$D$-Meson Mixing in 2+1 Lattice QCD

and Related Topics

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$B(s)$ mixing in Phys. Rev. D 93, 113016 (2016) [arXiv:1602.03560 [hep-lat]]
Neutral-Meson Mixing

- In the Standard Model, neutral mesons can oscillate into their antiparticles:

- In extensions of the SM, other particles could appear in the boxes;
  - could appear at the tree level: flavor-changing neutral current.

- Observed in the lab for all neutral mesons: $K^0, D^0, B^0, B_s$. 
One $\Delta F = 2$ or Two $\Delta F = 1$ Interactions

see, e.g., Artuso, Meadows, Petrov review

- Mixing originates in two kinds of processes:

\[
(M_{12} - \frac{i}{2} \Gamma_{12}) \propto \langle D^0 | \mathcal{L}^{\Delta C=2} | \bar{D}^0 \rangle + \sum_n \frac{\langle D^0 | \mathcal{L}^{\Delta C=1} | n \rangle \langle n | \mathcal{L}^{\Delta C=1} | \bar{D}^0 \rangle}{M_D - E_n + i\varepsilon}
\]

where the $\Delta F = 1$ interactions can be separated by hadronic distances.

- Second term is very difficult to estimate.

- With $D$ mesons (unlike $K, B, B_s$) it is also not negligible: $d, s, b$ in loop.

- Even so, some BSMs modify first term only [e.g., arXiv:0903.2830].
Effective Hamiltonian

- After integrating out heavy particles:

\[ \mathcal{L} = \mathcal{L}_{\text{kin}}[\ell, q, \gamma, g] + \sum_i \mathcal{C}_i(\alpha, \alpha_s, G_F, \sin^2 \theta, m_\ell, m_q, V; \text{NP}) \mathcal{L}_i[\ell, q, \gamma, g] \]

- For \( \Delta F = 2 \) processes, discrete symmetries and Fierz rearrangement reduces the list of operators to \( 8 = 5 + 3 \):

\[
\mathcal{O}_1 = \bar{c} \gamma^\mu Lu \bar{c} \gamma^\mu Lu \\
\mathcal{O}_2 = \bar{c} Lu \bar{c} Lu \\
\mathcal{O}_3 = \bar{c}^\alpha L u^\beta \bar{c}^\beta L u^\alpha \\
\mathcal{O}_4 = \bar{c} Lu \bar{c} Ru \\
\mathcal{O}_5 = \bar{c}^\alpha Lu^\beta \bar{c}^\beta Ru^\alpha c
\]

\[ \tilde{\mathcal{O}}_1 = \bar{c} \gamma^\mu Ru \bar{c} \gamma^\mu Ru \\
\tilde{\mathcal{O}}_2 = \bar{c} Ru \bar{c} Ru \\
\tilde{\mathcal{O}}_3 = \bar{c}^\alpha Ru^\beta \bar{c}^\beta Ru^\alpha \]

By parity in QCD:

\[ \langle D^0 | \tilde{\mathcal{O}}_i | \bar{D}^0 \rangle = \langle D^0 | \mathcal{O}_i | \bar{D}^0 \rangle \]
Fermilab/MILC $D$ vs $B_{(s)}$ Mixing

- Our (ongoing) $D$-meson and (published) $B_{(s)}$-meson analyses have:
  - same ensembles, same light valence masses;
  - same treatment of chiral perturbation theory;
  - same mostly nonperturbative matching to continuum QCD.

- Some differences:
  - ranges for correlator fits—better signal-to-noise for $D$;
  - quote renormalized matrix elements at 3 GeV for $D$ but $m_b$ for $B_{(s)}$. 
Lattice Operators

• Staggered light quarks $\chi_u$ and clover (a la Fermilab) heavy quarks $\Psi_c$:

$$O_1(x) = \bar{\Psi}_c(x) \gamma^\mu L\Omega(x) \chi_u(x) \bar{\Psi}_c(x) \gamma^\mu L\Omega(x) \chi_{\bar{u}}(x)$$
$$O_2(x) = \bar{\Psi}_c(x) L\Omega(x) \chi_u(x) \bar{\Psi}_c(x) L\Omega(x) \chi_{\bar{u}}(x)$$
$$O_3(x) = \bar{\Psi}_c(x)^\alpha L\Omega(x) \chi_{\bar{u}}(x)^\beta \bar{\Psi}_c(x)^\beta L\Omega(x) \chi_{\bar{u}}(x)^\alpha$$
$$O_4(x) = \bar{\Psi}_c(x) L\Omega(x) \chi_u(x) \bar{\Psi}_c(x) R\Omega(x) \chi_{\bar{u}}(x)$$
$$O_5(x) = \bar{\Psi}_c(x)^\alpha L\Omega(x) \chi_{\bar{u}}(x)^\beta \bar{\Psi}_c(x)^\beta R\Omega(x) \chi_{\bar{u}}(x)^\alpha$$

• Three-point correlators contain the desired terms, opposite-parity terms (as usual with staggered), and “wrong-spin” contributions.

