Progress on the lattice QCD calculation of the rare kaon decays: $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

The 34th International Symposium on Lattice Field Theory

July 28, 2016

N.H. Christ*, A. Juettner, A. Lawson, A. Portelli, C. Sachrajda, X. Feng

RBC and UKQCD Collaborations

The RBC & UKQCD collaborations

BNL and RBRC

Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Taichi Kawanai
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

CERN

Marina Marinkovic

Columbia University

Ziyuan Bai
Norman Christ
Luchang Jin
Christopher Kelly
Bob Mawhinney
Greg McGlynn
David Murphy
Jiqun Tu

University of Connecticut

Tom Blum

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Richard Kenway
Julia Kettle
Ava Khamseh
Brian Pendleton
Antonin Portelli
Oliver Witzel
Azusa Yamaguchi

KEK

Julien Frison

Peking University

Xu Feng

Plymouth University

Nicolas Garron

University of Southampton

Jonathan Flynn
Vera Guelpers
James Harrison
Andreas Juettner
Andrew Lawson
Edwin Lizarazo
Chris Sachrajda
Francesco Sanfilippo
Matthew Spraggs
Tobias Tsang

York University (Toronto)

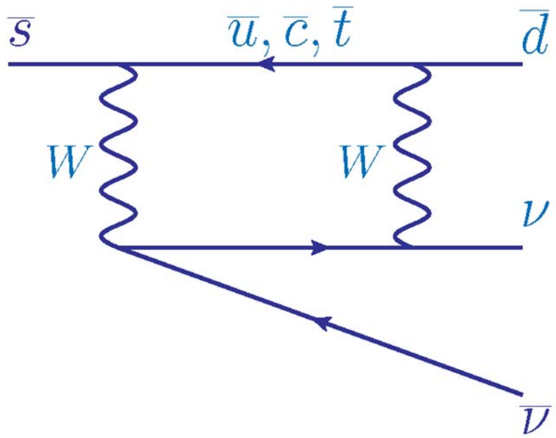
Renwick Hudspith

Outline

- $K^+ \to \pi^+ \nu \bar{\nu}$ phenomenology
  - Short distance: E-W perturbation theory
  - Long distance: Lattice QCD needed
- Overview of lattice calculation
  - Subtraction of exponentially growing terms
  - NPR for bilocal operators
- Preliminary lattice results
- Relation to present predictions

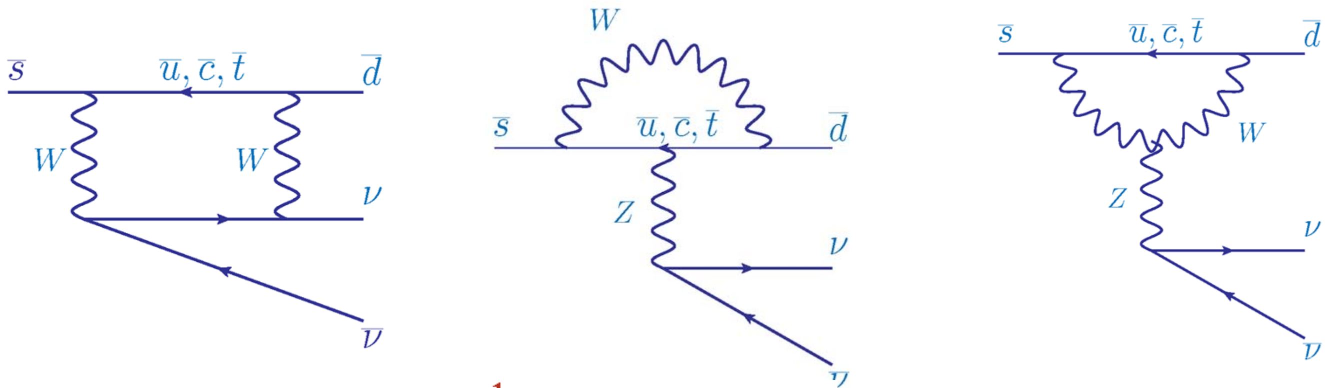
$K^+ \to \pi^+ \nu \bar{\nu}$

- Flavor changing neutral current
  - Allowed in the Standard Model only in second order
  - Short distance dominated



- Target of NA62 at CERN
  - 100 events in 2-3 years
  - Test Standard Model prediction at 10% level
  - Use lattice for long distance part: 5% effect ?

