Hadron Matrix Elements and the Feynman-Hellman Theorem

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Feynman-Hellman Theorem

The Feynman-Hellman Theorem (FHT) relates matrix elements to (variations in) the spectrum

$$\frac{\partial E_n}{\partial \lambda} = \langle n | H_\lambda | n \rangle$$

The FHT is often used to determine the scalar quark matrix elements in the nucleon (needed to interpret direct dark matter detection) both with Chiral Perturbation Theory and direct lattice QCD calculations

$$m_q \frac{\partial m_N}{\partial m_q}\bigg|_{m_q = m_q^{\rm phy}} = \langle N | m_q \bar{q} q | N \rangle$$

More recently, by A. Chambers et.al. to study spin structure of the nucleon PRD90 (2014) [1405.3019]
and in the next talk - we'll here about advances they made

I'll present an improved method of computing hadronic matrix elements based on the FHT

Consider a two point correlation function in the presence of some source

$$\begin{aligned} C_\lambda(t) &= \langle\lambda|\hat{O}(t)\hat{O}^\dagger(0)|\lambda\rangle \\ &= \frac{1}{\mathcal{Z}_\lambda}\int D\Phi e^{-S-S_\lambda} O(t)O^\dagger(0) \end{aligned}$$

$$|\lambda\rangle \equiv \lambda\text{-vacuum}$$

$$|\Omega\rangle \equiv \lim_{\lambda\to 0}|\lambda\rangle$$

$$S_\lambda = \lambda\int d^4x j(x)$$

$$j(x) = \text{ some bi-linear current density}$$

$$\text{e.g. } \lambda j(x) = \bar{q}(x)m_q q(x)$$

We can differentiate the correlator with respect to **λ**

$$-\frac{\partial C_\lambda}{\partial \lambda} = \frac{\partial_\lambda \mathcal{Z}_\lambda}{\mathcal{Z}_\lambda} C_\lambda(t) + \frac{1}{\mathcal{Z}_\lambda} \int D\Phi e^{-S-S_\lambda} \int d^4x' j(x')\ \mathcal{O}(t)\mathcal{O}^\dagger(0)$$

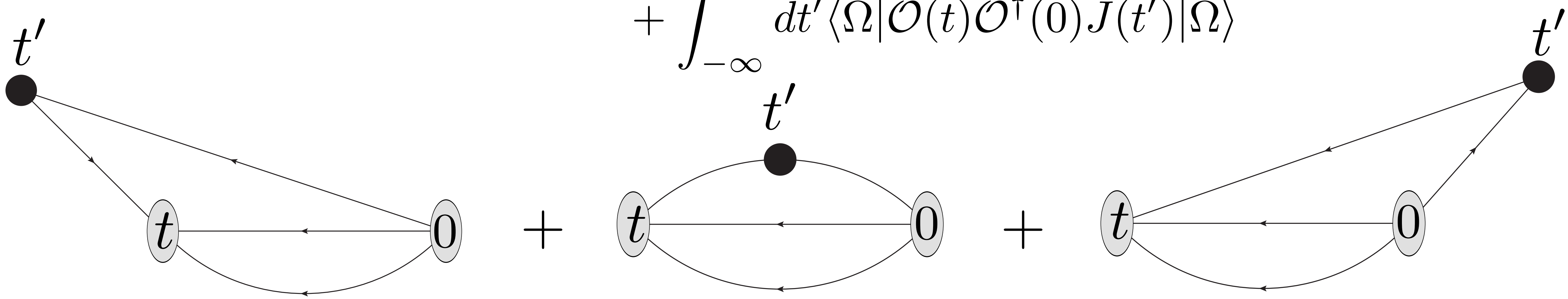
The first term is proportional to a vacuum matrix element and the second contains the matrix element we are interested in. We are really interested in the linear-response

$$-\frac{\partial C_\lambda(t)}{\partial \lambda}\bigg|_{\lambda=0} = -C_\lambda(t) \int d^4x' \langle\Omega|j(x')|\Omega\rangle + \int dt' \langle\Omega|T\{\mathcal{O}(t)J(t')\mathcal{O}^\dagger(0)\}|\Omega\rangle$$

$$J(t) = \int d^3x\ j(t, \mathbf{x})$$

Let us focus on the second term:

$$\int dt' \langle\Omega|T\{\mathcal{O}(t)J(t')\mathcal{O}^\dagger(0)\}|\Omega\rangle = \int_t^\infty dt' \langle\Omega|J(t')\mathcal{O}(t)\mathcal{O}^\dagger(0)|\Omega\rangle + \int_0^t dt' \langle\Omega|\mathcal{O}(t)J(t')\mathcal{O}^\dagger(0)|\Omega\rangle + \int_{-\infty}^t dt' \langle\Omega|\mathcal{O}(t)\mathcal{O}^\dagger(0)J(t')|\Omega\rangle$$



(+ disconnected pieces)

The middle contribution is the matrix element of interest, summed over all time slices between the src (0) and sank (t) operators. The other two terms we do not want, but must understand.

The FHT relates matrix elements to the spectrum. Can we find something similar in QFT?

Let us try the first obvious thing, take a derivative of the effective mass:

$$m^{eff}(t,\tau) = \frac{1}{\tau}\ln\left(\frac{C(t)}{C(t+\tau)}\right) \underset{t\to\infty}{\longrightarrow} \frac{1}{\tau}\ln(e^{E_0\tau})$$

$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C(t)}\right]$$

NOTE: even for currents with non-vanishing vacuum matrix elements, this contribution exactly cancels in this quantity

$$-\left.\frac{\partial C_\lambda(t)}{\partial\lambda}\right|_{\lambda=0} = -C_\lambda(t)\int d^4x' \langle\Omega|j(x')|\Omega\rangle + \int dt' \langle\Omega|T\{\mathcal{O}(t)J(t')\mathcal{O}^\dagger(0)\}|\Omega\rangle$$

We are then left with the following

$$\partial_\lambda m_\lambda^{eff}(t,\tau)\Big|_{\lambda=0} = \frac{R(t+\tau) - R(t)}{\tau}$$

$$R(t) = \frac{\int dt' \langle 0|T\{\mathcal{O}(t)J(t')\mathcal{O}(0)\}|0\rangle}{C(t)}$$

To understand this quantity, we begin by inserting complete set's of states where appropriate

$$\begin{aligned} C(t) &= \langle\Omega|\mathcal{O}(t)\mathcal{O}^\dagger(0)|\Omega\rangle \\ &= \sum_n \langle\Omega|e^{\hat{H}t}\mathcal{O}(0)e^{-\hat{H}t}\frac{|n\rangle\langle n|}{2E_n}\mathcal{O}^\dagger(0)|\Omega\rangle \\ &= \sum_n Z_n Z_n^\dagger \frac{e^{-E_n t}}{2E_n} \end{aligned}$$

$$Z_n \equiv \langle\Omega|\mathcal{O}|n\rangle$$

$$Z_n^\dagger \equiv \langle n|\mathcal{O}^\dagger|\Omega\rangle$$

The numerator term we can separate into the three regions:

$$t' < 0, \quad 0 \le t' \le t, \quad t < t'$$

I II III

The middle region, II, is the region we are interested in, where we have the matrix element of interest. The other two regions will contribute to systematics that must be controlled.

$$N(t) \equiv \int dt' \langle\Omega|T\{\mathcal{O}(t)J(t')\mathcal{O}^\dagger(0)|\Omega\rangle$$

$$N(t) = N_I(t) + N_{II}(t) + N_{III}(t)$$

$$
\begin{aligned}
N_{II}(t) &= \sum_{t'=0}^{t} \langle \Omega | \mathcal{O}(t) J(t') \mathcal{O}^\dagger(0) | \Omega \rangle \\
&= \sum_{t'=0}^{t} \sum_{n,m} \frac{\tilde{Z}_n^{\mathbf{0}} Z_m^\dagger}{4E_n E_m} \langle n | J | m \rangle e^{-E_n t} e^{-(E_m - E_n)t'} \\
&= \sum_{t'=0}^{t} \Bigg[\sum_n \frac{\tilde{Z}_n^{\mathbf{0}} Z_n^\dagger}{4E_n^2} \langle n | J | n \rangle e^{-E_n t} \\
&\quad + \sum_{n \neq m} \frac{\tilde{Z}_n^{\mathbf{0}} Z_m^\dagger}{4E_n E_m} \langle n | J | m \rangle e^{-E_n t} e^{-(E_m - E_n)t'} \Bigg]
\end{aligned}
$$

