Calculation of the Nucleon Axial Form Factor Using Staggered Lattice QCD

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July 28, 2016

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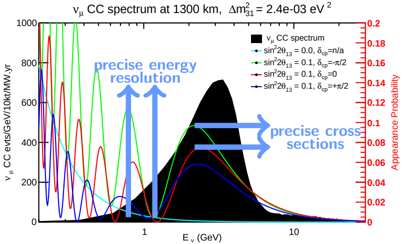
Fermilab Lattice & MILC Collaborations

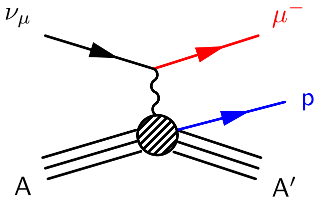
Motivation

- Next generation particle physics projects dedicated to measuring fundamental properties of neutrinos
  - Precision measurement of θ_{23}, discovery of δ_{CP}
- Fermilab host to a number of neutrino experiments:
  - DUNE, MicroBooNE, MINERνA, NOνA, SBND, ...
- To date, most experiments employ near/far detector paradigm
- New experiments will be more sensitive,
  need more precise nuclear/nucleon cross sections

Cross Sections



(Figure from LBNE, 1307.7335 [hep-ex])



Charge Current QE scattering

- Measurements of neutrino parameters require precise knowledge of cross sections
- Nuclear cross sections obtained using nucleon amplitudes as input to nuclear models
- Uncertainty on $F_A(Q^2)$ is primary contribution to systematic errors
  - F_{1V}, F_{2V} known from $e-p$ scattering
  - F_P suppressed by lepton mass in cross sections
- Focus on F_A, other form factors as consistency checks

Dipole Form Factor

Neutrino community typically assumes dipole form factor:

$$F_A(Q^2) = \frac{g_A}{(1 + Q^2/m_A^2)^2}$$

Introduced by Llewellyn-Smith in 1971 as an ansatz

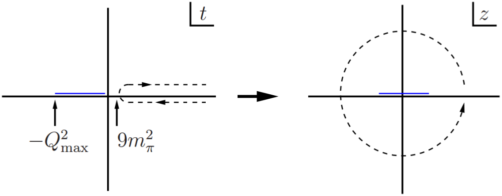
Unmotivated in interesting energy range

$\implies$ Uncontrolled systematics and underestimated uncertainties

z-Expansion

The z-Expansion (1108.0423 [hep-ph]) is a conformal mapping which takes the kinematically allowed region ($t = -Q^2 \leq 0$) to within $|z| < 1$

$$z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}} \qquad F_A(z) = \sum_{n=0}^{\infty} a_n z^n \qquad t_c = 9m_\pi^2$$



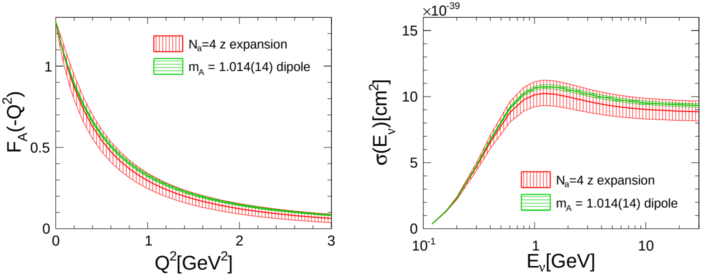
- Model independent: motivated by analyticity arguments from QCD
- Only few parameters needed: unitarity bounds
- Successful in B-meson physics

Deuterium Bubble Chamber - z Expansion

Analysis in Phys. Rev. D 93, 113015 (1603.03048 [hep-ph])
ASM, M. Betancourt, R. Gran, R. Hill

Reanalyzed deuterium bubble chamber data by replacing dipole with z expansion framework

Form factor fit to BNL, ANL, FNAL data sets of $\sim$ 1000 events



$\Longrightarrow$ Unfounded assumption of dipole form factor shape will severely underestimate systematic uncertainties

$\Longrightarrow$ A better determination of the form factor is needed to build sensible nuclear models

$\Longrightarrow$ Lots of room for LQCD to make significant contributions to cross section determinations essential for neutrino physics

Fermilab Lattice/MILC Effort

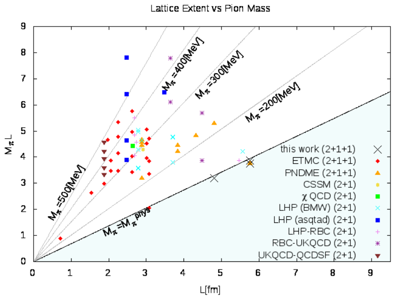
We are calculating the axial form factor $F_A(Q^2)$ using staggered quarks on the MILC HISQ 2+1+1 gauge ensembles

