Calculation of the Nucleon Axial Form Factor Using Staggered Lattice QCD

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People

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Also:

Fermilab Lattice & MILC Collaborations
Motivation

- Next generation particle physics projects dedicated to measuring fundamental properties of neutrinos
  - Precision measurement of $\theta_{23}$, discovery of $\delta_{CP}$
- Fermilab host to a number of neutrino experiments:
  - DUNE, MicroBooNE, MINER$\nu$A, NO$\nu$A, SBND, . . .
- To date, most experiments employ near/far detector paradigm
- New experiments will be more sensitive, need more precise nuclear/nucleon cross sections
Cross Sections

(Figure from LBNE, 1307.7335 [hep-ex])  Charge Current QE scattering

- Measurements of neutrino parameters require precise knowledge of cross sections
- Nuclear cross sections obtained using nucleon amplitudes as input to nuclear models
- Uncertainty on $F_A(Q^2)$ is primary contribution to systematic errors
  - $F_1V, F_2V$ known from $e - p$ scattering
  - $F_P$ suppressed by lepton mass in cross sections
- Focus on $F_A$, other form factors as consistency checks
Dipole Form Factor

Neutrino community typically assumes dipole form factor:

\[ F_A(Q^2) = \frac{g_A}{(1 + Q^2/m_A^2)^2} \]

Introduced by Llewellyn-Smith in 1971 as an ansatz

Unmotivated in interesting energy range

\[ \implies \text{Uncontrolled systematics and underestimated uncertainties} \]
The z-Expansion (1108.0423 [hep-ph]) is a conformal mapping which takes the kinematically allowed region \( t = -Q^2 \leq 0 \) to within \( |z| < 1 \)

\[
z(t; t_0, t_c) = \frac{\sqrt{t_c - t} - \sqrt{t_c - t_0}}{\sqrt{t_c - t} + \sqrt{t_c - t_0}}
\]

\[
F_A(z) = \sum_{n=0}^{\infty} a_n z^n \quad t_c = 9m_{\pi}^2
\]

- Model independent: motivated by analyticity arguments from QCD
- Only few parameters needed: unitarity bounds
- Successful in B-meson physics
Deuterium Bubble Chamber - $z$ Expansion

Analysis in Phys. Rev. D 93, 113015 (1603.03048 [hep-ph])

ASM, M. Betancourt, R. Gran, R. Hill

Reanalyzed deuterium bubble chamber data by replacing dipole with $z$ expansion framework

Form factor fit to BNL, ANL, FNAL data sets of $\sim 1000$ events

Unfounded assumption of dipole form factor shape will severely underestimate systematic uncertainties
A better determination of the form factor is needed to build sensible nuclear models.

Lots of room for LQCD to make significant contributions to cross section determinations essential for neutrino physics.
Fermilab Lattice/MILC Effort

We are calculating the axial form factor $F_A(Q^2)$ using staggered quarks on the MILC HISQ 2+1+1 gauge ensembles

- no explicit chiral symmetry breaking in $m \to 0$ limit
- no exceptional configurations
- physical pion mass at multiple lattice spacings
- large volumes
- exact renormalization
- high-statistics (computationally fast)

Effort is needed to handle:

- Complicated group theory
- Lots of baryon tastes in correlation functions
Gauge Ensembles

Current data:

- $a = 0.15\text{ fm}$, $32^3 \times 48$ ensemble only
- $m_{\text{valence}} = m_{\text{physical}}$
- $\sim 1000$ 2-point measurements, $\sim 500$ 3-point
Group Theory

- Irreps of group \(((T_M \times Q_8) \rtimes W_3) \rtimes D_4)/\mathbb{Z}_2\)
- Fermionic irreps: 8, 8', 16; Isospin: \(\frac{3}{2}, \frac{1}{2}\)
- Fundamental quark contained in the 8 representation, where 8 operators correspond to the 8 unit cube corners
- Because of symmetrization with taste quantum number, can generate “nucleon-like” states with either choice of isospin

Number of different taste states in lowest-order \((n = 0)\) multiplet:

<table>
<thead>
<tr>
<th>Irrep</th>
<th>(I = \frac{3}{2})</th>
<th>(I = \frac{1}{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>(3N + 2\Delta)</td>
<td>(5N + 1\Delta)</td>
</tr>
<tr>
<td>8'</td>
<td>(0N + 2\Delta)</td>
<td>(0N + 1\Delta)</td>
</tr>
<tr>
<td>16</td>
<td>(1N + 3\Delta)</td>
<td>(3N + 4\Delta)</td>
</tr>
</tbody>
</table>