• Undesired parts removed in correlator fits and chiral-continuum extrap’n.
asqtad 2+1 Ensembles from MILC

- Partially quenched data on 600–2200 gauge fields with (sea)

\[
177 \text{ MeV} \leq M_\pi \leq 555 \text{ MeV} \\
257 \text{ MeV} \leq M_{\pi}^{\text{rms}} \leq 670 \text{ MeV}
\]
Two-Point Functions: Priors and Posteriors

\[ B_q \]
\[ m_q = \frac{m_s}{10} \]
0.06 \text{ fm}

\[ D_q \]
\[ m_q = \frac{m_s}{5} \]
0.12 \text{ fm}
Three-Point Functions: S-to-N & Fit Regions

\[ B_q, \ m_q = m_s/10, \ 0.09 \ \text{fm}, \ O_2 \]

\[ D_q, \ m_q = m_s/5, \ 0.12 \ \text{fm}, \ O_4 \]
Matching and Renormalization

• Mostly nonperturbative (mNPR):
  \[ \tilde{O}_i = Z_{V_{cc}^4} Z_{V_{uu}^4} \rho_{ij} O_j = \mathcal{O}_i + O(\alpha_s a, a^2) \]
  where the nonperturbative \( Z \)'s remove wave-function factors, all tadpoles, and some vertex corrections.

• Remaining factor \( \rho_{ij} \) obtained at one loop in two independent calculations.

• Two-loop corrections incorporated into chiral-continuum fit.

• Checks by changing mNPR to tadpole-improved PT: \( Z_{ij} = u_0^2 \tilde{Z}_{ij} \), with \( u_0 \) from plaquette or Landau link.
Chiral-Continuum Extrapolation

\[ F_i = F_i^{\text{logs}} + F_i^{\text{analytic}} + F_i^{\text{HQ disc}} + F_i^{\alpha_s a^2 \text{ gen}} + F_i^\kappa + F_i^{\text{renorm}} \]

- **Nonanalytic terms** from NLO HMrS\(\chi\)PT aka “chiral logs”
- **Heavy-quark discretization effects** (derived in HQET)
- **Fine tune** \(c\)-quark hopping parameter

- **Analytic terms in** \(N^n\text{LO}\ \chi\)PT: base fit \(n = 2\)
- **Gluon & light-quark cutoff effects** a la Symanzik

- **Fit** \(\alpha_s^2 \rho_{ij}^{[2]}\) (alternatively \(\alpha_s^3 \rho_{ij}^{[3]}\))
Wrong-Spin Terms

- As noted above, the 3-point functions with staggered-clover 4-quark operators lead to contributions with the wrong spin.

- Schematically [C. Bernard, arXiv:1303.0435],

\[ F_{i}^{\text{logs}} = \beta_i (1 + \text{normal } \chi \text{ logs}) + \beta_j (\text{wrong-spin } \chi \text{ logs}), \quad j \neq i \]

  leading LEC for this operator \( i \)

  leading LEC for some other operator \( j, \Gamma_j \neq \Gamma_i \)

- Base fit: use single set of \( \beta_i \) for all five matrix elements.

- Alternate fit: fit each matrix element individually, allowing uninformed \( \beta_{j \neq i} \).
$D$ Mixing Operators 1, 2, 3
$D$ Mixing Operators 4, 5

- $B$-meson plots look similar
Stability under Fit Variations

- $f_K$ instead of $f_{\pi}$;
- different renorm’n;
- vary $\chi$PT data, priors;
- vary discretization effects included;
- “dumb” fits.
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Nearly Final (Published) Results for $D$ ($B$) Mesons