$K^+ \to \pi^+ \nu \bar{\nu}$ in the Standard Model

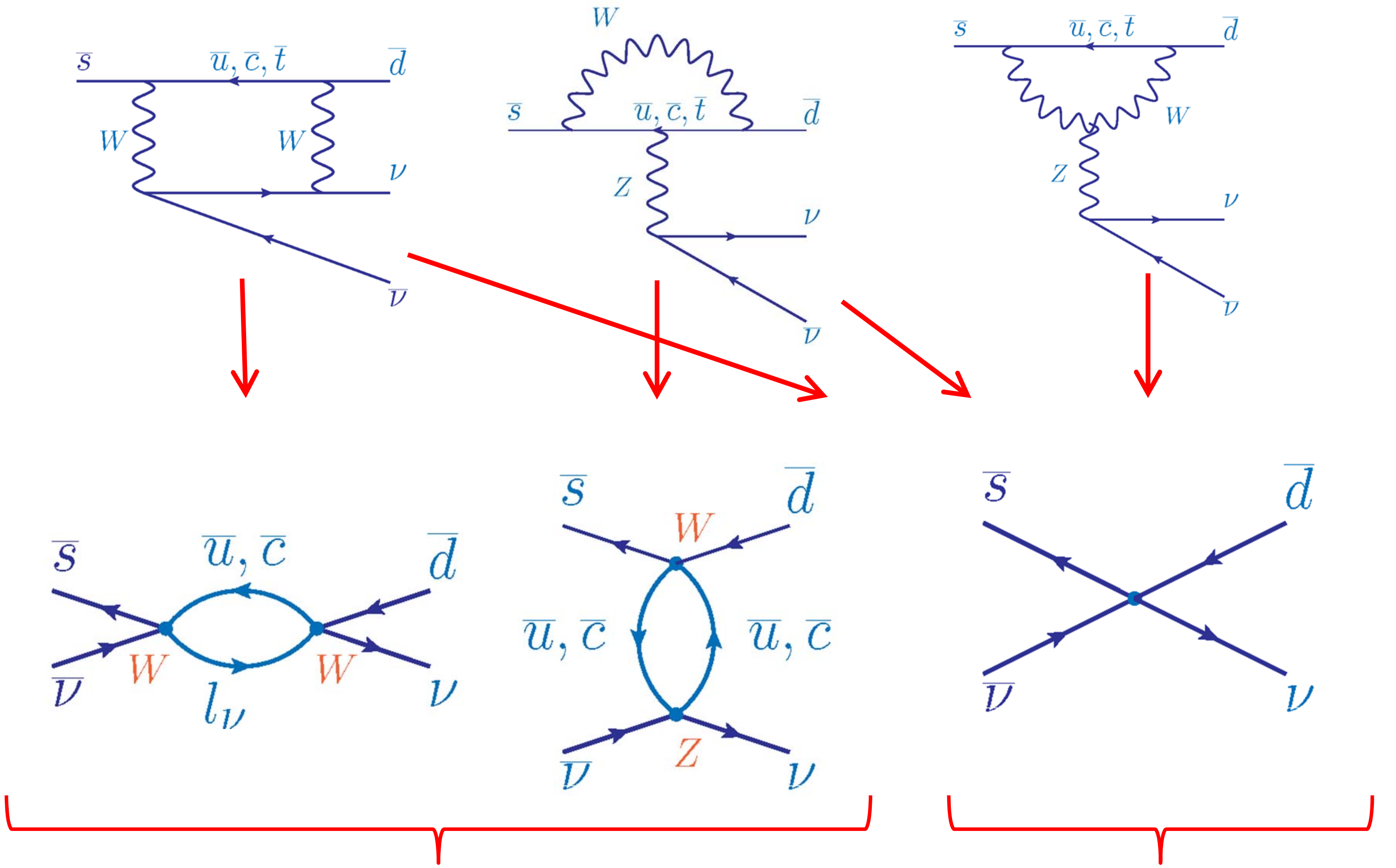


- Factors of $\frac{1}{M_W^4}$ or $\frac{1}{M_W^2 M_Z^2}$ force the largest contribution to come from short distance

Pert. Th. {
- Top quark contribution largest.
- GIM implies charm-up $\sim \frac{m_c^2 - m_u^2}{M_W^4} \ln(M_W^2 / m_c^2)$

Lattice {
- Long distance part $\sim \frac{m_c^2}{M_W^4}$

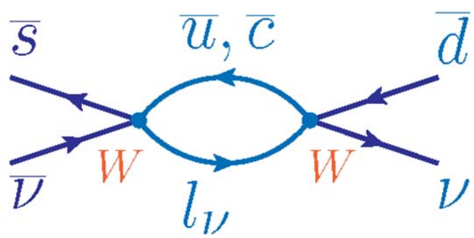
$K^+ \to \pi^+ \nu \bar{\nu}$ at long distance

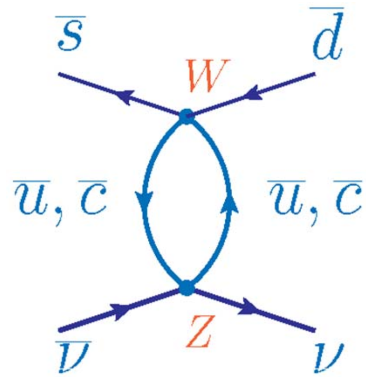


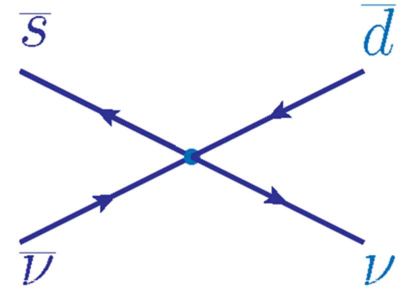
Use lattice QCD

Known $\langle \pi^+ | V_\mu | K^+ \rangle$

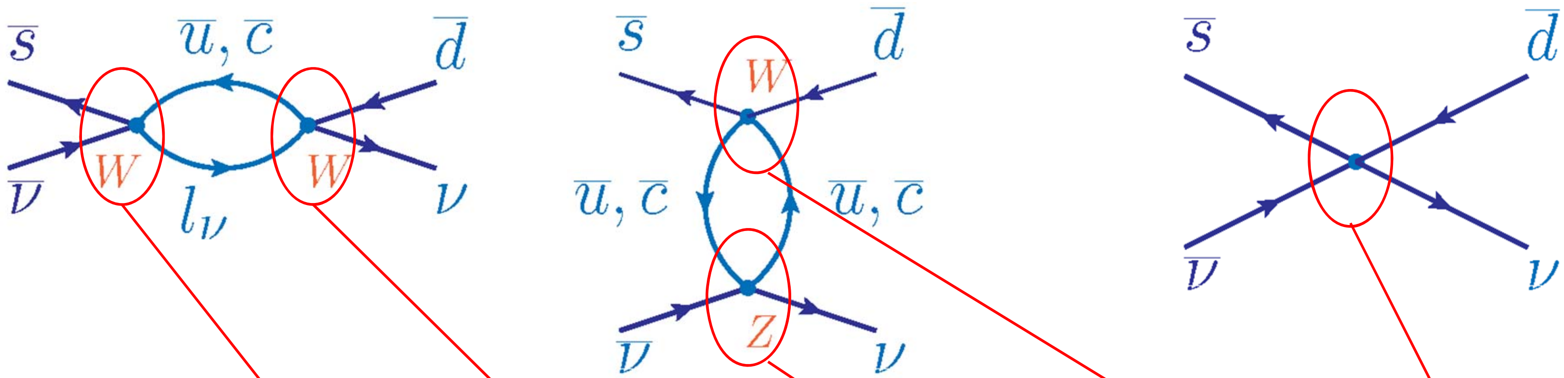
H_{eff} for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$





