Heavy-ion phenomenology and lattices

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Lattice 2016, South Hampton

Universitetet i Stavanger

Disclaimer

What this talk is not:

- Collection of recent experimental results from the heavy-ion experiments
  Quark Matter 15

- Collection of recent lattice computations of thermal field theory
  Heng-Tong Ding, Seyong Kim, many many parallels

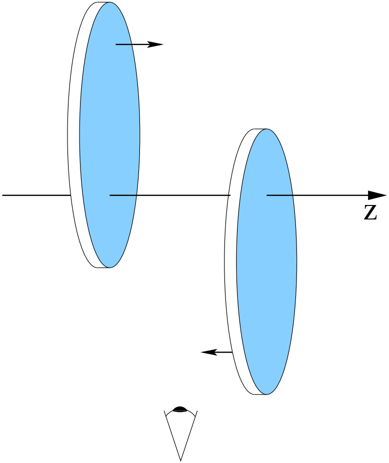
What this talk is:

- Few topics in heavy-ion community where LGT can make impact

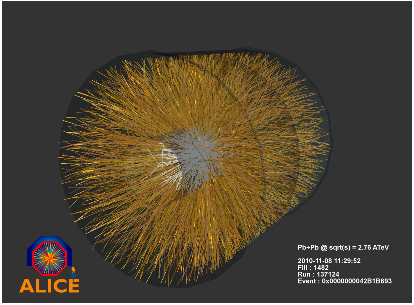
Geometry of heavy-ion collision

In:
Pb+Pb @ few TeV per nucleon

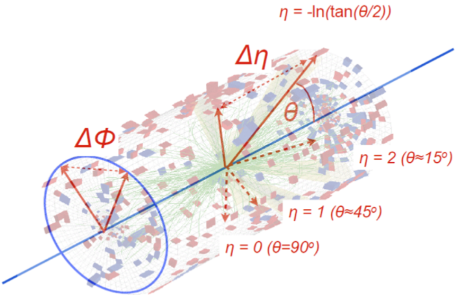
Out:
$\sim 3 \times 10^4$ hadrons



$\Rightarrow$

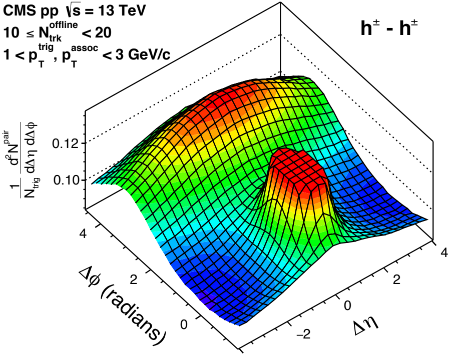


Pair correlation function



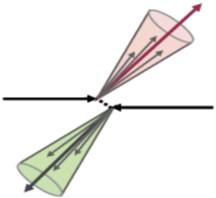
- In each collision, count all pairs of particles
- Count as a function of relative $\Delta\phi$, $\Delta\eta$
- Average over many collisions

Pair correlation function, proton-proton

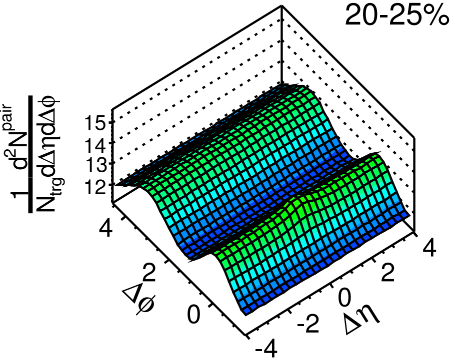


CMS 1606.06198

- Near-side jet peak: Collinear fragmentation of jets
- Away-side jet: Energy balance

