D meson semileptonic form factors with HISQ valence and sea quarks

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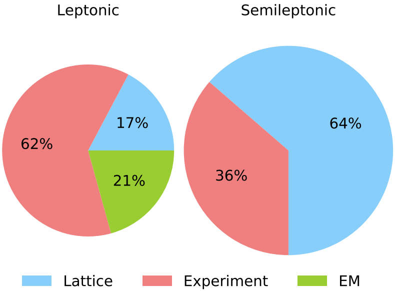
Motivation

GOAL: Calculate $D \to K(\pi)\ell\nu$ semileptonic form factors $f_+(q^2 = 0)$ for the purpose of determining the CKM matrix elements $|V_{cs(d)}|$:

- Requires a combination of lattice and experimental results.
- In leptonic decays lattice errors are smaller than experimental ones.
- In semileptonic decays, it is the other way around, as shown.



Comparison of contributions to $|V_{cs}|$ errors from the leading leptonic decay[1] and semileptonic decay[2] determinations. Radius is proportional to total error.

[1] Fermilab/MILC PRD (2014) [arXiv:1407.3772v2], Rosner, Stone and Van de Water, [arXiv:1509.02220]
[2] J. Koponen, *et. al* (HPQCD) [arXiv:1305.1462v1], HFAG [arXiv:1412.7515v1]

Calculation Method

Vector form factor $f_+(q^2)$ defined via vector current $V^\mu = \bar{q}\gamma^\mu c$,

$$\langle K|V^\mu|D\rangle = f_+(q^2)\left[p_D^\mu + p_K^\mu - \frac{M_D^2 - M_K^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_D^2 - M_K^2}{q^2}q^\mu. \qquad (1)$$

We instead calculate the scalar current $S = \bar{q}c$,

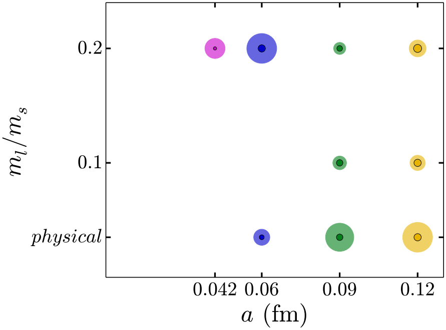
$$\langle K|S|D\rangle = \frac{M_D^2 - M_K^2}{m_c - m_s} f_0(q^2), \qquad (2)$$

- With staggered quarks the local scalar current yields an absolutely normalized f_0.
- Kinematic constraint requires that $f_+(0) = f_0(0)$.
- This approach of using the scalar current was introduced by HPQCD[1].

[1] HPQCD Phys.Rev.D82, 114506, (2010) [arXiv:1008.4562v2]

Simulation Details

- MILC $2+1+1$ flavor HISQ ensembles.
- Light, strange and charm valence quarks also use the HISQ action.
- Inner symbols radius indicates N_{conf}.
- Outer symbol radius indicates $N_{conf} \times N_{tsrc}$ which is at least 3000 for each ensemble.
- $M_\pi L > 3.5$ for all ensembles.
- The 0.06 fm, 0.2 m_s ensemble is new since last year.



Correlators

We employ twisted boundary conditions[1,2,3] in order to reach $q^2 = 0$ kinematics.

- Required momentum determined via $q^2 = M_K^2 + M_D^2 - 2E_K M_D$ and the dispersion relation $E_K = \sqrt{M_K^2 + p^2}$.
- Momentum up to integer multiples of $2\pi/L$ come from the usual Fourier transform.
- The rest comes from a twist of θ in each spatial direction giving $\vec{p} = \theta\frac{\pi}{L}(1,1,1)$.
- For $D \to K$ and $D \to \pi$ the momenta required to have $q^2 = 0$ are large.
- On the lattice the dispersion relation is not exact, with violations expected to scale with $\alpha_s^2(ap)^2$.

[1] P. F. Bedaque, J.-W. Chen, Phys. Lett. B 616, 208-214 (2005) [hep-lat/0412023]

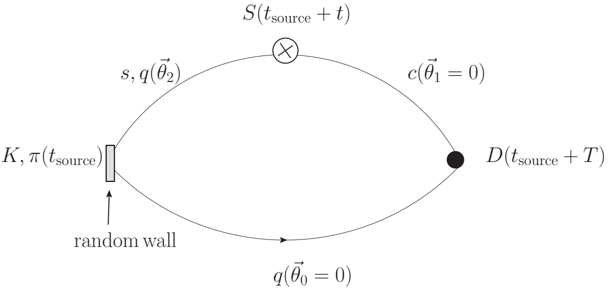
[2] C. T. Sachrajda and G. Villadoro, Phys. Lett. B 609, 73 (2005) [hep-lat/0411033]

[3] A. Bazavov *et al.*, Phys. Rev. Lett. **112**, no. 11, 112001 (2014) [arXiv:1312.1228]

Correlators

The diagram below describes the structure of our three-point correlators.

