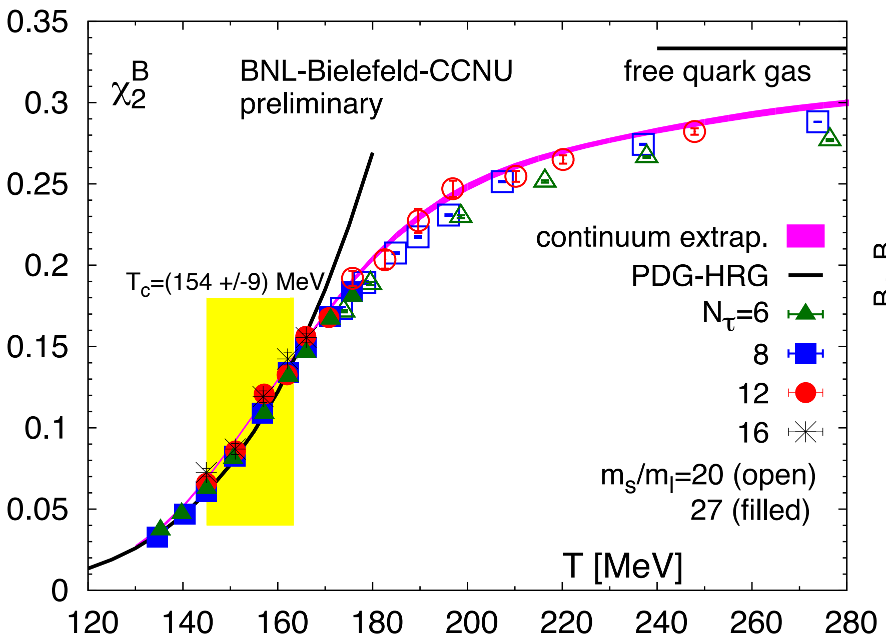
$$\frac{p}{T^4} = \sum_{m \in meson, baryon} \ln Z(T, V, \mu) \sim \exp(-m_H/T)\, \exp((B\mu_B + S\mu_s + Q\mu_Q)/T)$$

Pressure of QCD at nonzero muB

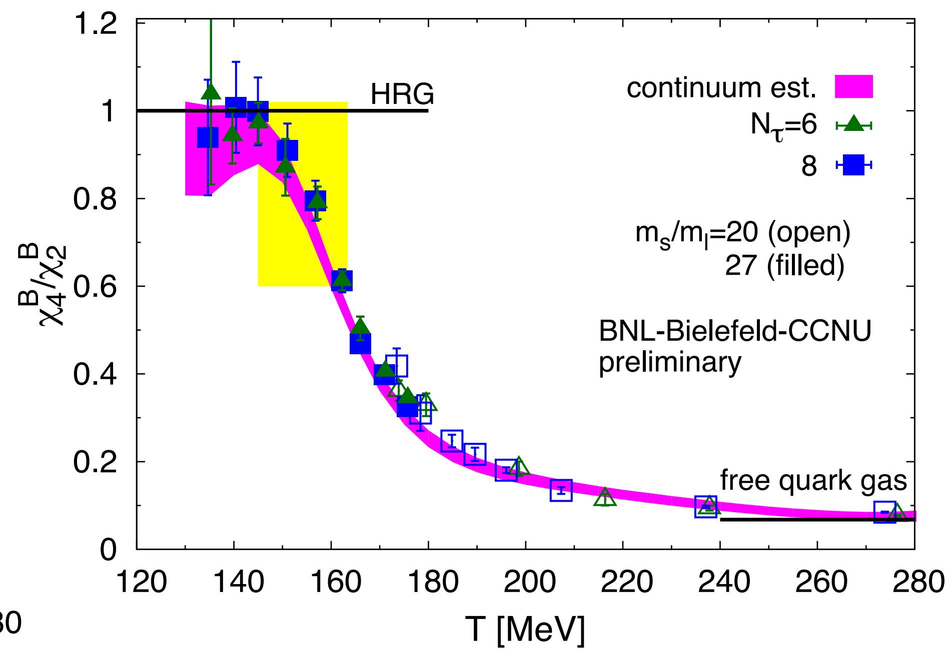
[Edwin Laermann, Monday]

$$\Delta(P/T^4) = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$
$$= \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2 \left(1 + \frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2 + \frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4 + \cdots\right)$$

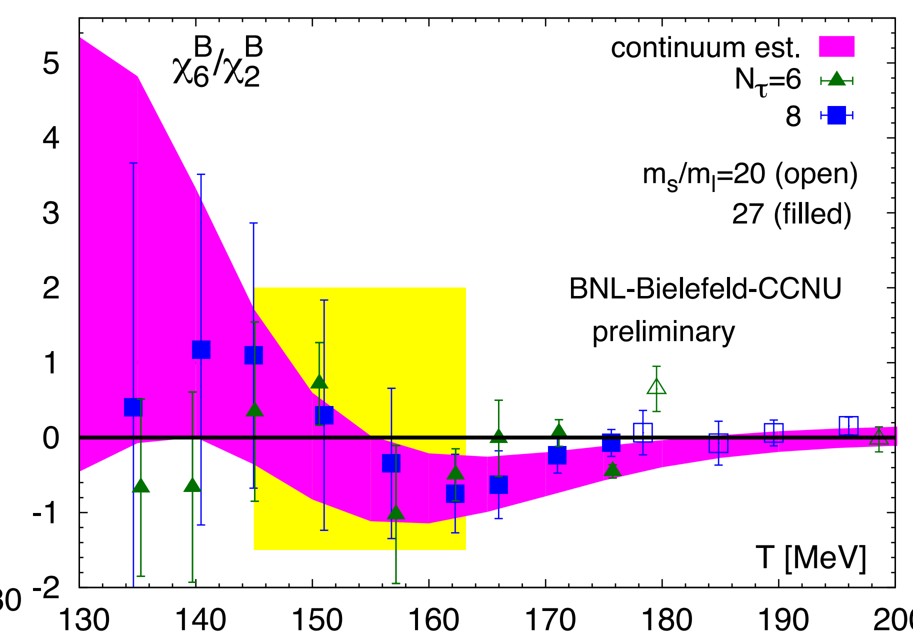
LO expansion coefficient
variance of net-baryon number distribution



NLO expansion coefficient
kurtosis * variance

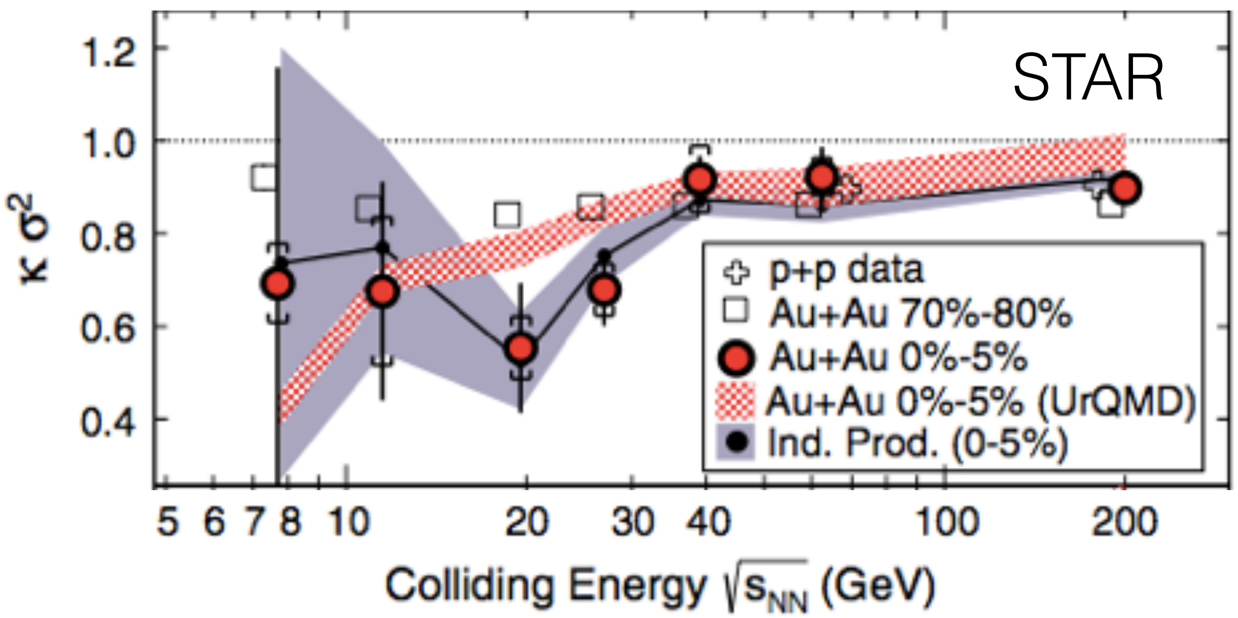


NNLO expansion coefficient



- HRG describes well on the LO expansion coefficient up to ~160 MeV while it deviates from NLO expansion coefficient ~ 40% in the crossover region
- For small muB/T the LO contribution dominates

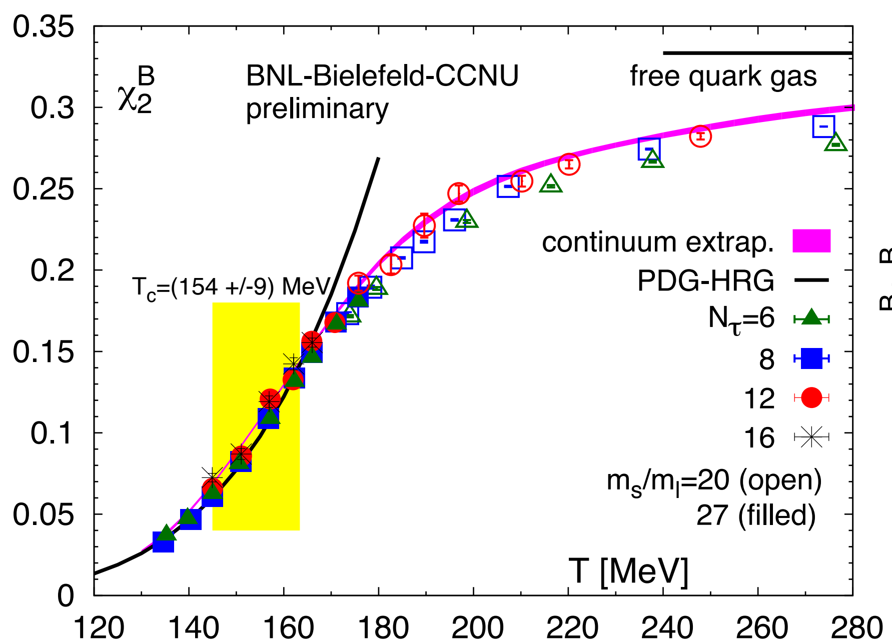
Explore the QCD phase diagram in Heavy Ion collisions

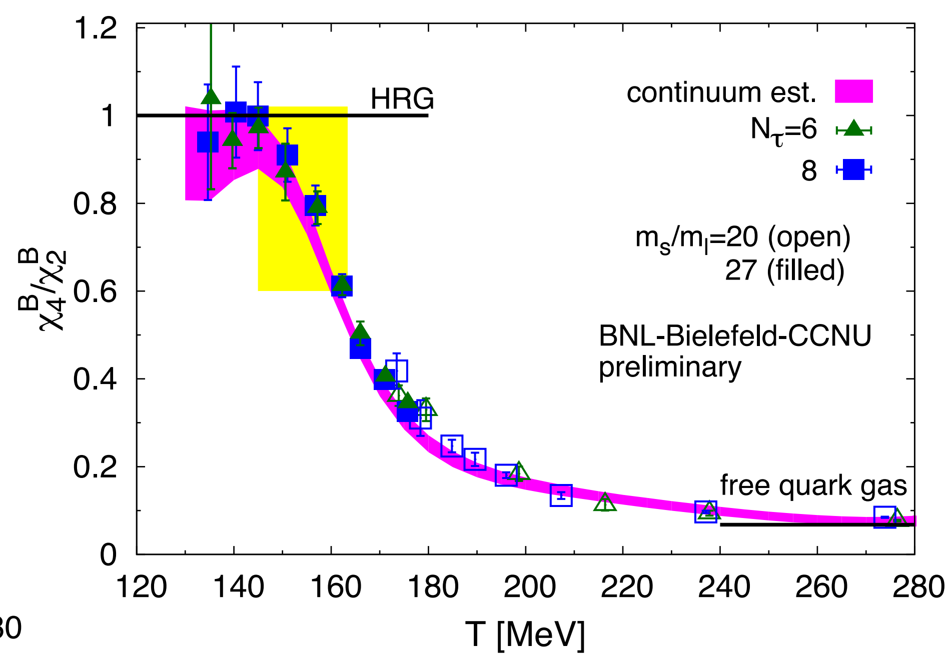


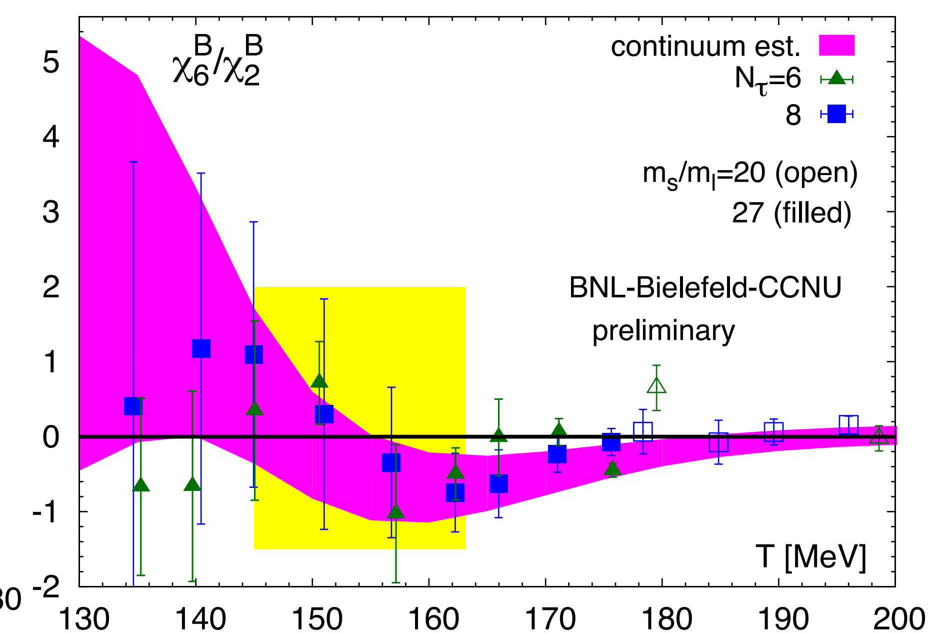
$$(\kappa\sigma^2)_B = \frac{\chi^B_{4,\mu}}{\chi^B_{2,\mu}} = \frac{\chi^B_4}{\chi^B_2}\left[1 + \left(\frac{\chi^B_6}{\chi^B_4} - \frac{\chi^B_4}{\chi^B_2}\right)\left(\frac{\mu_B}{T}\right)^2 + \cdots\right]$$

