D-Meson Mixing in 2+1 Lattice QCD and Related Topics

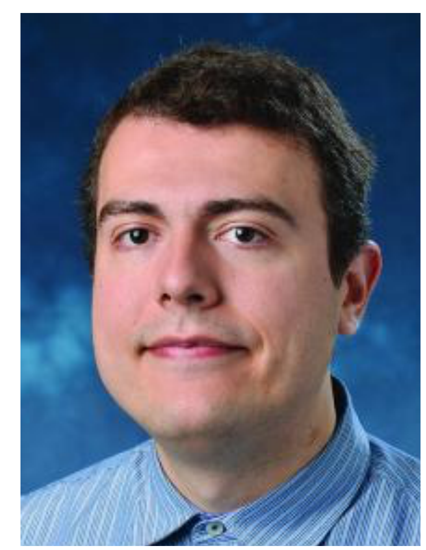
Andreas S. Kronfeld
Fermilab & IAS TU München

Lattice 2016
July 29, 2016
University of Southampton

Fermilab Lattice and MILC Collaborations

A. Bazavov, C. Bernard, C.M. Bouchard, C.C. Chang, C. DeTar, Daping Du, A.X. El-Khadra, E.D. Freeland, E. Gámiz, Steven Gottlieb, U.M. Heller, ASK, J. Laiho, P. B. Mackenzie, E. T. Neil, J. Simone, R. Sugar, D. Toussaint, R. S. Van de Water, Ran Zhou

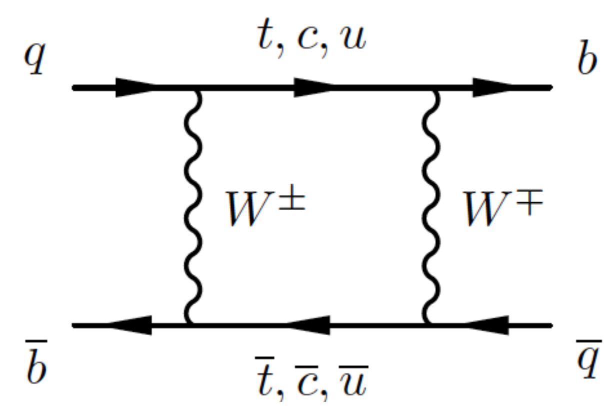


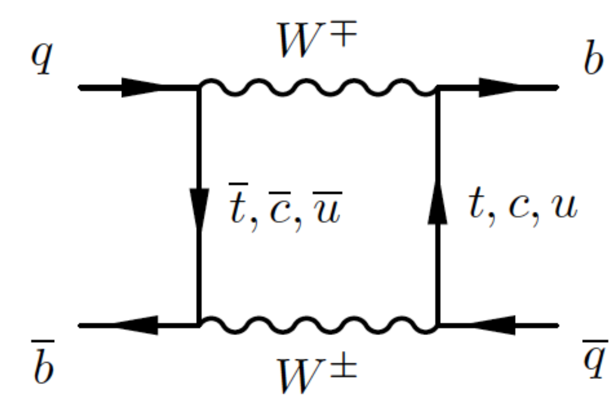



$B_{(s)}$ mixing in Phys. Rev. D 93, 113016 (2016) [arXiv:1602.03560 [hep-lat]]

Neutral-Meson Mixing

- In the Standard Model, neutral mesons can oscillate into their antiparticles:





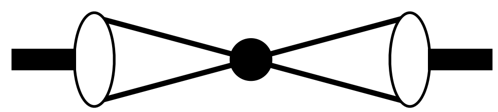
- In extensions of the SM, other particles
  - could appear in the boxes;
  - could appear at the tree level: flavor-changing neutral current.
- Observed in the lab for all neutral mesons: K^0, D^0, B^0, B_s.

One $\Delta F = 2$ or Two $\Delta F = 1$ Interactions

see, *e.g.*, Artuso, Meadows, Petrov review

- Mixing originates in two kinds of processes:

$$(M_{12} - \tfrac{i}{2}\Gamma_{12}) \propto \langle D^0|\mathscr{L}^{\Delta C=2}|\bar{D}^0\rangle + \sum_n \frac{\langle D^0|\mathscr{L}^{\Delta C=1}|n\rangle\langle n|\mathscr{L}^{\Delta C=1}|\bar{D}^0\rangle}{M_D - E_n + i\varepsilon}$$




where the $\Delta F = 1$ interactions can be separated by hadronic distances.

- Second term is very difficult to estimate.
- With D mesons (unlike K, B, B_s) it is also not negligible: d, s, b in loop.
- Even so, some BSMs modify first term only [*e.g.*, arXiv:0903.2830].

Effective Hamiltonian

- After integrating out heavy particles:

$$\mathscr{L} = \mathscr{L}_{\text{kin}}[\ell, q, \gamma, g] + \sum_i \mathscr{C}_i(\alpha, \alpha_s, G_F, \sin^2\theta, m_\ell, m_q, V; \text{NP}) \mathscr{L}_i[\ell, q, \gamma, g]$$

- For $\Delta F = 2$ processes, discrete symmetries and Fierz rearrangement reduces the list of operators to 8 = 5 + 3:

$$\mathscr{O}_1 = \bar{c}\gamma^\mu L u \, \bar{c}\gamma^\mu L u \qquad \tilde{\mathscr{O}}_1 = \bar{c}\gamma^\mu R u \, \bar{c}\gamma^\mu R u$$

$$\mathscr{O}_2 = \bar{c} L u \, \bar{c} L u \qquad \tilde{\mathscr{O}}_2 = \bar{c} R u \, \bar{c} R u$$

$$\mathscr{O}_3 = \bar{c}^\alpha L u^\beta \, \bar{c}^\beta L u^\alpha \qquad \tilde{\mathscr{O}}_3 = \bar{c}^\alpha R u^\beta \, \bar{c}^\beta R u^\alpha$$

$$\mathscr{O}_4 = \bar{c} L u \, \bar{c} R u$$

$$\mathscr{O}_5 = \bar{c}^\alpha L u^\beta \, \bar{c}^\beta R u^\alpha c$$

By parity in QCD: $\langle D^0 | \tilde{\mathscr{O}}_i | \bar{D}^0 \rangle = \langle D^0 | \mathscr{O}_i | \bar{D}^0 \rangle$

Fermilab/MILC D vs $B_{(s)}$ Mixing

- Our (ongoing) D-meson and (published) $B_{(s)}$-meson analyses have:
    - same ensembles, same light valence masses;
    - same treatment of chiral perturbation theory;
    - same mostly nonperturbative matching to continuum QCD.
- Some differences:
    - ranges for correlator fits—better signal-to-noise for D;
    - quote renormalized matrix elements at 3 GeV for D but m_b for $B_{(s)}$.

Lattice Operators

- Staggered light quarks χ_u and clover (a la Fermilab) heavy quarks Ψ_c:

$$O_1(x) = \bar{\Psi}_c(x)\gamma^\mu L\Omega(x)\underline{\chi}_u(x)\,\bar{\Psi}_c(x)\gamma^\mu L\Omega(x)\underline{\chi}_u(x)$$
$$O_2(x) = \bar{\Psi}_c(x)L\Omega(x)\underline{\chi}_u(x)\,\bar{\Psi}_c(x)L\Omega(x)\underline{\chi}_u(x)$$
$$O_3(x) = \bar{\Psi}_c(x)^\alpha L\Omega(x)\underline{\chi}_u(x)^\beta\,\bar{\Psi}_c(x)^\beta L\Omega(x)\underline{\chi}_u(x)^\alpha$$
$$O_4(x) = \bar{\Psi}_c(x)L\Omega(x)\underline{\chi}_u(x)\,\bar{\Psi}_c(x)R\Omega(x)\underline{\chi}_u(x)$$
$$O_5(x) = \bar{\Psi}_c(x)^\alpha L\Omega(x)\underline{\chi}_u(x)^\beta\,\bar{\Psi}_c(x)^\beta R\Omega(x)\underline{\chi}_u(x)^\alpha$$

