From spin models to lattice QCD – The scientific legacy of Péter Hasenfratz

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Péter Hasenfratz

Budapest, Sept. 22, 1946 – Bern, April 9, 2016

Péter was a very gentle and extremely modest person

He had a very generous personality

"*If I would not know Péter's profession, I would have guessed he must be a pediatrician.*"

quote from a friend

"*If I would not have become a scientist, I would be a poet.*"

quote from Péter

He would tolerate only the highest scientific standards

We are sad to have lost an extraordinary person and scientist

Péter's CV

- B.Sc. in Physics, 1971, Eötvös University, Budapest
- Ph.D. in Physics, 1973, Eötvös University, Budapest
- 1973 - 1975 Member of the Central Research Institute for Physics, Budapest
- 1975 - 1976 Postdoc with Gerard 't Hooft, Utrecht
- 1977 - 1979 Return to CRIP, Budapest
- 1979 - 1980 Postdoc at CERN
- 1981 - 1984 Staff member Theory Division at CERN
- 1984 - 1991 Associate professor at the University of Bern
- 1991 - 2011 Full professor at the University of Bern

Péter's scientific contributions

Over 125 publications:

- many of them plenary talks, review articles and lecture notes

Excellent academic teacher, also at numerous international schools

Advisor to 14 Ph.D. students

He initiated the first lattice conference at CERN in 1982:

- handwritten personal invitations
- the seed for a new scientific community
- 33 conferences so far, 420 participants today

Péter's scientific contributions

Ability to analytically calculate seemingly uncalculable things:

- the scale parameter of QCD on the lattice $\Lambda_{\mathrm{QCD}}^{\mathrm{latt}}$
- exact mass gaps in several 2-dim. asymptotically free QFTs

Very creative and original thinker, numerous seminal contributions to lattice QFT:

- FP actions, index theorem
- understanding of chiral symmetry
- chemical potential
- ...

Other major research topics

- quark bag model
- topological excitations
- spin models
- hopping expansion
- higgs physics (upper bound, top quark condensate, ...)
- finite size effects from Goldstone bosons
- finite temperature phase transition in QCD

Overview

The connection between the lattice and the continuum

The mass gaps

The renormalization group and FP actions

Quotes from a 1981 review paper

"*MC simulations did not help us to obtain a better physical understanding, a deeper insight into the theory.*"

"*Is g small enough? [...] there is reason to worry: the approach to asymptotic scaling might be very slow.*"

"*In spite of the intense work, there is no real progress one can report on.*"

"*The whole program is faced with unexpected and unpleasant difficulties at this moment.*"

"*Clarification is needed.*"

"*One should consider these numbers with some reservations.*"

"*[...] although it is not clear whether every part of the calculation is under control.*"

Quotes from a 1982 review paper

"*[. . .] the reliability of this procedure is really questionable.*"

"*I sense a big change concerning the expectations of the physics community. Actually I believe this change is too big.*"

"*Please, have your own healthy doubts [. . .]. Solving QCD is not <u>so</u> easy.*"

"*[. . .] admit clearly the defects of our methods and make serious efforts to improve them. This path is less spectacular, but, perhaps, worth following.*"

The connection between the lattice and the continuum

- lattice QCD contains only the dimensionless coupling g and implicitly the lattice spacing a as parameters

- for a physical mass m or a length ξ one has

$$m = f(g) \cdot \frac{1}{a} \qquad \xi = h(g) \cdot a$$

- continuum limit reached when $1/m$ or $\xi \gg a$:
  - system approaches continuous PT (statistical physics)
  - in asymptotically free theories: $a \to 0$ for $g \to 0$

- physical quantities should become independent of a in the continuum limit:

$$\frac{d}{da} m = 0 \quad (a \to 0) \qquad \Longleftrightarrow \qquad \text{renormalizability}$$