In the O(4) universality class:

$$\chi^B_6 < 0\,, \qquad T \sim T_c$$





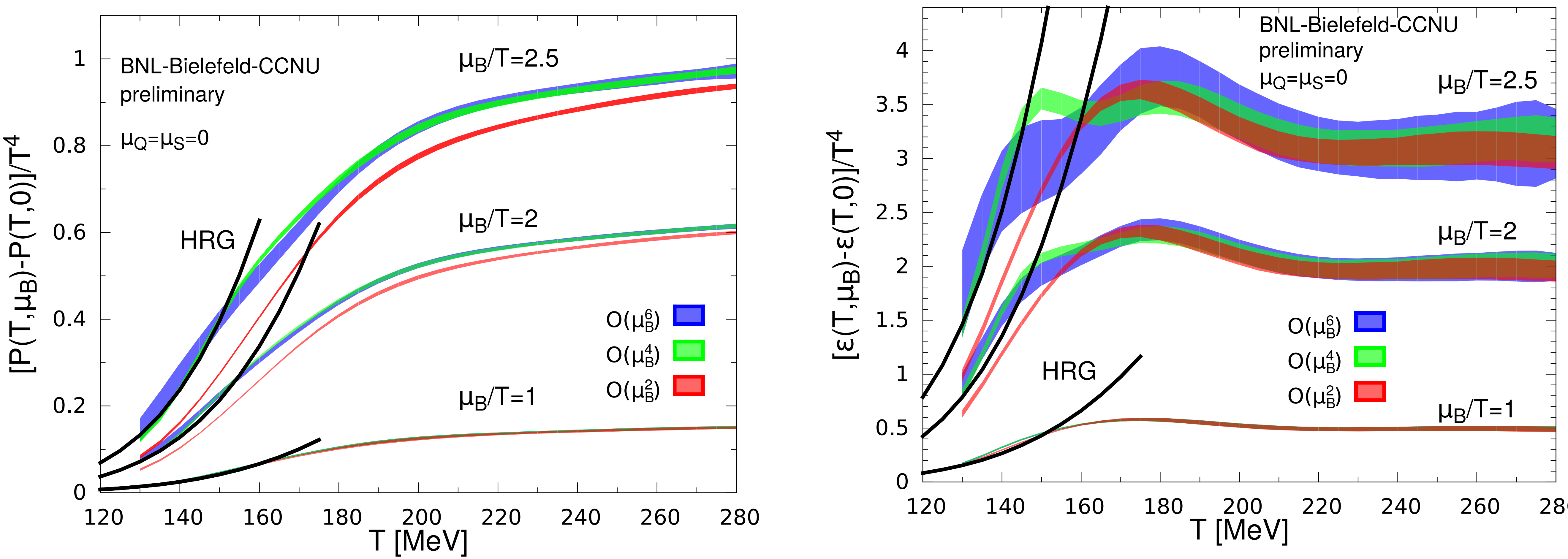


Pressure of QCD at nonzero μ_B

[Edwin Laermann, Monday]

$$\Delta(P/T^4) = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$
$$= \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2 \left(1 + \frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2 + \frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4 + \cdots\right)$$

$\mu_Q=\mu_S=0$

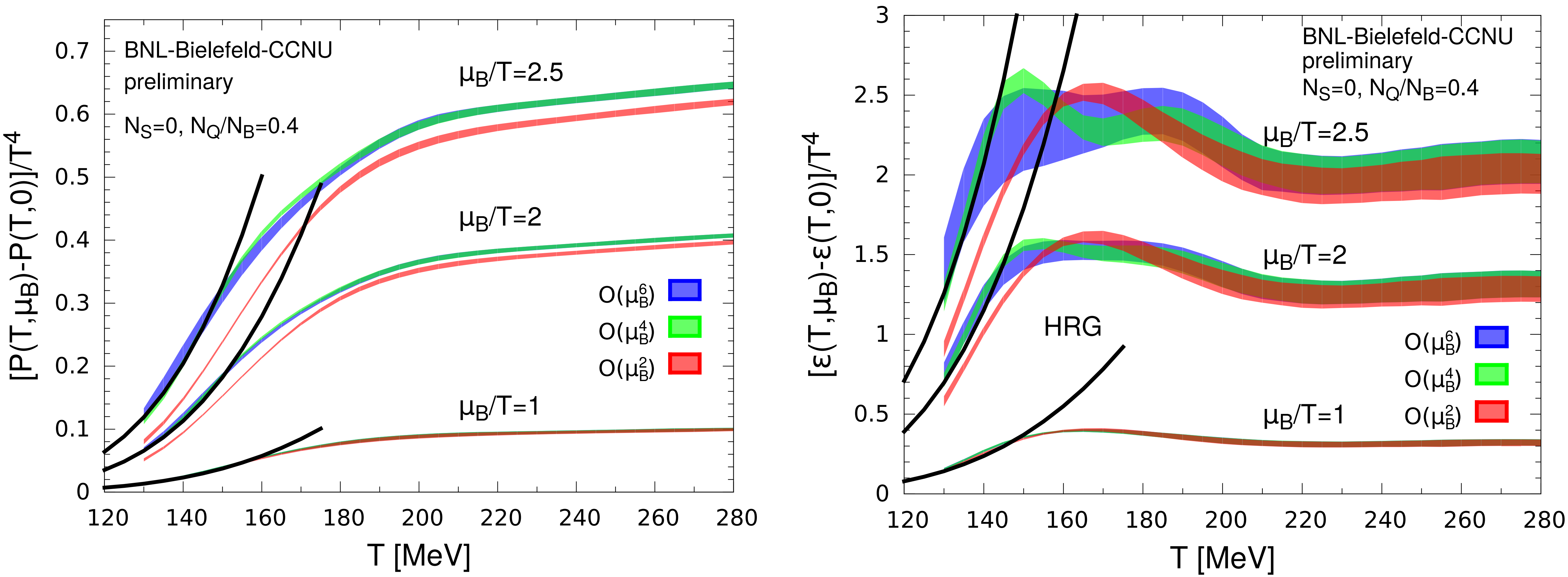


Equation of State well under control at $\mu_B/T \leq 2$

EoS in the strangeness neutral case: conditions in Heavy Ion Collisions

[Edwin Laermann, Monday]

At LHC and RHIC: <n_S>=0, <N_Q>/<N_B>=0.4



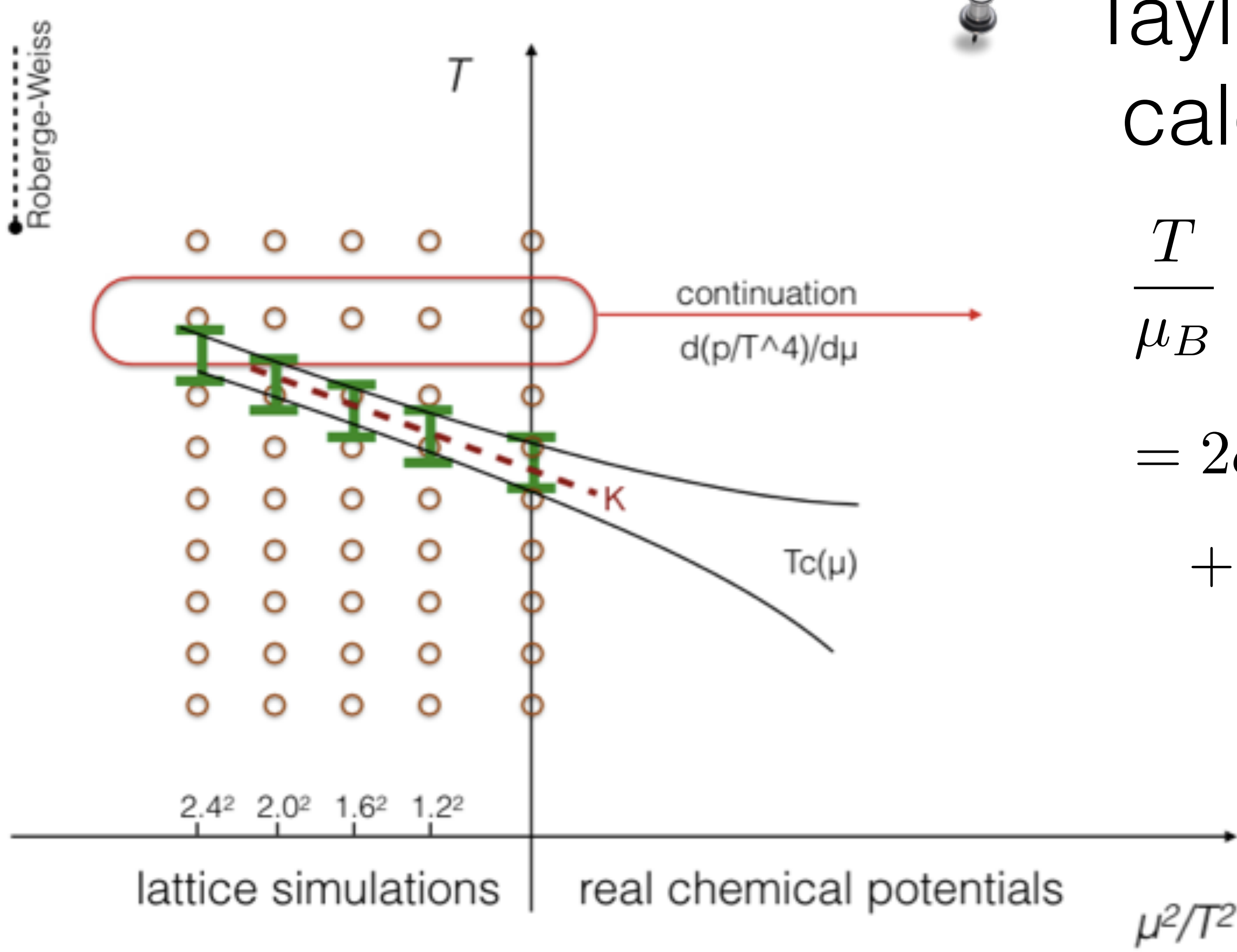
Equation of State well under control at $\mu_B/T \leq 2$, i.e. sqrt(S_{NN}) > 12 GeV in Heavy Ion Collisions

Taylor expansion coefficients from analytic continuation

[Jana Günther, Wednesday]

- Taylor expansion of pressure in real μ_B:

$$\frac{p(\mu_B)}{T^4} = c_0(T) + c_2(T)\left(\frac{\mu_B}{T}\right)^2 + c_4(T)\left(\frac{\mu_B}{T}\right)^4 + c_6(T)\left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8).$$