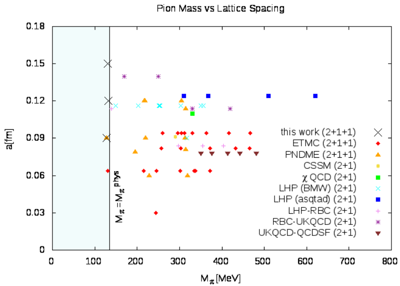
- no explicit chiral symmetry breaking in $m \to 0$ limit
- no exceptional configurations
- physical pion mass at multiple lattice spacings
- large volumes
- exact renormalization
- high-statistics (computationally fast)

Effort is needed to handle:

- Complicated group theory
- Lots of baryon tastes in correlation functions

Gauge Ensembles





Current data:

- $a = 0.15$ fm, $32^3 \times 48$ ensemble only
- $m_{\text{valence}} = m_{\text{physical}}$
- ~ 1000 2-point measurements, ~ 500 3-point

Group Theory

- Irreps of group $(((\mathcal{T}_M \times \mathcal{Q}_8) \rtimes W_3) \times D_4)/\mathbb{Z}_2$
- Fermionic irreps: 8, 8′, 16; Isospin: $\frac{3}{2}$, $\frac{1}{2}$
- Fundamental quark contained in the 8 representation, where 8 operators correspond to the 8 unit cube corners
- Because of symmetrization with taste quantum number, can generate "nucleon-like" states with either choice of isospin

Number of different taste states in lowest-order ($n = 0$) multiplet:

Irrep	$I = \frac{3}{2}$	$I = \frac{1}{2}$
8	$3N + 2\Delta$	$5N + 1\Delta$
8′	$0N + 2\Delta$	$0N + 1\Delta$
16	$1N + 3\Delta$	$3N + 4\Delta$

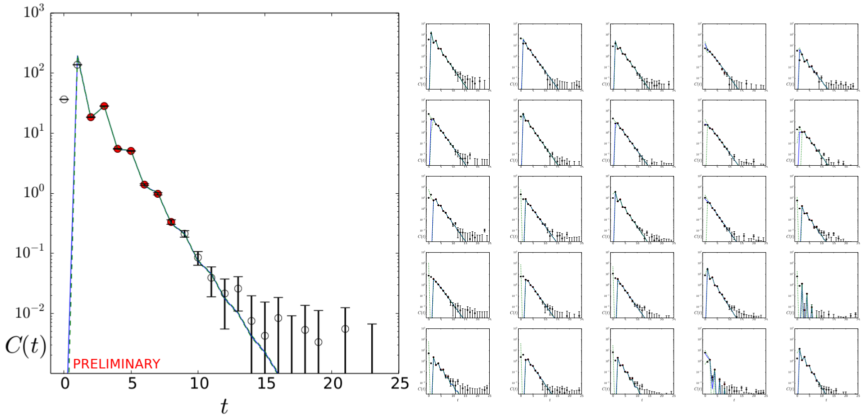
- $I = \frac{3}{2}$: $N_{\text{ops}} = N_{\text{states},n=0}$
- $I = \frac{1}{2}$: $N_{\text{ops}} = 2 \times N_{\text{states},n=0}$

Can construct large operator basis of $N_{\text{ops}} \times N_{\text{ops}}$ correlators for each irrep, isospin;

$\Longrightarrow$ always expect to extract all $n = 0$ states from variational method!

2-Point Functions: Correlators

Nucleon 2-point function $\langle N_i | N_j \rangle$:



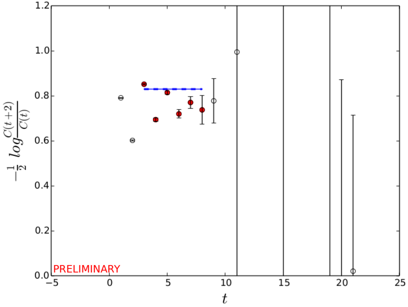
Matrix of correlation functions shown (5 sources $\times$ 5 sinks)

$\Longrightarrow$ Get good t range, at least up to $t = 10$

$\Longrightarrow$ Wrong-parity oscillating states clearly visible

2-Point Functions: Effective Mass

Effective mass as a demonstration



Effective mass prohibitively noisy at $t = 10$

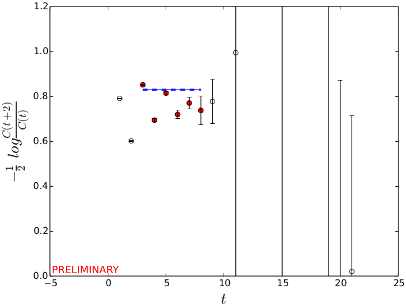
Presence of wrong-parity (oscillating) excited state clearly visible

From effective mass alone, not clear that we could get a reliable spectrum
$\implies$ can we benefit from using correlations?

