Renormalisation of the scalar energy-momentum tensor with the Wilson flow

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In collaboration with
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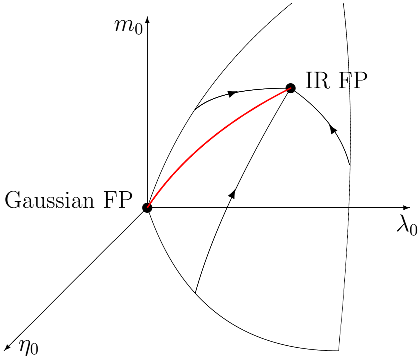
Lattice 2016 - Southampton

Motivation

- EMT relates to β-function

  $\langle \int d^D x \, T_{\mu\mu} \, \phi(x_1) ... \phi(x_n) \rangle$
  $$= -\left(\sum_k \beta_k \frac{\partial}{\partial g_k} + n(\gamma_\phi + d_\phi) \right) \langle \phi(x_1) ... \phi(x_n) \rangle$$

- ϕ^4-theory in 3D, $m_0^2 < 0$: toy model for theories with IR fixed point



Energy-momentum tensor and Ward identity

- Euclidean action

$$S = \int d^D x \, \left(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} m_0^2 \phi^2 + \frac{\lambda_0}{4!} \phi^4 \right)$$

- Energy-momentum tensor

$$T_{\mu\rho}(x) = \partial_\mu \phi \partial_\rho \phi - \delta_{\mu\rho} \left(\frac{1}{2} \sum_\sigma \partial_\sigma \phi \partial_\sigma \phi + \frac{1}{2} m_0^2 \phi^2 + \frac{\lambda_0}{4!} \phi^4 \right)$$

- Translation Ward identity

$$\langle \, \delta_{x,\rho} P \, \rangle = - \langle \, P \, \partial_\mu \, T_{\mu\rho}(x) \, \rangle$$

- Local operator of translation

$$\delta_{x,\rho} P = \frac{\delta P}{\delta \phi(x)} \, \partial_\rho \phi(x)$$

Translation Ward identity on the lattice

- Lattice action

$$\hat{S} = a^D \sum_n \left(\frac{1}{2}(\hat{\partial}_\mu \phi)^2 + \frac{1}{2} m_0^2 \phi^2 + \frac{\lambda_0}{4!}\phi^4 \right)$$

- Lattice regularisation breaks translation symmetry explicitly

$$\langle\, \hat{\delta}_{x,\rho} \hat{P} \,\rangle = -\langle\, \hat{P} \left(\hat{\partial}_\mu \hat{T}_{\mu\rho} + \hat{R}_\rho \right) \rangle$$

- Renormalised lattice TWI

$$\langle\, Z_\delta \; \hat{\delta}_{x,\rho} \hat{P} \,\rangle = -\langle\, \hat{P} \left(\hat{\partial}_\mu [\hat{T}_{\mu\rho}] + \hat{\bar{R}}_\rho \right) \rangle$$

- Renormalised $\hat{T}_{\mu\rho}(x)$

$$[\hat{T}_{\mu\rho}(x)] = \sum_i c_i \left\{ \hat{T}^{(i)}_{\mu\rho} - \langle \hat{T}^{(i)}_{\mu\rho} \rangle \right\}$$

Renormalisation of the EMT

$$\hat{T}_{\mu\rho}(x) = \hat{\partial}_\mu\phi\hat{\partial}_\rho\phi - \delta_{\mu\rho}\left(\frac{m_0^2}{2}\phi^2 + \frac{1}{2}\sum_\sigma \hat{\partial}_\sigma\phi\hat{\partial}_\sigma\phi + \frac{\lambda_0}{4!}\phi^4\right)$$

- Possible mixing: $D \leq 3$, Lorentz, $\phi \to -\phi$, $x \to -x$

$$\hat{\partial}_\mu\phi\hat{\partial}_\rho\phi,\ \ \phi\hat{\partial}_\mu\hat{\partial}_\rho\phi,$$

$$\delta_{\mu\rho}\left(\phi^2,\ \ \phi^4,\ \ \phi^6,\ \ \sum_\sigma \hat{\partial}_\sigma\phi\hat{\partial}_\sigma\phi,\ \ \sum_\sigma \phi\hat{\partial}_\sigma\hat{\partial}_\sigma\phi,\ \ \hat{\partial}_\mu\phi\hat{\partial}_\mu\phi,\ \ \phi\hat{\partial}_\mu\hat{\partial}_\mu\phi\right)$$