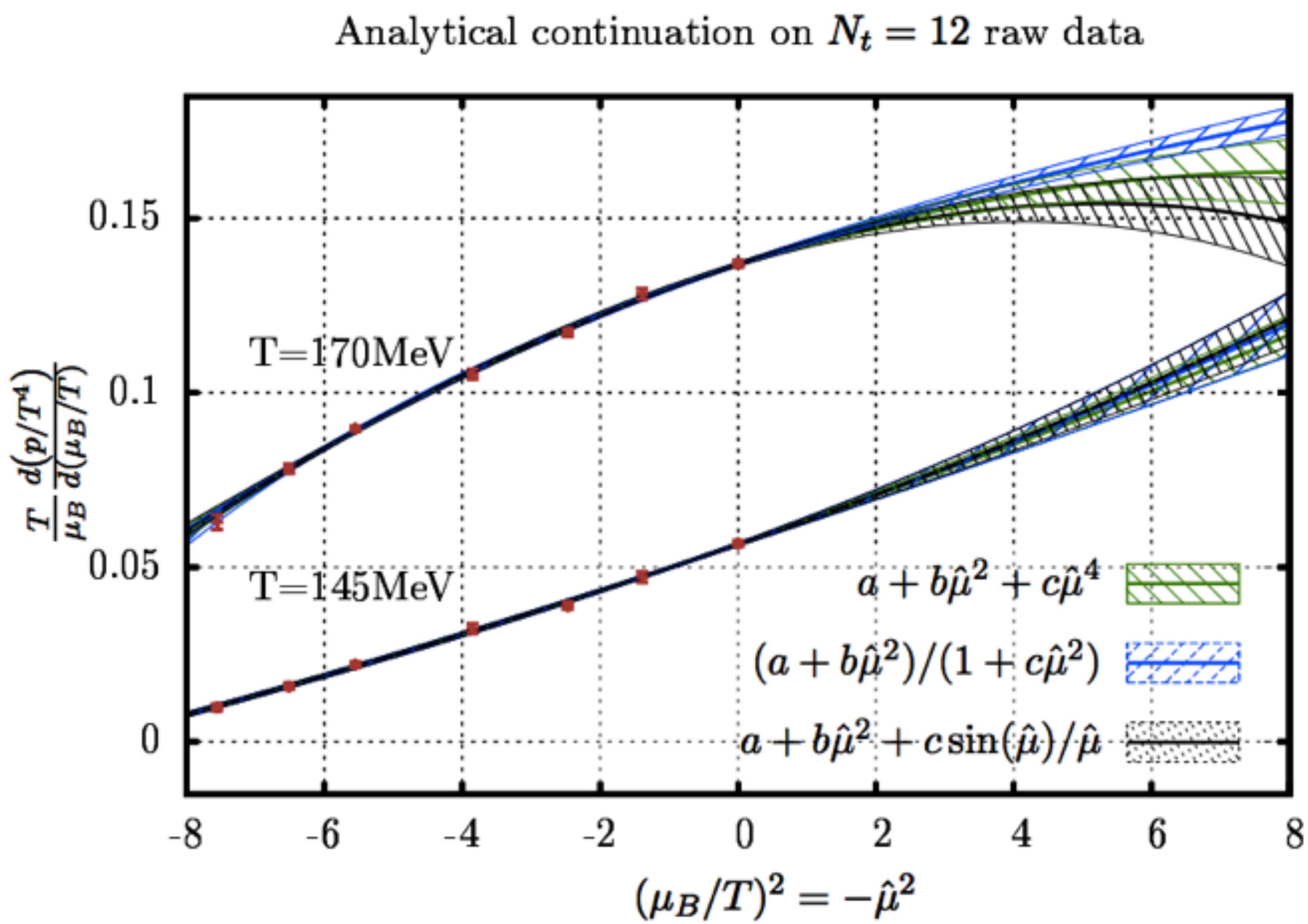


- Taylor expansion of pressure calculated in imaginary μ_B:

$$\frac{T}{\mu_B}\frac{\mathrm{d}(p/T^4)}{\mathrm{d}(\mu_B/T)}\Big|_{<n_S>=0, r=0.4, T=\mathrm{const.}}$$
$$= 2\tilde{c}_2(T) + 4\tilde{c}_4(T)\left(\frac{\mu_B}{T}\right)^2 + 6\tilde{c}_6(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}(\mu_B^8)$$

J. Günther et al.[WB collaboration], 1607.02493

Analytic continuation from analytic continuation



[Jana Günther, Wednesday]

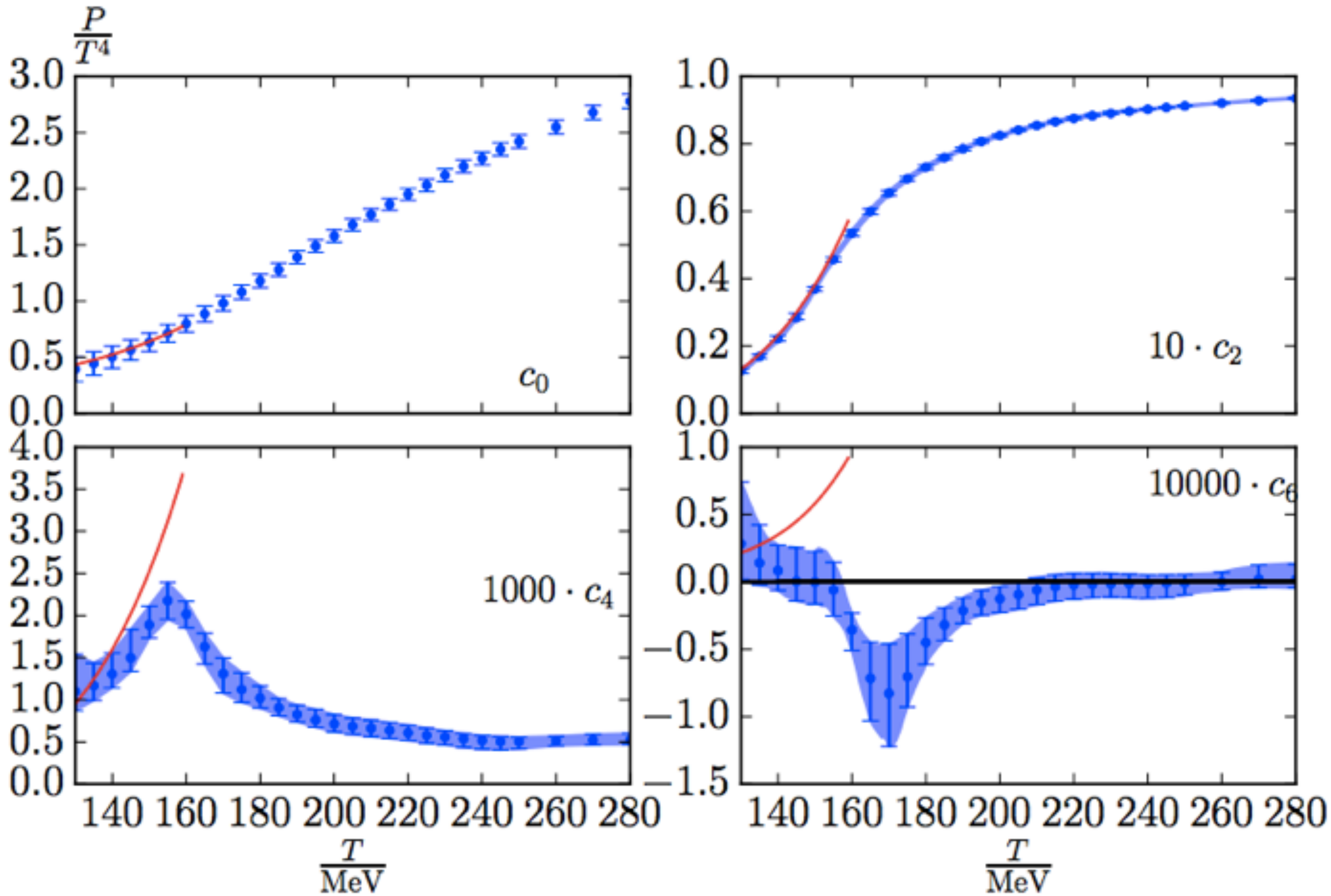
Intercept ->c2
slope -> c4
curvature ->c6
c8 ?

$$\frac{T}{\mu_B}\frac{\mathrm{d}(p/T^4)}{\mathrm{d}(\mu_B/T)}\Big|_{<n_S>=0, r=0.4, T=\mathrm{const.}} = 2\tilde{c}_2(T) + 4\tilde{c}_4(T)\left(\frac{\mu_B}{T}\right)^2 + 6\tilde{c}_6(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}(\mu_B^8)$$

Taylor expansion coefficients from analytic continuation

[Jana Günther Wednesday]

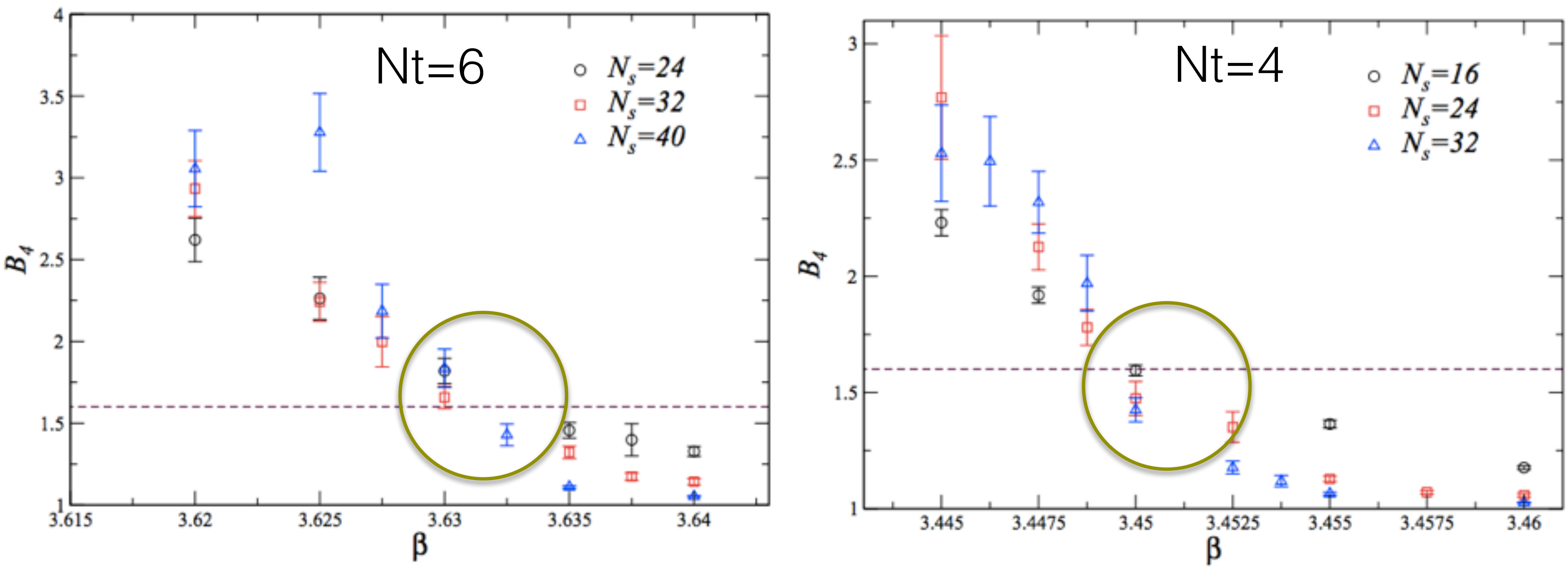
continuum extrapolated results from 4stout results on Nt=10,12,16



Compatible with the preliminary results of Taylor expansion coefficients from direct calculations (see Laermann's talk, Monday)

Roberge-Weiss phase transition temperature

[Michele Mesiti, Monday]

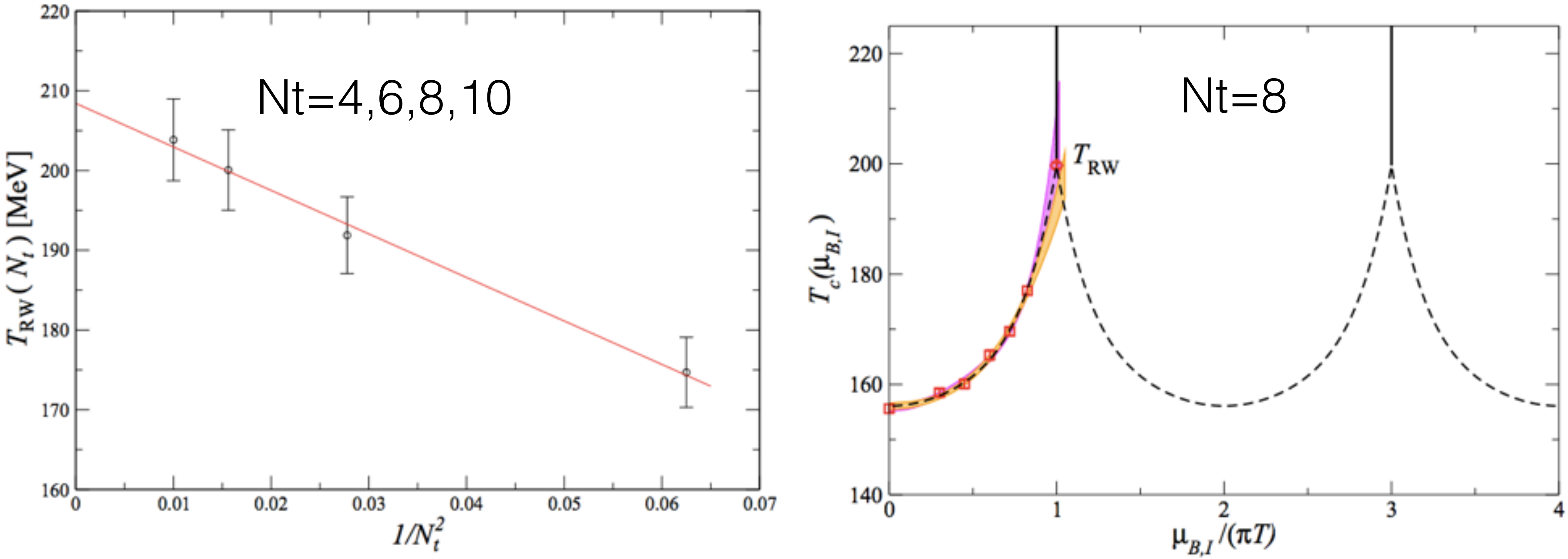


Bonati et al., Phys.Rev. D93 (2016) no.7, 074504

- Nf=2+1 QCD, stout fermions with physical pion mass
- Evidence found for $m_L^{tri} < m_l^{phy} < m_H^{tri}$ on Nt=4 & 6 lattices

Roberge-Weiss phase transition temperature

[Michele Mesiti, Monday]