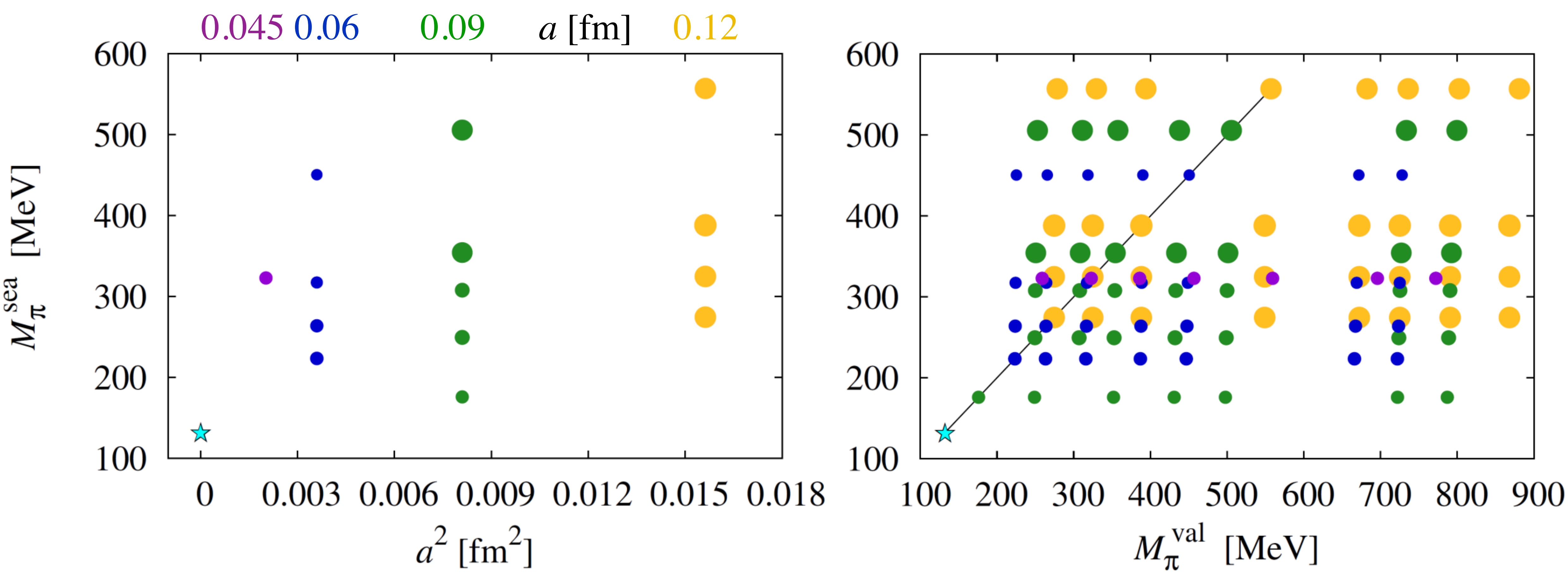
- Three-point correlators contain the desired terms, opposite-parity terms (as usual with staggered), and “wrong-spin” contributions.

- Undesired parts removed in correlator fits and chiral-continuum extrap'n.

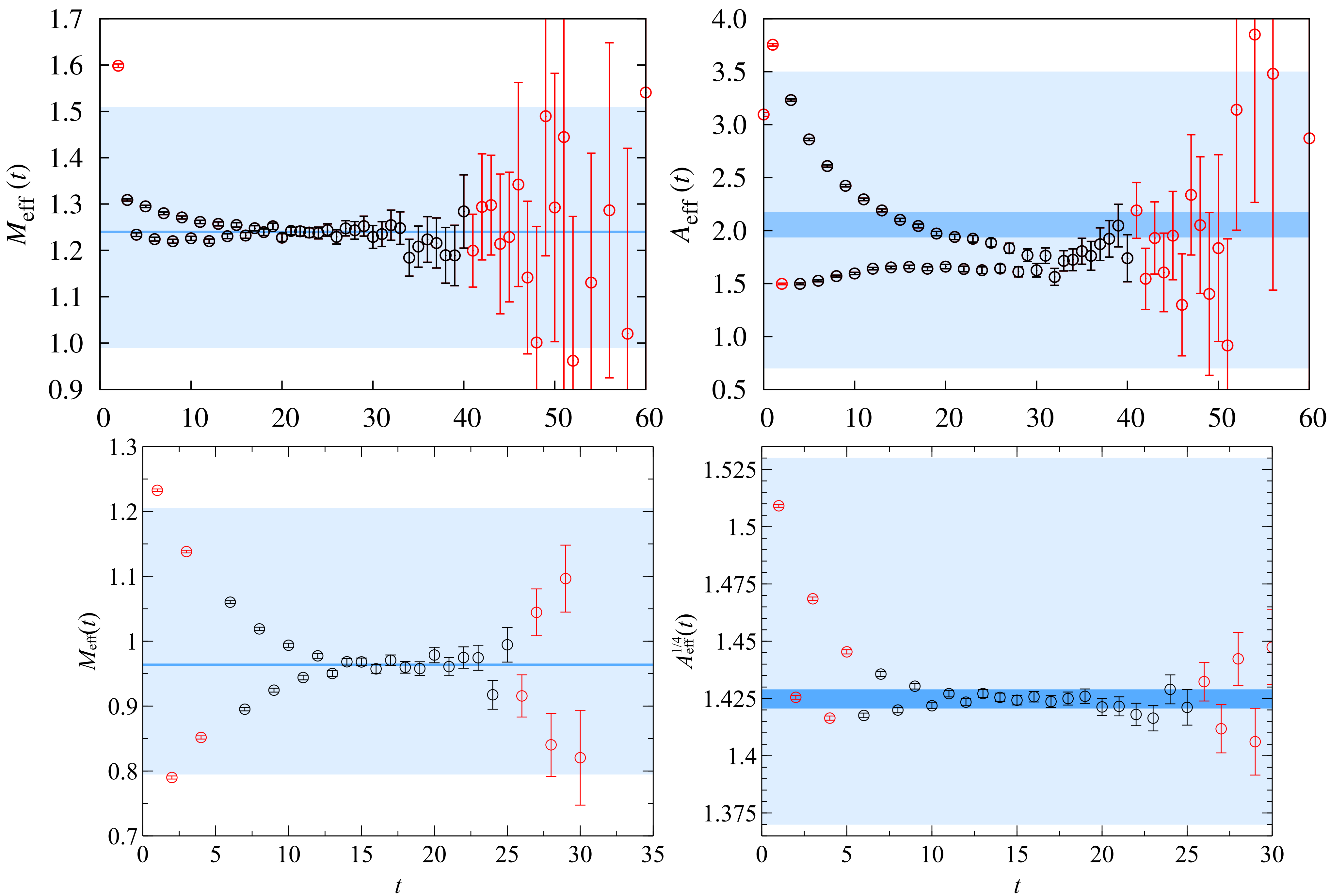
asqtad 2+1 Ensembles from MILC

- Partially quenched data on 600–2200 gauge fields with (sea)

$$177\text{ MeV} \leq \quad M_\pi \quad \leq 555\text{ MeV}$$
$$257\text{ MeV} \leq M_\pi^{\text{rms}} \leq 670\text{ MeV}$$