The connection between the lattice and the continuum

- this yields a differential equation for $f(g)$:

$$-f(g) + f'(g)\left(a\frac{d}{da}g\right) = 0$$

where

$$\beta(g) \equiv a\frac{d}{da}g = -b_0 g^3 - b_1 g^5 - \dots$$

- every physical quantity can be expressed in terms of a single, RG-invariant mass parameter Λ^{latt}, e.g. $m = c_m \cdot \Lambda^{\mathrm{latt}}$

$$\Lambda^{\mathrm{latt}} = \frac{1}{a}\, e^{-1/2b_0 g^2} \left(b_0 g^2\right)^{-b_1/2b_0^2} \cdot [1 + \mathcal{O}(g^2)]$$

- analogously in a continuum ren. scheme one has

$$\Lambda = M\, e^{-1/2b_0 g(M)^2} \left(b_0 g(M)^2\right)^{-b_1/2b_0^2} \cdot [1 + \mathcal{O}(g(M)^2)]$$

The connection between the lattice and the continuum

- to set the scale (and to make sense), better connect the two

- in 1980 Peter and Anna Hasenfratz obtained this connection:

$$\begin{aligned}\Lambda^{\mathrm{MOM}}_{\text{Feynman gauge}} &= 83.5\,\Lambda^{\mathrm{latt}} \quad SU(3)\\ &= 57.5\,\Lambda^{\mathrm{latt}} \quad SU(2)\end{aligned}$$

  - long 1-loop lattice PT calculation of 2- and 3-point functions
  - explicit demonstration that there are no unwanted divergences
  - all non-covariant terms cancel
  - first to get it correct

- it took 15 more years until the 2-loop calculation [Lüscher, Weisz '95]

The connection between the lattice and the continuum

- of course, the Λ parameter is nonperturbatively defined:

$$\Lambda = M\, e^{-1/2b_0 g(M)^2} \left(b_0 g(M)^2\right)^{-b_1/2b_0^2} \times \left[1 + \mathcal{O}(g(M)^2)\right]$$

- lattice QCD is the ideal method to relate it nonperturbatively to the low-energy properties of QCD

The connection between the lattice and the continuum

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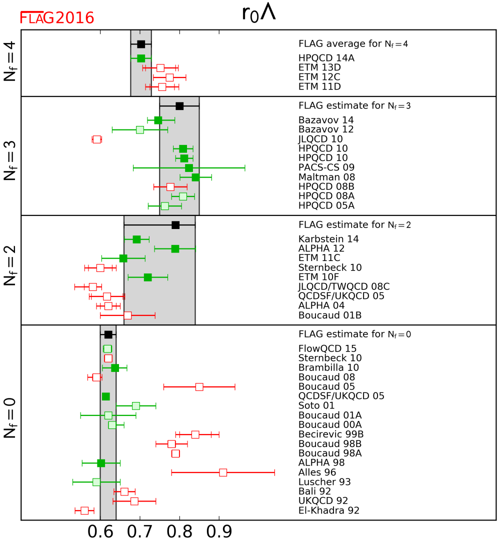
$$\Lambda = M\, e^{-1/2b_0 g(M)^2} \left(b_0 g(M)^2\right)^{-b_1/2b_0^2} \times \exp\left[-\int_0^{g(M)} dx \left(\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x}\right)\right]$$

- lattice QCD is the ideal method to relate it nonperturbatively to the low-energy properties of QCD

The connection between the lattice and the continuum

- Λ parameter in the $\overline{\mathrm{MS}}$-scheme in units of r_0:



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- lattice QCD is the ideal method to relate it nonperturbatively to the low-energy properties of QCD
- closely related is the running coupling α_s at scale M

$$\alpha_s(M) = \frac{g^2(M)}{4\pi}$$

- measure a short distance quantity $\mathcal{Q}$ at scale M and match with perturbative expansion

$$\mathcal{Q}(M) = c_1 \alpha_{\overline{\mathrm{MS}}}(M) + c_2 \alpha_{\overline{\mathrm{MS}}}(M)^2 + \ldots$$

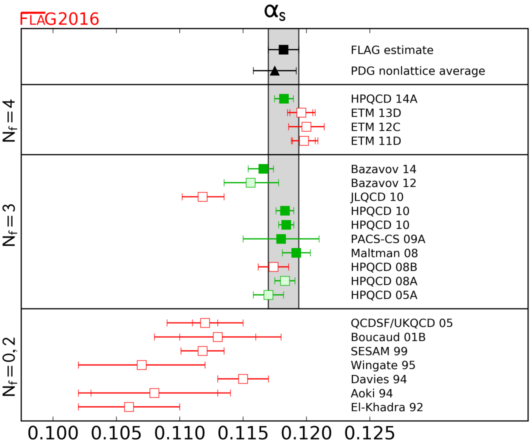
The connection between the lattice and the continuum