- Perturbative analysis shows that divergencies are $\propto \phi^2$

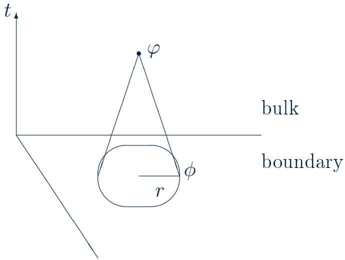
$$[\hat{T}_{\mu\rho}] = \frac{c}{2}\hat{\partial}_\mu\phi\hat{\partial}_\rho\phi + \delta_{\mu\rho}\left(\frac{c_2}{2}\phi^2 + \frac{c'}{2}\sum_\sigma \hat{\partial}_\sigma\phi\hat{\partial}_\sigma\phi + \frac{c_4}{4!}\phi^4\right)$$

$$c_2 = -m_0^2 - 0.0215\frac{\lambda_0}{a}$$

Wilson flow - gradient flow on the lattice

- Flow equation [*Monahan,Orginos 2014*]

$$\partial_t \varphi(t,x) = \hat{\partial}^2 \varphi(t,x), \qquad \varphi(t,x)|_{t=0} = \phi(x)$$



- Smoothing effect, radius $r = \sqrt{2Dt}$

Renormalisation of the EMT using the Wilson flow

- Renormalised TWI

$$\langle\, Z_\delta \; \hat{\delta}_{x,\rho} \hat{P} \,\rangle = -\langle\, \hat{P} \left(\hat{\partial}_\mu [\, \hat{T}_{\mu\rho}] + \hat{\hat{R}}_\rho \right) \rangle$$

- Renormalisation condition [*Del Debbio,Patella,Rago 2013*]
  - Choose probe $\hat{P}_t$: function of fields at $t > 0$, then:
  - Coefficients c_i can be tuned such that EMT is finite
  - $\hat{\hat{R}}_\rho \to 0$

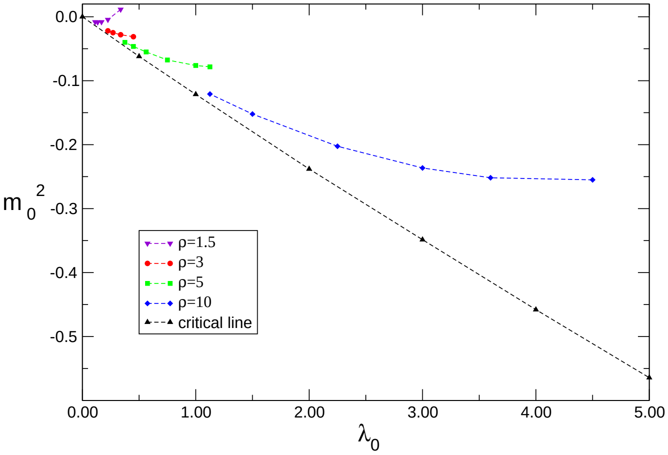
$$Z_\delta \,\langle\, \hat{\delta}_{x,\rho} \hat{P}_t \,\rangle = -\sum_i c_i \,\langle\, \hat{P}_t \; \hat{\partial}_\mu \, \hat{T}^{(i)}_{\mu\rho}(x) \,\rangle$$