- We report continuum results for two choices of evanescent operators in dimensional regularization: BBGLN and BMU:

| BBGLN | $M_D^{-1} \langle D^0| \mathcal{O}_i| \bar{D}^0 \rangle$ (GeV$^3$) | $f_{Bq}^2 B_{Bq}$ (GeV$^2$) |
|-------|-------------------------------------------------|------------------|
|       | $q = d$                                          | $q = s$           |
| $\mathcal{O}_1$ | 0.0432(29)(9)                                   | 0.0342(29)(7)    | 0.0498(30)(10) |
| $\mathcal{O}_2$ | $-0.0833(38)(17)$                               | 0.0303(27)(6)    | 0.0449(29)(9)  |
| $\mathcal{O}_3$ | 0.0248(16)(5)                                   | 0.0399(77)(8)    | 0.0571(77)(11) |
| $\mathcal{O}_4$ | 0.1469(69)(30)                                  | 0.0390(28)(8)    | 0.0534(30)(11) |
| $\mu$    | 0.0554(38)(11)                                  | 0.0361(35)(7)    | 0.0493(36)(10) |

where second error is an estimate (2%) for the omission of the charm sea.

- Papers (will) also give correlations among these quantities, bag factors, etc.
Comparison with ETM Collaboration

- **ETM:**
  \[ n_f = 2+1+1 \]
  [arXiv:1505.06639]

- **Fermilab/MILC:**
  \[ n_f = 2+1 \]

- **ETM:**
  \[ n_f = 2 \]
  [arXiv:1403.7302]
Decay Constants & Bag Factors

- Fermilab/MILC also calculating decay constants on the same ensembles.

- Status of chiral-continuum extrapolation:

- Can be used to get bag factors

\[ B_D \propto \frac{\langle \mathcal{O}_i \rangle}{f_D^2} \]

with cancellations in errors.
Phenomenology

• Matrix elements for $D$ mixing ⇒ constraints on new physics (in progress).

• Oscillation frequencies for $B_{(s)}$ mesons:

$$
\Delta M^{SM}_d = 0.639(50)(36)(5)(13) \text{ ps}^{-1} \quad \Delta M^{expt}_d = (0.5055 \pm 0.0020) \text{ ps}^{-1}
$$
$$
\Delta M^{SM}_s = 19.8(1.1)(1.0)(0.2)(0.4) \text{ ps}^{-1} \quad \Delta M^{expt}_s = (17.757 \pm 0.021) \text{ ps}^{-1}
$$
$$
\frac{\Delta M^{SM}_d}{\Delta M^{SM}_s} = 0.0323(9)(9)(0)(3)
$$

using tree-level inputs for CKM factors in SM formula.

• These amount to discrepancies of $2.1\sigma$, $1.3\sigma$, and $2.9\sigma$, respectively.

• Alternatively, stronger constraint on CKM unitarity triangle.
CKM Unitarity Triangle 2016


plot by E.Lunghi
Summary & Outlook

- Fermilab/MILC and ETM errors for $D$ mixing suffice until long-distance parts become available.

- Fermilab/MILC results for $B_{(s)}$ mixing:
  - treat wrong-spin contributions rigorously;
  - have overall error significantly smaller than earlier work;
  - but need sub-% to match $B_{(s)}$ mixing experiments.

![Graph showing results for $f_{B_d}^2 B_d^{(i)}$ and $f_{B_s}^2 B_s^{(i)}$]
Backup
Bag Factors

\[
\langle \bar{B}^0 | \mathcal{O}_1 | B^0 \rangle = \frac{2}{3} f_B^2 M_B^2 B_B^{(1)} \\
\langle \bar{B}^0 | \mathcal{O}_2 | B^0 \rangle = -\frac{5}{12} \left( \frac{M_B}{m_b(\mu) + m_q(\mu)} \right)^2 f_B^2 M_B^2 B_B^{(2)} \\
\langle \bar{B}^0 | \mathcal{O}_3 | B^0 \rangle = \frac{1}{12} \left( \frac{M_B}{m_b(\mu) + m_q(\mu)} \right)^2 f_B^2 M_B^2 B_B^{(3)} \\
\langle \bar{B}^0 | \mathcal{O}_4 | B^0 \rangle = \left[ \frac{1}{12} + \frac{1}{2} \left( \frac{M_B}{m_b(\mu) + m_q(\mu)} \right)^2 \right] f_B^2 M_B^2 B_B^{(4)} \\
\langle \bar{B}^0 | \mathcal{O}_5 | B^0 \rangle = \left[ \frac{1}{4} + \frac{1}{6} \left( \frac{M_B}{m_b(\mu) + m_q(\mu)} \right)^2 \right] f_B^2 M_B^2 B_B^{(5)}
\]