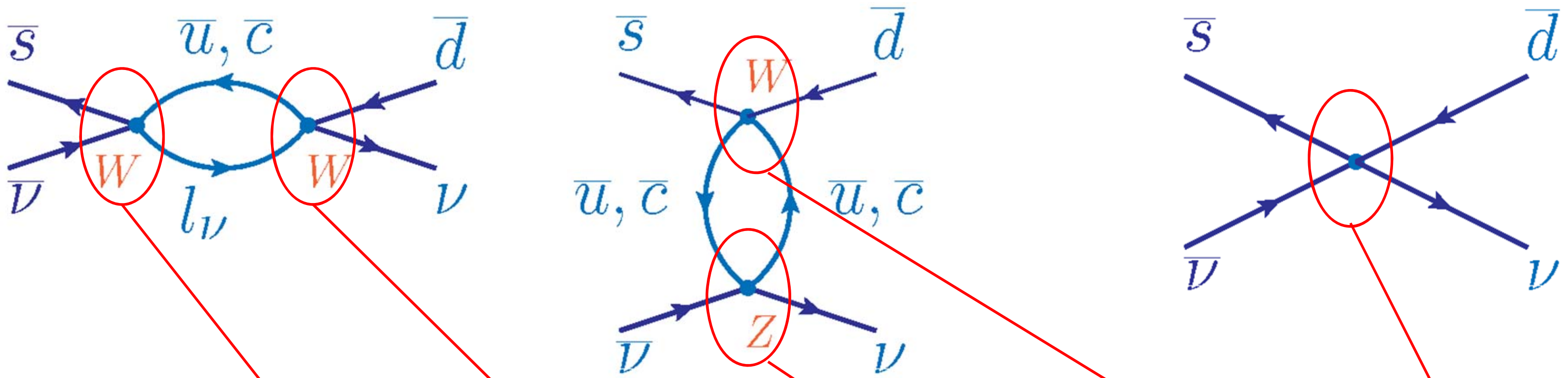


H_{eff} for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$



$$\mathcal{H}_{\text{eff}} = +\frac{G_F}{\sqrt{2}}\left\{\sum_{\substack{q=u,c\\ \ell=e,\mu,\tau}}\left(V^*_{qs}O^{\Delta S=1}_{q\ell} + V_{qd}O^{\Delta S=0}_{q\ell}\right) + \sum_{\ell=e,\mu,\tau} O^Z_\ell + \sum_{q=u,c}\lambda_q O^W_q\right\} + O_0$$

$H_{\rm eff}$ for $K^+ \to \pi^+ \nu \bar{\nu}$



$$\mathcal{H}_{\rm eff} = +\frac{G_F}{\sqrt{2}} \left\{ \sum_{\substack{q=u,c \\ \ell=e,\mu,\tau}} \left(V_{qs}^* O_{q\ell}^{\Delta S=1} + V_{qd} O_{q\ell}^{\Delta S=0} \right) + \sum_{\ell=e,\mu,\tau} O_\ell^Z + \sum_{q=u,c} \lambda_q O_q^W \right\} + O_0$$

$$O^{\Delta S=1} = C_{\Delta S=1} (\bar{s}q)_{V-A} (\bar{\ell}\nu_\ell)_{V-A}$$

$$O_q^W = C_1 (\bar{s}_a q_b)_{V-A} (\bar{q}_b d_a)_{V-A} + C_2 (\bar{s}_a q_a)_{V-A} (\bar{q}_b d_b)_{V-A}$$

$$O^{\Delta S=0} = C_{\Delta S=0} (\bar{q}d)_{V-A} (\bar{\ell}\nu_\ell)_{V-A}$$

$$O_0 = C_0 \sum_{\ell=e,\mu,\tau} (\bar{s}d)_{V-A} (\bar{\nu}_\ell \nu_\ell)_{V-A}$$

$$O_\ell^Z = C_Z \sum_{q=u,c,d,s} \left(T_3^q \bar{q} \gamma_\mu (1-\gamma_5) q - Q_{{\rm em},q} \sin^2\theta_W \bar{q} \gamma_\mu q \right) \overline{\nu_\ell} \gamma_\mu (1-\gamma_5) \ell$$

Unphysical terms growing exponentially with time

- Encountered previously for $M_{K_L} - M_{K_S}$

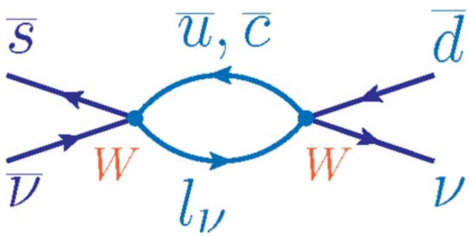
$$\int_{-T}^{T} dt \langle \pi \nu \overline{\nu} | T\Big(O_A(t) O_B(0)\Big) | K \rangle$$

$$= \sum_n \left\{ \frac{\langle \pi \nu \overline{\nu} | O_A | n \rangle \langle n | O_B | K \rangle}{M_K - E_n} + \frac{\langle \pi \nu \overline{\nu} | O_B | n \rangle \langle n | O_A | K \rangle}{M_K - E_n} \right\} \left(1 - e^{(M_K - E_n)T}\right)$$

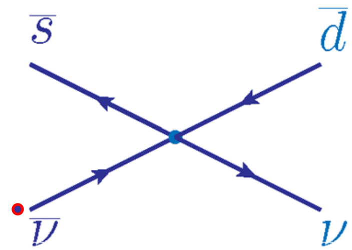
- Terms with $M_K > E_n$ must be removed.
- Possibly large finite volume corrections: replace principal part by a finite volume sum. (N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda, arXiv:1504.01170)