Collaboration	Ref.	N_f	publication status	renormalization scale	perturbative behaviour	continuum extrapolation	$\alpha_{\overline{\mathrm{MS}}}(M_Z)$	Method	Table
HPQCD 14A	[12]	2+1+1	A	○	★	○	0.11822(74)	current two points	45
ETM 13D	[121]	2+1+1	A	○	○	■	0.1196(4)(8)(16)	gluon-ghost vertex	46
ETM 12C	[122]	2+1+1	A	○	○	■	0.1200(14)	gluon-ghost vertex	46
ETM 11D	[123]	2+1+1	A	○	○	■	$0.1198(9)(5)(^{+0}_{-5})$	gluon-ghost vertex	46
Bazavov 14	[10]	2+1	A	○	★	○	$0.1166(^{+12}_{-8})$	Q-$\bar{Q}$ potential	42
Bazavov 12	[70]	2+1	A	○	○	○	$0.1156(^{+21}_{-22})$	Q-$\bar{Q}$ potential	42
HPQCD 10	[91]	2+1	A	○	★	○	0.1183(7)	current two points	45
HPQCD 10	[91]	2+1	A	○	★	★	0.1184(6)	Wilson loops	44
JLQCD 10	[79]	2+1	A	■	■	■	$0.1118(3)(^{+16}_{-17})$	vacuum polarization	43
PACS-CS 09A	[53]	2+1	A	★	★	○	0.118(3)#	Schrödinger functional	41
Maltman 08	[92]	2+1	A	○	○	★	0.1192(11)	Wilson loops	44
HPQCD 08B	[99]	2+1	A	■	■	■	0.1174(12)	current two points	45
HPQCD 08A	[83]	2+1	A	○	★	★	0.1183(8)	Wilson loops	44
HPQCD 05A	[82]	2+1	A	○	○	○	0.1170(12)	Wilson loops	44
QCDSF/UKQCD 05	[93]	$0,2 \to 3$	A	★	■	★	0.112(1)(2)	Wilson loops	44
Boucaud 01B	[116]	$2 \to 3$	A	○	○	■	0.113(3)(4)	gluon-ghost vertex	46
SESAM 99	[89]	$0,2 \to 3$	A	★	■	■	0.1118(17)	Wilson loops	44
Wingate 95	[90]	$0,2 \to 3$	A	★	■	■	0.107(5)	Wilson loops	44
Davies 94	[88]	$0,2 \to 3$	A	★	■	■	0.115(2)	Wilson loops	44
Aoki 94	[87]	$2 \to 3$	A	★	■	■	0.108(5)(4)	Wilson loops	44
El-Khadra 92	[86]	$0 \to 3$	A	★	■	○	0.106(4)	Wilson loops	44

Result with a linear continuum extrapolation in a.

- critical assessment of the situation is necessary
- dominant source of uncertainty from discretization errors and truncation of continuum/lattice PT

The connection between the lattice and the continuum



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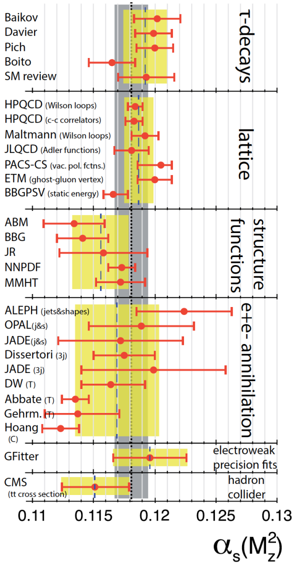

- FLAG 16 estimate yields [arXiv:1607.00299]

  $$\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.1182(12)$$

- to be compared with PDG 16 values

  $$\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.1175(17) \quad (\textit{phen. only})$$
  $$\alpha_{\overline{\text{MS}}}^{(5)}(M_Z) = 0.1181(13)$$

- still room for systematic improvement (smaller lattice spacing,. . .)