The first term we are interested is independent of t’, and becomes enhanced

$$
(t+1) \sum_n \frac{\tilde{Z}_n^{\mathbf{0}} Z_n^\dagger}{4E_n^2} \langle n | J | n \rangle e^{-E_n t}
$$

$$
\begin{aligned}
N_{II}(t) &= \sum_{t'=0}^{t} \langle \Omega | \mathcal{O}(t) J(t') \mathcal{O}^\dagger(0) | \Omega \rangle \\
&= \sum_{t'=0}^{t} \sum_{n,m} \frac{\tilde{Z}_n^{\mathbf{0}} Z_m^\dagger}{4E_n E_m} \langle n | J | m \rangle e^{-E_n t} e^{-(E_m - E_n)t'} \\
&= \sum_{t'=0}^{t} \Bigg[\sum_{n} \frac{\tilde{Z}_n^{\mathbf{0}} Z_n^\dagger}{4E_n^2} \langle n | J | n \rangle e^{-E_n t} \\
&\quad + \sum_{n \neq m} \frac{\tilde{Z}_n^{\mathbf{0}} Z_m^\dagger}{4E_n E_m} \langle n | J | m \rangle e^{-E_n t} e^{-(E_m - E_n)t'} \Bigg]
\end{aligned}
$$

The t' dependence of the second term is contained entirely in the exponenent

$$
\sum_{t'=0}^{t} e^{-(E_m - E_n)t'} = \frac{1 - e^{-\Delta_{mn}(t+1)}}{1 - e^{-\Delta_{mn}}}
$$

$$
\Delta_{mn} \equiv E_m - E_n
$$

The contribution from the region of interest is then

$$N_{II}(t) = (t+1) \sum_n \frac{\tilde{Z}_n^{\mathbf{0}} Z_n^\dagger}{4E_n^2} e^{-E_n t} J_{nn} + \sum_{n \neq m} \frac{\tilde{Z}_n^{\mathbf{0}} Z_m^\dagger}{4E_n E_m} e^{-E_n t} \frac{1 - e^{-\Delta_{mn}(t+1)}}{1 - e^{-\Delta_{mn}}} J_{nm}$$

NOTE: the contribution from ALL terms depends upon t AND the t dependence of the excited states and transition matrix elements are different from the t dependence of the ground state

The contributions from regions I and III must have some symmetry. The easiest way to evaluate these terms is to consider a shifted coordinate system, and a symmetric correlation function about the origin

$$-\frac{T}{2} \leq t \leq \frac{T}{2}$$

I: $$\sum_{t'=-T/2}^{-t/2-1} \langle\Omega|\mathcal{O}(t/2)\mathcal{O}^\dagger(-t/2)J(t')|\Omega\rangle$$

II: $$\sum_{t'=t/2+1}^{T/2} \langle\Omega|J(t')\mathcal{O}(t/2)\mathcal{O}^\dagger(-t/2)|\Omega\rangle$$

It is straightforward to show this is equivalent to summing over just the first lattice and none of it’s images

The sum to the matrix element from these two regions is

$$\frac{\left[\sum_{t'=-T/2}^{-t/2-1} + \sum_{t'=t/2+1}^{T/2}\right] \langle\Omega|\mathcal{O}(t/2)J(t')\mathcal{O}^\dagger(-t/2)|\Omega\rangle}{\frac{Z_0 Z_0^\dagger e^{-E_0 t}}{2E_0}}$$

$$= \sum_{n,m_J} e^{-\Delta_{n0}t}\frac{E_0}{2E_n E_{m_J}} \frac{1-e^{-E_{m_J}(T/2-t/2)}}{e^{E_{m_J}}-1}\left(\frac{Z_n Z^\dagger_{nm_J}}{Z_0 Z_0^\dagger}\langle m_J|J|\Omega\rangle + \frac{Z_n^\dagger Z_{m_J n}}{Z_0 Z_0^\dagger}\langle\Omega|J|m_J\rangle\right)$$

$$E_{m_J} = \text{ (mesonic) states which couple to the current, J}$$

$$\langle m_J|J|\Omega\rangle$$

$$Z_{nm_J} \equiv \langle n|\mathcal{O}|m_J\rangle$$

These terms are also not enhanced by t

Putting it all together, we are left with a somewhat horrible looking expression

$$
\begin{aligned}
R(t) = \frac{1}{1+\sum_{n>0}\frac{E_0}{E_n}\frac{Z_nZ_n^\dagger}{Z_0Z_0^\dagger}e^{-\Delta_{n0}t}} \Bigg\{ &(t+1)\left(\frac{J_{00}}{2E_0} + \sum_{n>0}\frac{Z_nZ_n^\dagger}{Z_0Z_0^\dagger}\frac{E_0}{E_n}\frac{J_{nn}}{2E_n}e^{-\Delta_{n0}t}\right) \\
&+ \sum_{m\neq n}\frac{Z_nZ_m^\dagger}{Z_0Z_0^\dagger}\frac{E_0}{\sqrt{E_nE_m}}\frac{e^{-\Delta_{n0}t-\frac{\Delta_{nm}}{2}} - e^{-\Delta_{m0}t-\frac{\Delta_{mn}}{2}}}{e^{+\frac{-\Delta_{nm}}{2}} - e^{-\frac{\Delta_{mn}}{2}}}\frac{J_{nm}}{2\sqrt{E_nE_m}} \\
&+ \sum_{m_J}\frac{1-e^{-E_{m_J}(T-t)/2}}{(e^{E_{m_J}}-1)}\left[\frac{J_{m_J\Omega}}{2E_{m_J}}\frac{Z_{0m_J}^\dagger}{Z_0^\dagger}\left(1+\sum_{n>0}e^{-\Delta_{n0}t}\frac{E_0}{E_n}\frac{Z_n}{Z_0}\frac{Z_{nm_J}^\dagger}{Z_{0m_J}^\dagger}\right)\right. \\
&\left.\left. + \frac{J_{\Omega m_J}}{2E_{m_J}}\frac{Z_{0m_J}}{Z_0}\left(1+\sum_{n>0}e^{-\Delta_{n0}t}\frac{E_0}{E_n}\frac{Z_n^\dagger}{Z_0^\dagger}\frac{Z_{nm_J}}{Z_{0m_J}}\right)\right]\right\}
\end{aligned}
$$

$$J_{mn} \equiv \langle m|J|n\rangle$$

While this looks horrid, all the unknown quantities in this expression are determined from the standard 2-point functions, except for the matrix elements of interest,

$$J_{nn}, J_{mn}$$

Putting it all together, we are left with a somewhat horrible looking expression

$$
\begin{aligned}
R(t) = \frac{1}{1+\sum_{n>0}\frac{E_0}{E_n}\frac{Z_n Z_n^\dagger}{Z_0 Z_0^\dagger}e^{-\Delta_{n0}t}} \Bigg\{ & (t+1)\left(\frac{J_{00}}{2E_0} + \sum_{n>0}\frac{Z_n Z_n^\dagger}{Z_0 Z_0^\dagger}\frac{E_0}{E_n}\frac{J_{nn}}{2E_n}e^{-\Delta_{n0}t}\right) \\
& + \sum_{m\neq n}\frac{Z_n Z_m^\dagger}{Z_0 Z_0^\dagger}\frac{E_0}{\sqrt{E_n E_m}}\frac{e^{-\Delta_{n0}t-\frac{\Delta_{nm}}{2}} - e^{-\Delta_{m0}t-\frac{\Delta_{mn}}{2}}}{e^{+\frac{-\Delta_{nm}}{2}} - e^{-\frac{\Delta_{mn}}{2}}}\frac{J_{nm}}{2\sqrt{E_n E_m}} \\
& + \sum_{m_J}\frac{1-e^{-E_{m_J}(T-t)/2}}{(e^{E_{m_J}}-1)}\left[\frac{J_{m_J\Omega}}{2E_{m_J}}\frac{Z_{0m_J}^\dagger}{Z_0^\dagger}\left(1+\sum_{n>0}e^{-\Delta_{n0}t}\frac{E_0}{E_n}\frac{Z_n}{Z_0}\frac{Z_{nm_J}^\dagger}{Z_{0m_J}^\dagger}\right)\right. \\
& \left.\left. + \frac{J_{\Omega m_J}}{2E_{m_J}}\frac{Z_{0m_J}}{Z_0}\left(1+\sum_{n>0}e^{-\Delta_{n0}t}\frac{E_0}{E_n}\frac{Z_n^\dagger}{Z_0^\dagger}\frac{Z_{nm_J}}{Z_{0m_J}}\right)\right]\right\}
\end{aligned}
$$