New short-distance divergence

- Second order effective theory requires new counter terms



$+ X^{\text{Lat}\to\text{RI}}$



$\Big\{ \ldots \Big\} = 0$

$p^2 = \mu^2$

- Use NPR for bilocal operator

$$\left\{\int d^4x T\left(Q_A^{\overline{\text{MS}}}(x) Q_B^{\overline{\text{MS}}}(0)\right)\right\}^{\overline{\text{MS}}}$$

$$= Z_A Z_B \left\{\int d^4x T\left(Q_A^{\text{Lat}}(x) Q_B^{\text{Lat}}(0)\right)\right\}^{\text{Lat}} + \left(Z_A Z_B X^{\text{Lat}\to\text{RI}} + Y^{\text{RI}\to\overline{\text{MS}}}\right) Q_0(0)$$

Exploratory Lattice Calculation

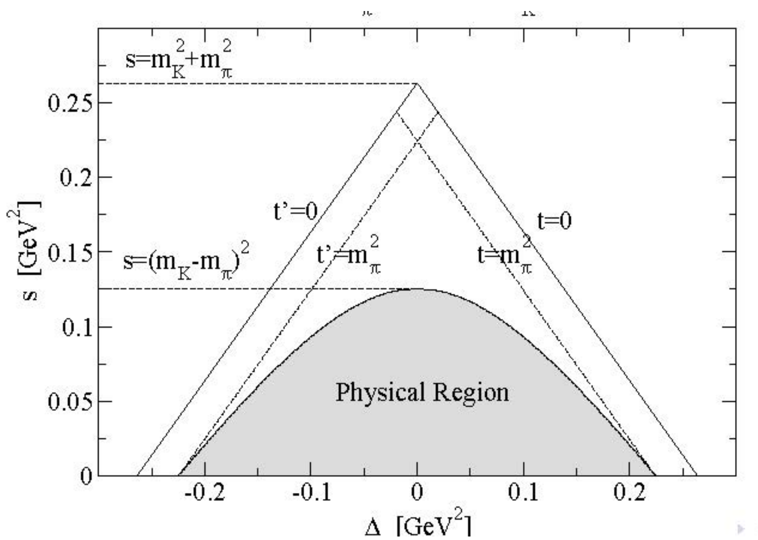
- 16^3 x 32, RBC-UKQCD ensemble
  - 2+1 flavor DWF, $1/a$ = 1.73 GeV
  - M_π = 420 MeV, M_K = 540 MeV,
  - $m_c(2\ \mathrm{GeV})^{\overline{\mathrm{MS}}}$ = 863 GeV
- Calculate all diagrams
- 800 configurations
- Low-mode deflation with 100 modes
- Place sources on 32 time slices
- Treat internal lepton as an overlap fermion moving in ∞ time.

Exploratory Lattice Calculation

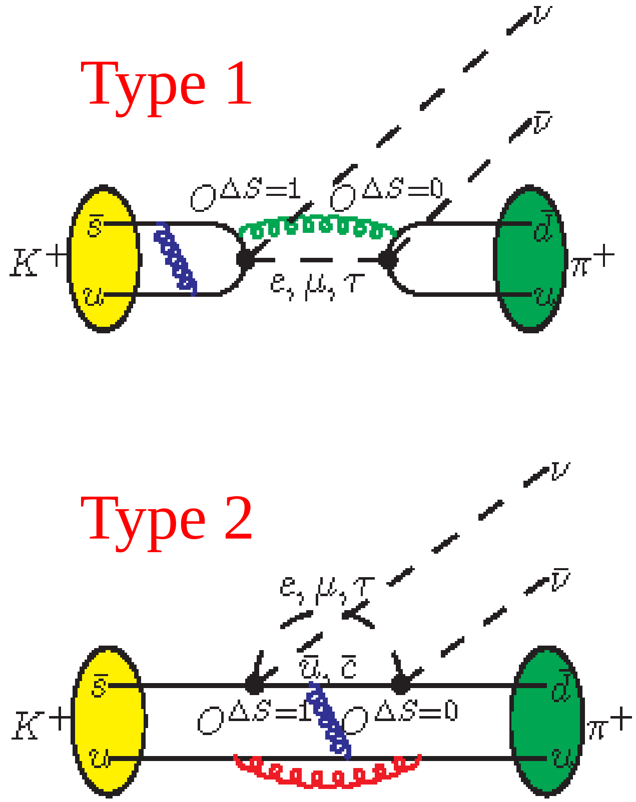
- All results given as scalar amplitudes
  - *W*-exchange diagram determines $F_{WW}(s, \Delta)$ for Dalitz plot variables:

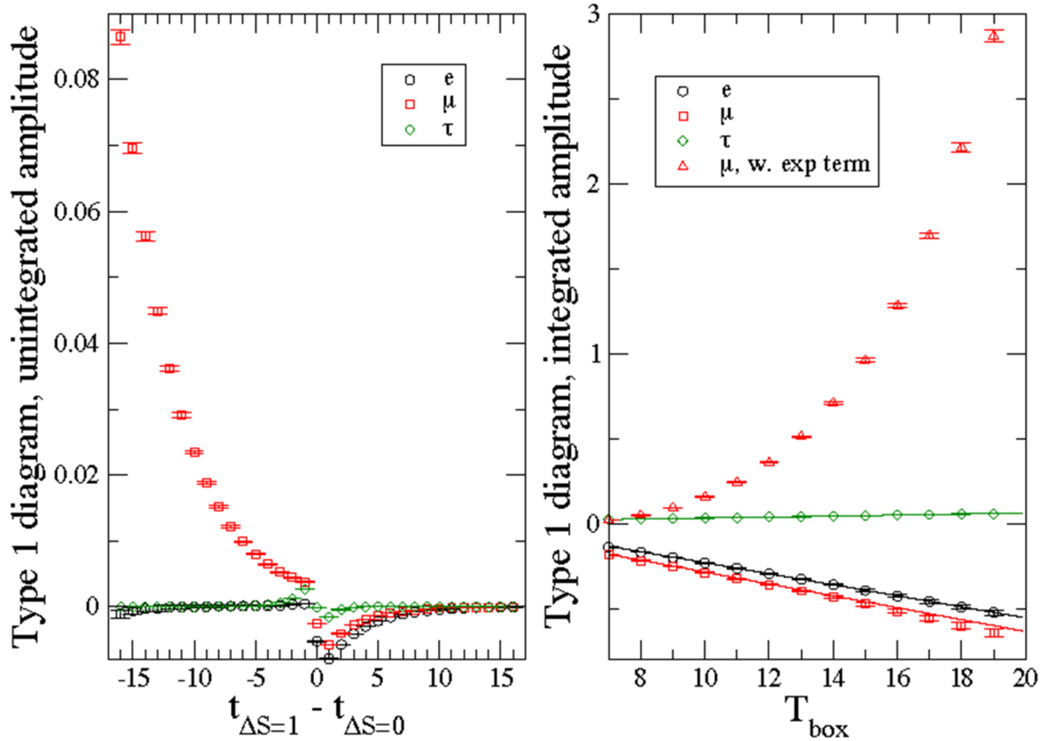
    $$s = (p_K - p_\pi)^2, \qquad \Delta = (p_K - p_\nu)^2 - (p_K - p_{\bar{\nu}})^2$$