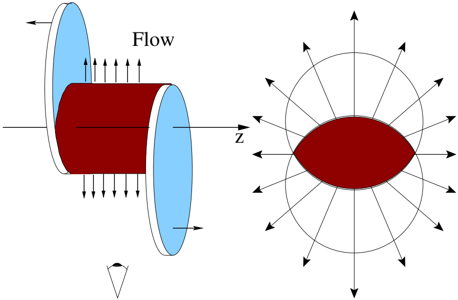


Pair correlation function, nucleus-nucleus



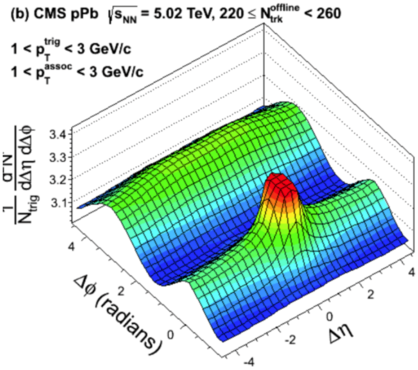
CMS 1201.3158

- Near-side ridge: particles at all longitudinal momenta, all pushed by same pressure gradient
- Away-side ridge: due to geometry, away-side correlated



Flow in small systems?

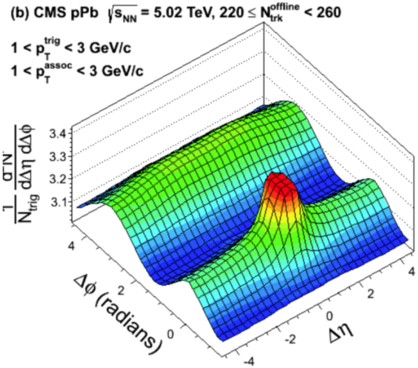
p - Pb



- p-A collisions display a pronounced ridge an flow-like behaviour

Flow in small systems?

p - Pb

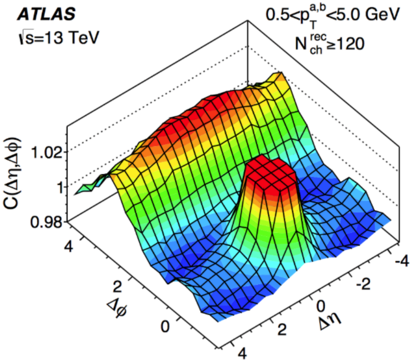


- p-A collisions display a pronounced ridge an flow-like behaviour

Creation of a tiny droplet of plasma?

Flow in small systems?

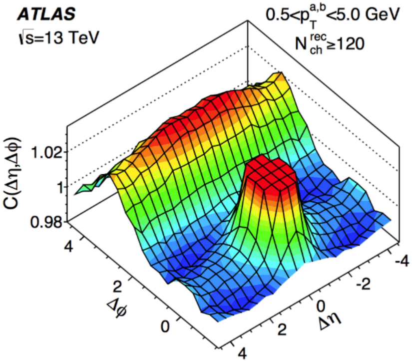
p - p



- High multiplicity p-p flow-like signature!

Flow in small systems?

p - p



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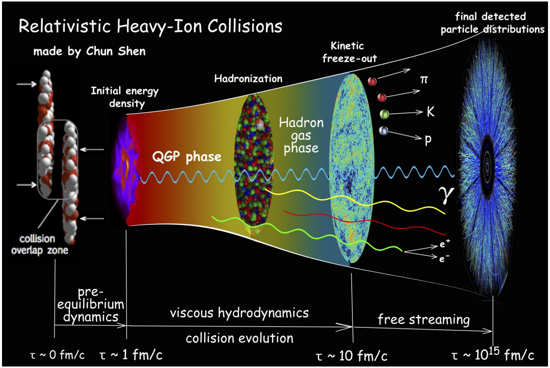
?????