$$J_{mn} \equiv \langle m|J|n\rangle$$

Recall: what we are interested in is the quantity

$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{R(t+\tau)-R(t)}{\tau}$$

Putting it all together, we are left with a somewhat horrible looking expression

$$R(t) = \frac{1}{1+\sum_{n>0}\frac{E_0}{E_n}\frac{Z_n Z_n^\dagger}{Z_0 Z_0^\dagger}e^{-\Delta_{n0}t}}\Bigg\{(t+1)\left(\frac{J_{00}}{2E_0}+\sum_{n>0}\frac{Z_n Z_n^\dagger}{Z_0 Z_0^\dagger}\frac{E_0}{E_n}\frac{J_{nn}}{2E_n}e^{-\Delta_{n0}t}\right)$$

$$+\sum_{m\neq n}\frac{Z_n Z_m^\dagger}{Z_0 Z_0^\dagger}\frac{E_0}{\sqrt{E_n E_m}}\frac{e^{-\Delta_{n0}t-\frac{\Delta_{nm}}{2}}-e^{-\Delta_{m0}t-\frac{\Delta_{mn}}{2}}}{e^{+\frac{-\Delta_{nm}}{2}}-e^{-\frac{\Delta_{mn}}{2}}}\frac{J_{nm}}{2\sqrt{E_n E_m}}$$

$$+\sum_{m_J}\frac{1-e^{-E_{m_J}(T-t)/2}}{(e^{E_{m_J}}-1)}\Bigg[\frac{J_{m_J\Omega}}{2E_{m_J}}\frac{Z_{0m_J}^\dagger}{Z_0^\dagger}\left(1+\sum_{n>0}e^{-\Delta_{n0}t}\frac{E_0}{E_n}\frac{Z_n}{Z_0}\frac{Z_{nm_J}^\dagger}{Z_{0m_J}^\dagger}\right)$$

$$+\frac{J_{\Omega m_J}}{2E_{m_J}}\frac{Z_{0m_J}}{Z_0}\left(1+\sum_{n>0}e^{-\Delta_{n0}t}\frac{E_0}{E_n}\frac{Z_n^\dagger}{Z_0^\dagger}\frac{Z_{nm_J}}{Z_{0m_J}}\right)\Bigg]\Bigg\}$$

The leading contribution from is the ground state matrix element of interest.

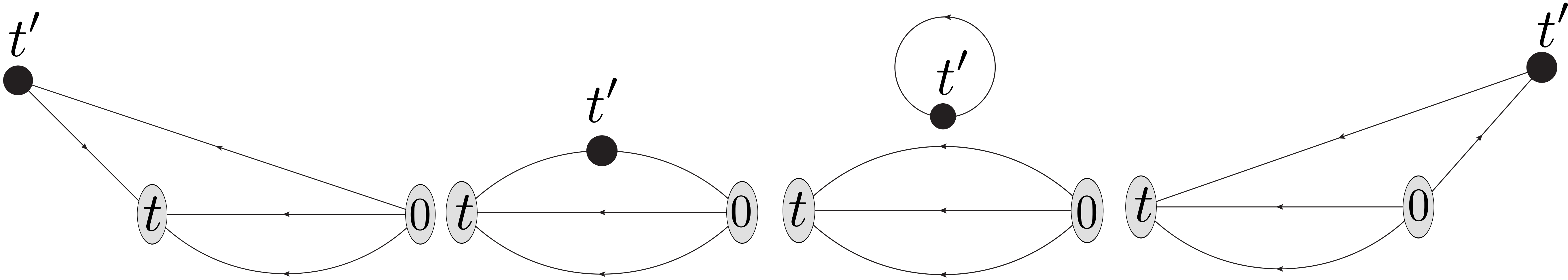
$$\frac{R(t+\tau)-R(t)}{\tau}$$

$$\frac{J_{00}}{2E_0} = g_0^J$$

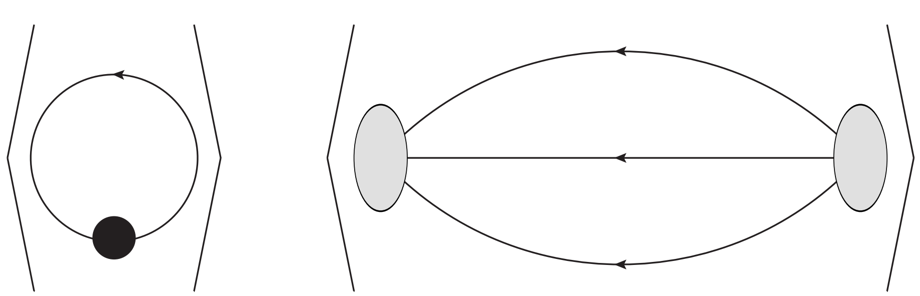
All other terms are suppressed and time-dependent. The time-dependence is critical as it allows for the corrections to be controlled systematically with a single calculation

Recall the differentiation of the correlator with respect to **λ**

$$-\frac{\partial C_\lambda}{\partial \lambda}\bigg|_{\lambda=0} = \frac{\partial_\lambda \mathcal{Z}_\lambda}{\mathcal{Z}_\lambda} C(t) + \frac{1}{\mathcal{Z}_\lambda}\int D\Phi e^{-S}\int d^4x' j(x')\mathcal{O}(t)\mathcal{O}^\dagger(0)$$

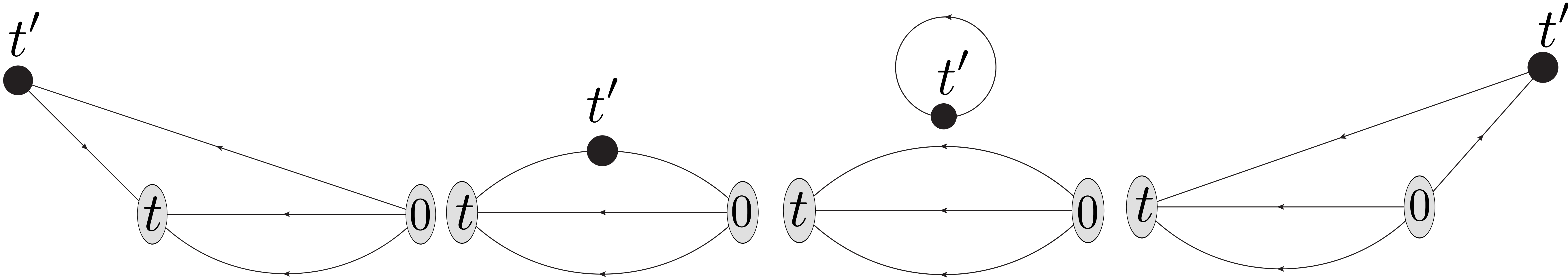


Vacuum term:

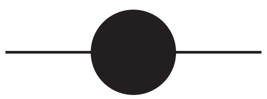


Recall the differentiation of the correlator with respect to **λ**

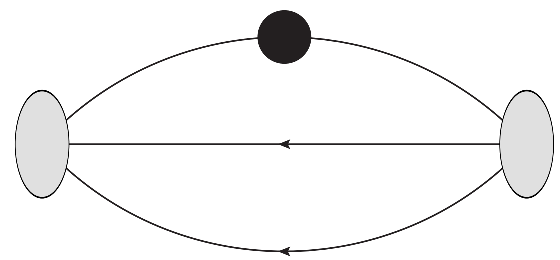
$$-\frac{\partial C_\lambda}{\partial \lambda}\bigg|_{\lambda=0} = \frac{\partial_\lambda \mathcal{Z}_\lambda}{\mathcal{Z}_\lambda} C(t) + \frac{1}{\mathcal{Z}_\lambda}\int D\Phi e^{-S}\int d^4x' j(x')\mathcal{O}(t)\mathcal{O}^\dagger(0)$$



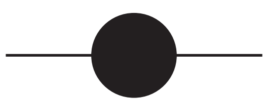
What are these "Feynman-Hellman propagators"?