Two-Point Functions: Priors and Posteriors



B_q

$m_q = m_s/10$

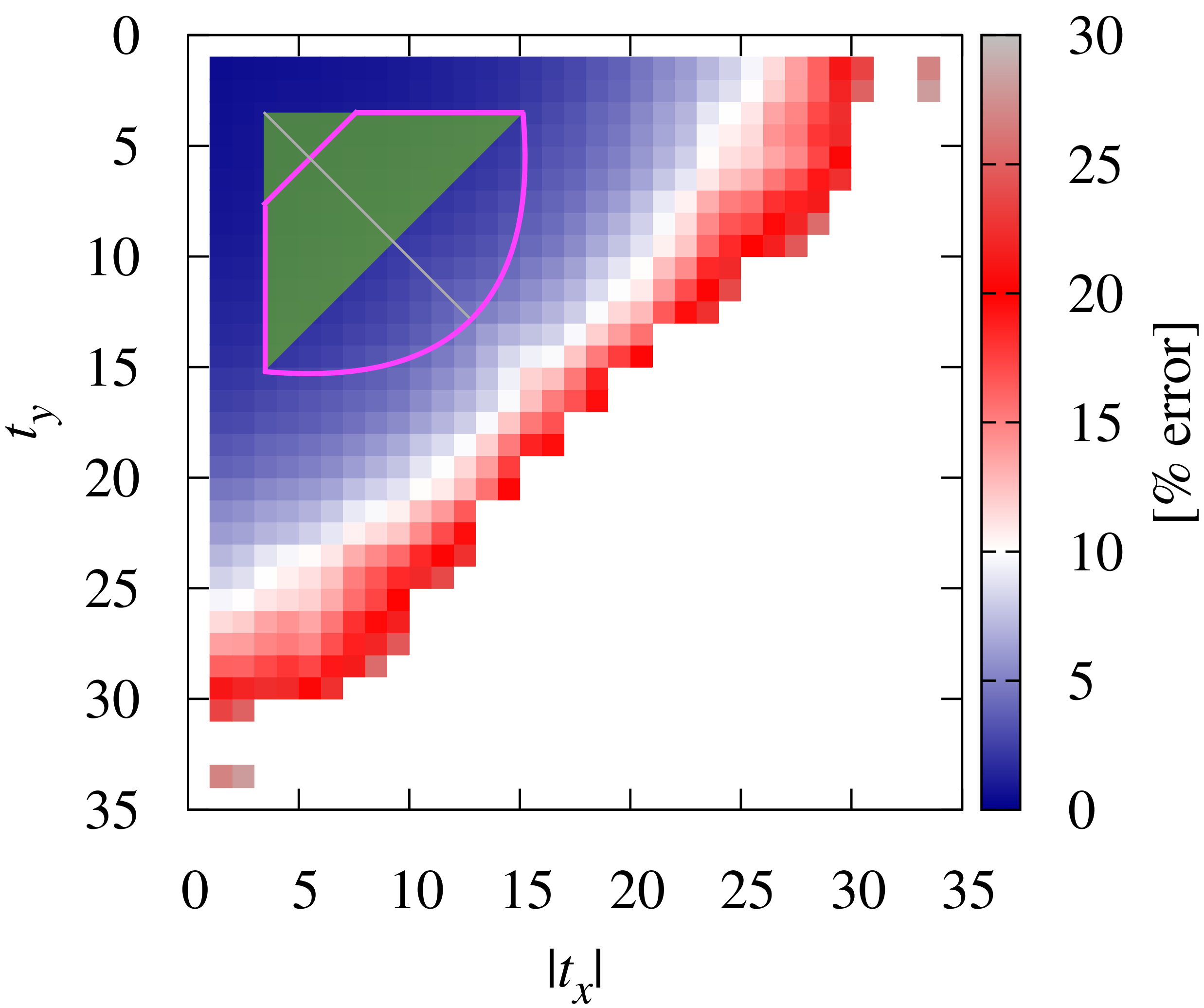
0.06 fm

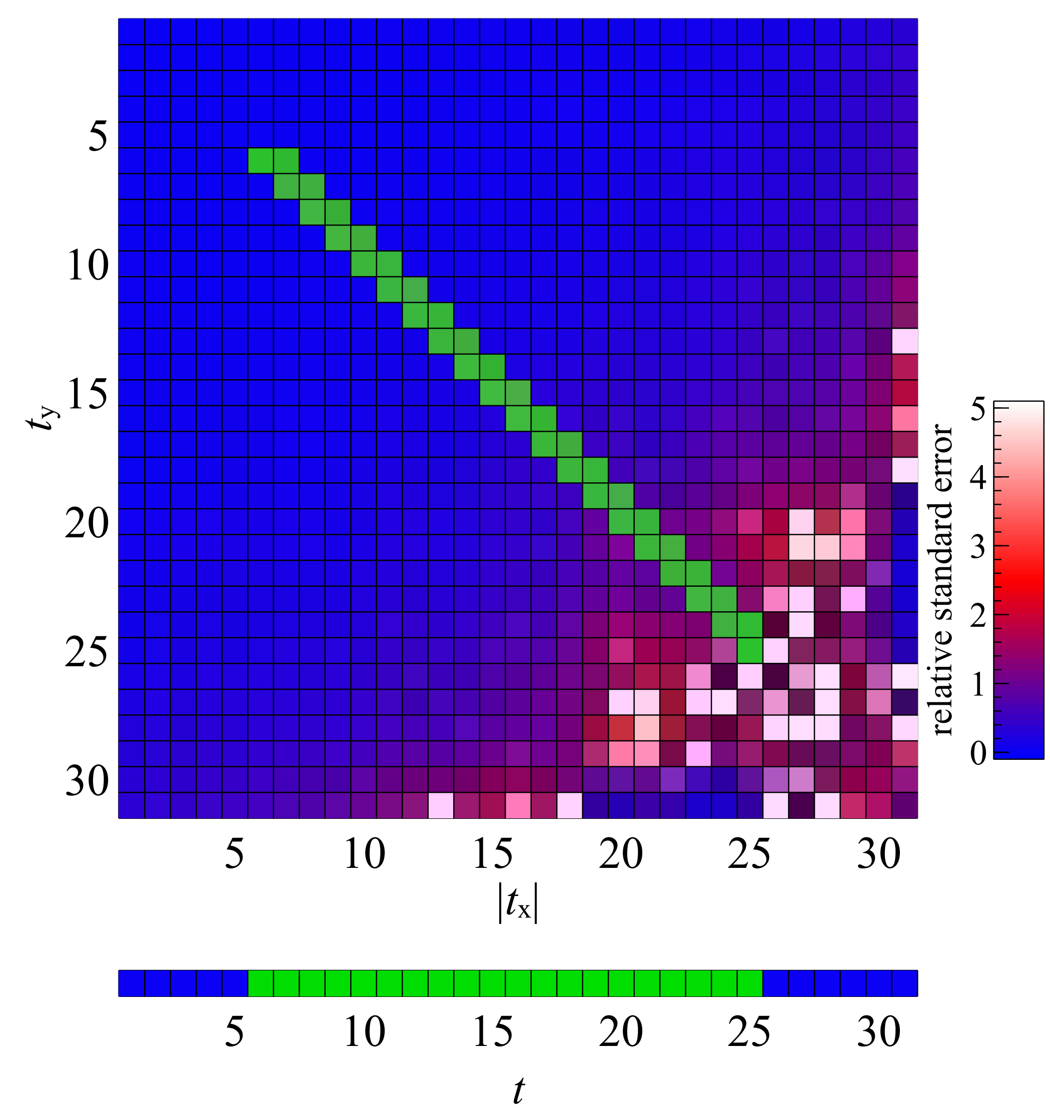
D_q

$m_q = m_s/5$

0.12 fm

Three-Point Functions: S-to-N & Fit Regions





B_q, $m_q = m_s/10$, 0.09 fm, O_2

D_q, $m_q = m_s/5$, 0.12 fm, O_4

Matching and Renormalization

- Mostly nonperturbative (mNPR):

$$\bar{O}_i = Z_{V^4_{cc}} Z_{V^4_{uu}} \rho_{ij} O_j \doteq \mathscr{O}_i + \mathrm{O}(\alpha_s a, a^2)$$

  where the nonperturbative Z_Vs remove wave-function factors, all tadpoles, and some vertex corrections.

- Remaining factor ρ_{ij} obtained at one loop in two independent calculations.

- Two-loop corrections incorporated into chiral-continuum fit.

- Checks by changing mNPR to tadpole-improved PT: $Z_{ij} = u_0^2 \tilde{Z}_{ij}$, with u_0 from plaquette or Landau link.