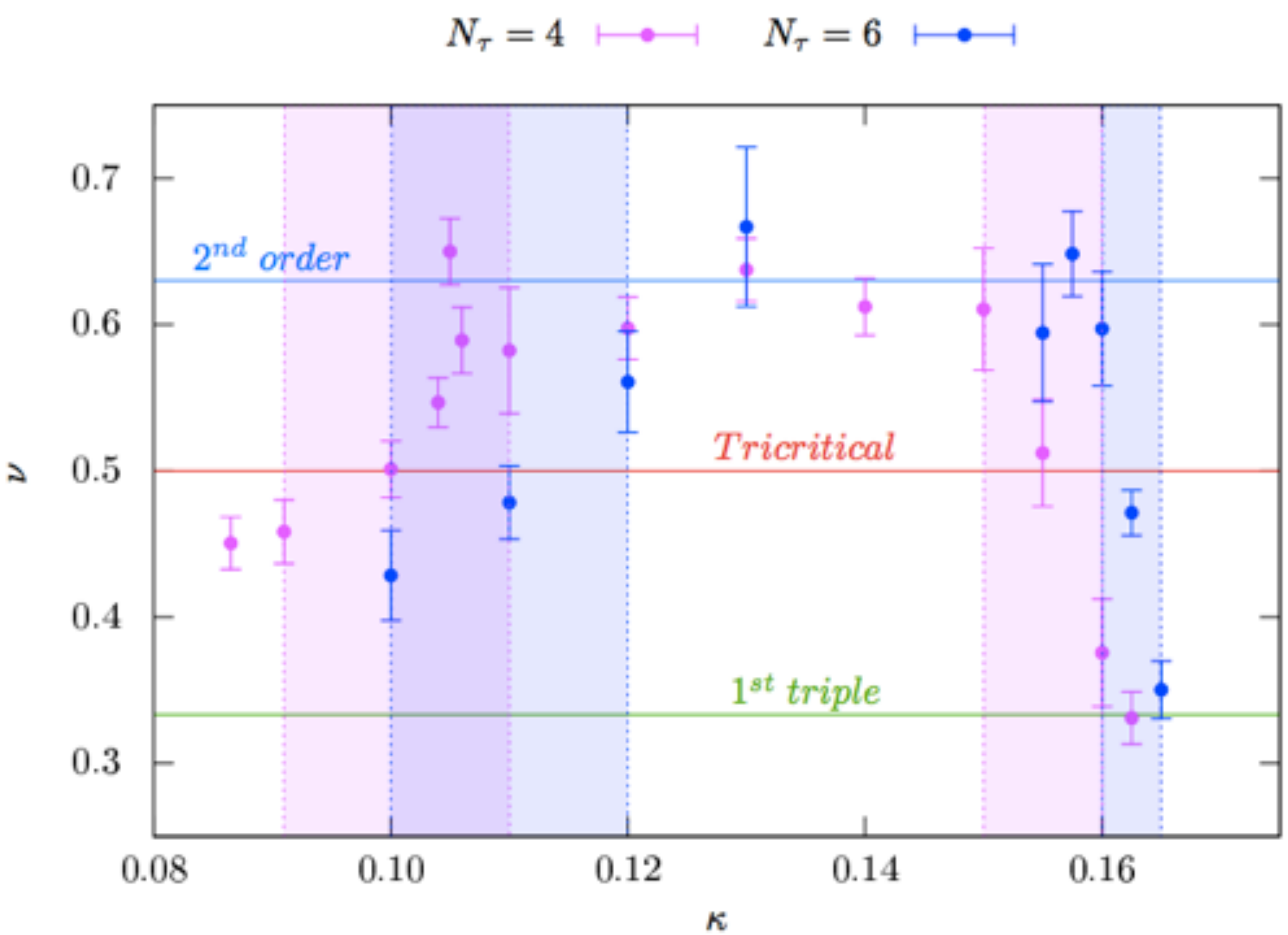


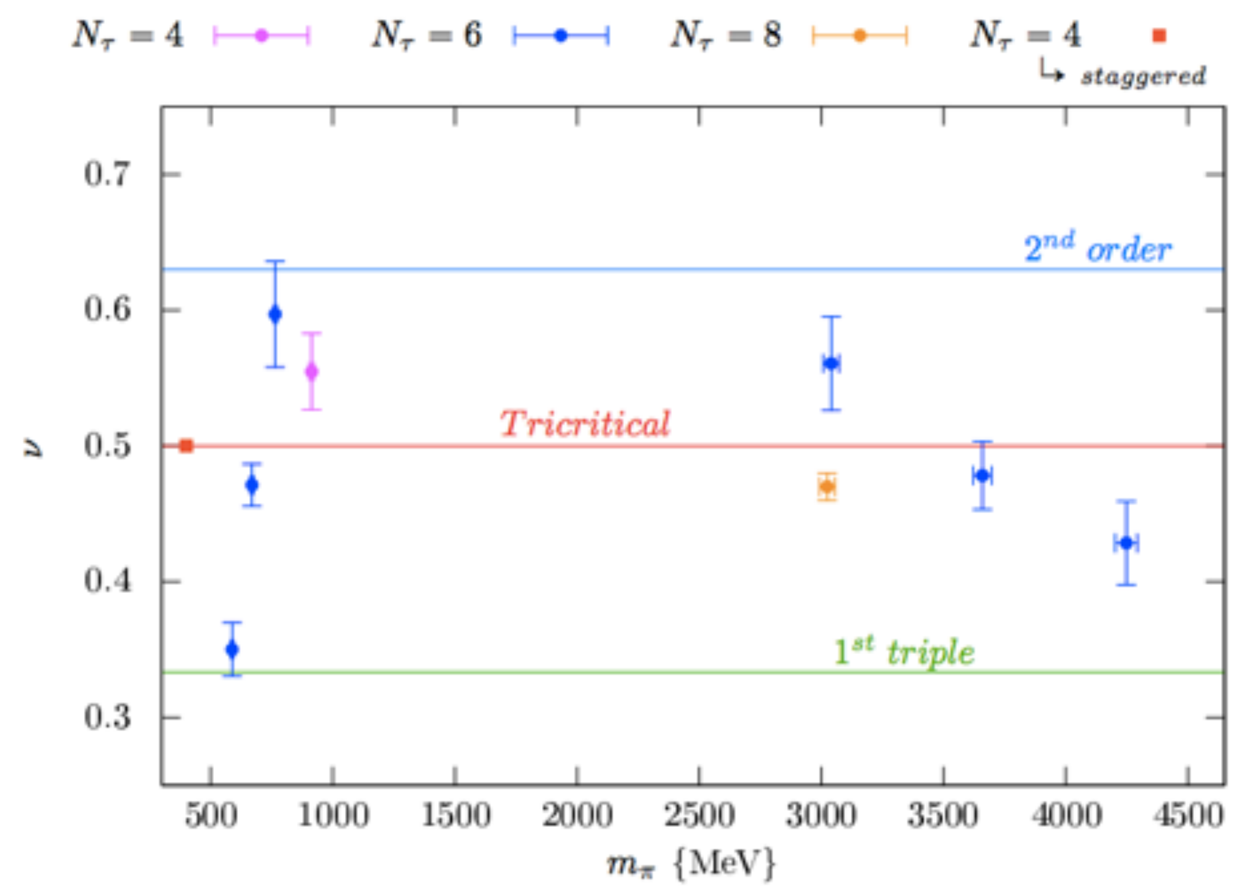
Bonati et al., Phys.Rev. D93 (2016) no.7, 074504

- Nf=2+1 QCD, stout fermions with physical pion mass
- Continuum extrapolated: T_{RW} = 208(5) MeV
- The location of RW endpoint is obtained

Roberge-Weiss phase transition temperature

[Christopher Czaban, Monday]



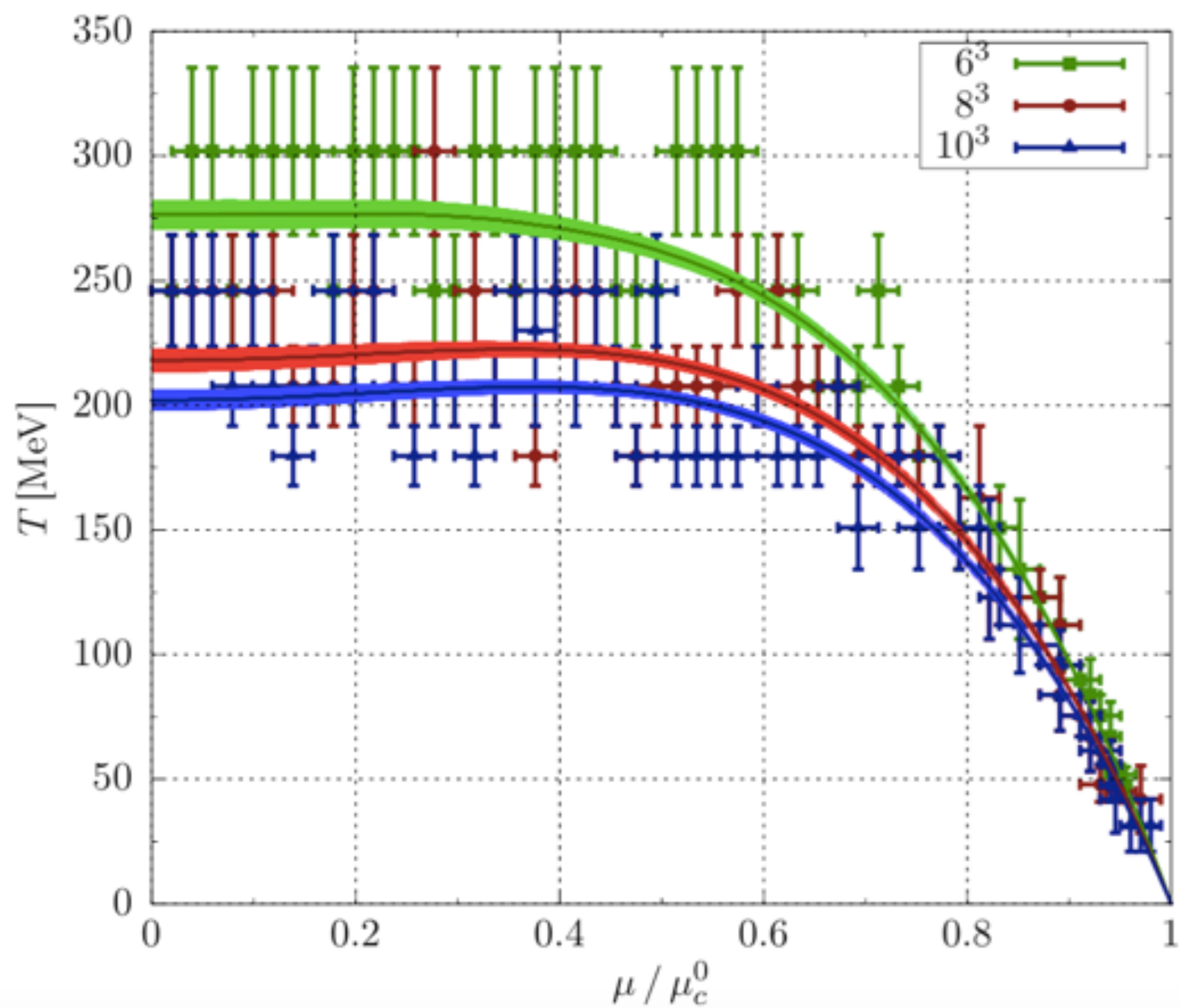


Czaban et al., Phys.Rev. D93 (2016) no.5, 054507

- Nf=2 QCD, standard Wilson fermions, Nt=6,8
- tri-critical pion mass values shift considerably when lattice cutoff is reduced

Phase diagram of QCD with heavy quarks (HDQCD) from Complex Langevin

[Felipe Attanasio, Tuesday]



Simulation setup

- Gauge coupling $\beta = 5.8$
  - Lattice spacing (approximate) $a \sim 0.15$ fm
- Hopping parameter $\kappa = 0.04$
  - Critical chemical potential $\mu_c^0 = 2.53$
- Lattice volumes $V = 6^3, 8^3, 10^3$
- Number of flavours $N_f = 2$
- Temporal extents/temperatures