Numerical Implementation:



the "Feynman-Hellman" propagator is given by



$$= S_{FH}(y,x) = \sum_z S(y,z)\Gamma(z)S(z,x)$$

$S(z,x)$ standard quark propagator off some source at x, to all z

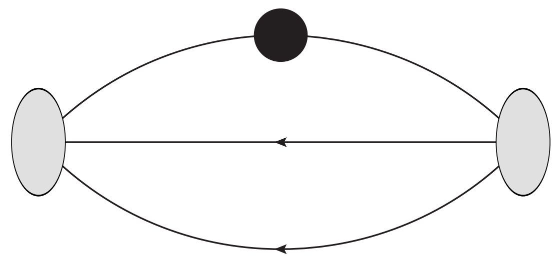
$\Gamma(z)$ some bi-linear operator (can be constant)
e.g., γ_4 for the vector current

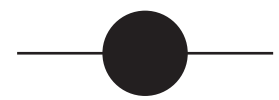
$\Gamma(z)S(z,x)$ treat like a source to invert off of

NOTE: this is the same equation as appears in
de Divitiis, Petronzio, Tantalo, PLB718 (2012)

Numerical Implementation:

$$-\partial_\lambda C_\lambda(t)\Big|_{\lambda=0} =$$

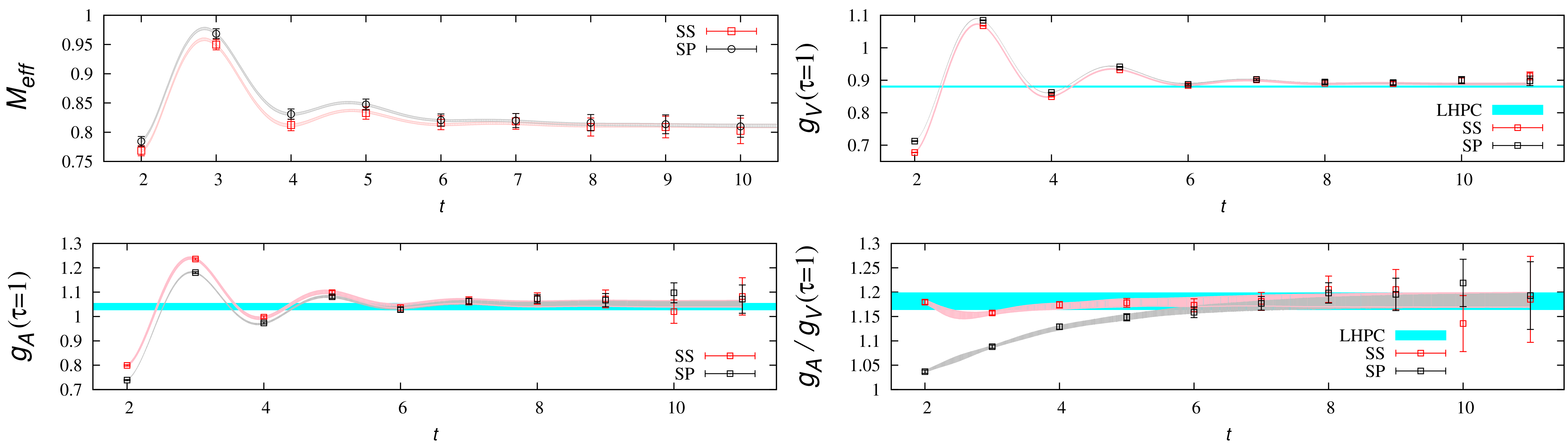


1. Compute Feynman-Hellman propagator  with current $\bar{q}\,\Gamma\,q$
2. Add FH propagator to two-point function with all relevant combinatorics
3. Construct $$\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial \lambda}\Big|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C(t)}\right]$$
4. Fit!

Test case: nucleon axial charge, LHPC comparison

there are old LHPC calculations of the nucleon axial charge with moderate pion masses using DWF on asqtad MILC ensembles

the “regular” propagators were on disk at JLab, so we could simply make the Feynman-Hellman propagators

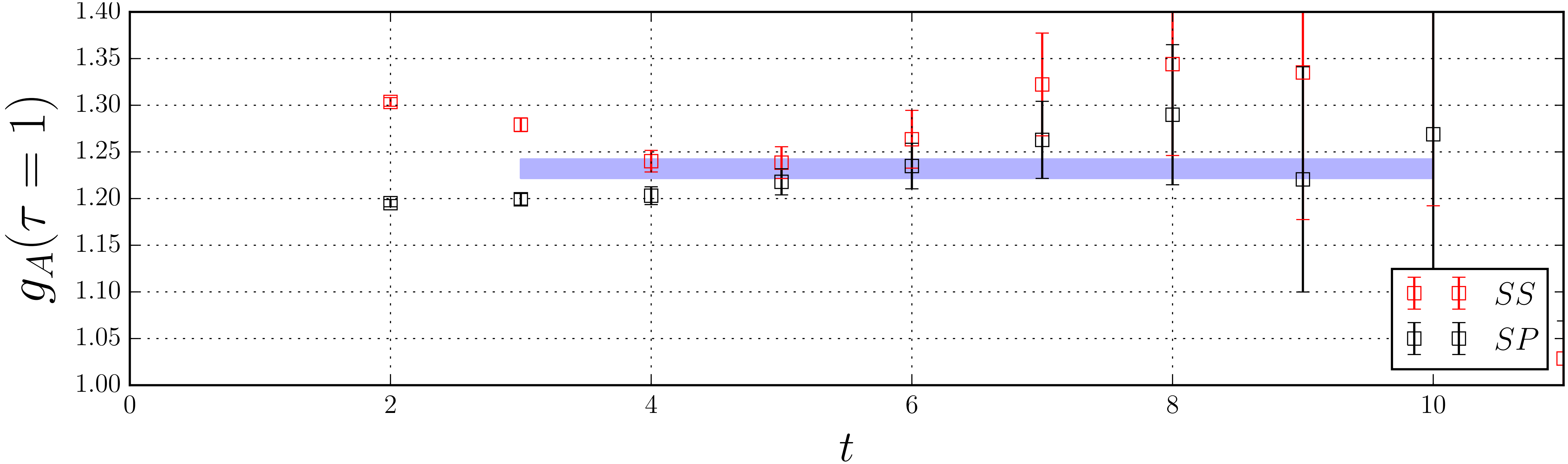


(the oscillations are from a large domain wall mass, $M_5 = 1.7$)

Test case: nucleon axial charge, Möbius DWF on HISQ

$m_\pi \simeq 310$ MeV

$a \simeq 0.15$ fm $\quad 16^3 \times 48 (L_5 = 12)$



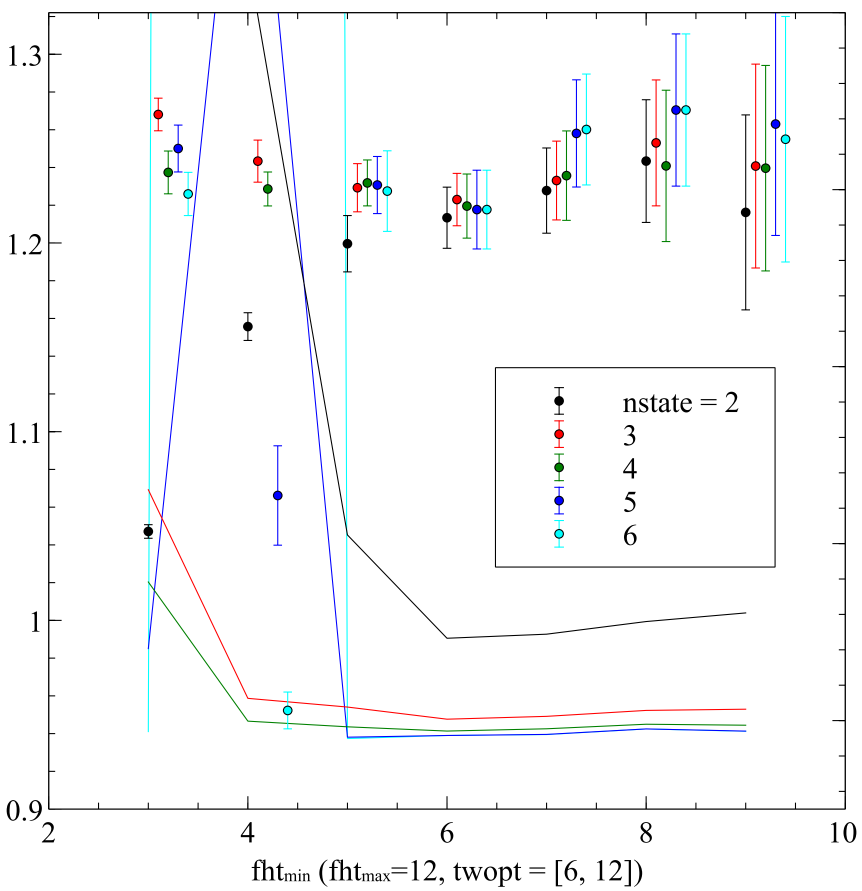
$$\mathring{g}_A(m_\pi \simeq 310, a \simeq 0.15) = 1.232(10)$$