- Non-zero twist is given only to the daughter quark ($\vec{\theta_2}$).
- Putting some or all of the momentum on the D meson is possible but was found to lead to much larger statistical errors.
- 5 different external source times (T) for each three-point correlator.
- Also calculate two-point kaons and pions with and without momentum and D mesons with no momentum only.

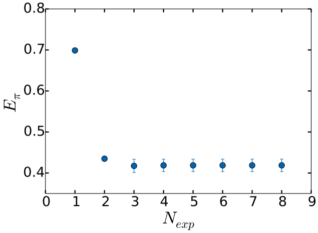


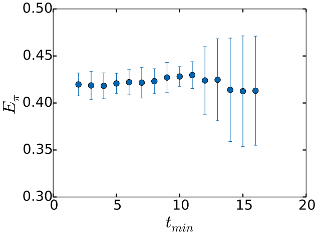
Correlator Fitting procedure

Two-point correlators are fit to a multi-state fit function:

$$C_P(t) = \sum_j^{N_{exp}} (a_j^P)^2 (e^{-E_j^P t} + e^{-E_j^P (N_t - t)}) - \sum_k^{N_{exp}} (-1)^t (b_k^P)^2 (e^{-E_k'^P t} + e^{-E_k'^P (N_t - t)}) \quad (3)$$

- N_{exp} is increased until the fit result becomes stable.
- Fit t_{min} taken as earliest choice with a good p-value and consistent fit result.
- Fit t_{max} is chosen as late as possible while there is good signal ($< 30\%$ error).
- Bayesian priors with broad widths are used to help fit stability only.





Pion energy values from the 0.09 fm physical quark mass ensemble as a function of the number of states of each parity (left) and as a function of t_{min} (right). p-value is ≈ 1 for every fit shown except $N_{exp} = 1$.

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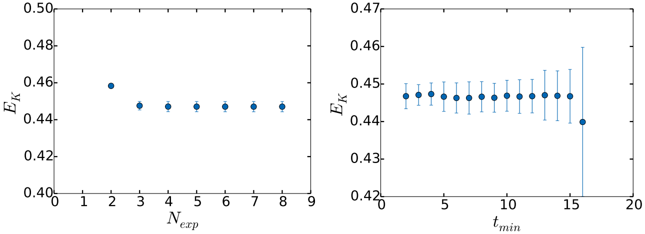
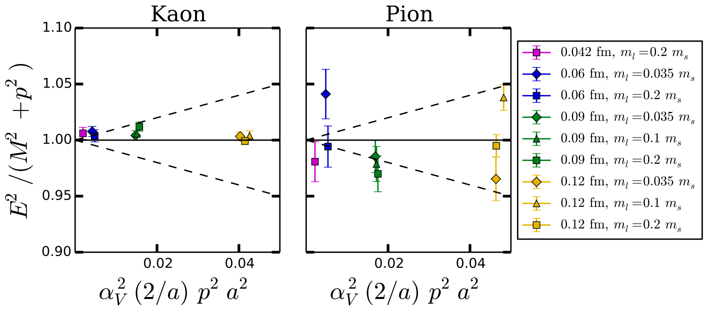


Kaon energy values from the 0.09 fm physical quark mass ensemble as a function of the number of states of each parity (left) and as a function of t_{min} (right). p-value is ≈ 1 for every fit shown except $N_{exp} = 1$.

Dispersion relation

Fitted energies of moving pions and kaons do not agree perfectly with the expected value from the dispersion relation.

- Dispersion relation errors are expected to scale like $\alpha_S^2 (ap)^2$.
- Violations appear random in our data.
- Pion and kaon have similar momenta but pion energies have much larger errors.
- We correct for $q^2 \neq 0$ with a term in the chiral-continuum fit.

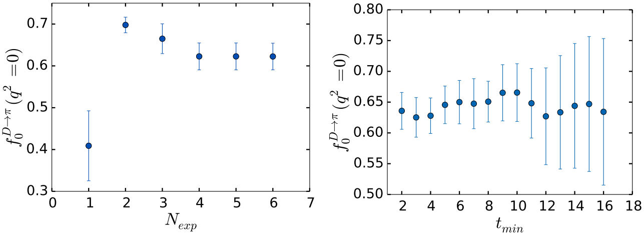


Three-point fits

The three-point correlator fit functions have the form:

$$C_{D\to P}(t) = \sum_{j}^{N_{exp}} \sum_{k}^{N_{exp}} V_{jk}(a_j^P)(a_k^D)(e^{-E_j^P t} e^{-E_k^D (T-t)}) + \{\text{Other parity comb.}\} \quad (4)$$

- Tested fitting two-point correlators then three-points sequentially or both simultaneously, with the former giving better stability.
- Three of the five available T are included in each fit, more than this shows no improvement in errors or stability and can make the fit more difficult.