  - Vector and axial from *Z*-exchange determine familiar $Kl3$ $f_+(s)$
  - Assume these are constants
  - Evaluate at $\vec{p}_K=0$ and $\vec{p}_\pi= (0.0414, 0.0414, 0.0414)$
  - For vector *Z*-exchange also use $\vec{p}_\pi = 0$



W W diagrams

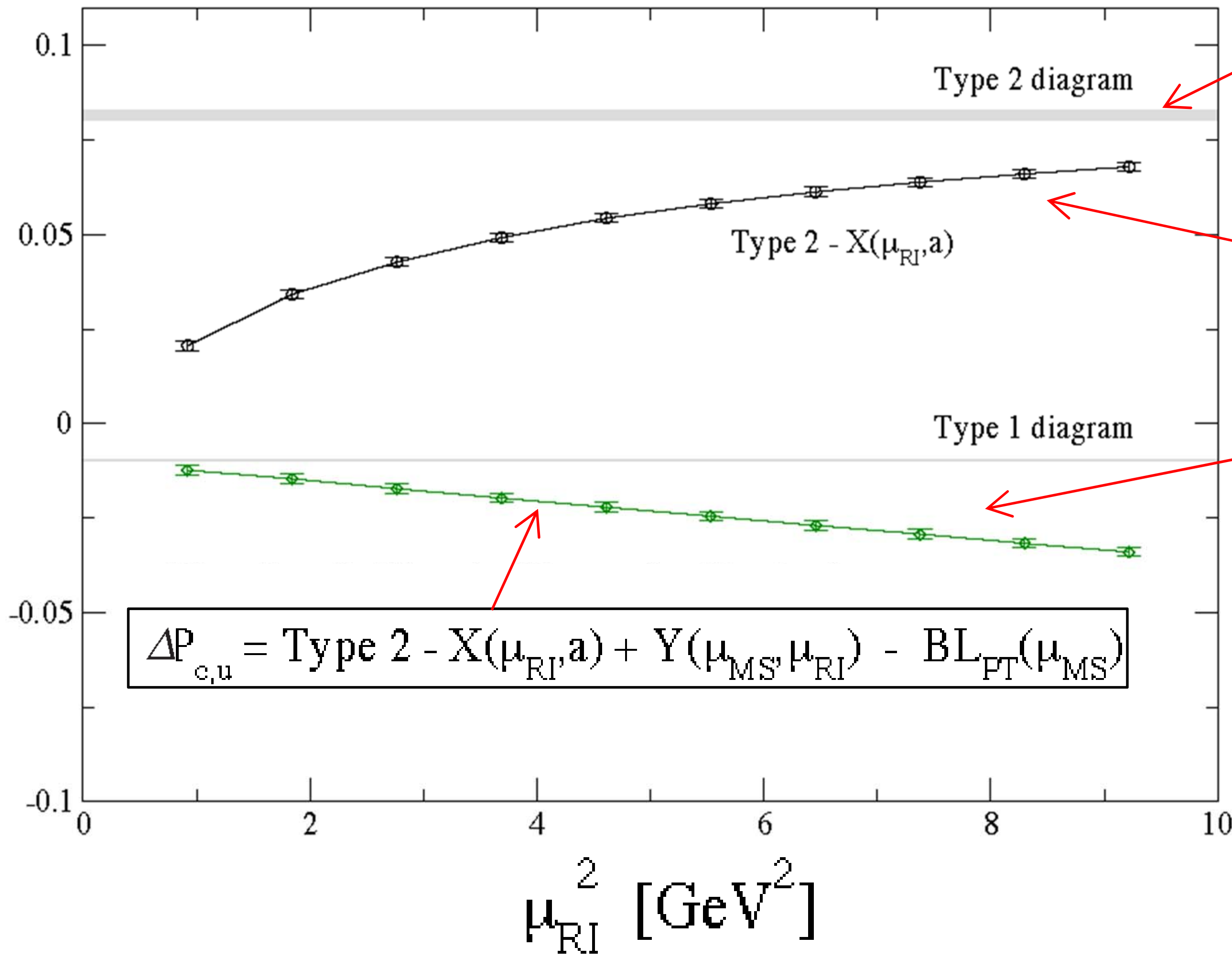




F_{WW}	Type 1	model
e	$-1.685(47)\times 10^{-2}$	$-1.740(6)\times 10^{-2}$
μ	$-1.818(40)\times 10^{-2}$	$-1.822(6)\times 10^{-2}$
τ	$1.491(36)\times 10^{-3}$	$1.471(5)\times 10^{-3}$