Chiral-Continuum Extrapolation

nonanalytic terms from NLO HMrSχPT aka "chiral logs"

heavy-quark discretization effects (derived in HQET)

fine tune c-quark hopping parameter

$$F_i = F_i^{\text{logs}} + F_i^{\text{analytic}} + F_i^{\text{HQ disc}} + F_i^{\alpha_s a^2 \text{ gen}} + F_i^{\kappa} + F_i^{\text{renorm}}$$

analytic terms in N^nLO χPT: base fit $n = 2$

gluon & light-quark cutoff effects a la Symanzik

fit $\alpha_s^2 \rho_{ij}^{[2]}$ (alternatively $\alpha_s^3 \rho_{ij}^{[3]}$)

Wrong-Spin Terms

- As noted above, the 3-point functions with staggered-clover 4-quark operators lead to contributions with the wrong spin.

- Schematically [C. Bernard, arXiv:1303.0435],

$$F_i^{\text{logs}} = \beta_i(1 + \text{normal } \chi \text{ logs}) + \beta_j(\text{wrong-spin } \chi \text{ logs}), \quad j \neq i$$

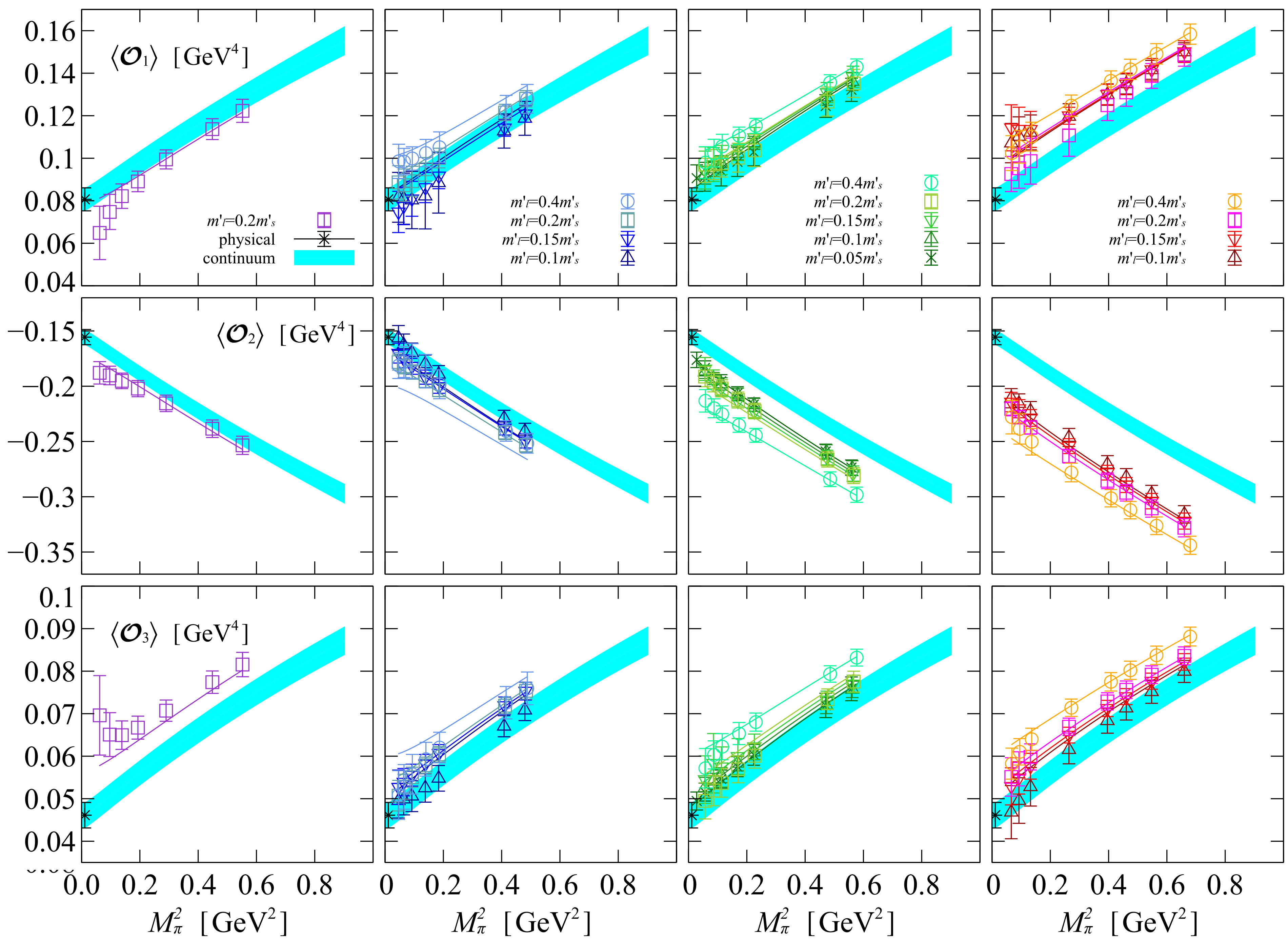
leading LEC for this operator i

leading LEC for some other operator j, $\Gamma_j \neq \Gamma_i$

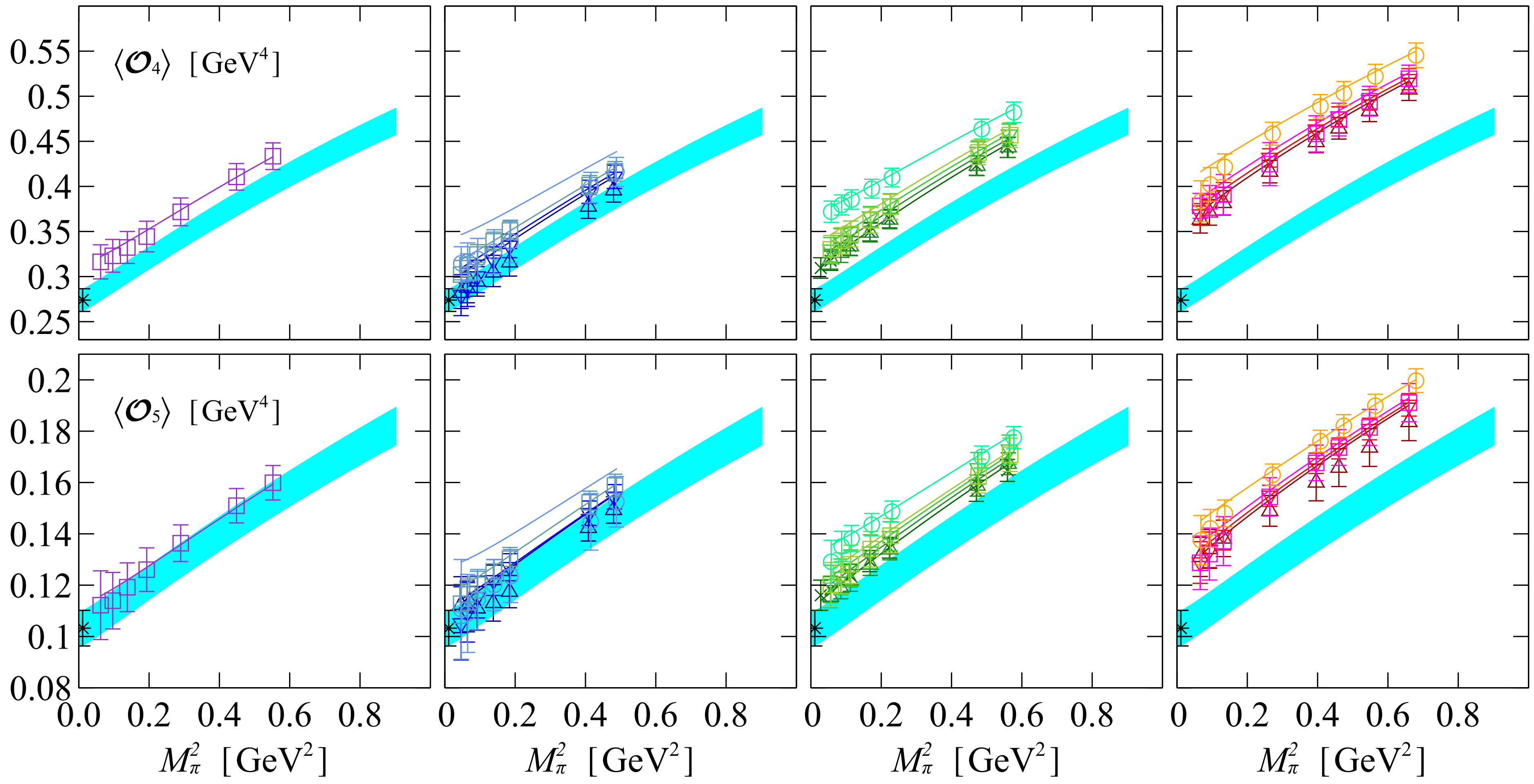
- Base fit: use single set of β_i for all five matrix elements.