"Standard model" of heavy-ion collisions



- Pre-equilibrium evolution, thermalization
- Hydrodynamical evolution
- Hadronization, freeze-out

"Standard model" of heavy-ion collisions

- AA: Beautifully description of wealth of heavy-ion data
  - Major uncertainties from poor understanding from the pre-equilibrium evolution
- pA: No quantitative description of proton-nuclus collisions
  - Even if the system becomes hydrodynamical, large part of the evolution pre-equilibrium

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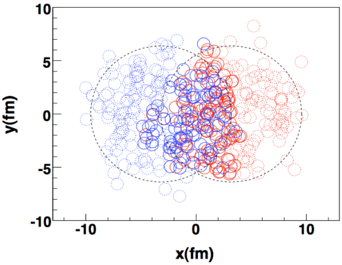
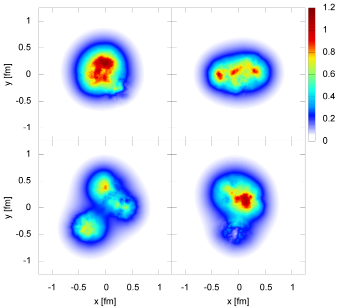
- pp: ??????

Pre-equilibrium evolution absolutely essential!

Event-by-event shape of the proton?

Importance of fluctuating initial conditions

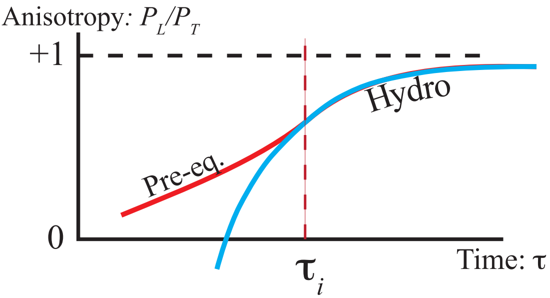




Mäntysaari, Schenke PRL 117 (2016) no.5, 052301

What can lattice say, beyond radius of p?

Pre-equilibrium dynamics



- Hydrodynamics is gradient expansion around thermal equilibrium

$$\partial_\mu T^{\mu\nu} = 0$$

$$T^{\mu\nu} = T^{\mu\nu}_{\text{Loc. therm. eq.}} - \eta(\epsilon)\sigma^{\mu\nu} - \zeta(\epsilon)\{g^{\mu\nu} + u^\mu u^\nu\}(\nabla \cdot u) + \dots$$

- At early times gradients diverge

Pre-equilibrium dynamics

Methods:

- Strong coupling $\mathcal{N} = 4$, $N_c \to \infty$, $\lambda = g^2 N_c \to \infty$
  - Holography, classical simulations in 5d gravity

- Weak coupling QCD, $\alpha_s(Q_s) \to 0$
  - Even at weak coupling, non-perturbative physics: Lattices needed
  - Physics often classical: Simulations possible
  - Recently: semi-quantitative understanding of the process

Overoccupied fields

- In statistical field theory, loops arise from *field fluctuations*

$$\alpha_s \omega_k \langle A(-\mathbf{k},t) A(\mathbf{k},t) \rangle \sim \alpha_s \left(\frac{1}{2} + f(|\mathbf{k}|) \right)$$

- Fluctuations quantum or statistical:
  - Vacuum not eigenstate of field operator: $\frac{1}{2}$
  - The state of the system is not vacuum: f

In T-eq. $f(T) = n_B(T) \sim 1$

- If $f \gg 1$, or $\langle AA \rangle \gg \frac{1}{\omega_{\mathbf{k}}}$, stat. fluct. overwhelm quantum fluct.

  $\Rightarrow$ Classical field theory

  Aarts, Smit Nucl.Phys. B511 (1998) 451-478; Son, Mueller PLB582 (2004) 279-287;

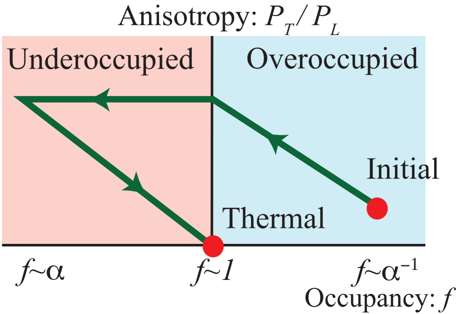
- If $f \sim 1/\alpha_s$, nonpertubative

  Spatial string tension, classical confinement, etc.