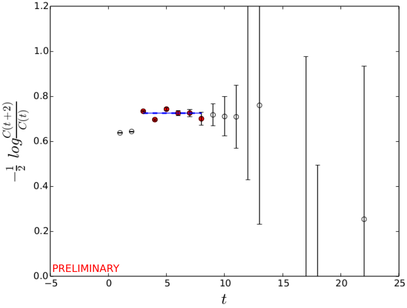
2-Point Functions: S/N Optimization

Optimize a metric related to the signal to noise by varying v, w to visualize statisical power hidden in correlations:

$$\frac{S^2}{N^2} = \sum_{ii} \sum_{t=t_{min}}^{t_{max}} \frac{[v_i C_{ij}(t) w_j]^2}{\delta [v_i C_{ij}(t) w_j]^2}$$



unoptimized



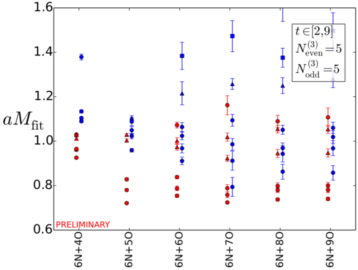
optimized

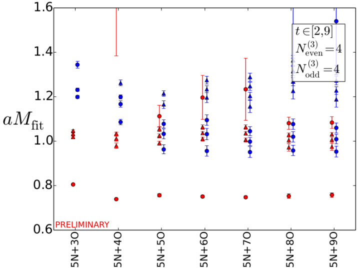
Resulting correlators are cleaner

$\Longrightarrow$ Statistical power hidden in correlations

$\Longrightarrow$ Oscillating states still visible, no choice but to do excited state fits

2-Point Functions: Stability





8 representation $(3N + 2\Delta)$ $\rightarrow$ 16 representation $(1N + 3\Delta)$

Fully correlated fit with Bayesian priors, many excited states

Using fit results from 8 representation as priors to fit to 16 representation improves precision on mass determinations

8′ w/ $0N + 2\Delta \rightarrow \Delta$ mass, taste splitting
8 w/ $3N + 2\Delta \rightarrow N$ mass, $N - \Delta$ mass splitting, better taste splitting
16 w/ $1N + 3\Delta$
$\rightarrow$ “Golden channel”: precise measurement of nucleon properties

3-Point Functions: Normalization of A_μ/Blinding

Calculate form factor:

$$\left.\frac{\langle N|\, Z_A A_\mu \,|N\rangle}{\langle 0|\, Z_A A_\mu \,|\pi^a\rangle}\right|_{q=0} \propto \frac{g_A}{f_\pi}$$

Benefits from statistical cancellation, exact renormalization

Normalize with f_π computed from MILC computation of f_π,
Phys. Rev. D 90, 074509 (1407.3772 [hep-lat])

F_A at nonzero momentum computed as ratio of nuclear matrix elements:

$$\frac{\langle N(0)|\, Z_A A_{\perp\mu}(q) \,|N(q)\rangle}{\langle N(0)|\, Z_A A_\mu(0) \,|N(0)\rangle} \propto \frac{F_A(Q^2)}{g_A}$$

3-Point Functions: Blinding

Value of g_A well-known from neutron beta decay experiments

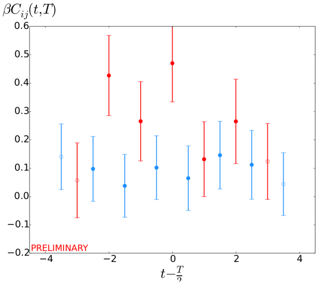
$\implies$ Blinding implemented as a factor multiplying 3-point function

$$\beta \langle N(0)| Z_A A_\mu(q) |N(q)\rangle \sim \beta F_A(Q^2)$$

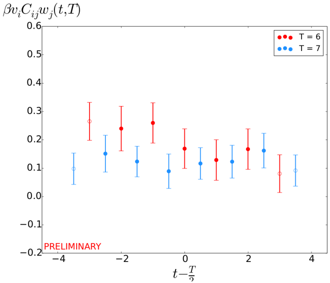
Blinding known only to few members of collaboration, not to me

3-Point Functions: First Look

$$\text{optimize:} \quad \frac{S^2}{N^2} = \sum_{ij} \sum_{\tau=1}^{t-1} \frac{\left[v_i C_{ij}(\tau, t) w_j \right]^2}{\delta \left[v_i C_{ij}(\tau, t) w_j \right]^2}$$



raw correlator →



optimized correlator

- raw 3-point functions have no visible plateau
- prominent oscillating states
- errors improved by S/N optimization $\implies$ strong correlations
- many currents available (local, point split)

Future Prospects

- $6\times$ propagators for 32×48 lattice computed, computation of tie-ups in progress
- Still have not included pion 2-point function in ratio, expect statistical cancellation to improve results
- Can disentangle more excited states than implied by variational method alone
- Have USQCD resources for inversions on $a = 0.12, 0.09\mathrm{fm}$ ensembles
- Will compute full error budget for form factor

Conclusions

- Axial form factor is essential for the success of future neutrino oscillation experiments
- Staggered baryons have the potential to weigh in on g_A puzzle
- Preliminary data for 2- and 3-point functions have been calculated
- Spectrum calculation for staggered baryons is feasible
- We are optimistic that our g_A calculation will be competitive with other collaborations

Thank you for your attention!