- Determine Z_δ separately, $Z_\delta = 1$
- System of (at least) 4 equations with 4 different operators $P_t^{(k)}$

$$V^{(k)} = -\sum_i c_i \; M^{(k,i)}$$

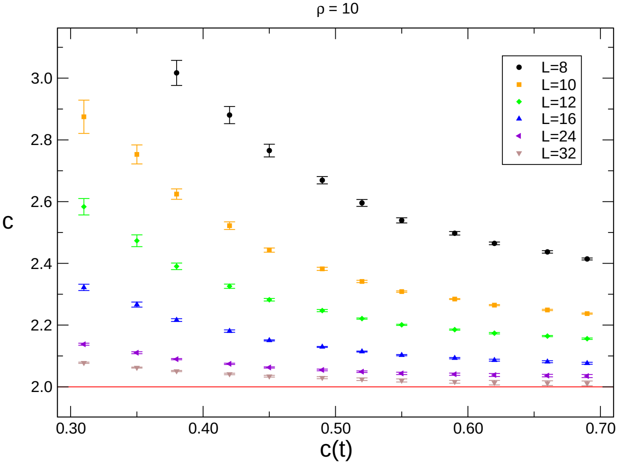
Phase diagram

- Interested in staying close to critical line, $m_0^2 < 0$
- Lines of constant physics defined by $\rho = \lambda_0 / m_R$



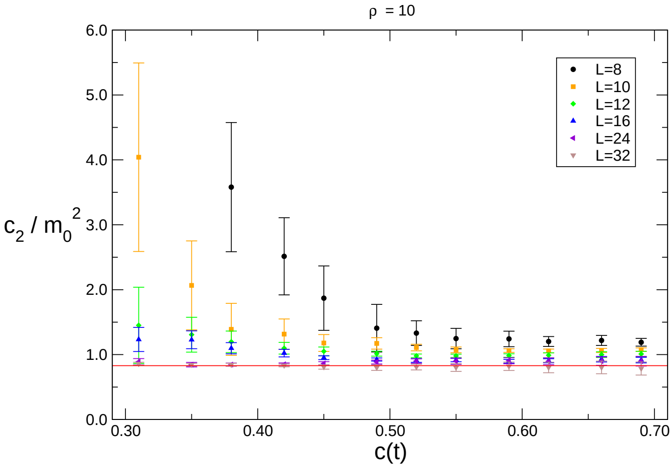
Results - c

- $c\,\frac{1}{2}\hat{\partial}_\mu\phi\hat{\partial}_\rho\phi$, expected: $c = 2$
- $c(t) = \sqrt{6t}/L$



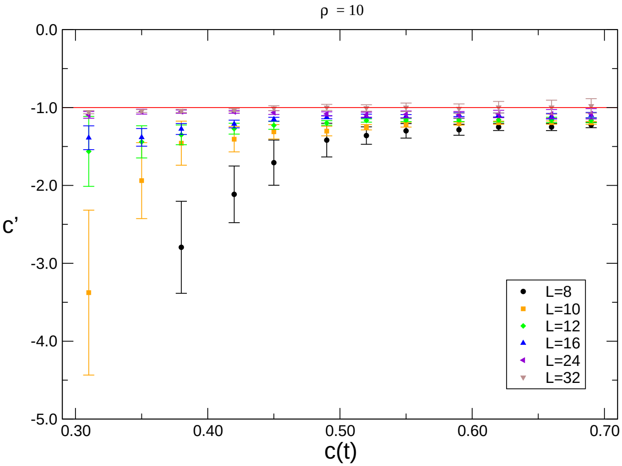
Results - c_2

- $c_2 \, \frac{1}{2}\phi^2\delta_{\mu\rho}$
- Expected from PT: $c_2/m_0^2 = 0.83$



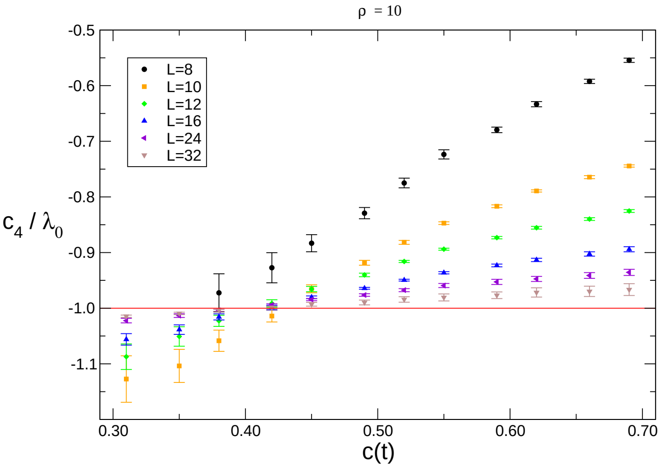
Results - c'