- in the long term, it pays off to be conservative

The connection between the lattice and the continuum

Critical assessment and summary table by FLAG 16:

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Result with a linear continuum extrapolation in a.

The connection between the lattice and the continuum

Critical assessment and summary table by Peter in 1981:

Reference	Gauge group fermion formulation	Virtual quark loops included?	Lattice size No. of iterations	The scope of the investigation	~ computer time in CDC hours
11)	SU(2) subgroup Kogut-Susskind	NO	$8^4 + 8^3.16$ 4 confs.	$<\bar{\psi}\psi>$, f_π ; RG. $m_\pi, m_\rho, m_\delta; m_u$ $\beta = 2.2$ fixed	~ 10
12)	SU(2) subgroup Wilson	NO	12^4 8 confs.	$m_\pi, m_\rho, K_u(g^2), a(g^2)$ $\beta = 2.05, 2.25, 2.44$	~ 100
13)	SU(3) Wilson	NO	$6^4 + 6^3.10$ ~15 confs./K val.	$<\bar{\psi}\psi>$, f_π ; RG. Mesons and baryons with u and d quarks m_u $1/g^2 = 1.0$ fixed	~ 100
14)	SU(2) subgroup Kogut-Susskind	NO	$8^3.32$ 8 confs.	$c\bar{c}$ mesons $\eta_c, \psi, \chi, \psi'$ $\beta = 2.7$ fixed	~ 2
The first Ref. in 8)	SU(3) Wilson	YES	Hopping exp.: ∞^4 MC : 8^4 15-20 confs./g val.	Vector and PS mesons of u, d, s and c quarks Quark masses, $K_i(q^2)$, $a(g^2)$ $1/g^2 = 0.0, 0.7, 0.9, 0.925, 0.95, 1.0$	~ 5
15)	SU(3) Wilson	NO	5^3x10 32 confs./K val.	$1/g^2 = 1.0$ fixed mesons and baryons ; quark masses	~ 100
The second Ref. in 8)	SU(3) Wilson	NO	∞^4 MC : 8^4 3-10 confs./g val.	Baryon masses $1/g = 0.0, 0.4, 0.7, 0.925$ Preliminary results on the P wave mesons created by point-splitted operators	~ 10

- Table 2 -

The connection between the lattice and the continuum

Critical assessment and summary table by Peter in 1981:

Reference	13)	8)	15)	8)
PS and vector meson masses	m_ρ = 800±100 MeV	m_ϕ = 950 MeV m_{K*} = 860 MeV	m_ϕ = 990±50 MeV m_{K*} = 890±70 MeV	
P wave meson masses	m_δ = 1000±100 MeV M_{A_1} = 1200±100 MeV	no sensible results with local operators	no sensible results with local operators	the p wave meson poles could be identified by using spatially extended operators
Baryon masses	m_p = 950±100 MeV m_Δ = 1300±100 MeV		m_p = 1270±440 MeV m_Δ = 1370±600 MeV	essentially the strong coupling spectrum
Quark masses	$m_{u,d}$ = 4 MeV	$m_{u,d}$ = 7.5 MeV m_s = 190 MeV m_c = 1600 MeV	$m_{u,d}$ = 5.9±0.8 MeV m_s = 150±10 MeV	

- Table 3 -

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- Table 3 -

"*We are able to obtain non-perturbative numbers in a four-dimensional, relativistic, relevant theory. We are proud of it.*"

The mass gaps

- The determination of the mass-coupling relation belongs to the most difficult problems in a QFT:
  - relation between renormalized couplings from the Lagrangian to the physical masses
  - e.g. nucleon mass in the chiral limit of QCD in units of $\Lambda^{\overline{\mathrm{MS}}}$

$$m_N = c_{m_N} \cdot \Lambda^{\overline{\mathrm{MS}}}$$

- difficulty lies in the fact that
  - Lagrangian defined at short distances (UV-scale)
  - masses are parameters at large distances (IR-scale)

- one family of models where this relation can be found *exactly*:

O(N) nonlinear sigma model in $d = 2$

  - asympt. free, with massive O(N) isovector multiplets

The mass gaps

- in 1990 Peter (with M. Maggiore and F. Niedermayer) calculated this relation *exactly* for $N = 3$ and 4:

$$m = \frac{8}{e} \cdot \Lambda^{\overline{\mathrm{MS}}} \qquad N = 3$$

$$m = \sqrt{\frac{32}{\pi e}} \cdot \Lambda^{\overline{\mathrm{MS}}} \qquad N = 4$$

- in the same year, Peter (with F. Niedermayer) extended the calculation to arbitrary $N \geq 3$:

$$m = \left(\frac{8}{e}\right)^{1/(N-2)} \frac{1}{\Gamma(1 + 1/(N-2))} \cdot \Lambda^{\overline{\mathrm{MS}}}$$

  - at the time, over 30 nonperturbative determinations differed wildly from each other

The mass gaps

- rather involved calculation, based on a beautiful idea:
  - introduce a chemical potential h coupled to a Noether charge
  - observe that the *change* of free energy and h is RG invariant
  - calculate the free energy in PT for $h \gg m$ (asympt. freedom)

$$f(h) - f(0) = -(N-2)\frac{h^2}{4\pi}\left[\ln\frac{h}{e^{1/2}\Lambda_{\overline{\mathrm{MS}}}} + \frac{1}{N-2}\ln\ln\frac{h}{\Lambda_{\overline{\mathrm{MS}}}} + \mathcal{O}\left(\frac{\ln\ln(h/\Lambda_{\overline{\mathrm{MS}}})}{\ln(h/\Lambda_{\overline{\mathrm{MS}}})}\right)\right]$$

  - calculate the free energy from the S-matrix:

$$f(h) - f(0) = -\frac{m}{2\pi}\int \cosh\theta\,\varepsilon(\theta)d\theta$$

  - use generalised Wiener-Hopf technique to express integral equation in terms of $\ln h/m$ for $h \gg m$

The mass gaps

- application of the same idea yields the exact mass gap
  - in the GN model [Forgács, Niedermayer, Weisz '91]
  - in the antiferromagnetic $d = 2 + 1$ Heisenberg model at low T

    [P. Hasenfratz, Niedermayer '91; P. Hasenfratz cond-mat/9901355]

- another interesting application is for matching chiral Lagrangians with different regularizations:

  [Niedermayer, Weisz arXiv:1601.00614, arXiv:1602.03159]
  - related to the QCD rotator in the δ-regime where $m_\pi L_s \ll 1$ and $F_\pi L_s \gg 1$ [Leutwyler '87]
  - provides promising new way to determine LECs

    [P. Hasenfratz arXiv:0909:3419]
  - finite box introduces an IR-cutoff, study FS scaling in the chiral limit

The mass gaps

- χPT for massless 2-flavour QCD has $\mathrm{SU}(2) \times \mathrm{SU}(2) \simeq \mathrm{O}(4)$:
  - for general $\mathrm{O}(N)$ spectrum is given by quantum mechanical rotator [Leutwyler '87]

$$E(l) = l(l+N-2)/2\Theta \qquad l = 0, 1, 2, \ldots$$

$$\Theta = F^2 L_s^3 \quad \text{moment of inertia}$$

  - NLO term of expansion in $1/(F^2 L_s^2)$ [P. Hasenfratz, Niedermayer '93]
  - NNLO terms in DR scheme and on the lattice

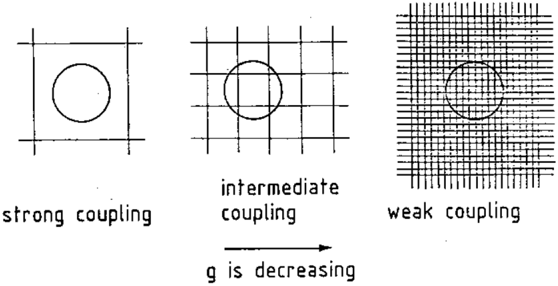
    [P. Hasenfratz '09 ; Niedermayer, Weiermann '10]