  - Related to $m_g \sim g^2 T$ in eq.

  Linde problem, dimensional reduction, etc.

Bottom-up thermalization at weak coupling



- Color Glass Condensate: Initial condition overoccupied

McLerran, Venugopalan PRD49 (1994) 2233-2241 , PRD49 (1994) 3352-3355 ; Gelis et. al Int.J.Mod.Phys. E16 (2007) 2595-2637 , Ann.Rev.Nucl.Part.Sci. 60 (2010) 463-489

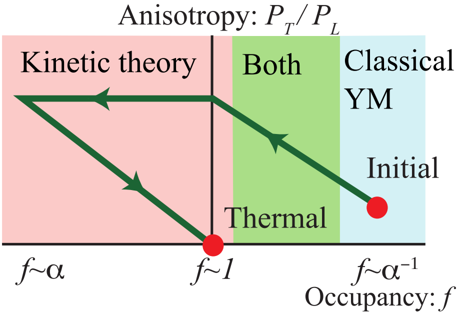
$$f(Q_s) \sim 1/\alpha_s, \qquad Q_s \sim 2\text{GeV}$$

- Expansion makes system underoccupied before thermalizing

Baier et al Phys.Lett. B502 (2001) 51-58; AK, Moore JHEP 1111 (2011) 120

$$f(Q_s) \ll 1$$

Bottom-up thermalization at weak coupling

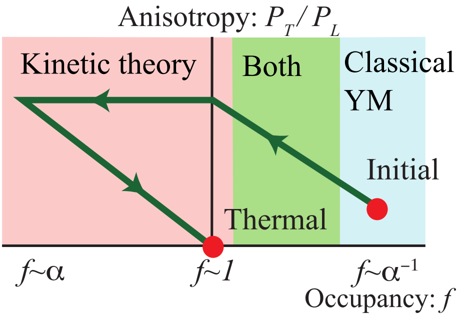


- Degrees of freedom:
  - $f \gg 1$: Classical Yang-Mills theory (CYM)
  - $f \ll 1/\alpha_s$: (Semi-)classical particles, Eff. Kinetic Theory (EKT)

- Transmutation of fields to particles: Field-particle duality
  Son, Mueller PLB582 (2004) 279-287; Jeon PRC72 (2005) 014907; Mathieu et al EPJ. C74 (2014) 2873 ; AK, Moore PRD89 (2014) 7, 074036

$$1 \ll f \ll 1/\alpha_s$$

Outline



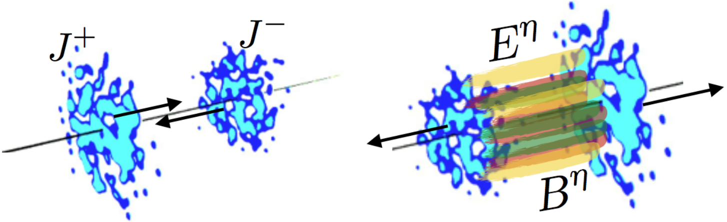
1. Initial non-pertrubative CYM evolution

$$1 \gg f \sim 1/\alpha_s$$

2. Approach to hydrodynamics in kinetic theory

$$f \ll 1/\alpha_s$$

Initial conditions at weak coupling



- Systematic EFT framework: "Color-glass-condensate"
  Gelis et al. Ann.Rev.Nucl.Part.Sci.60:463-489,2010
  - fast partons with small occupancies, classical color charges

$$J^\mu = \delta^{\mu +}\delta(x^-)\rho(\mathbf{x}_T)$$

  - soft partons with large occupancies: classical Yang-Mills fields

$$[D_\mu, F^{\mu\nu}] = J^\nu, \qquad [D_i, E^i] = 0$$

- Varying cutoff between particles and fields gives renormalization group equations JIMWLK,BK