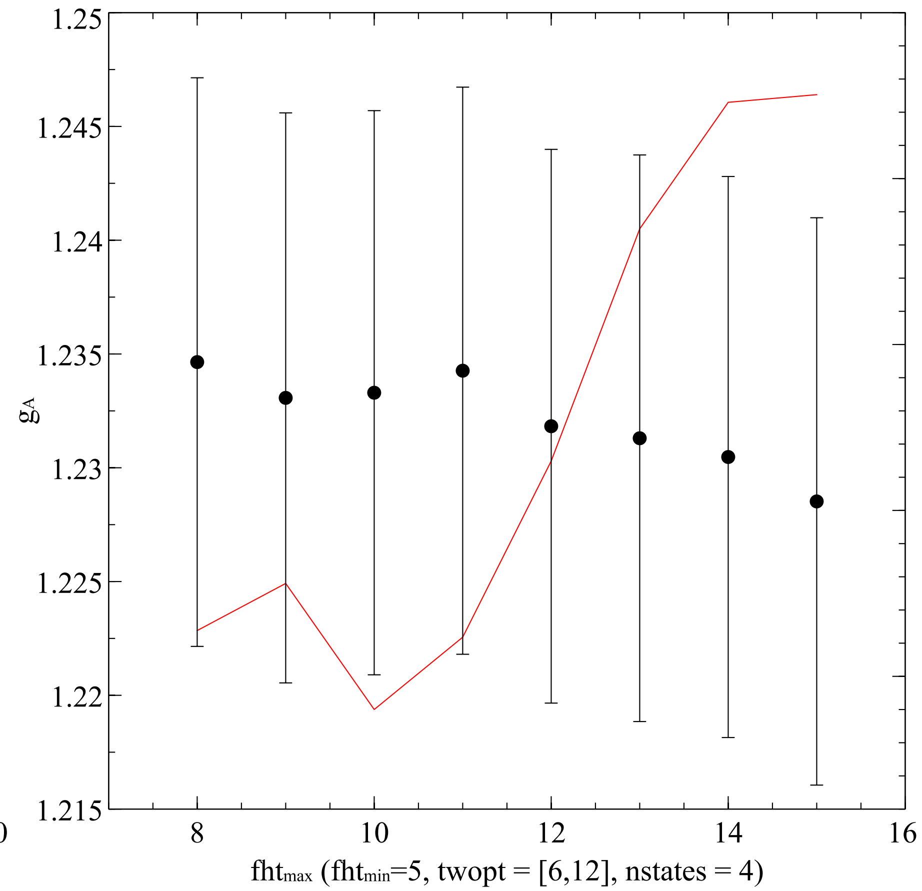
$N_{cfg} = 1960$

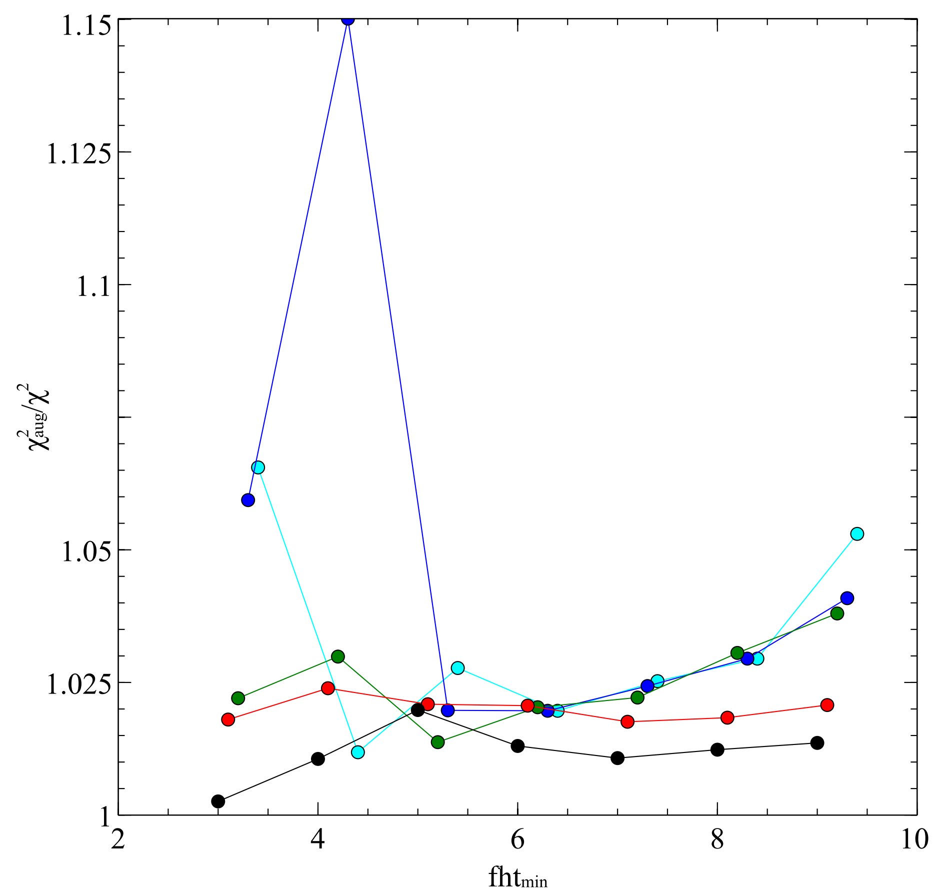
$N_{src} = 6$

Möbius DWF inverter in QUDA: achieves *ridiculous* performance, about 1TFlop/box

Möbius DWF on HISQ $16^3 \times 48(L_5 = 12)$ $a \simeq 0.15$ fm $m_\pi \simeq 310$ MeV
$\mathring{g}_A(m_\pi \simeq 310, a \simeq 0.15) = 1.232(10)$ $N_{cfg} = 1960$ $N_{src} = 6$