$D \to \pi$ form factor fit values from the 0.09 fm physical quark mass ensemble as a function of the number of states of each parity (left) and as a function of t_{min} (right). p-value is ≈ 1 for every fit shown except $N_{exp} < 4$.

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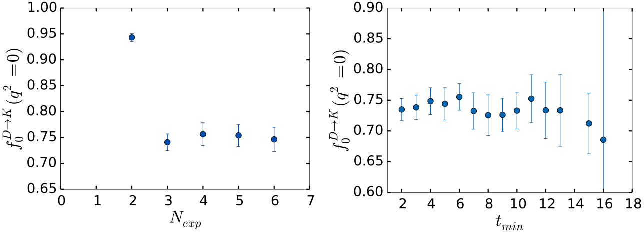

$D \to K$ form factor fit values from the 0.09 fm physical quark mass ensemble as a function of the number of states of each parity (left) and as a function of t_{min} (right). p-value is ≈ 1 for every fit shown except $N_{exp} < 4$.

Chiral perturbation theory

- We apply Heavy Meson Staggered χPT expressions calculated by Aubin and Bernard[1,2], in the hard pion/kaon[3] limit.

$$f_0(q^2) \quad = \quad \frac{C_0}{f_\pi}\left[(1+\delta f_{logs}) + C_v\chi_v + C_s\chi_{sea} + C_a\chi_{a^2} + C_q\chi_{q^2}\right] \qquad (5)$$

- χ_v, χ_{sea}, χ_{a^2} and χ_{q^2} capture the dependence on the daughter quark mass, sea quark masses, light quark discretization effects and q^2 respectively.
- g_π appears in the chiral logs and is included as a fit parameter with a prior of 0.52 ± 0.07.
- Define dimensionless parameters χ_i such that fit parameters are expected to be of order one and use priors $C_i = 0 \pm 2$.
- Other terms capturing dependence on $(ap)^2$, a^4 and higher order terms are not included in the central fit but are considered as a part of our systematic error analysis.
- The central values of the fits are stable under inclusion of such terms.

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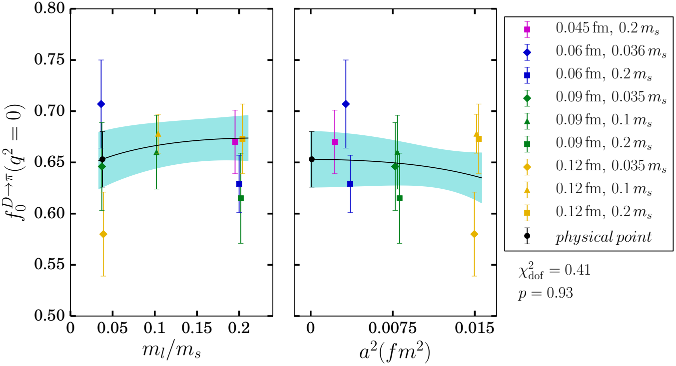
[1] C. Aubin and C. Bernard, Phys. Rev. D **76**, 014002 (2007) [arXiv:0704.0795]

[2] D. Becirevic, S. Prelovsek and J. Zupan, Phys. Rev. D **68**, 074003 (2003) [hep-lat/0305001]

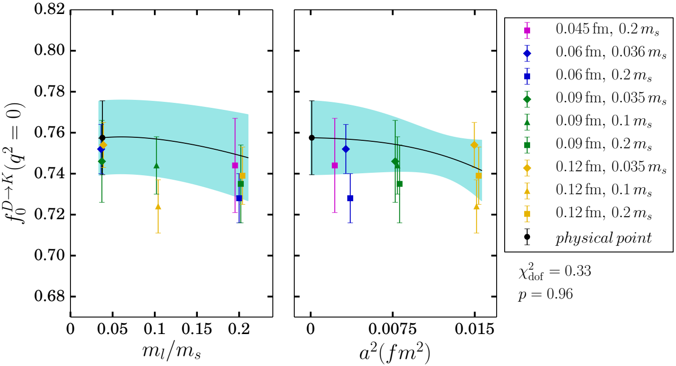
[3] J. Bijnens and I. Jemos, Nucl. Phys. B **840**, 54 (2010) [arXiv:1006.1197v2].