Exponentially growing term comes from $\mu^+\nu$ state

W W diagrams

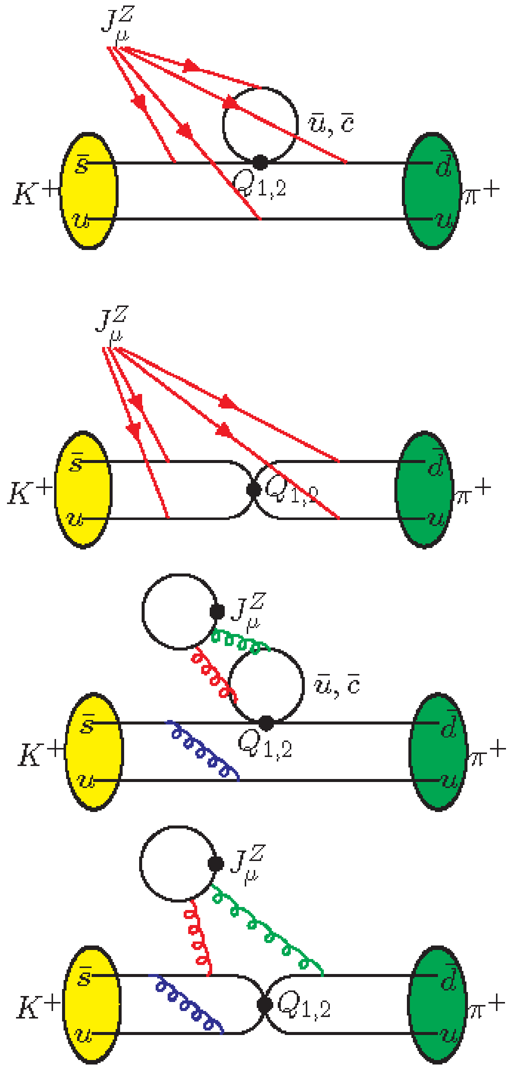


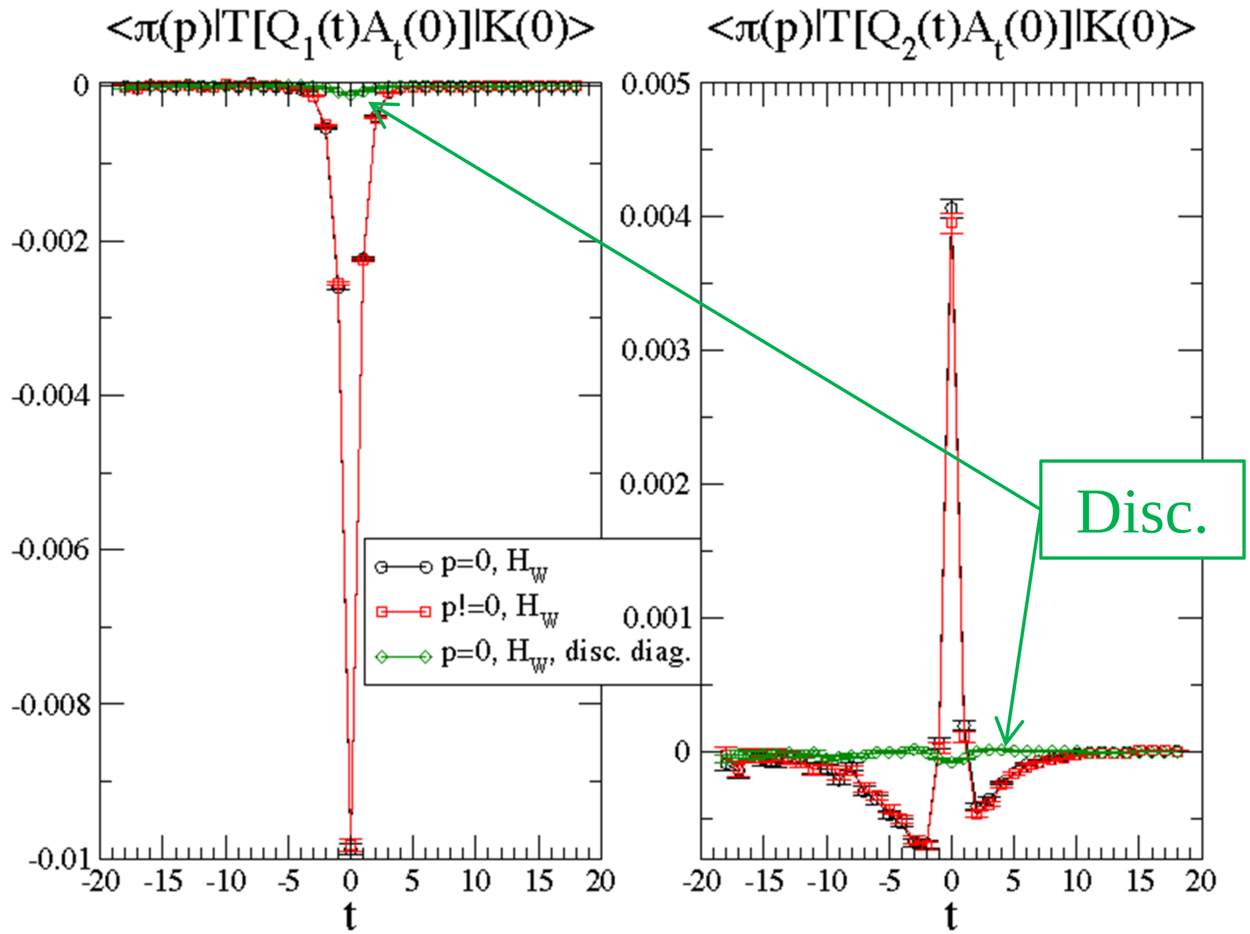
"Bare" lattice

RI - normalized

Difference between lattice and perturbative $\overline{\text{MS}}$ operators, $\mu_{\overline{\text{MS}}}$=2GeV