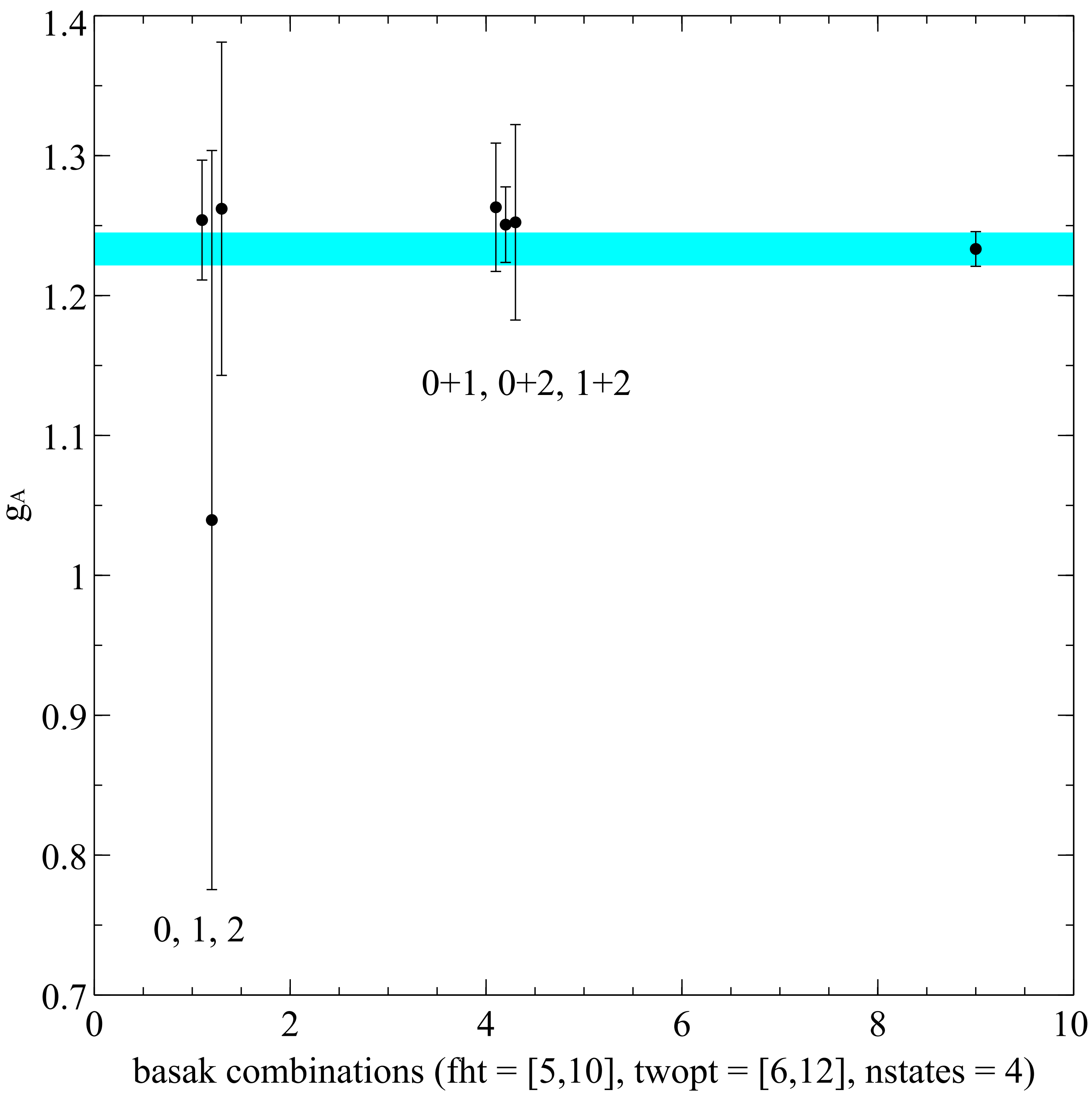




Möbius DWF on HISQ $16^3 \times 48(L_5 = 12)$ $a \simeq 0.15$ fm $m_\pi \simeq 310$ MeV

$\mathring{g}_A(m_\pi \simeq 310, a \simeq 0.15) = 1.232(10)$ $N_{cfg} = 1960$ $N_{src} = 6$

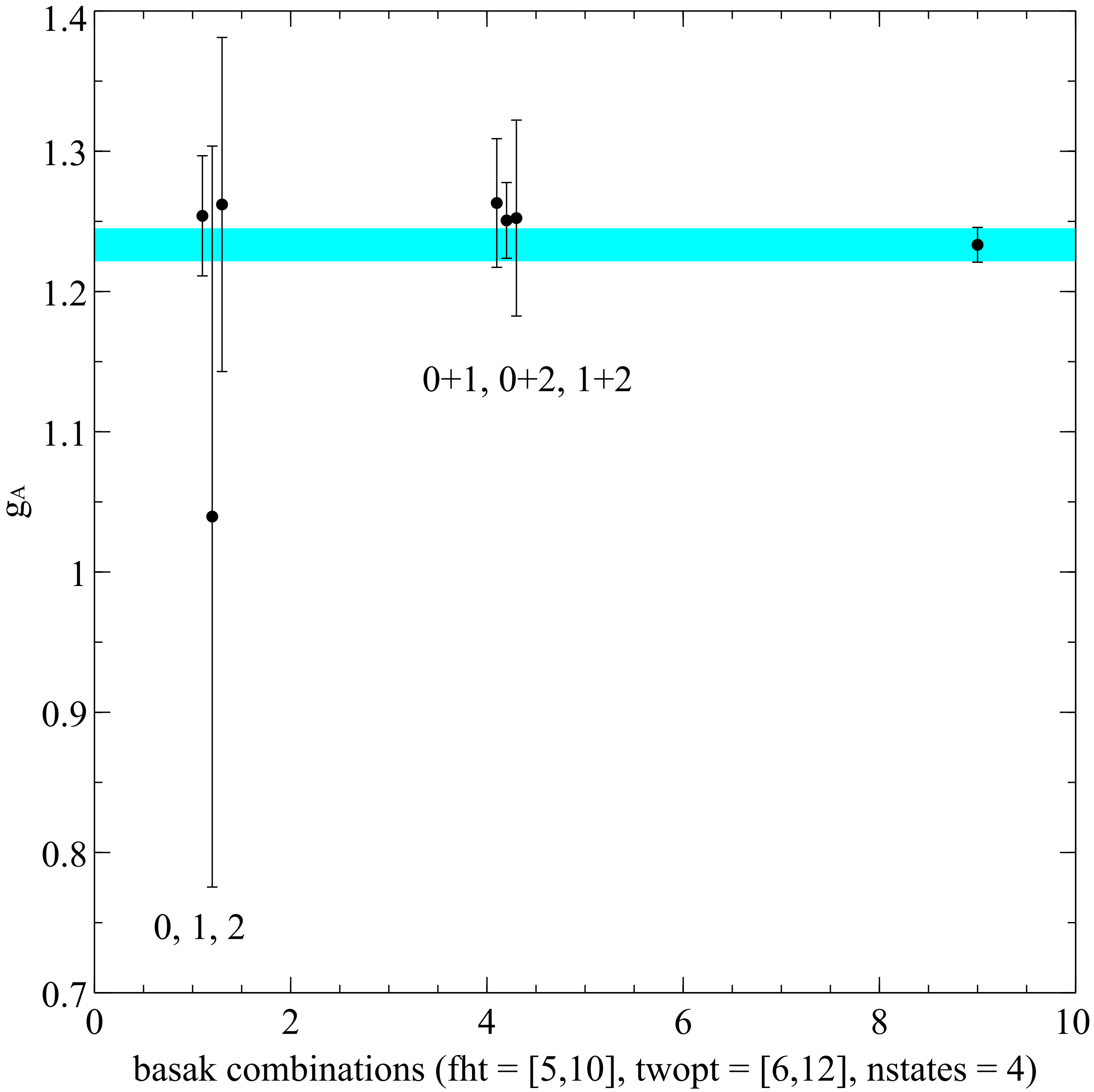
$2 \times (1 \times 1)$ $2 \times (2 \times 2)$ $2 \times (3 \times 3)$



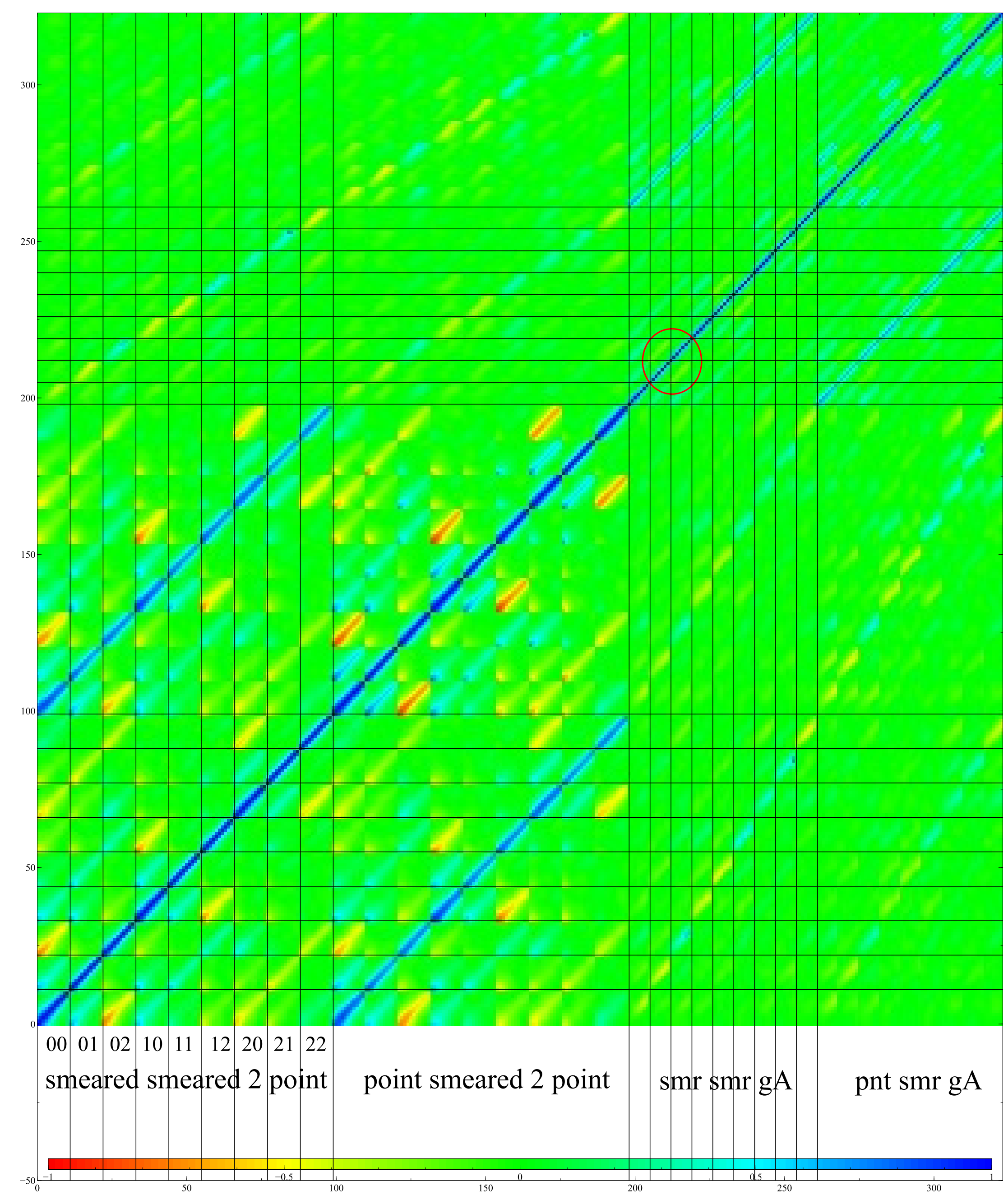
We find the 3 different local Basak operators of the nucleon provide significant improvement in the uncertainty when fit together. The main (all "upper" component quarks) is most important for the central value and the other two Basak operators help control the uncertainty. The quarks are inverted from Gaussian-smeared sources with both smeared and point sinks.