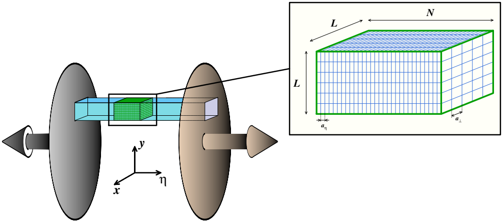
Classical Yang-Mills evolution on the lattice

- In expanding (Milne) coordinates

$$\tau = \sqrt{t^2 + z^2}, \quad \eta = \operatorname{atanh}(z/t), \quad a_\eta = \tau \Delta\eta$$

- Microcanonical evolution

$$\frac{\mathrm{d}}{\mathrm{d}\tau} U^i_{\hat{\eta},\hat{\mathbf{n}}} = i E^i_{\hat{\eta},\hat{\mathbf{n}}} U^i_{\hat{\eta},\hat{\mathbf{n}}}, \qquad \frac{\mathrm{d}}{\mathrm{d}\tau} \frac{a_\eta}{a_i^2} E^i_{\hat{\eta},\hat{\mathbf{n}}} = \mathrm{ad}\left[\sum_{j\neq i} \frac{-i a_\eta}{a_i^2 a_j^2} \left(\underline{\Box}^{i,j}_{\hat{\eta},\hat{\mathbf{n}}} + \overline{\Box}^{i,j}_{\hat{\eta},\hat{\mathbf{n}}} \right) \right],$$



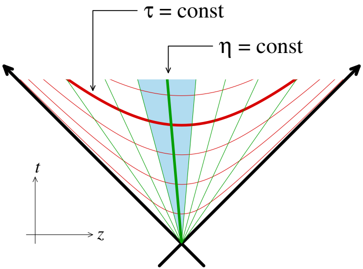
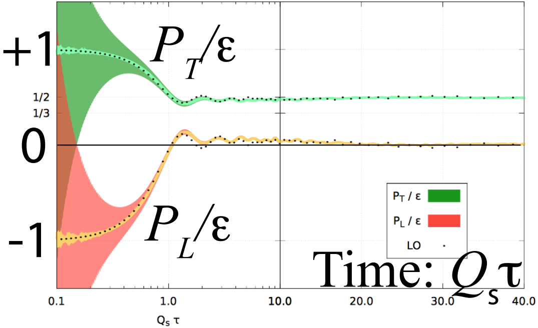


Fig: 1307.2214

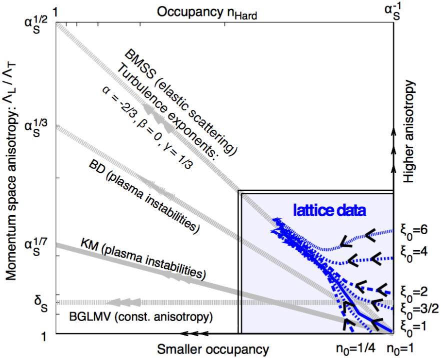
Nonperturbative classical evolution

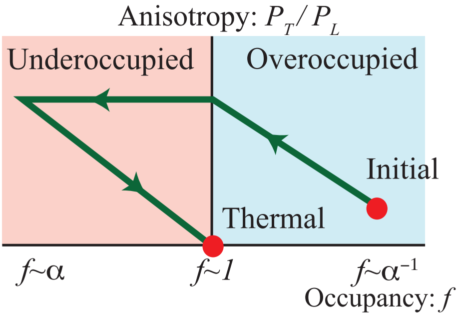


Epelbaum & Gelis, PRL. 111 (2013) 23230

- Initially boost invariant coherent chromomagetic and -electric fields
- After $\tau \sim 1/Q_s$, fields decohere, $P_L > 0$