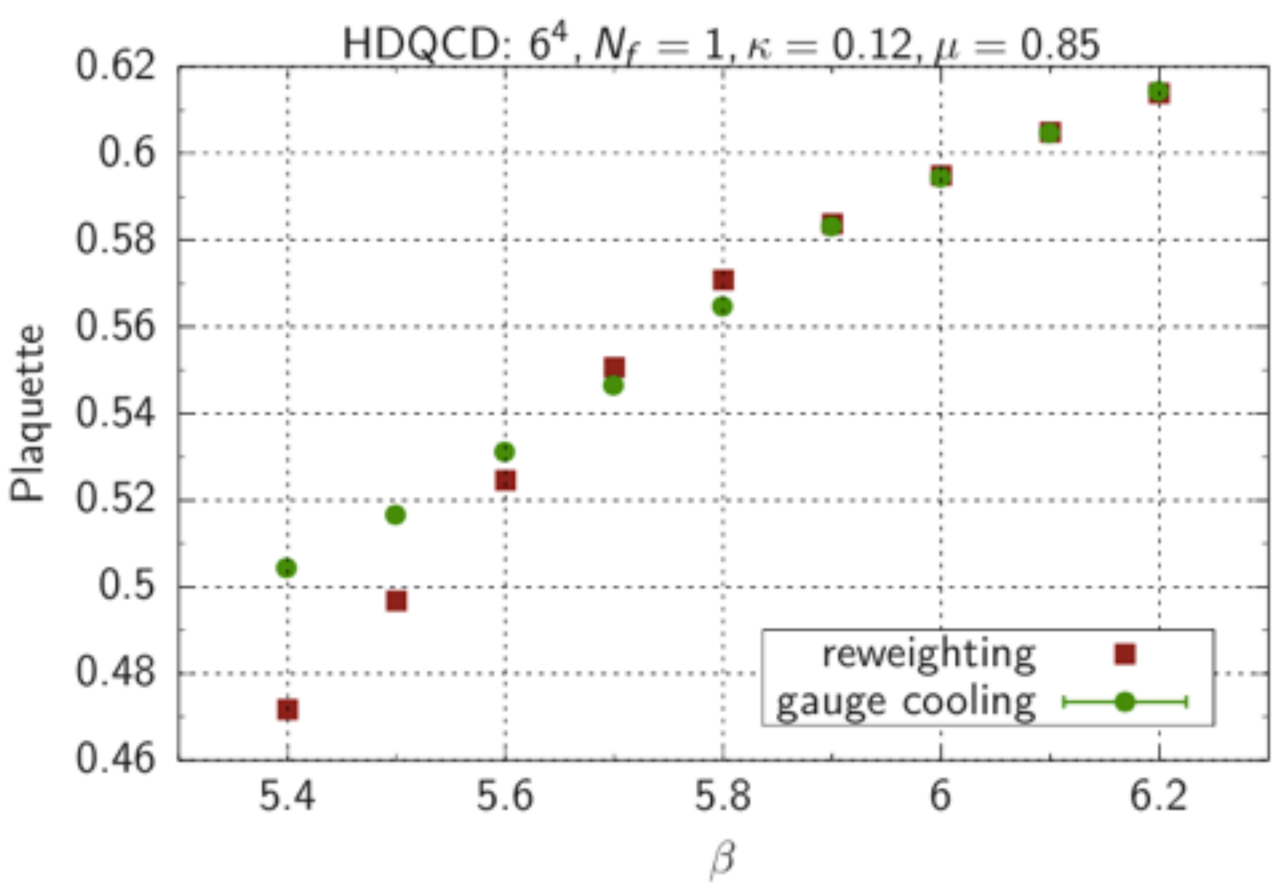
N_τ	28	24	20	16	14	12	10
T [MeV]	48	56	67	84	96	112	134
N_τ	8	7	6	5	4	3	2
T [MeV]	168	192	224	268	336	447	671

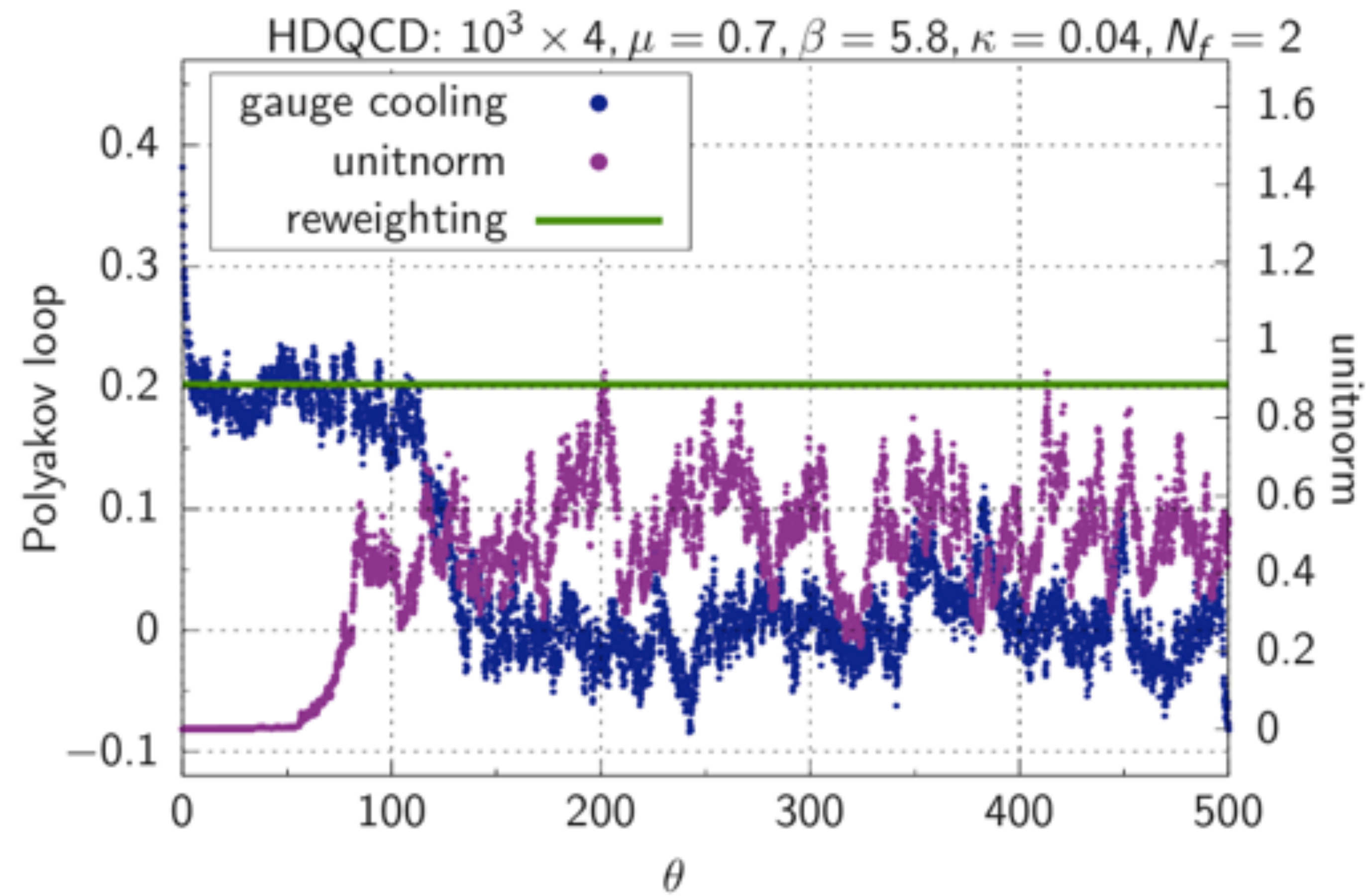
- To cure the sign problem: Gauge links SU(3) -> SL(3,ℂ)
- Gauge cooling: Gauge transformations between Langevin updates to minimize the distance from SU(3)

Instabilities in Complex Langevin simulations for Heavy Dense QCD

[Benjamin Jager, Tuesday]

See also [Felipe Attanasio, Tuesday]





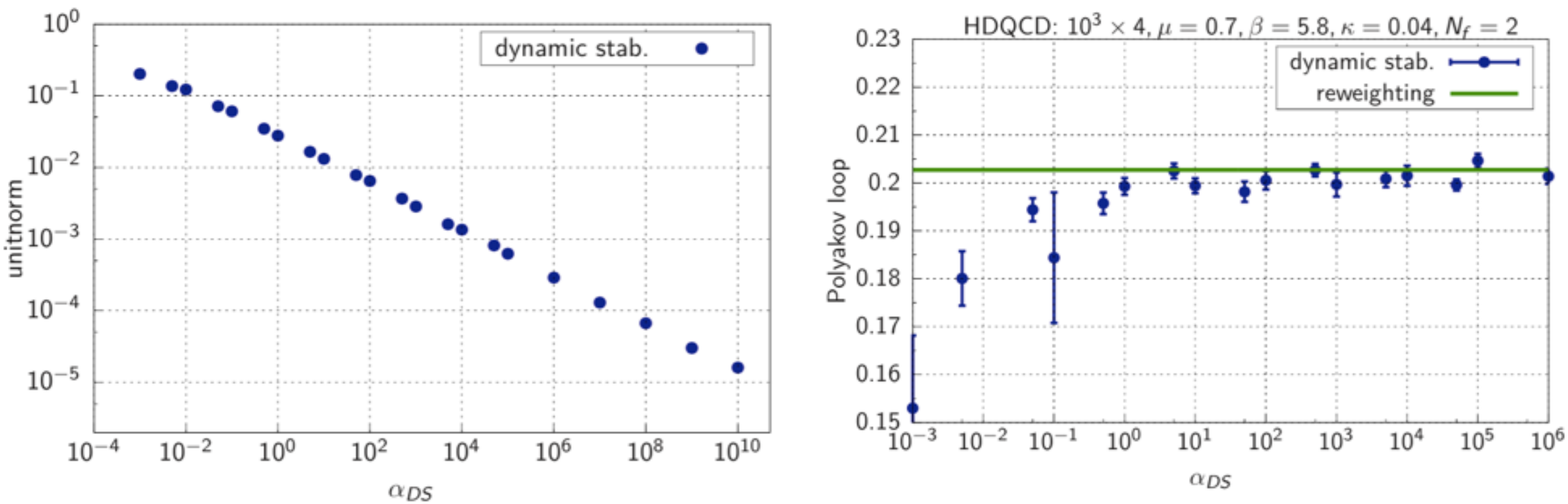
Gauge cooling is essential, however, it fails at some circumstances

Dynamic stabilization

[Benjamin Jager, Tuesday]

$$U_{x,\nu}(\theta+\varepsilon) = \exp\left[i\lambda^a\left(\varepsilon\, K^a_{x,\nu} - \varepsilon\,\alpha_{DS}\, M^a_x + \sqrt{\varepsilon}\,\eta^a_{x,\nu}\right)\right] U_{x,\nu}(\theta)$$

M: SU(3) gauge invariant, ~ a^7



Dynamic stabilization

[Benjamin Jager, Tuesday]

$$U_{x,\nu}(\theta+\varepsilon) = \exp\left[i\lambda^a\left(\varepsilon\, K^a_{x,\nu} - \varepsilon\,\alpha_{DS}\, M^a_x + \sqrt{\varepsilon}\,\eta^a_{x,\nu}\right)\right]U_{x,\nu}(\theta)$$

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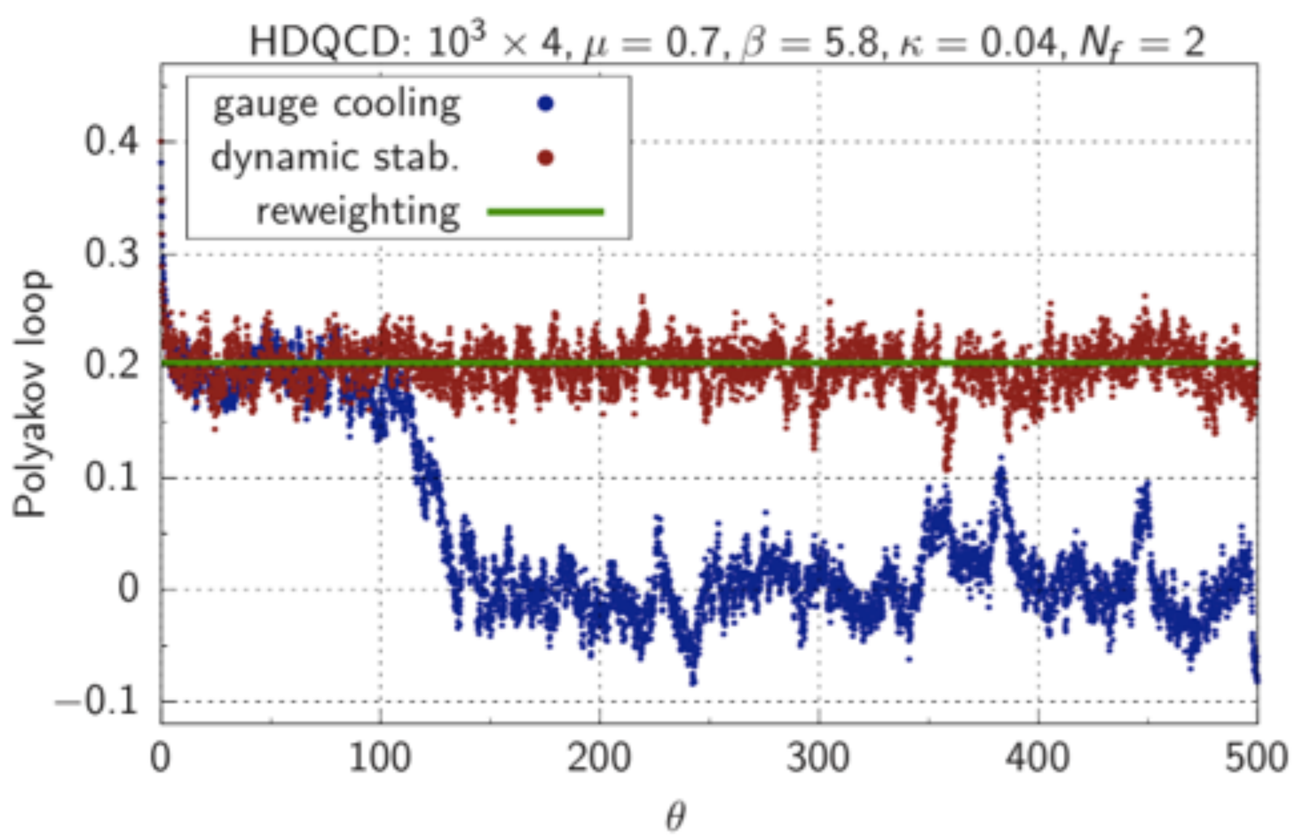
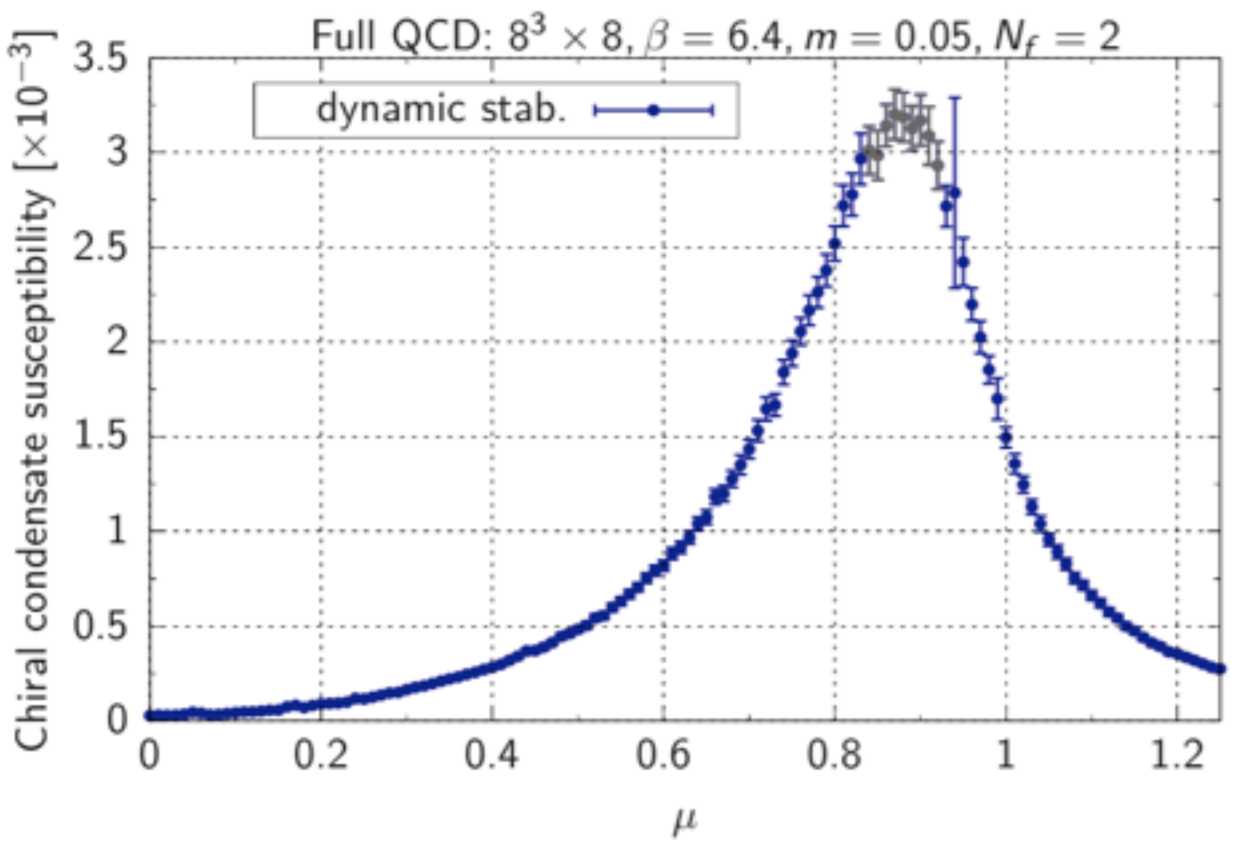