- \(I = \frac{3}{2}\): \(N_{\text{ops}} = N_{\text{states},n=0}\)
- \(I = \frac{1}{2}\): \(N_{\text{ops}} = 2 \times N_{\text{states},n=0}\)

Can construct large operator basis of \(N_{\text{ops}} \times N_{\text{ops}}\) correlators for each irrep, isospin;

\[\Rightarrow\text{ always expect to extract all } n = 0 \text{ states from variational method!}\]
Nucleon 2-point function $\langle N_i | N_j \rangle$:

Matrix of correlation functions shown (5 sources $\times$ 5 sinks)

$\Rightarrow$ Get good $t$ range, at least up to $t = 10$

$\Rightarrow$ Wrong-parity oscillating states clearly visible
Effective mass as a demonstration

Effective mass prohibitively noisy at $t = 10$

Presence of wrong-parity (oscillating) excited state clearly visible

From effective mass alone, not clear that we could get a reliable spectrum

$\implies$ can we benefit from using correlations?
2-Point Functions: S/N Optimization

Optimize a metric related to the signal to noise by varying $v$, $w$ to visualize statistical power hidden in correlations:

$$\frac{S^2}{N^2} = \sum_{ii} \sum_{t=t_{\text{min}}}^{t_{\text{max}}} \frac{[v_i C_{ij}(t) w_j]^2}{\delta [v_i C_{ij}(t) w_j]^2}$$

Resulting correlators are cleaner

$\Rightarrow$ Statistical power hidden in correlations

$\Rightarrow$ Oscillating states still visible, no choice but to do excited state fits
2-Point Functions: Stability

8 representation \((3N + 2\Delta)\) \rightarrow 16 representation \((1N + 3\Delta)\)

Fully correlated fit with Bayesian priors, many excited states

Using fit results from 8 representation as priors to fit to 16 representation improves precision on mass determinations

8’ w/ \(0N + 2\Delta\) \rightarrow \(\Delta\) mass, taste splitting
8 w/ \(3N + 2\Delta\) \rightarrow \(N\) mass, \(N - \Delta\) mass splitting, better taste splitting
16 w/ \(1N + 3\Delta\)

\rightarrow “Golden channel”: precise measurement of nucleon properties
3-Point Functions: Normalization of $A_\mu$/Blinding

Calculate form factor:

$$\frac{\langle N | Z_A A_\mu | N \rangle}{\langle 0 | Z_A A_\mu | \pi^a \rangle} \bigg|_{q=0} \propto \frac{g_A}{f_\pi}$$

Benefits from statistical cancellation, exact renormalization

Normalize with $f_\pi$ computed from MILC computation of $f_\pi$, Phys. Rev. D 90, 074509 (1407.3772 [hep-lat])

$F_A$ at nonzero momentum computed as ratio of nuclear matrix elements:

$$\frac{\langle N(0) | Z_A A_{\perp \mu}(q) | N(q) \rangle}{\langle N(0) | Z_A A_\mu(0) | N(0) \rangle} \propto \frac{F_A(Q^2)}{g_A}$$
3-Point Functions: Blinding

Value of $g_A$ well-known from neutron beta decay experiments

⇒ Blinding implemented as a factor multiplying 3-point function

$$\beta \langle N(0) | Z_A A_{\mu}(q) | N(q) \rangle \sim \beta F_A(Q^2)$$

Blinding known only to few members of collaboration, not to me
3-Point Functions: First Look

Optimize:

\[
\frac{S^2}{N^2} = \sum_{ij} \sum_{\tau=1}^{t-1} \left[ \frac{\nu \cdot C_{ij}(\tau, t) \cdot w_j}{\delta} \right]^2
\]

- raw correlator \( \rightarrow \) optimized correlator

- raw 3-point functions have no visible plateau
- prominent oscillating states
- errors improved by S/N optimization \( \Rightarrow \) strong correlations
- many currents available (local, point split)
Future Prospects

- 6× propagators for 32 × 48 lattice computed, computation of tie-ups in progress
- Still have not included pion 2-point function in ratio, expect statistical cancellation to improve results
- Can disentangle more excited states than implied by variational method alone
- Have USQCD resources for inversions on \( a = 0.12, 0.09 \text{fm} \) ensembles
- Will compute full error budget for form factor
Conclusions

- Axial form factor is essential for the success of future neutrino oscillation experiments
- Staggered baryons have the potential to weigh in on $g_A$ puzzle
- Preliminary data for 2- and 3-point functions have been calculated
- Spectrum calculation for staggered baryons is feasible
- We are optimistic that our $g_A$ calculation will be competitive with other collaborations

Thank you for your attention!