Nonperturbative classical evolution

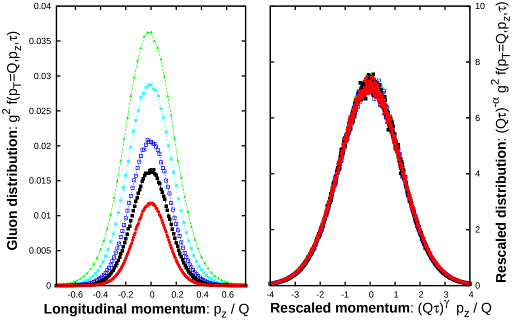




Berges et al. Phys.Rev. D89 (2014) 7, 074011

- Expansion leads to dilute system $\Rightarrow$ perturbative dynamics
- When $f \sim 1$, classical approximation fails

Particle spectrum



Berges et al. Phys.Rev. D89 (2014) 7, 074011

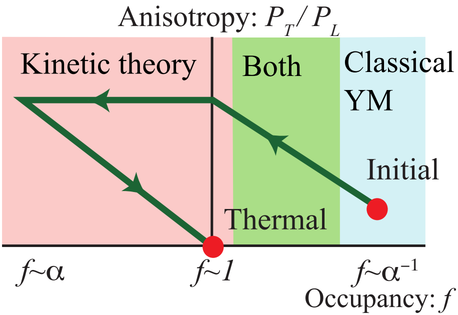
- Once the system becomes perturbative, the fields can be decomposed in quasiparticle excitations

$$f(|\mathbf{p}|) = \frac{\delta_{ij}\delta_{ab}}{2(N_c^2-1)}|\mathbf{p}| \int d^3x e^{i\mathbf{p}\cdot\mathbf{x}} \langle A_a^i(x) A_a^j(x)\rangle_{\text{coul}}$$

- The evolution eventually settles to a scaling solution

$$f(p_T, p_z, \tau) = (Q_s\tau)^\alpha f_S\left((Q_s\tau)^\beta p_T, (Q_s\tau)^\gamma p_z\right)$$

Outline



1. Initial non-pertrubative CYM evolution

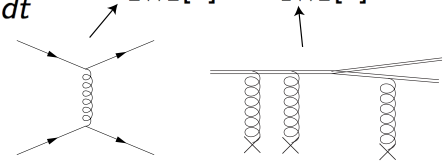
$$1 \gg f \sim 1/\alpha_s$$

2. Approach to hydrodynamics in kinetic theory

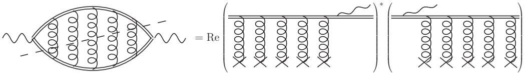
$$f \ll 1/\alpha_s$$

QCD kinetic theory Arnold et al. JHEP 0301 (2003) 030

$$\frac{df}{dt} = -C_{2\leftrightarrow 2}[f] - C_{1\leftrightarrow 2}[f]$$

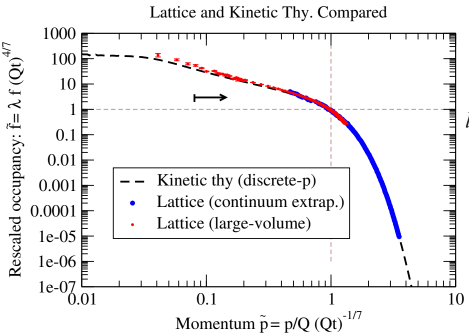


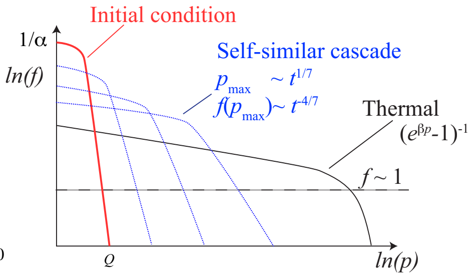
- Soft and collinear divergences lead to nontrivial matrix elements
  soft: screening, Hard-loop; collinear: LPM, ladder resum