$f_0^{D\to\pi}$ Chiral and continuum extrapolation



Chiral/continuum fit of $f_0^{D\to\pi}(0)$ as a function of the light quark mass ratio and a^2.

$f_0^{D\to K}$ Chiral and continuum extrapolation



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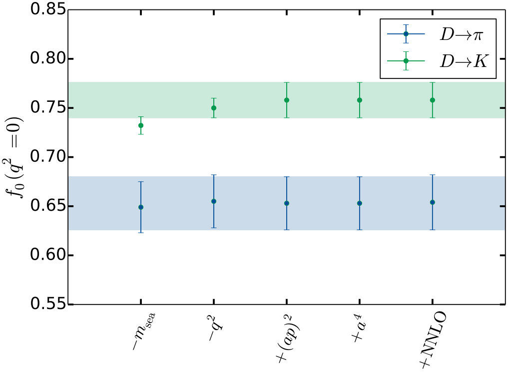
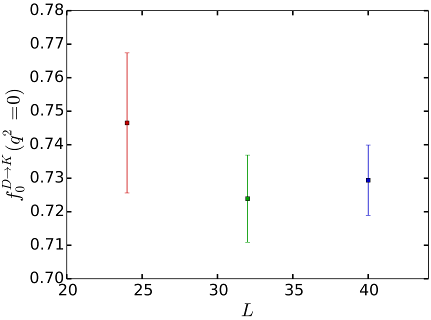
Chiral and continuum extrapolation



- The above plot shows the change in form factor values due to variations in the chiral fit function, compared to the central fit (colored band).
- The central values of each variation agree within errors, except when the sea quark mass term is removed, in which case the *p*-value of the fit is poor.

Finite volume checks



- $D \to K$ form factor values from three different volumes agree within errors.
- Finite volume effects obscured by dispersion relation violations in $D \to \pi$ case.
- Bijnens and Relefors[1] provide a method for calculating finite volume effects in the presence of twisted boundary conditions via χPT.
- Using this method we find an estimate of $\sim 0.06\%$ corrections for $D \to \pi$, which are negligible, and we expect corrections for $D \to K$ to be of the same order.

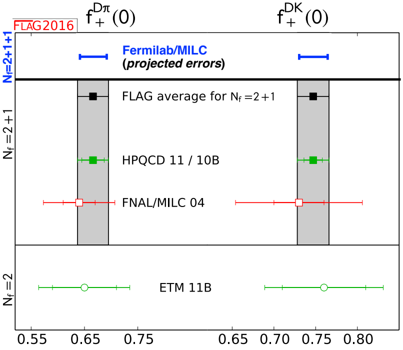
[1] J. Bijnens and J. Relefors, JHEP **1405**, 015 (2014) [arXiv:1402.1385]

Preliminary Error Budget

Source of uncertainty	% Error $f_+^{D\to\pi}(0)$	% Error $f_+^{D\to K}(0)$
Chiral-continuum fit	4.1	2.4
(Statistics)		
(Truncation of chiral model)		
(discretization errors)		
Finite volume	0.06	(0.06)
Scale a	0.2	0.2
Total	4.1	2.4

- Scale setting uncertainty effects determined by rerunning the chiral fit with each a varied by $\pm\sigma$, stated uncertainty is the largest change.
- These errors are comparable to those from HPQCD who used 2+1 flavor *asqtad* ensembles with HISQ valence quarks.

Conclusion



← Projected errors from this project overlaid on the central value of the FLAG average.

- In this work we are calculating $D \to K(\pi)$ semileptonic form factors at $q^2 = 0$.
  - χPT extrapolation to the physical point and continuum limit.
  - Anticipate total errors of $\sim 2.4\%$ ($\sim 4.1\%$) for $f_+^{D\to K}(0)$ ($f_+^{D\to\pi}(0)$).
- Future work:
  - Calculations of scalar & vector form factors at multiple q^2 values, employing a z-expansion to get the normalization and shapes.
  - Combine with experiment improve the result for the CKM matrix elements.
- Both of these projects use MILC 2+1+1 flavor HISQ ensembles.

EXTRA SLIDES

Chiral fit parameter definitions

$$\chi_v = \frac{\mu(2m_v)}{8\pi^2 f_\pi^2} \tag{6}$$

$$\chi_{sea} = \frac{\mu(2m_l + m_s)}{8\pi^2 f_\pi^2} \tag{7}$$

$$\chi_{a^2} = \frac{a^2\overline{\Delta}}{8\pi^2 f_\pi^2} \tag{8}$$

$$\chi_{q^2} = \frac{q^2}{8\pi^2 f_\pi^2} \tag{9}$$