- change of free energy due to a chemical potential coupled to Noether charge in $\mathrm{O}(N)$ nonlinear σ model [Niedermayer, Weisz '16]
  - connects the two regularizations of the effective theory
  - converts physical quantities on the lattice to those in DR
  - in particular, it relates the mass gap on the lattice to DR

The renormalization group and FP actions

Peter had a deep appreciation and understanding of the Wilson RG description:

- lattice gauge theory is just a statistical system
- critical limit corresponds to the continuum limit of the QFT



- the lattice provides a full non-perturbative description

The renormalization group and FP actions

- continuum physics is recovered in the lattice system at long distances (close to the phase transition):
  - integrate out variables describing short distance lattice physics
  - obtain effective action for the relevant, long distance variables

$$\left\{K_\alpha^{(1)}\right\} \quad \xrightarrow{\text{RG}} \quad \left\{K_\alpha^{(2)}\right\} \quad \xrightarrow{\text{RG}} \quad \ldots \quad \left\{K_\alpha^{(n)}\right\} \quad \xrightarrow{\text{RG}} \quad \ldots$$

- sequence of RG transformations might have a FP:

$$\{K_\alpha^*\} \quad \xrightarrow{\text{RG}} \quad \{K_\alpha^*\}$$

  - interested in FP where $\xi = \infty$
  - for gauge theories RG transformations complicated due to requirement of gauge invariance

The renormalization group and FP actions

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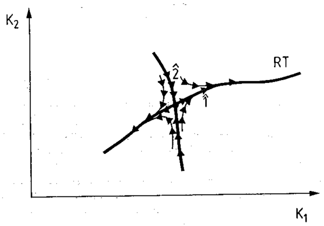
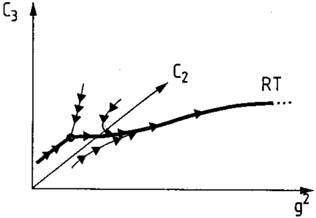
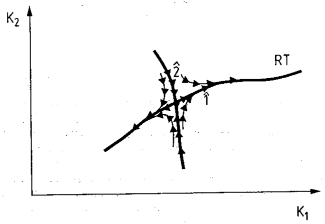
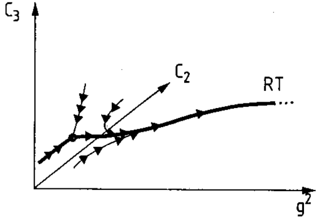




The renormalization group and FP actions

- basic starting point in expecting *renormalizability* and *universality* along the RT





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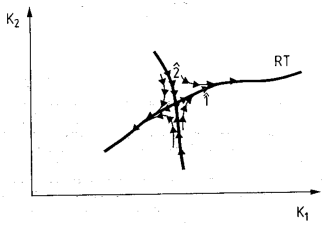
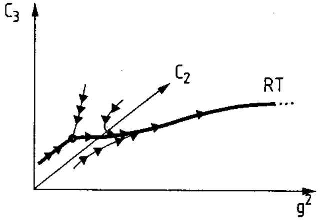


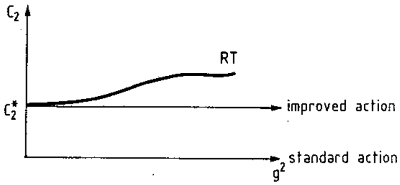


- warning for investigations of BSM models with conformal FP:
  - the IR FP is not perturbative
  - perturbative intuition could be misleading

remember: the lattice provides a fully nonperturbative description

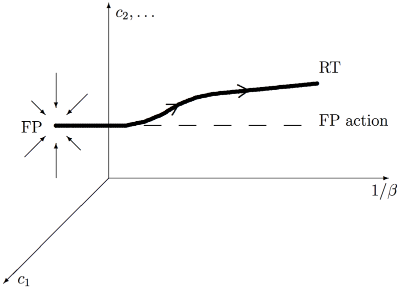
The renormalization group and FP actions

- basic starting point in expecting *renormalizability* and *universality* along the RT
- already in 1983 Peter thought about 'RG improved' actions:



The renormalization group and FP actions

- basic starting point in expecting *renormalizability* and *universality* along the RT
- and 10 years later Peter and Ferenc became more specific:



- the path integral for RG transformations in AF theories is reduced to classical saddle point equations

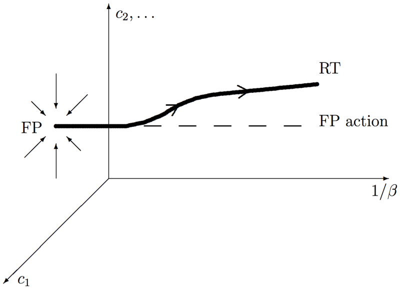
The renormalization group and FP actions

- the FP gauge action is defined by

$$S_G^{\mathrm{FP}}[V] = \min_{\{U\}} \left[S_G^{\mathrm{FP}}[U] + T_G[V,U] \right]$$

  and the FP Dirac operator by

$$D^{\mathrm{FP}}[V]^{-1} = R[V] + \omega[U] \cdot D^{\mathrm{FP}}[U]^{-1} \cdot \omega[U]$$



- the action $\beta S_G^{\mathrm{FP}} + \overline{\psi} D^{\mathrm{FP}} \psi$ is classically perfect

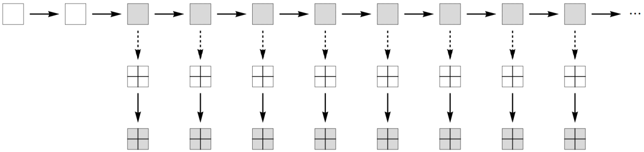
The renormalization group and FP actions

- the minimizing gauge field $U(x, V)$ defines a FP field in the continuum:
  - all symmetries of the continuum are well defined
  - representation of infinitesimal transformations on the lattice

$$V_n \quad \xrightarrow{\text{minimize}} \quad U(x, V) \quad \xrightarrow{\text{transform}} \quad U^{\varepsilon}(x^{\varepsilon}, V) \quad \xrightarrow{\text{block}} \quad V_n^{\varepsilon}$$

  - possibility to define SUSY algebra on the lattice

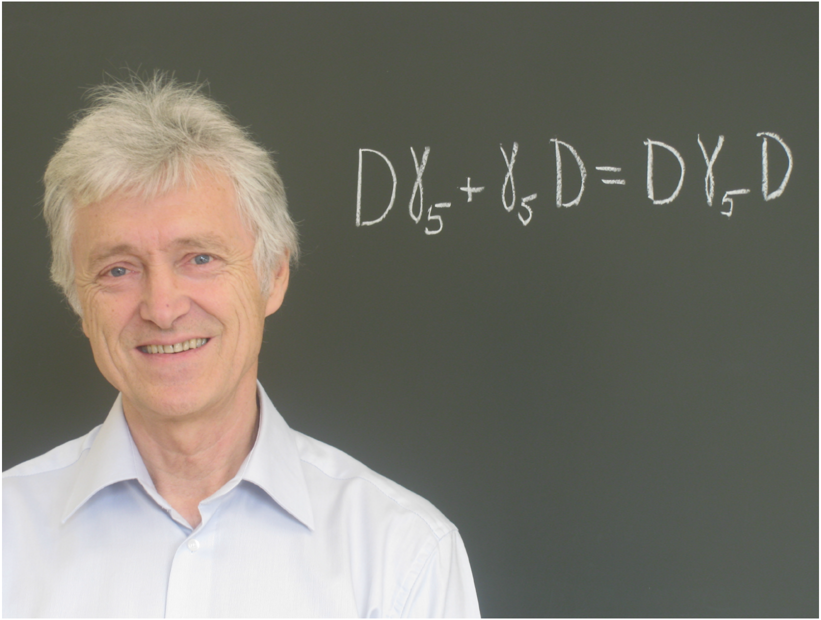
- FP equations define a scheme to relate coarse and fine lattice configurations:
  - recent multiscale thermalization algorithm [Endres at al. arXiv:1510.04675]



The renormalization group and FP actions

Then, in 1997 Peter made a truly groundbreaking observation. . .