- No free parameters; LO accurate in the $\alpha_s \to 0$, $\alpha_s f \to 0$ limit.
  For anisotropic systems further nonpert. input needed

Comparison between CYM and kinetic theory





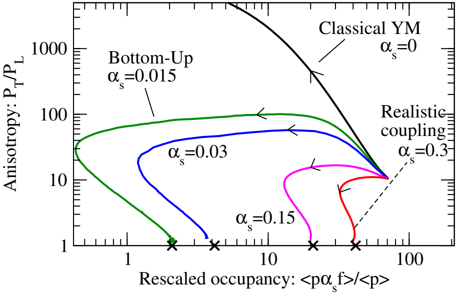
- Quantitative agreement between CYM and kinetic theory for

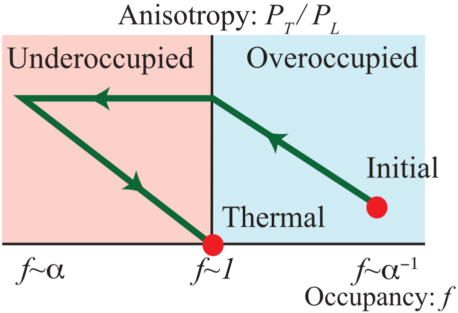
$$1 \ll f \ll 1/\alpha_s$$

Abraao et al. PRD89 (2014) 7, 074036; AK, Lu PRL 113 (2014) 18, 182301

Route to equilibrium in EKT

AK, Zhu, PRL. 115 (2015) 18, 182301



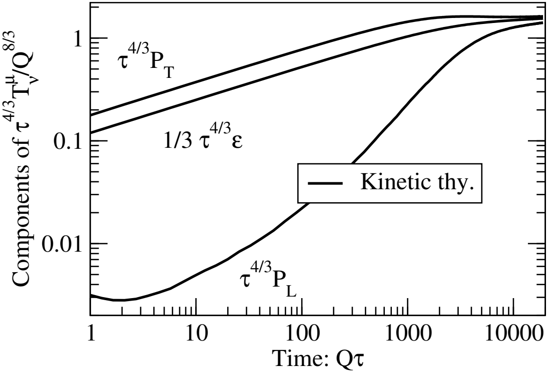


- Initial condition ($f \sim 1/\alpha_s$) from classical field theory calculation

  Lappi PLB703 (2011) 325-330
- In the classical limit ($\alpha_s \to 0, \alpha_s f$ fixed), no thermalization
- At small values of couplings, clear Bottom-Up behaviour
- Features become less defined as α_s grows

Smooth approach to hydrodynamics AK, Zhu, PRL 115 (2015) 18, 182301

$$\alpha_s = 0.03$$



- Kinetic theory converges to hydro smoothly and automatically

Smooth approach to hydrodynamics AK, Zhu, PRL 115 (2015) 18, 182301

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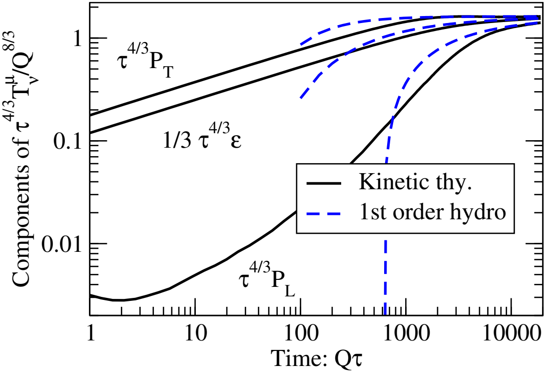