- $c' \, \frac{1}{2} \sum_\sigma \hat{\partial}_\sigma \phi \hat{\partial}_\sigma \phi \delta_{\mu\rho}$
- Expected: $c' = -1$



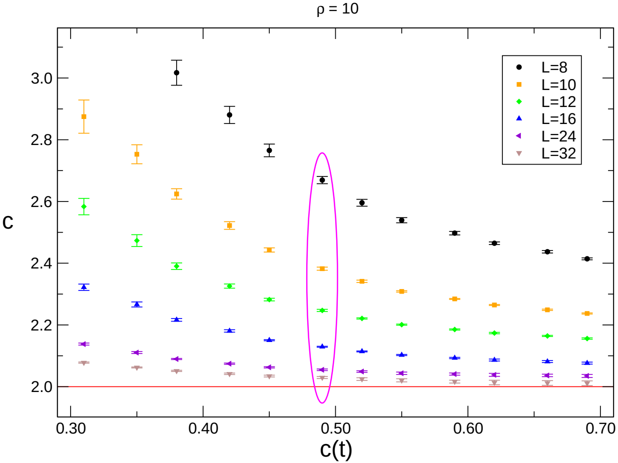
Results - c_4

- $c_4 \, \frac{1}{4!}\phi^4\delta_{\mu\rho}$
- Expected: $c_4/\lambda_0 = -1$



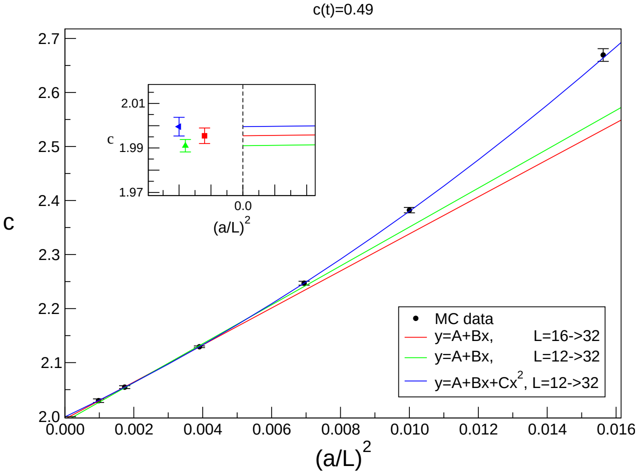
Continuum limit EMT

- $c\,\frac{1}{2}\hat{\partial}_\mu\phi\hat{\partial}_\rho\phi$



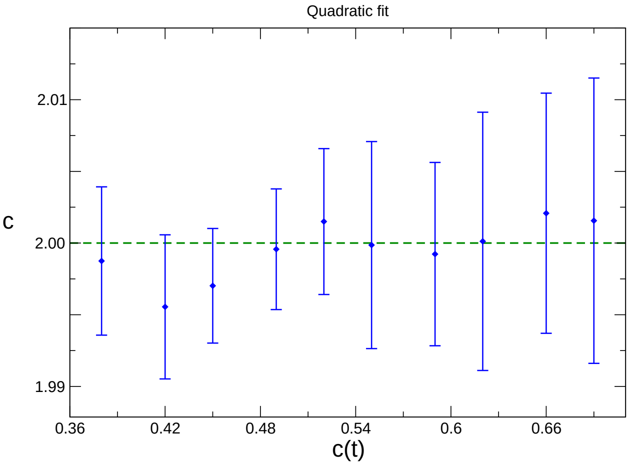
Continuum limit EMT

- c at $c(t) = 0.49$ along line of constant physics



Continuum limit EMT

- Extrapolated c for continuum limit of EMT at different t



Summary

- We studied the non-perturbative renormalisation of EMT
- The Wilson flow provides a new way to implement Ward identities free from contact terms
- We are able to define a properly renormalised EMT on lattice
  - Find coefficients of renormalised EMT at finite a
  - Each t gives a different definition of the EMT
  - Reproduce correct continuum limit of EMT
- Next step: study scaling behaviour in IR

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Thank you!