- Alternate fit: fit each matrix element individually, allowing uninformed $\beta_{j\neq i}$.

D Mixing Operators 1, 2, 3



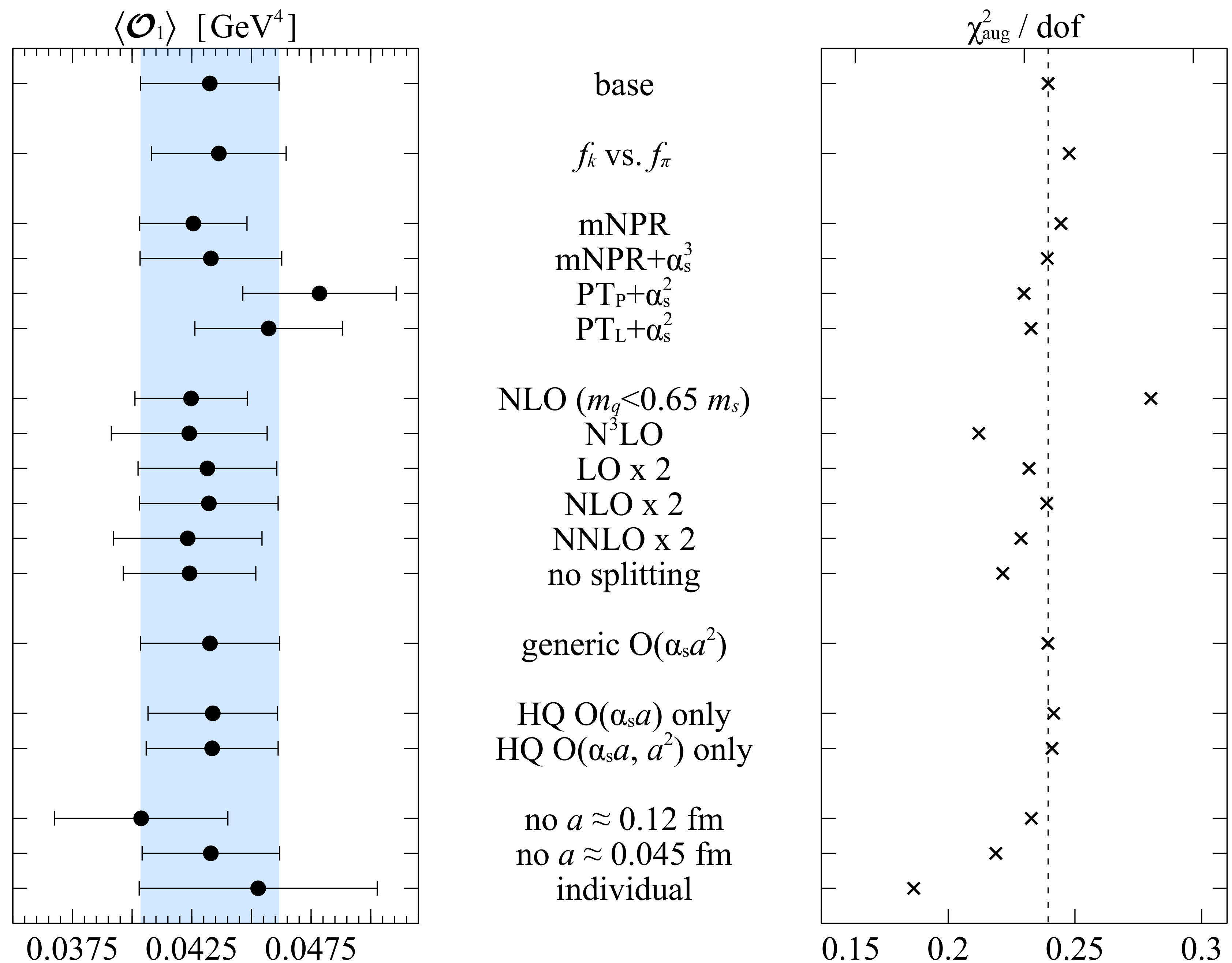
D Mixing Operators 4, 5



- B-meson plots look similar

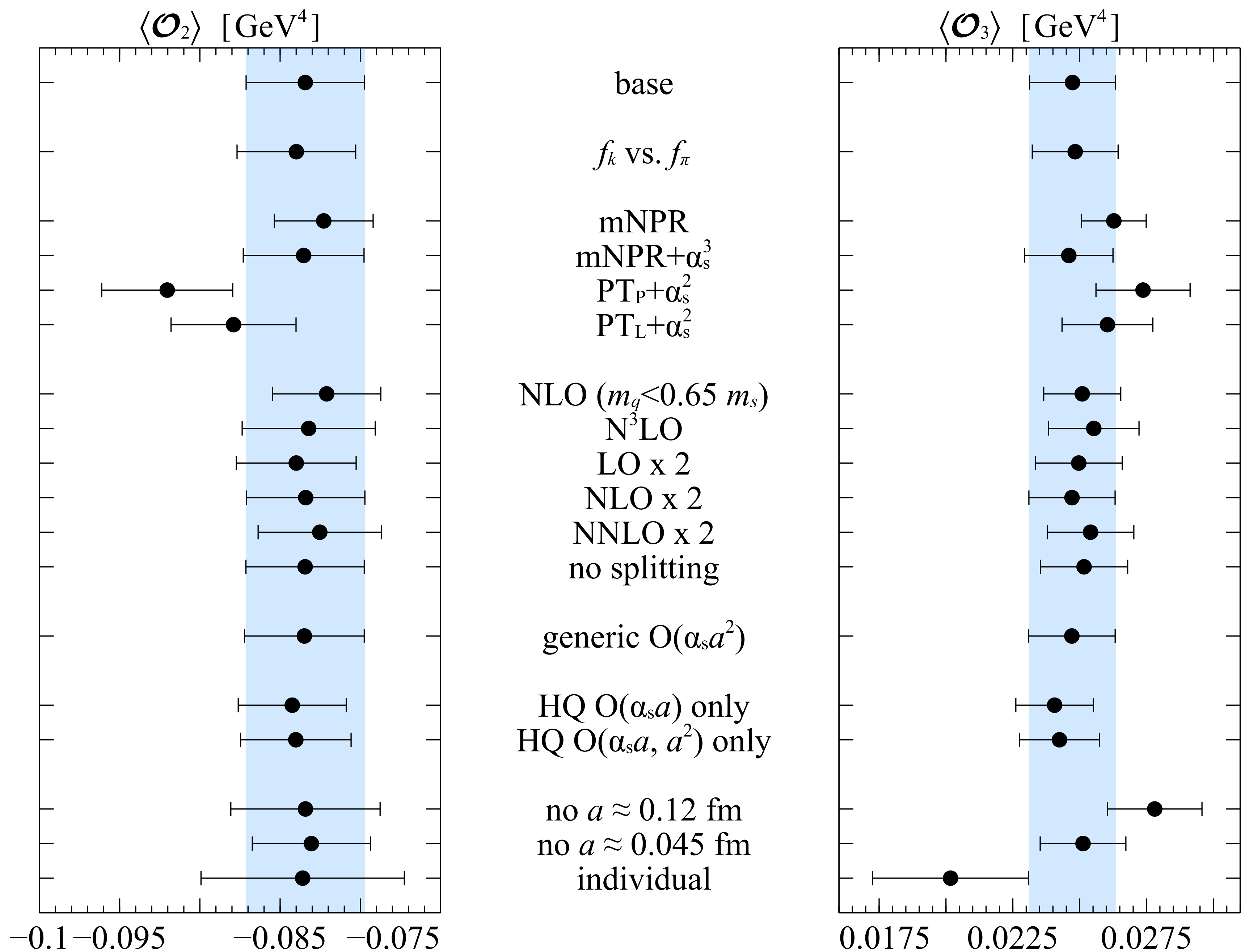
Stability under Fit Variations

- f_K instead of f_π;
- different renorm'n;
- vary χPT data, priors;
- vary discretization effects included;
- "dumb" fits.