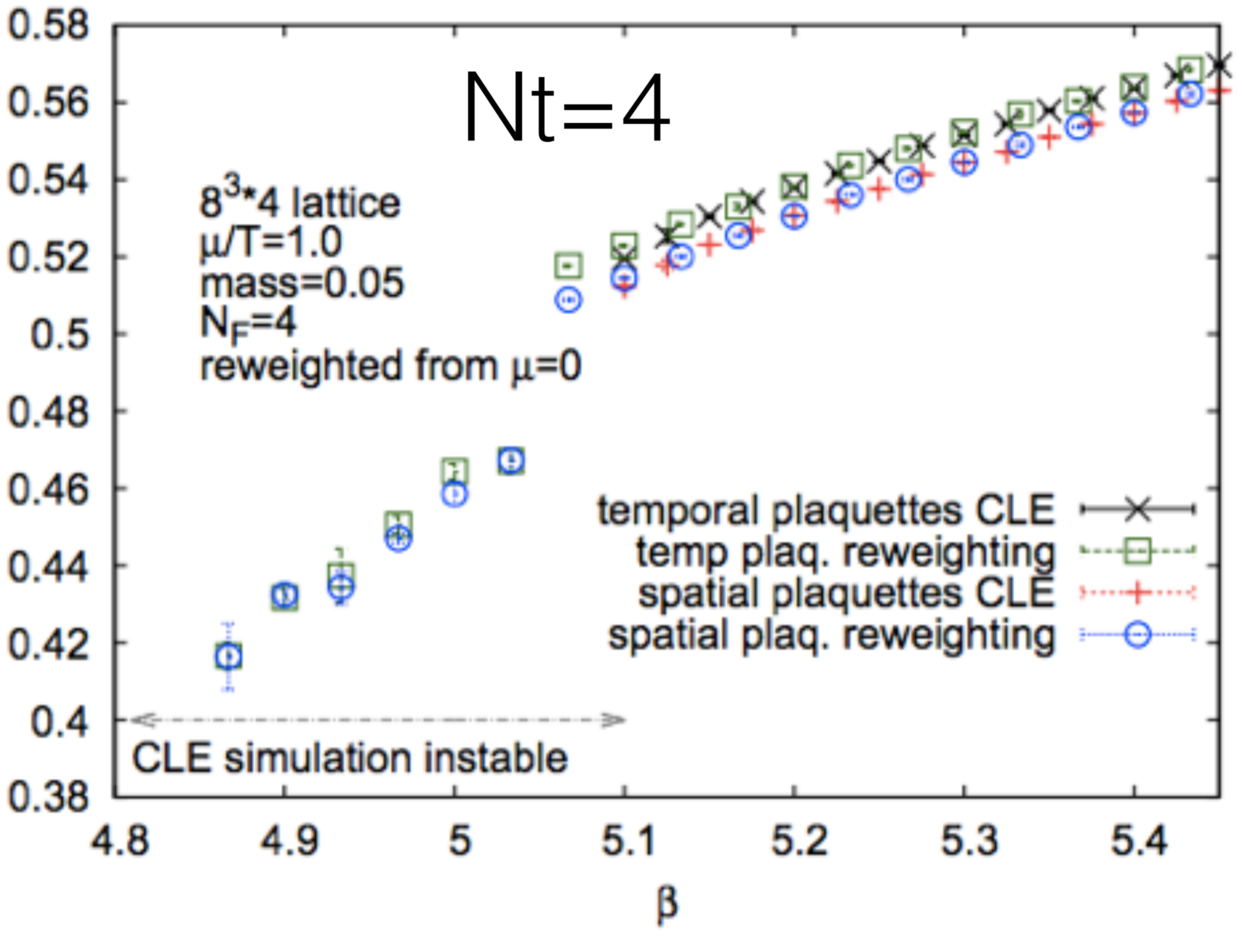


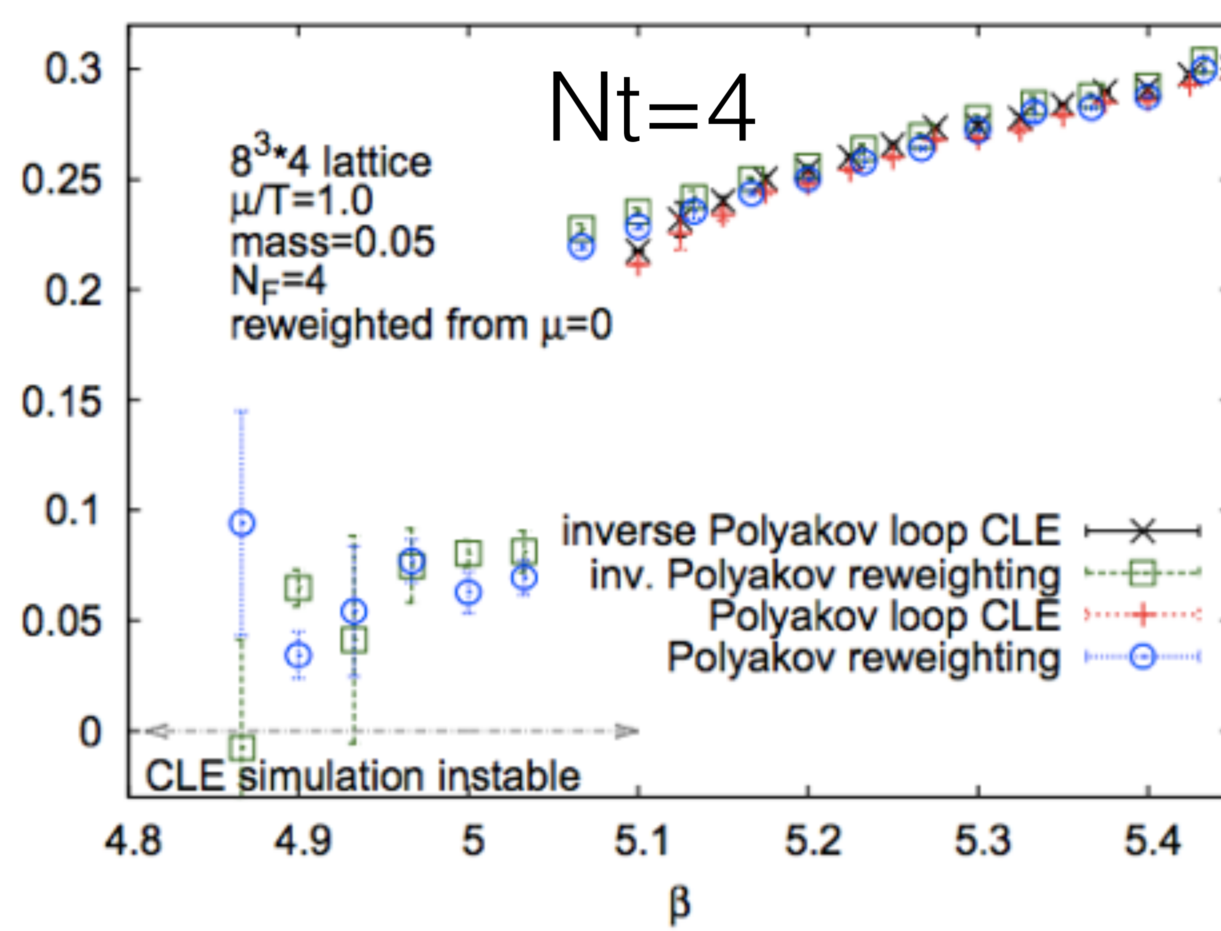
Dynamic stabilization improves convergence

More tests need for full QCD

Comparisons of Complex Langevin with reweighting for full QCD

[D. Sexty, Tuesday]