The renormalization group and FP actions



The renormalization group and FP actions

- in the summer of '97 Peter looked through a pile of old preprints while travelling
- he realized that the FP Dirac operator D^{FP} fulfills the Ginsparg-Wilson relation: [P. Hasenfratz hep-lat/9709110]

$$D\gamma_5 + \gamma_5 D = D\gamma_5 D$$

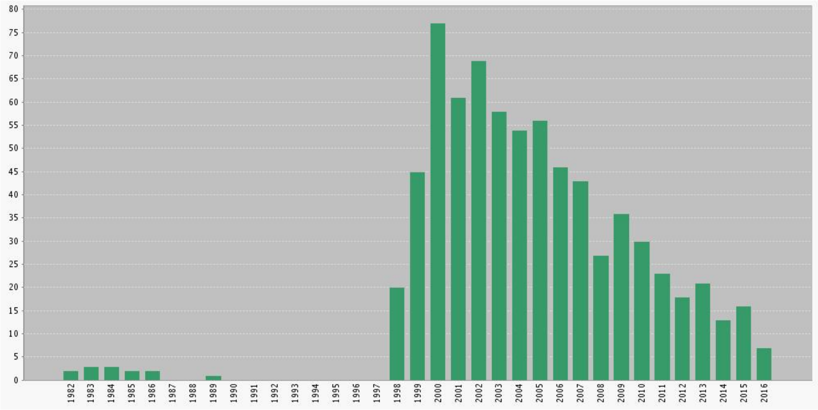
- obtained from RG transformations for free fermions

  [Ginsparg, Wilson '82]
  - avoids Nielsen-Ninomyia no-go theorem
  - implies correct triangle anomaly on the lattice
  - all the soft-pion theorems are expected to be valid
- no tuning, no mixing, no current renormalization on the lattice

  [P. Hasenfratz hep-lat/9802007]

The renormalization group and FP actions

- this set off an avalanche of developments:
  - citation history of GW paper



  - by now the third most cited lattice paper (963 citations)

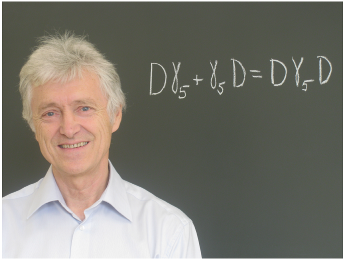
The renormalization group and FP actions

- this set off an avalanche of developments:
  - Index theorem in QCD on the lattice [P. Hasenfratz, Laliena, Niedermayer '98]
  - Overlap operator as a solution to GW [Neuberger '98]
  - Exact chiral symmetry on the lattice [Lüscher '98]
  - Abelian chiral gauge theories on the lattice [Lüscher '98]
  - Axial anomaly and topology [Adams; Chiu; Kikukawa, Yamada; Lüscher; ... all '98]
  - Chiral Jacobian on the lattice [Fujikawa '98; Suzuki '98]
  - Lattice supersymmetry [So, Ukita '98;]
  - ...
- implications for phenomenological and theoretical applications can hardly be overestimated
  - cf. parallel sessions *"Weak decays and matrix elements"*, *"Chiral symmetry"*, *"Theoretical developments"*, ...

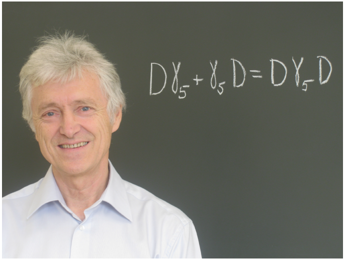
The renormalization group and FP actions

- chiral symmetry regularized on the lattice provides a true solution to a *hierarchy problem*
  - often unappreciated outside the lattice community
- FP fermions and overlap/domain wall fermions stem from completely different approaches
  - is there a connection? how are they related?
- GW relation is a specific form of an *algebraic Riccati equation*
  - sign function appears naturally in its solution
- the gauge field dependent chiral projectors $\hat{P}_\pm = 1/2(1 \pm \hat{\gamma}_5)$ with $\hat{\gamma}_5 = \gamma_5(1 - D)$
  - are responsible for fermion number violation
  - but break CP symmetry
  - cf. David Kaplan's talk on Saturday

Looking forward to an exciting new lattice conference. . .



Looking forward to an exciting new lattice conference...



...would make Peter very happy!