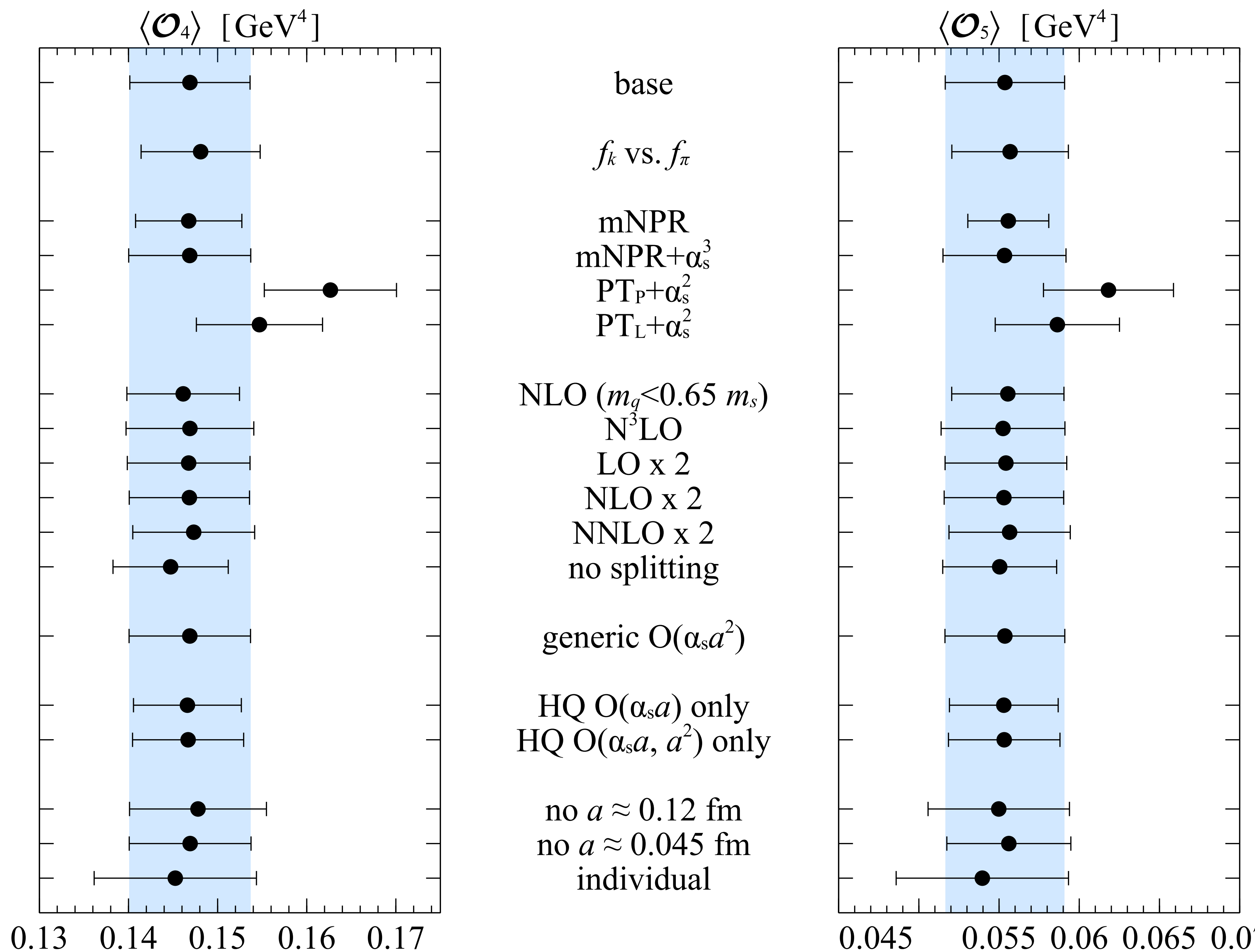
Stability under Fit Variations

- f_K instead of f_π;
- different renorm'n;
- vary χPT data, priors;
- vary discretization effects included;
- "dumb" fits.



Stability under Fit Variations

- f_K instead of f_π;
- different renorm'n;
- vary χPT data, priors;
- vary discretization effects included;
- "dumb" fits.



Nearly Final (Published) Results for D (B) Mesons

- We report continuum results for two choices of evanescent operators in dimensional regularization: BBGLN and BMU:

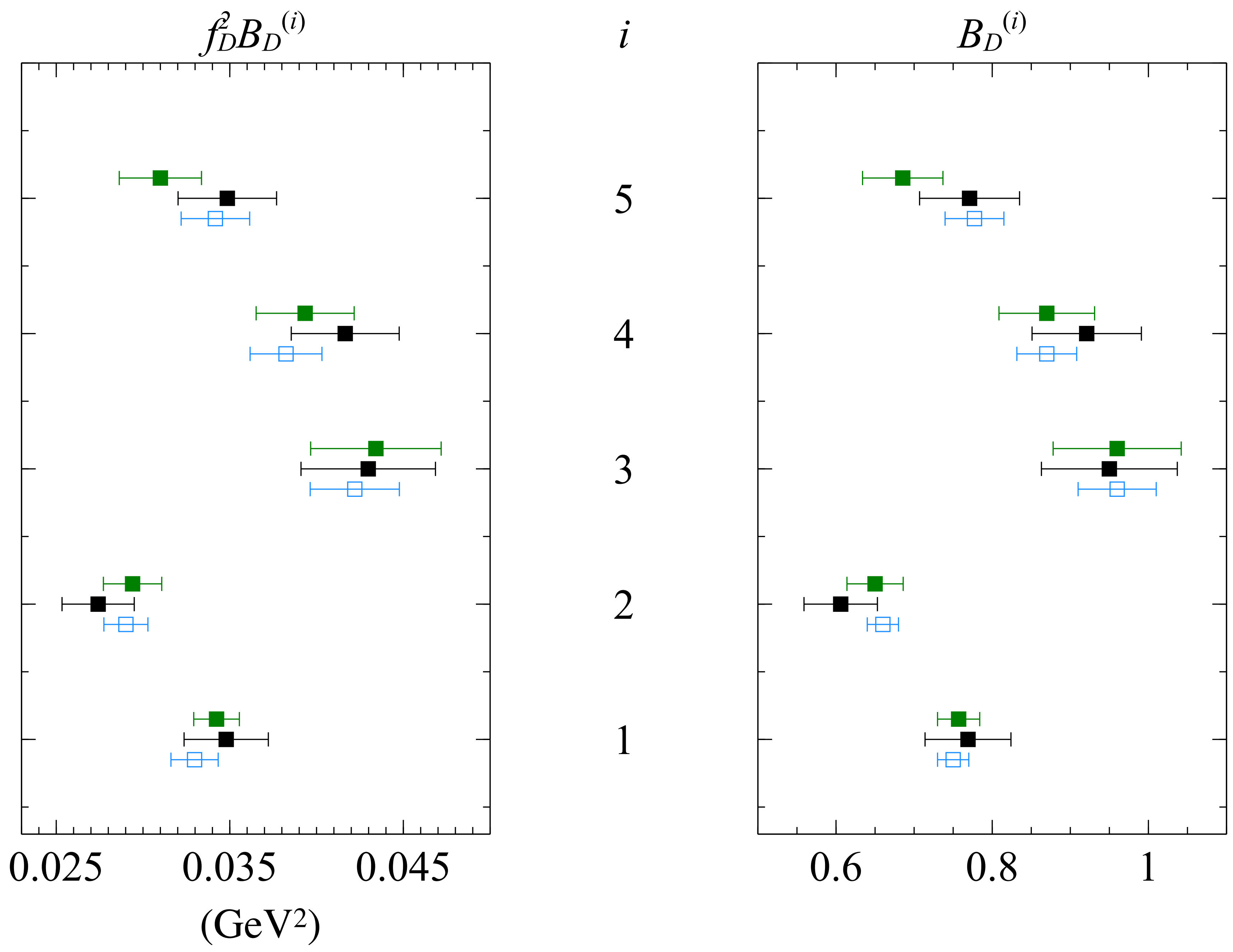
BBGLN	$M_D^{-1}\langle D^0\vert\mathcal{O}_i\vert\bar{D}^0\rangle$ (GeV^3)	$f_{B_q}^2 B_{B_q}$ (GeV^2)	
		$q=d$	$q=s$
$\mathcal{O}_1$	0.0432(29)(9)	0.0342(29)(7)	0.0498(30)(10)
$\mathcal{O}_2$	−0.0833(38)(17)	0.0303(27)(6)	0.0449(29)(9)
$\mathcal{O}_3$	0.0248(16)(5)	0.0399(77)(8)	0.0571(77)(11)
$\mathcal{O}_4$	0.1469(69)(30)	0.0390(28)(8)	0.0534(30)(11)
$\mathcal{O}_5$	0.0554(38)(11)	0.0361(35)(7)	0.0493(36)(10)
μ	3 GeV	m_b	m_b