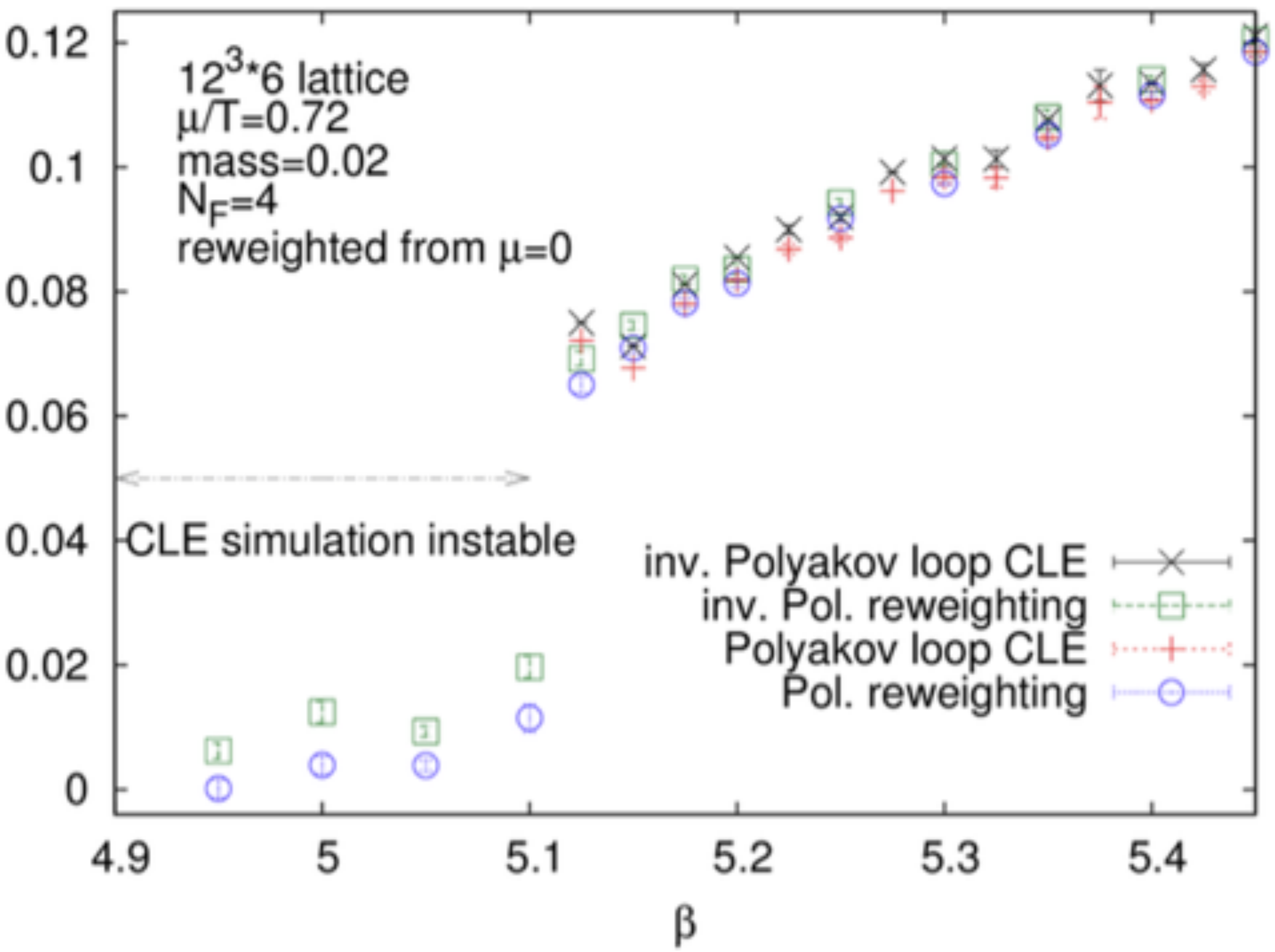
Fodor et al., PRD92 (2015) no.9, 094516

Similar to HDQCD, in the low temperate region
CLE simulation instable

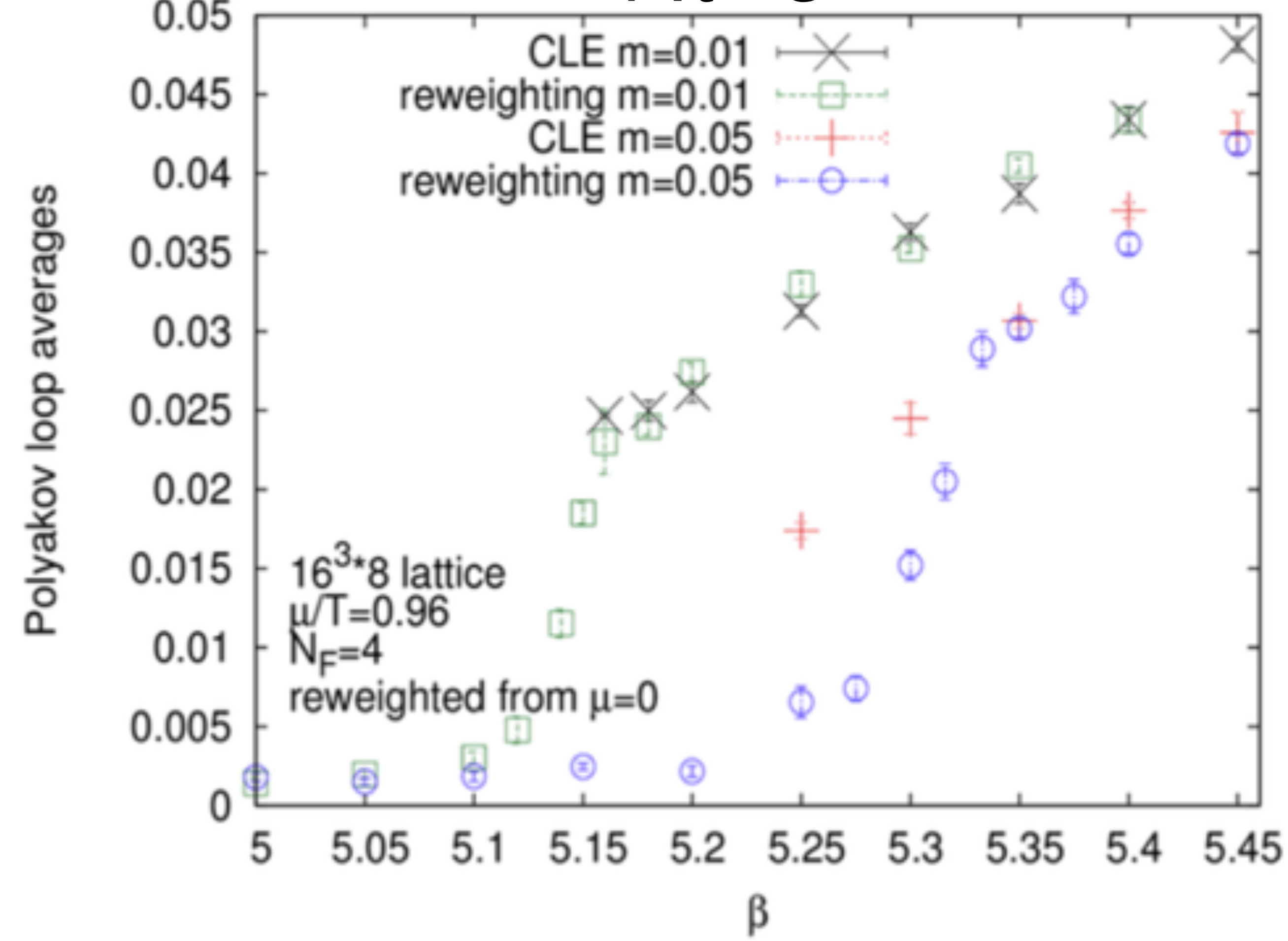
Comparisons of Complex Langevin with reweighting for full QCD

[D. Sexty, Tuesday]

Nt=6



Nt=8



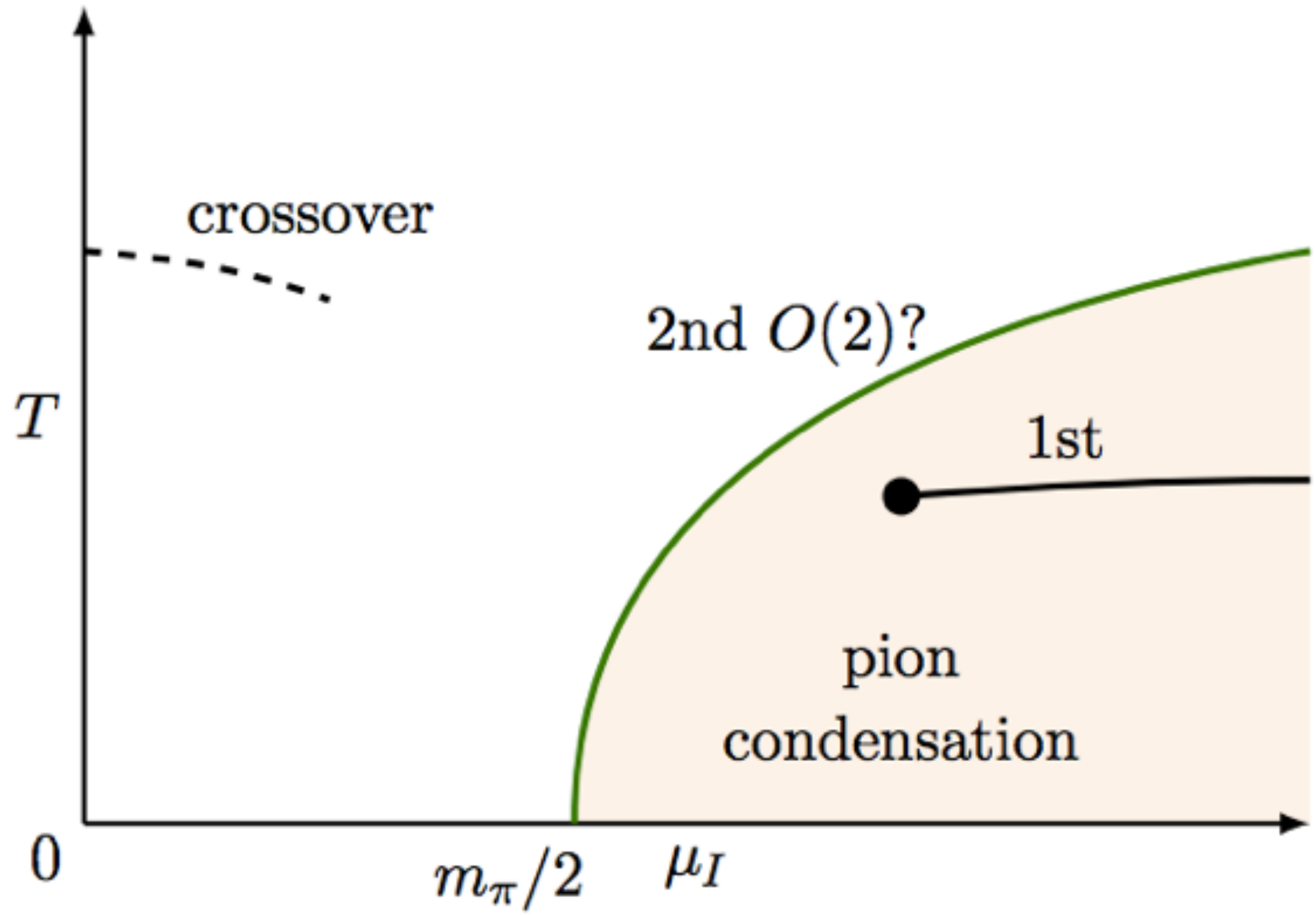
Fodor et al., PRD92 (2015) no.9, 094516

Similar to HDQCD, in the low temperate region
CLE simulation instable

Issues in singularities of the drift force of Langevin dynamics:
see talks on Tuesday by e.g. Gert Aarts, Keitaro Nagata

QCD at nonzero isospin density

Bastian Brandt & Gergely Endrodi (Thursday)



Son & Stephanov, PRL86 (2001)

First lattice simulations Nt=4 with m_π larger than physical one:

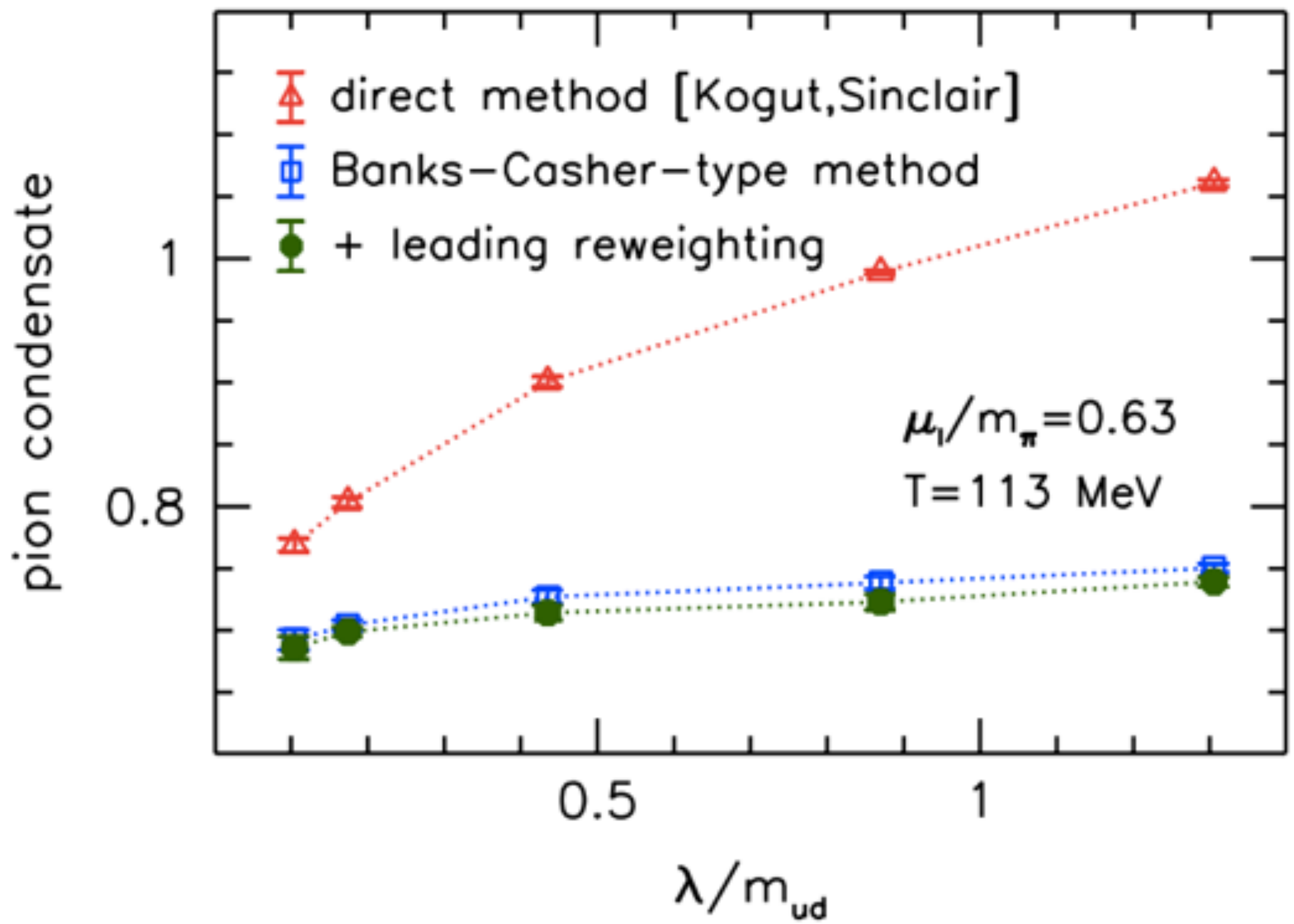
Kogut, Sinclair, PRD66 (2002); PRD70 (2004)

1st order deconfinement and 2nd curve join?