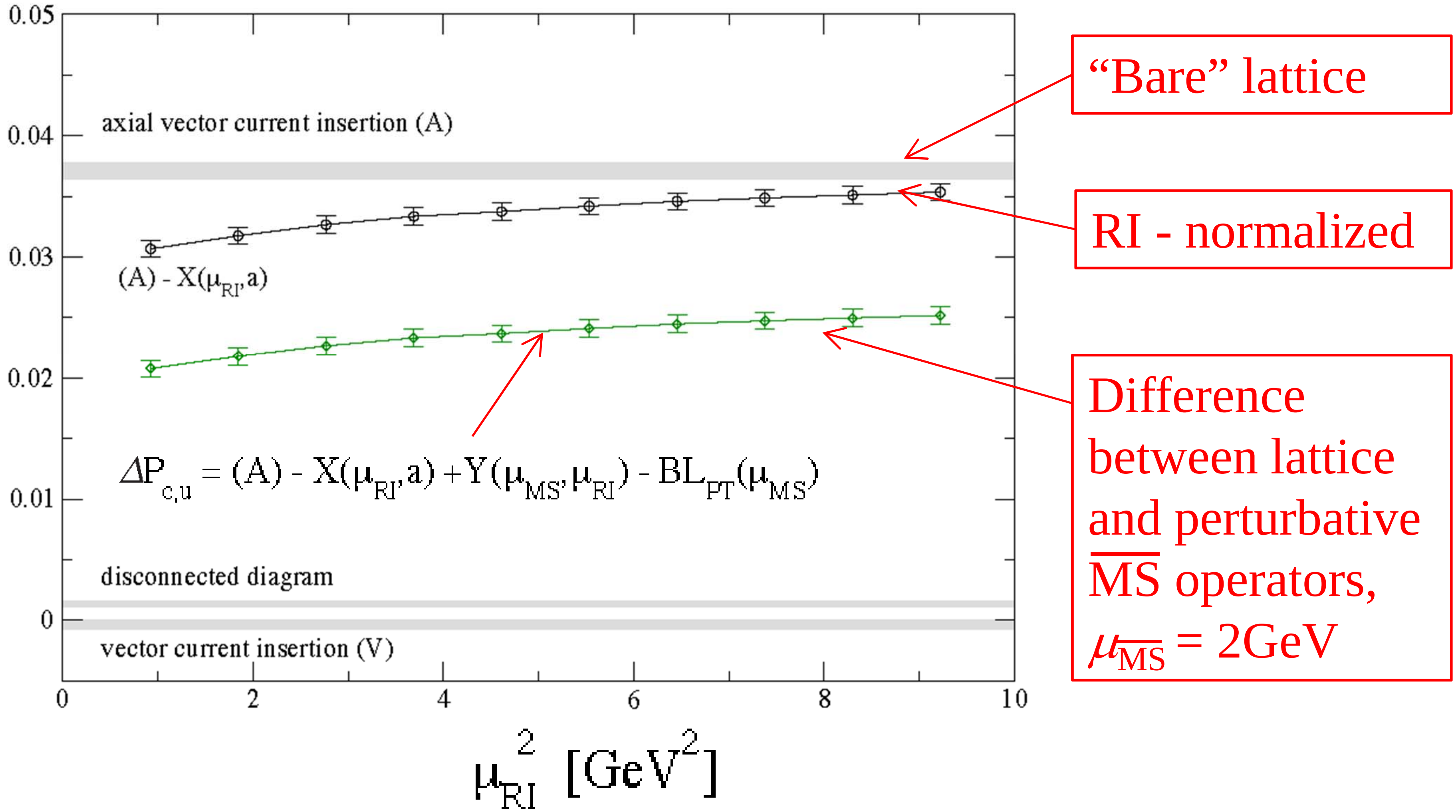
Z – Exchange Diagrams





Results from axial *Z* coupling

Z – Exchange diagrams



How does lattice QCD contribute?

- Decay rate is short distance dominated:

$$\mathrm{Br} = \kappa_+(1+\Delta_{\mathrm{EM}})\left[\left(\underbrace{\frac{\mathrm{Im}\lambda_t}{\lambda^4}X(x_t)}_{0.270\ \mathrm{x}1.481}\right)^2 + \left(\underbrace{\frac{\mathrm{Re}\lambda_c}{\lambda}P_c}_{-0.974\ \mathrm{x}\ 0.405} + \underbrace{\frac{\mathrm{Re}\lambda_t}{\lambda^5}X(x_t)}_{-0.533\ \mathrm{x}1.481}\right)^2\right]$$

$$P_c^{\mathrm{SD}} = \frac{1}{\lambda^4}\frac{X_c^e + X_c^\mu + X_c^\tau}{3} \qquad \lambda = |V_{us}|$$

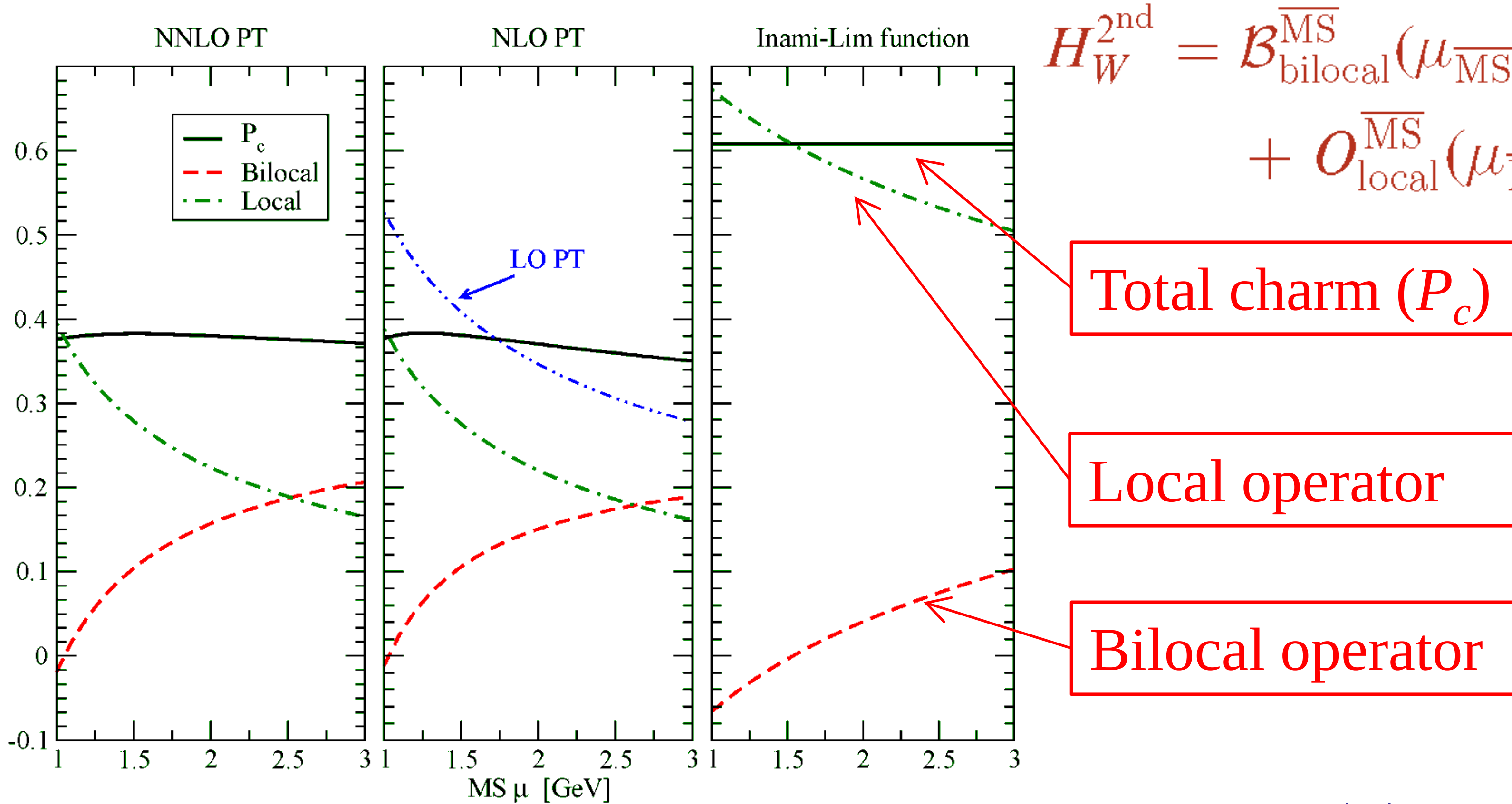
- Charm contribution is less than top but is significant (removing charm lowers BR by 50%)