where second error is an estimate (2%) for the omission of the charm sea.

- Papers (will) also give correlations among these quantities, bag factors, etc.

Comparison with ETM Collaboration

- ETM:
  $n_f = 2{+}1{+}1$ ■
  arXiv:1505.06639
- Fermilab/MILC:
  $n_f = 2{+}1$ ■
- ETM:
  $n_f = 2$ □
  arXiv:1403.7302



Decay Constants & Bag Factors

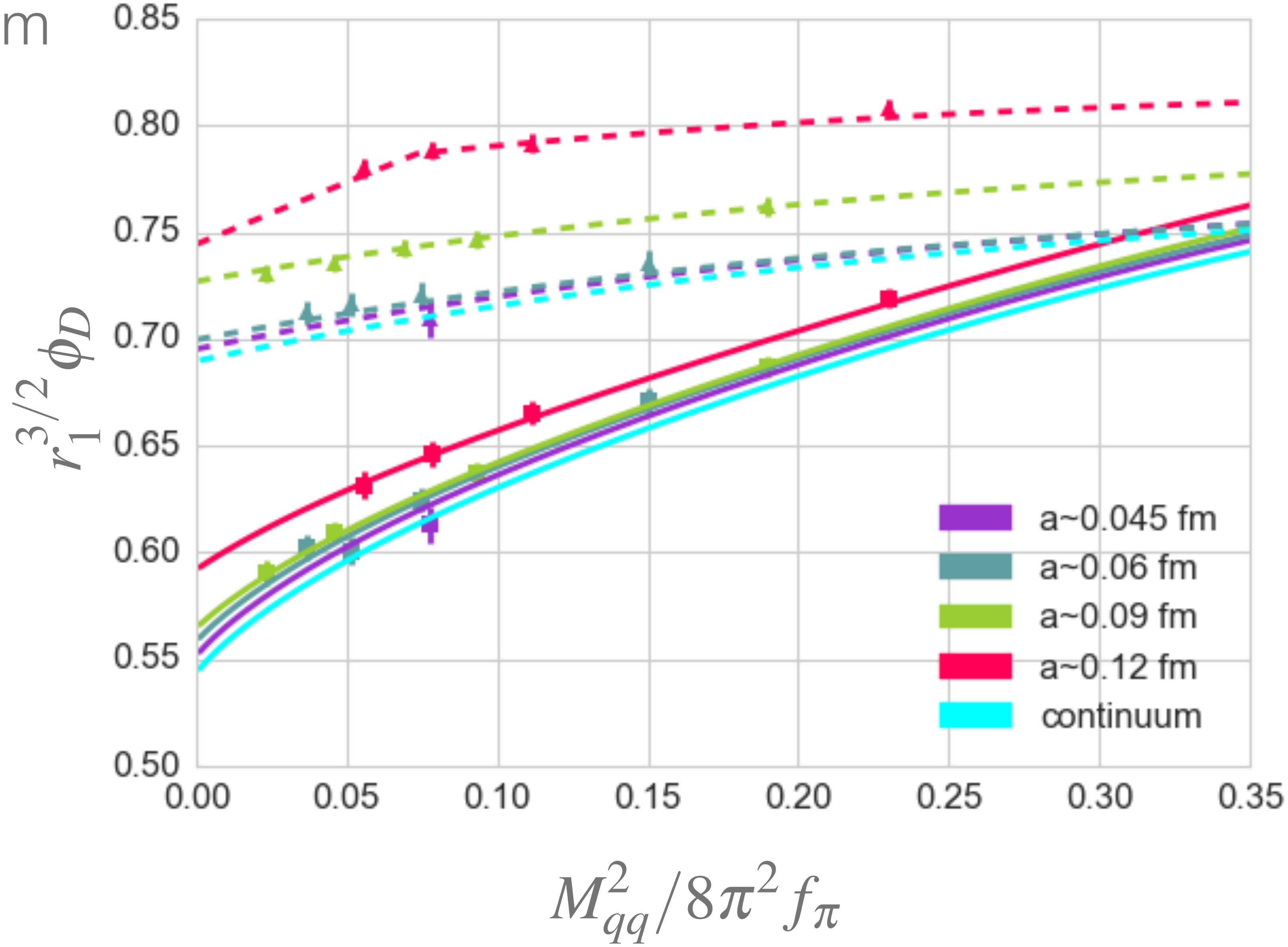
- Fermilab/MILC also calculating decay constants on the same ensembles.

- Status of chiral-continuum extrapolation:

- Can be used to get bag factors

$$B_D \propto \frac{\langle \mathcal{O}_i \rangle}{f_D^2}$$

with cancellations in errors.



Phenomenology

- Matrix elements for D mixing ⇒ constraints on new physics (in progress).

- Oscillation frequencies for $B_{(s)}$ mesons:

$$\Delta M_d^{\mathrm{SM}} = 0.639(50)(36)(5)(13)\ \mathrm{ps}^{-1} \qquad \Delta M_d^{\mathrm{expt}} = (0.5055 \pm 0.0020)\ \mathrm{ps}^{-1}$$
$$\Delta M_s^{\mathrm{SM}} = 19.8(1.1)(1.0)(0.2)(0.4)\ \mathrm{ps}^{-1} \qquad \Delta M_s^{\mathrm{expt}} = (17.757 \pm 0.021)\ \mathrm{ps}^{-1}$$
$$\frac{\Delta M_d^{\mathrm{SM}}}{\Delta M_s^{\mathrm{SM}}} = 0.0323(9)(9)(0)(3)$$