Existence of a tri-critical point

QCD at nonzero isospin density

Bastian Brandt & Gergely Endrodi (Thursday)



Direct method: $\Sigma_\pi \propto \left\langle \mathrm{Tr} M^{-1} \eta_5 \tau_2 \right\rangle$

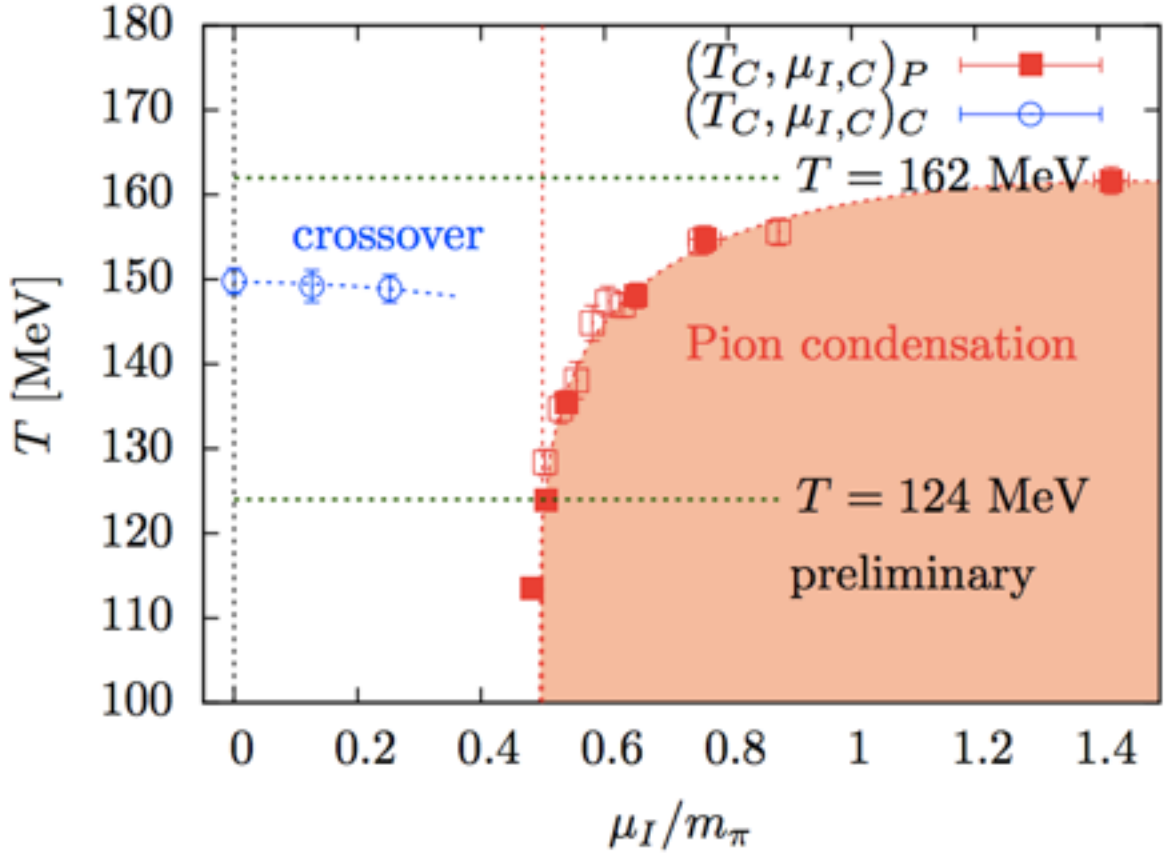
Banks-Casher-type method: $\pi = \frac{T}{V} \sum_i \frac{2\lambda}{\xi_i^2 + \lambda^2} = \int \mathrm{d}\xi \, \rho(\xi) \, \frac{2\lambda}{\xi^2 + \lambda^2} \xrightarrow{\lambda \to 0} \pi \rho(0)$

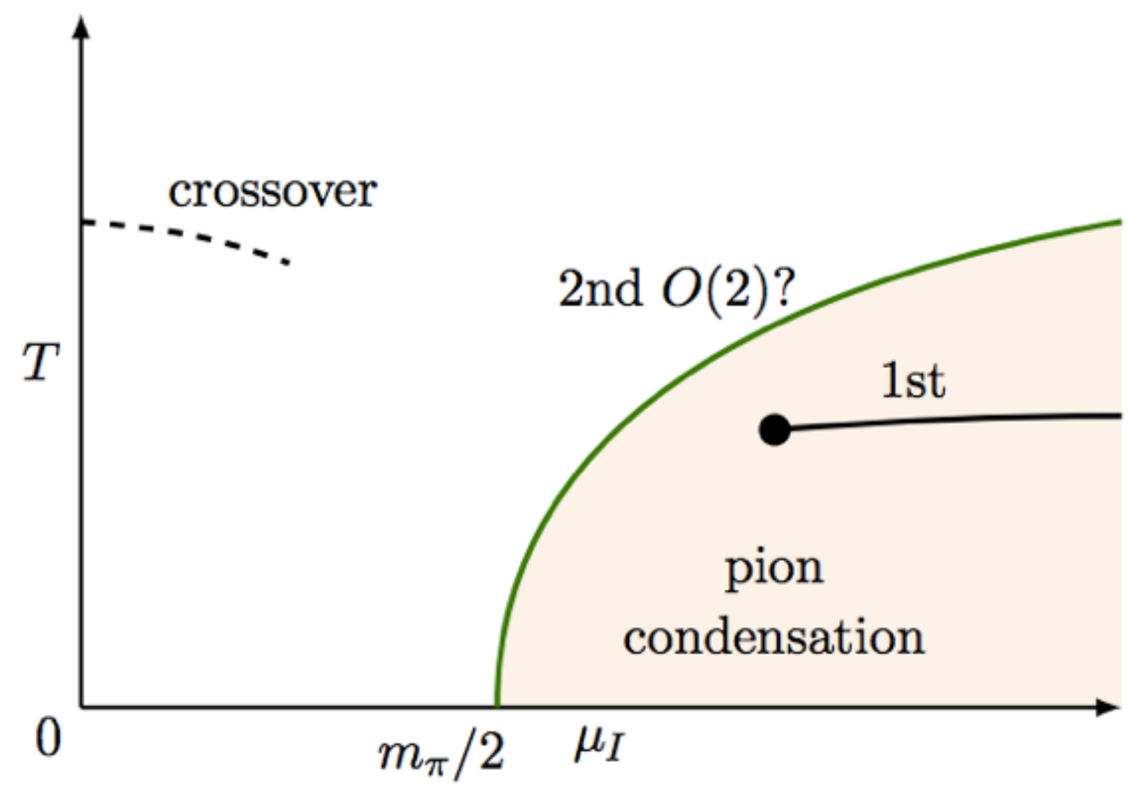
Kanazawa, Wettig, Yamamoto '11

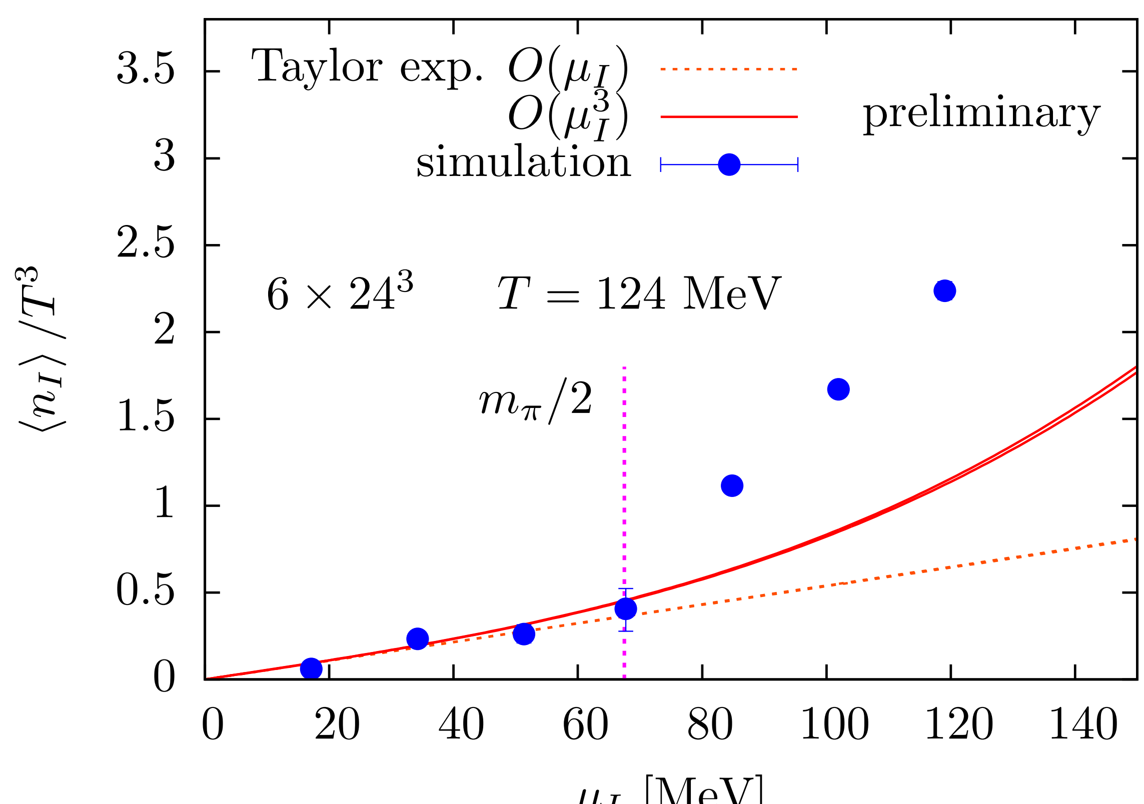
Leading reweighting: $\langle \pi \rangle_{\mathrm{rew}} = \langle \pi W_\lambda \rangle / \langle W_\lambda \rangle \quad W_\lambda = \exp[-\lambda V_4 \pi + \mathcal{O}(\lambda^2)]$

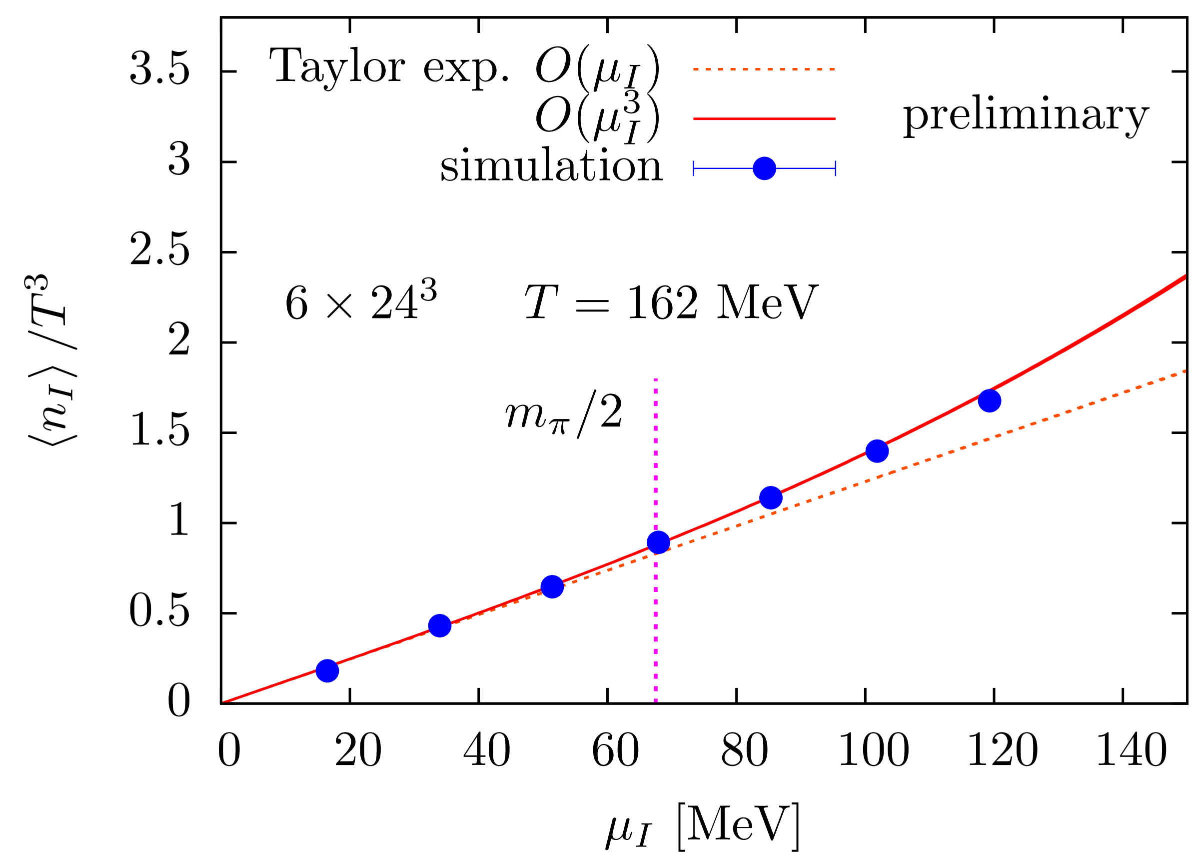
QCD at nonzero isospin density

Bastian Brandt & Gergely Endrodi (Thursday)









Thanks to

- Members of Bielefeld-BNL-CCNU collaboration
- People who sent me materials/plots: Pedro Bicudo, Jacques Bloch, Szabolcs Borsanyi, Bastian Brandt, Falk Bruckmann,Guido Cossu, Gergely Endrodi, Philippe de Forcrand, Yoichi Iwasaki, Kazuyuki Kanaya, Sandor Katz, Masakiyo Kitazawa, Yu Maezawa, Atsushi Nakamura, Yoshifumi Nakamura, Alexander Rothkopf, Jonivar Skullerud, Hélvio Vairinhos, Takashi Umeda

Apologies to those whose achievements were not mentioned in my talk

many talks on the sign problem, e.g. Lefschetz thimbles, canonical method, complex Langevin, subsets etc and QC_2D, strong coupling as well as some topics in the sessions of this afternoon are not covered