$$\lambda_t X_t(x_t) \ : \ \lambda_c X_c^\ell \quad :: \quad \lambda_t \frac{m_t^2}{M_W^2} \ : \ \lambda_c \frac{m_c^2}{M_W^2} \ln\frac{M_W}{m_c}$$

- How important is the charm energy scale?

Importance of charm energy scale

- Presence of $\ln(M_W^2/m_c^2) = 8.4$ suggests charm scale may be 12% of charm contribution?
- However, the log term is suppressed at NLO:



$$H_W^{2\text{nd}} = \mathcal{B}^{\overline{\text{MS}}}_{\text{bilocal}}(\mu_{\overline{\text{MS}}}) + O^{\overline{\text{MS}}}_{\text{local}}(\mu_{\overline{\text{MS}}})$$

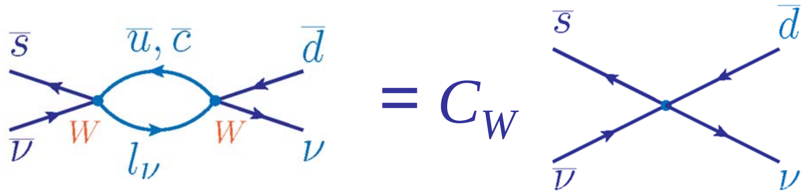
Conventional treatment of $p \leq m_c$

- Electroweak and QCD perturbation theory provides:

$$H_W^{2^{\rm nd}} = \mathcal{B}^{\overline{\rm MS}}_{\rm bilocal}(\mu_{\overline{\rm MS}}) + O^{\overline{\rm MS}}_{\rm local}(\mu_{\overline{\rm MS}})$$

- Integrate out charm:

$$\mathcal{B}^{\overline{\rm MS}}_{\rm bilocal}(\mu_{\overline{\rm MS}}) \approx C_W(\mu_{\overline{\rm MS}}) \cdot Q_0^{\overline{\rm MS}}(\mu_{\overline{\rm MS}}) \qquad Q_0 = (\overline{s}d)_{V-A}(\overline{\nu}\nu)_{V-A}$$



- Long distance effect of up quark is missing, represented by δP_{cu} : $P_c = P_c^{\rm SD} + \delta P_{cu}$
  - $P_c^{\rm SD} = 0.365(12)$
  - $\delta P_{cu} = 0.040(20)$ [Isidori *et. al*, hep-ph/0503107]

Lattice result (unphysical kinematics)

- Replace: $P_c = P_c^{SD} + \delta P_{cu}$ $\quad \delta P_{cu} = 0.040\ (20)$
- By: $P_c = P_c^{SD} + \Delta P_{cu}$

where

Evaluate bilocal matrix element

$$\Delta P_{cu}(\mu_{\overline{\rm MS}}) \propto \langle \pi \nu \bar{\nu} | \left\{ \mathcal{B}^{\overline{\rm MS}}_{\rm bilocal}(\mu^{\overline{\rm MS}}) - C_W(\mu_{\overline{\rm MS}}) \cdot Q_0^{\overline{\rm MS}}(\mu_{\overline{\rm MS}}) \right\} | K^+ \rangle$$

Remove conventional approximation to matrix element

WW:-0.032(1) +Z:0.025(1)

$$\Delta P_{cu}\left(\mu_{\overline{\rm MS}} = 2.0\ {\rm GeV}\right) = -0.007(2) + \left(\begin{array}{c} +7 \\ -11 \end{array}\right)_{\rm RI} + \left(\begin{array}{c} +5 \\ -21 \end{array}\right)_{\overline{\rm MS}}$$

Conclusion

- Use lattice methods to compute the QCD contribution to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ from $E \leq m_c$
- Exponentially growing terms and bilinear operator normalization can be controlled.
- Demonstrated by a 16^3 x 32 exploratory lattice calculation with $m_\pi = 420$ MeV
- Next steps:
  - Use a larger volume 32^3 x 64 with $m_\pi = 170$ MeV but $m_c = 750$ MeV – now being analyzed
  - Move to $1/a = 2.38$ GeV, 64^3 x 128 and physical m_c – currently a USQCD Incite proposal