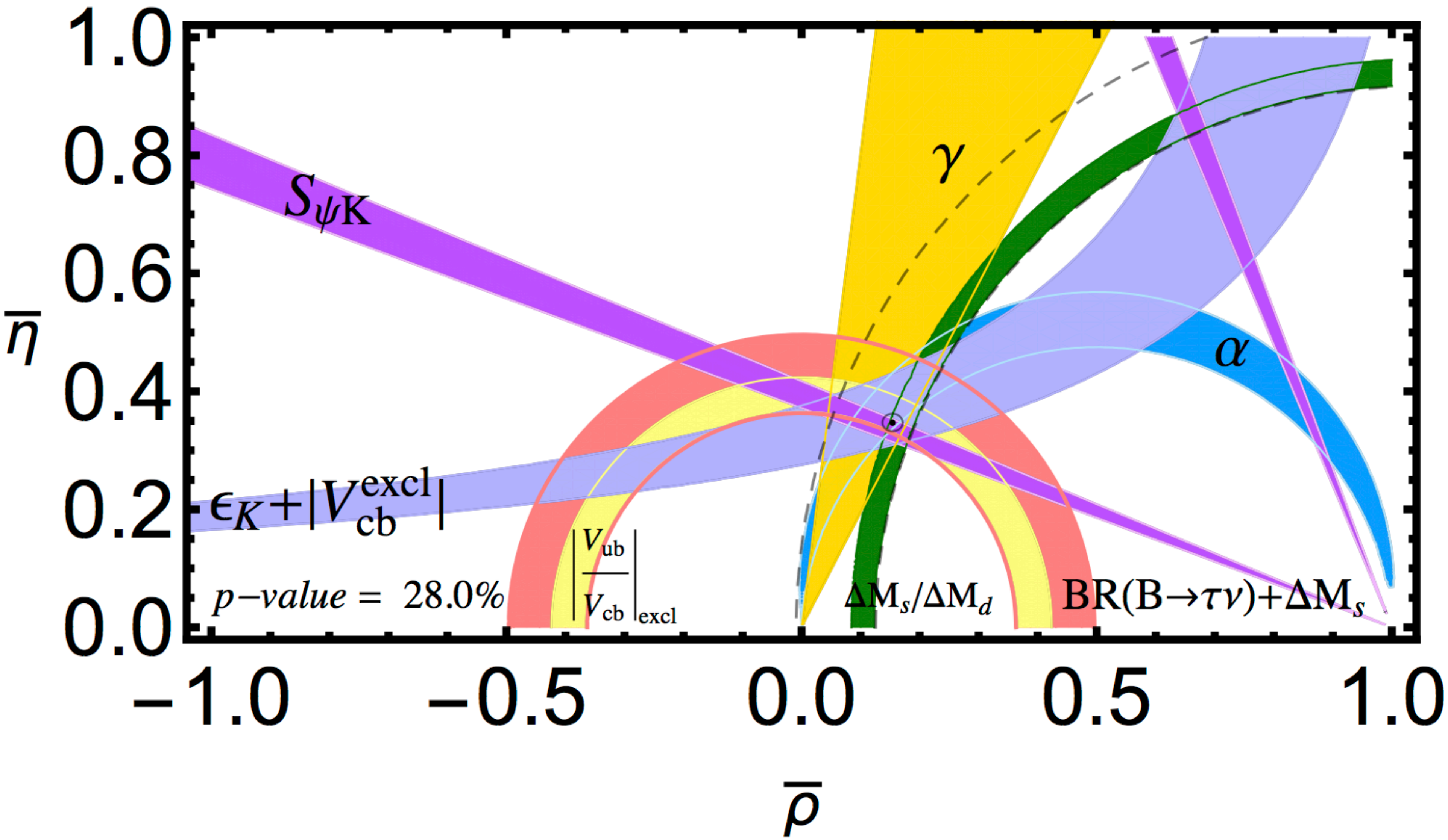
using tree-level inputs for CKM factors in SM formula.

- These amount to discrepancies of 2.1σ, 1.3σ, and 2.9σ, respectively.

- Alternatively, stronger constraint on CKM unitarity triangle.

CKM Unitarity Triangle 2016

- Using Fermilab/MILC results for $B_{(s)}$-mixing [arXiv:1602.03560] and for $|V_{cb}|$ [arXiv:1403.0635, arXiv:1503.07237] & $|V_{ub}|$ [arXiv:1503.07839].



plot by E.Lunghi

Summary & Outlook

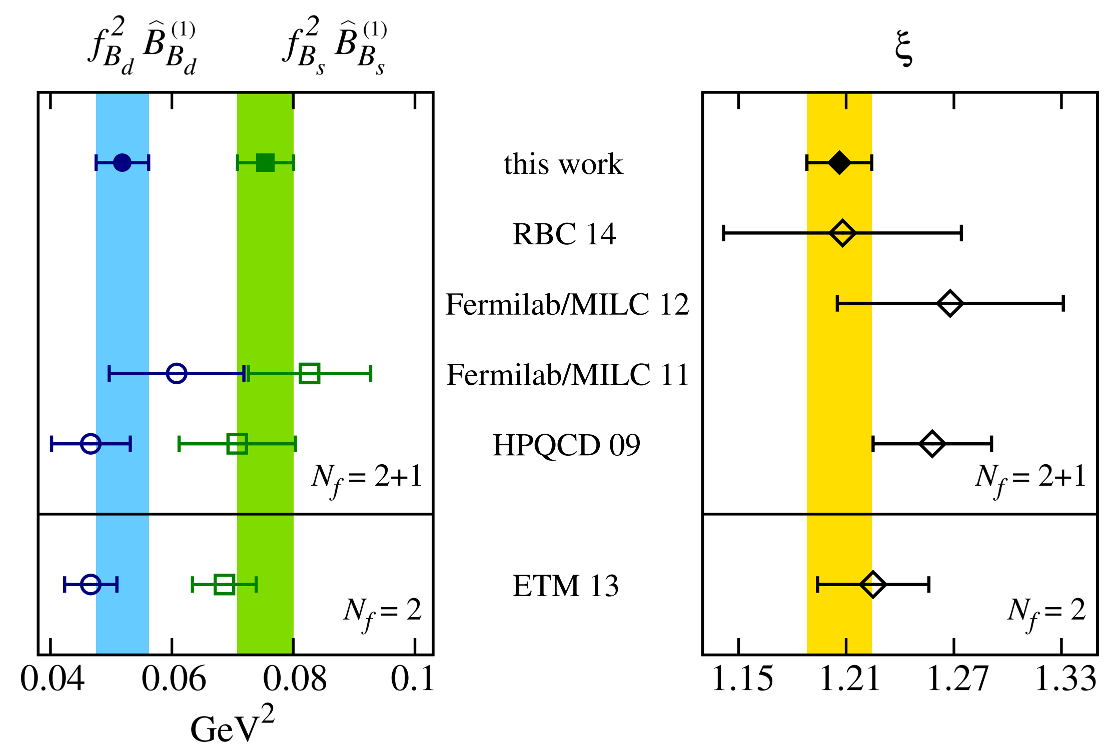
- Fermilab/MILC and ETM errors for D mixing suffice until long-distance parts become available.

- Fermilab/MILC results for $B_{(s)}$ mixing:

    - treat wrong-spin contributions rigorously;

    - have overall error significantly smaller than earlier work;

    - but need sub-% to match $B_{(s)}$ mixing experiments.



Backup

Bag Factors

$$\langle \bar{B}^0 | \mathcal{O}_1 | B^0 \rangle = \tfrac{2}{3} f_B^2 M_B^2 B_B^{(1)}$$

$$\langle \bar{B}^0 | \mathcal{O}_2 | B^0 \rangle = -\tfrac{5}{12} \left(\frac{M_B}{m_b(\mu) + m_q(\mu)} \right)^2 f_B^2 M_B^2 B_B^{(2)}$$

$$\langle \bar{B}^0 | \mathcal{O}_3 | B^0 \rangle = \tfrac{1}{12} \left(\frac{M_B}{m_b(\mu) + m_q(\mu)} \right)^2 f_B^2 M_B^2 B_B^{(3)}$$

$$\langle \bar{B}^0 | \mathcal{O}_4 | B^0 \rangle = \left[\tfrac{1}{12} + \tfrac{1}{2} \left(\frac{M_B}{m_b(\mu) + m_q(\mu)} \right)^2 \right] f_B^2 M_B^2 B_B^{(4)}$$

$$\langle \bar{B}^0 | \mathcal{O}_5 | B^0 \rangle = \left[\tfrac{1}{4} + \tfrac{1}{6} \left(\frac{M_B}{m_b(\mu) + m_q(\mu)} \right)^2 \right] f_B^2 M_B^2 B_B^{(5)}$$