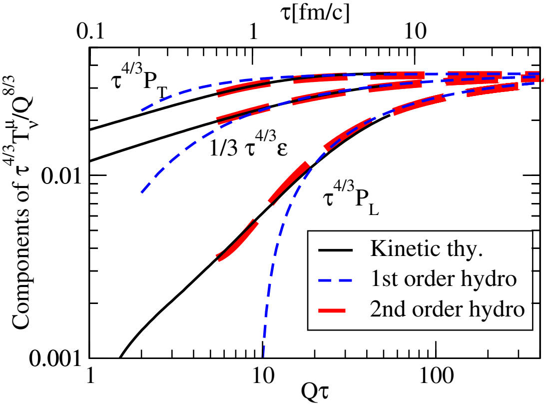
- Kinetic theory converges to hydro smoothly and automatically
- Approach to hydro fixed by perturbative η/s

Arnold et al. JHEP 0305 (2003) 051

$$\partial_\tau \epsilon = -\frac{4}{3}\frac{\epsilon}{\tau} + \frac{4\eta}{3\tau^2}, \qquad P_L = \frac{\epsilon}{3} - \frac{4\eta}{3\tau}$$

Smooth approach to hydrodynamics AK, Zhu, PRL. 115 (2015) 18, 182301

$$\alpha_s = 0.3$$



- For realistic couplings, hydrodynamics reached around $\lesssim 1$fm/c.
- Hydro seems to give a good description even when $P_L/P_T \sim 1/5$

Outstanding questions

Qualitative $\Rightarrow$ Quantitative

- All simulations so far exploratory
- Numerical challenge from the extreme anisotropies
- Poorly understood physics of non-perturbative plasma-instabilities

  Non-perturbative input needed for the kinetic theory

  Mrowczynski Phys.Lett. B314 (1993) 118-121, Kurkela, Moore JHEP 1112 (2011) 044
- Fermions, chemical equilibration

  Fermion production Gelfand et al. Phys. Rev. D 93, 085001 (2016)

  Anomalous charge production Tanji et al. Phys.Rev. D93 (2016) no.7, 074507
- Meaningfulness of weak coupling framework?

  How perturbative is the medium at $T \sim 3T_c$

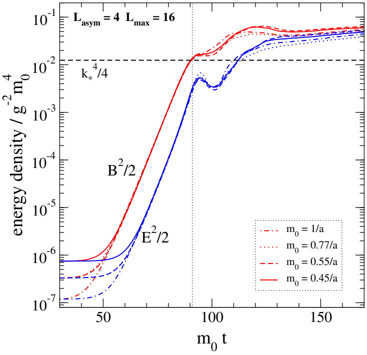
Plasma instabilities

- Even in kinetic regime, there are low-momentum modes that are nonperturbative
  - In T-equilibrium $f(g^2T) \sim 1/g^2$
- In isotropic systems NNLO corrections to kinetic theory
- Accessible in simulations in EQCD
  Simon Caron-Huot JHEP 0904 (2009) 004, Panero et al. Phys.Rev.Lett. 112 (2014) no.16, 162001
- In anisotropic systems, *plasma-instabilities* make more modes to become non-perturbatively large

Unknown leading order correction from plasma unstable modes to kinetic theory

Simulations of plasma instabilities

- Lattice simulations of non-abelian vlasov equations Hard-Loop theory
  - Soft nonperturbative modes: classical fields: $[D_\mu, F^{\mu\nu}] = 0$
  - Hard perturbatie modes: classical particles: $[D_\mu, J^\mu] = 0$



Arnold, Moore PRD76 (2007) 045009; Bodeker and Rummukainen JHEP 0707 (2007) 022

How do the large magnetic fields affect particle propagation?

Conclusions?

- Evidence of flow like behaviour in small systems (pA, pp) compels to study the pre-equilibrium evolution

- Strong nonperturbative fields are abundant at early times and require lattice input

- Combination of classical Yang-Mills simulations and effective kinetic theory allows to follow the time evolution from highly occupied initial condition to thermal equilibrium.

- Weak coupling thermalization extrapolated to realistic couplings is at least not in contraction with phenomenology

$$\tau_{\mathrm{i}} \sim 1\mathrm{fm/c}.$$