Möbius DWF on HISQ $16^3 \times 48(L_5 = 12)$ $a \simeq 0.15$ fm $m_\pi \simeq 310$ MeV

$\mathring{g}_A(m_\pi \simeq 310, a \simeq 0.15) = 1.232(10)$ $N_{cfg} = 1960$ $N_{src} = 6$

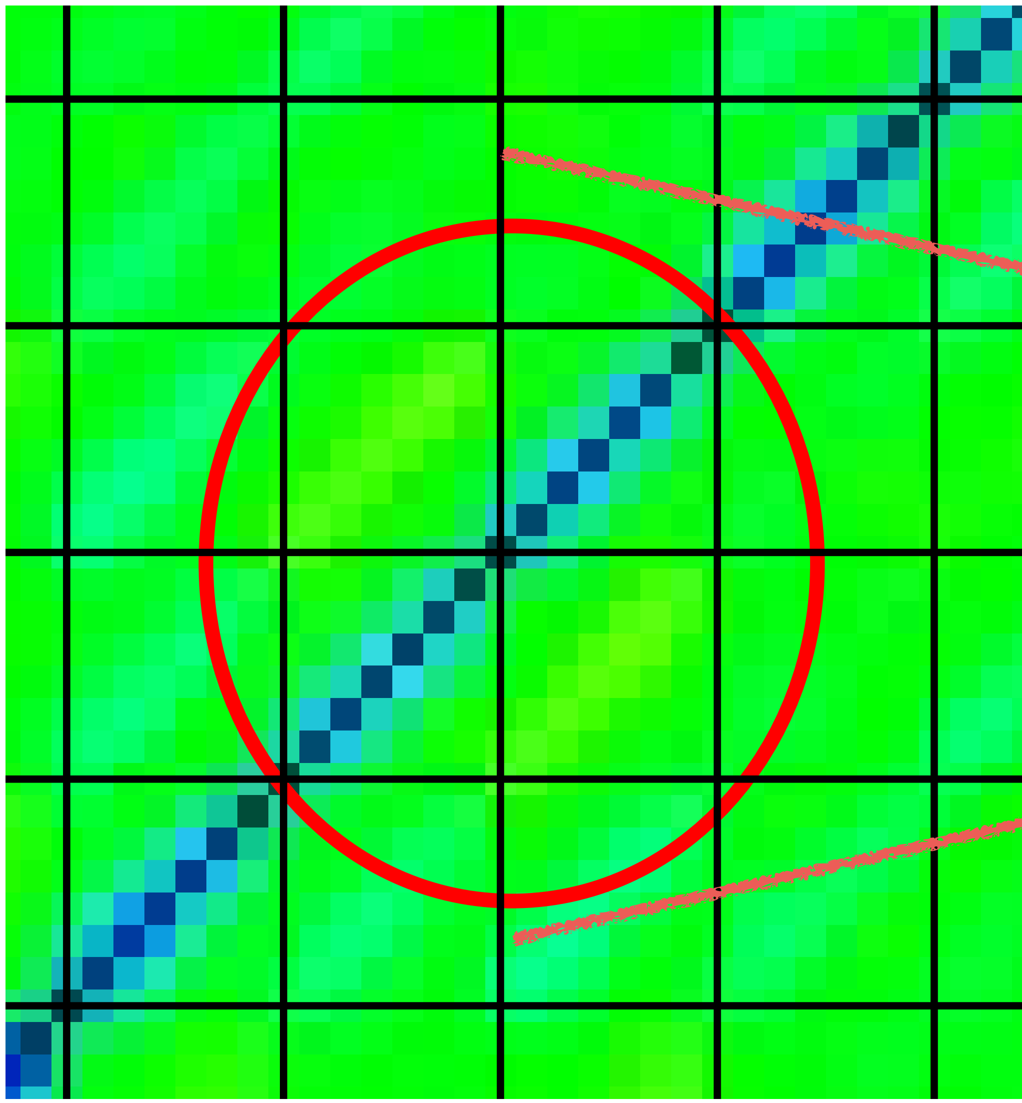


Correlation matrix

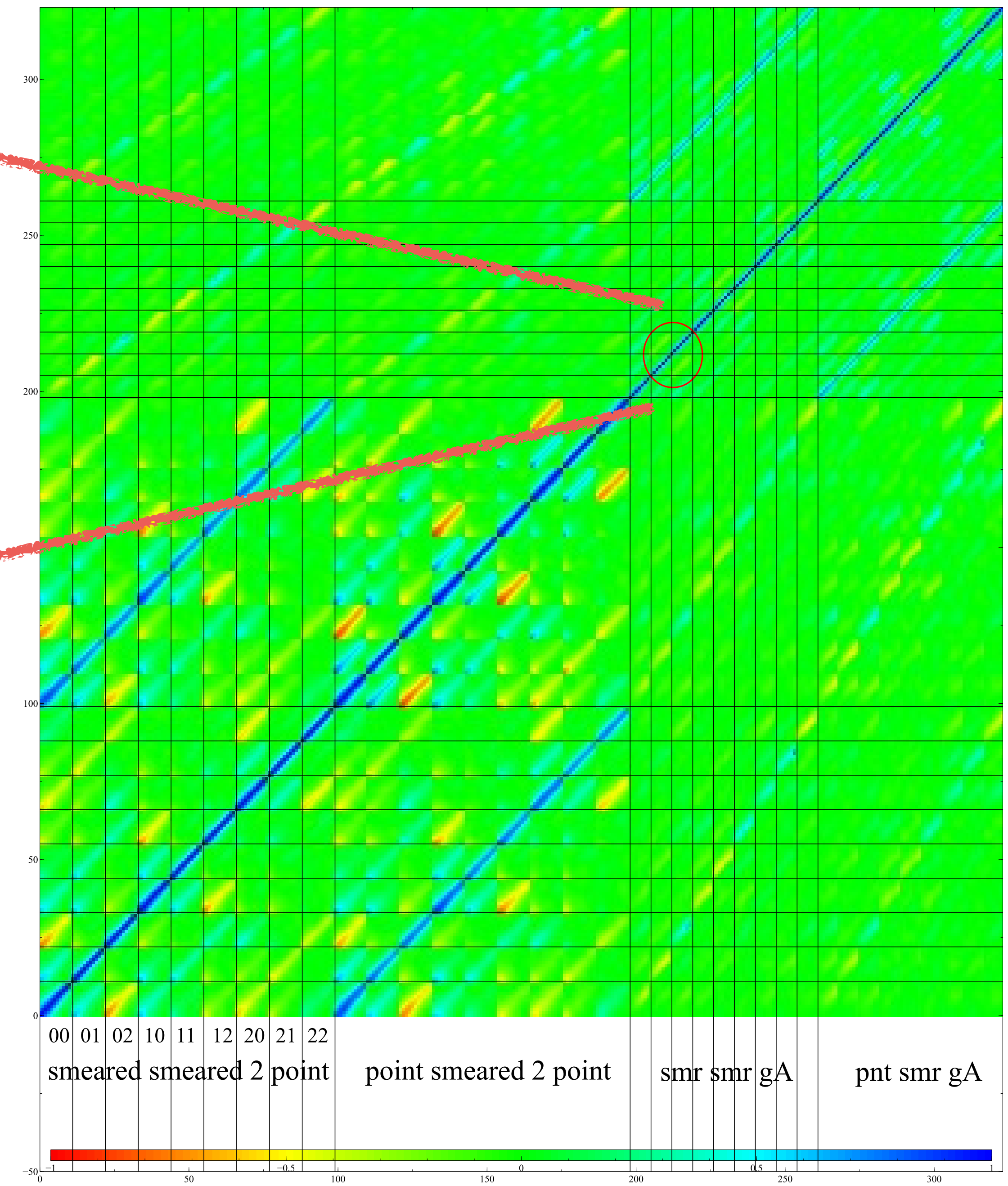


Möbius DWF on HISQ $16^3 \times 48 (L_5 = 12)$ $a \simeq 0.15$ fm $m_\pi \simeq 310$ MeV

$\mathring{g}_A(m_\pi \simeq 310, a \simeq 0.15) = 1.232(10)$ $N_{cfg} = 1960$ $N_{src} = 6$



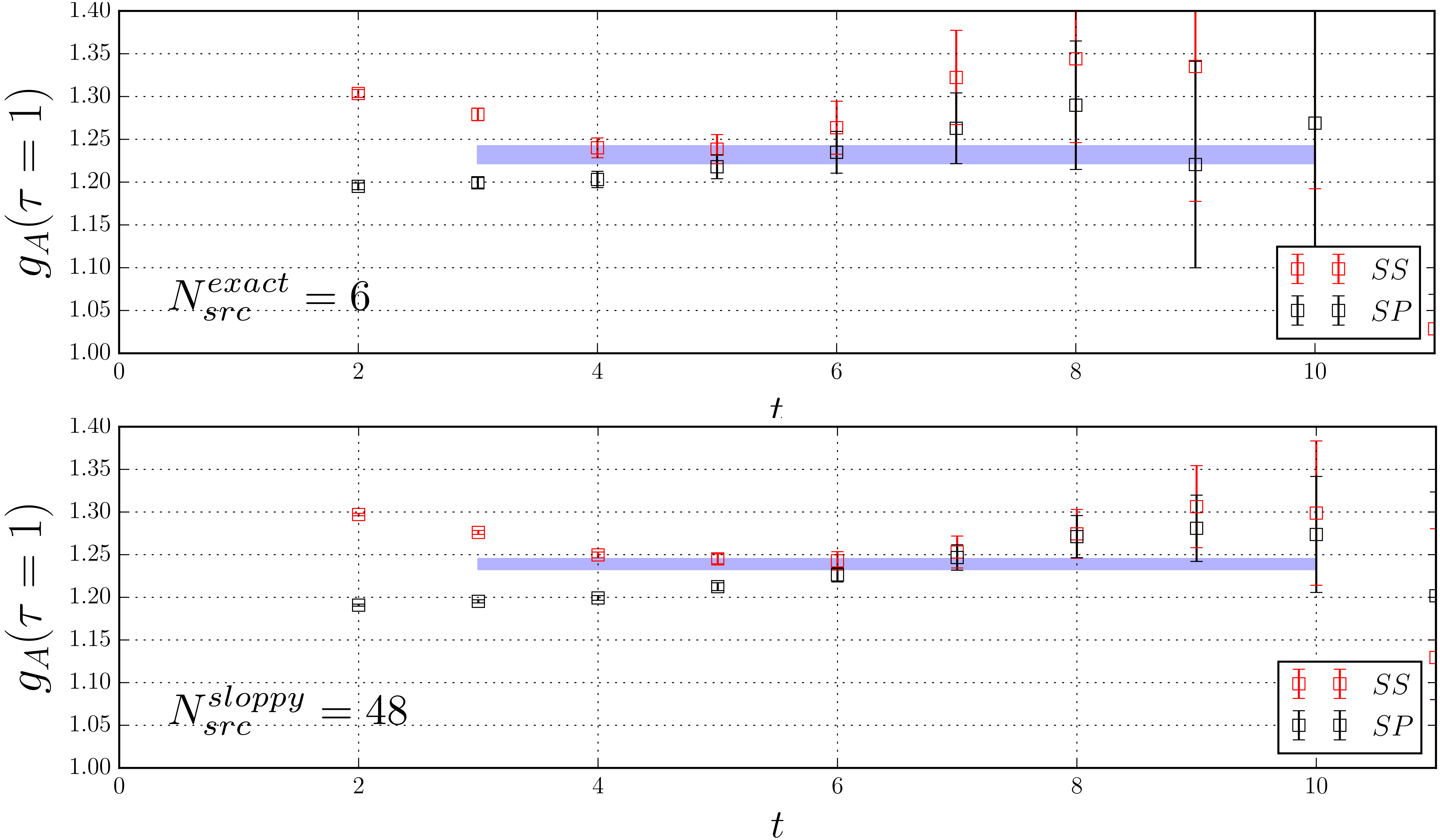
Correlation matrix



Difficult to see: the gA matrix elements have the same anti-correlations as the spectrum

Möbius DWF on HISQ $16^3 \times 48 (L_5 = 12)$ $a \simeq 0.15$ fm $m_\pi \simeq 310$ MeV

$\mathring{g}_A(m_\pi \simeq 310, a \simeq 0.15) = 1.232(10)$ $N_{cfg} = 1960$



Möbius DWF on Gradient-flowed HISQ

Mixed-Action LQCD with DW valence-fermions poses many good properties for calculations involving static quantities (not multi-particle). e.g.:

- retains good chiral symmetry. Using gradient-flowed HISQ cfgs allows us to keep $m_{res} < 0.1\ m_l$ for all light quark masses including physical, with small to moderate values of L_5 with $1.0 < M_5 < 1.3$
- determination of Z_A is simple: use 5d ward-identity to get f_π and 4d axial current to determine f_π/Z_A

We are using this setup to compute pion-nucleon couplings from BSM CP-violating quark chromo-EDM operators.

We can provide an independent determination of g_A addressing standard systematics and an alternate, improved means to control excited state contamination

	a[fm] : m_π[MeV]	310	220	135
g_A	0.15	1.239(6)	1.230(25)	–
	0.12	–	–	–
	0.09	–	–	?
	0.06	?	?	NA

This Feynman-Hellman Method

$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C(t)}\right]$$

is very general. We use the FHT to determine the linear response correlation function to compute.

It can be applied to any quark bi-linear operator for any hadronic correlation function, including non-zero momentum transfer and flavor changing interactions…

The big advantage is that for this quantity, the only time-independent quantity is the ground state matrix element of interest. This allows for much better systematic control of the excited state contributions, allowing for a robust determination of the g.s. matrix elements.

The numerical cost is the same as one source-sink separation with the sequential propagator method.

This Feynman-Hellman Method

$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C(t)}\right]$$

For disconnected diagrams, one begins the calculation as usual: compute the disconnected quark loop. BUT THEN, sum this quark loop over all time, and then multiply the standard two-point correlation function by this number, cfg-by-cfg. This generates $\left.-\partial_\lambda C_\lambda(t)\right|_{\lambda=0}$

For slightly more post processing: one can construct a further improved correlation function by summing ONLY over $0 < t' < t$ and then multiply the standard two-point function by this time-dependent number. This removes the contamination from the “outer” time regions.

This Feynman-Hellman Method

$$\left.\frac{\partial m_\lambda^{eff}(t,\tau)}{\partial\lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[\frac{-\partial_\lambda C_\lambda(t+\tau)}{C(t+\tau)} - \frac{-\partial_\lambda C_\lambda(t)}{C(t)}\right]$$

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For further developments of new methods for nucleon structure calculations - see talk by
Chia Cheng (Jason) Chang THUR 14:40 HADRON STRUCTURE

Thank You