Using Wilson flow to study the deconfinement transition

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Introduction

- Wilson flow: popular tool for scale setting, renormalization of composite operators.

  M. Lüscher, 1006.4518; Lüscher & Weisz, 1101.0963

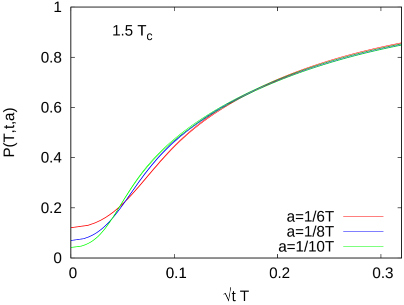
- Use of Wilson flow to study deconfinement transition.
- Equation of state: Asakawa et al.(FlowQCD), 1312.7492
- Polyakov loop: order parameter for deconfinement in pure gauge theory.

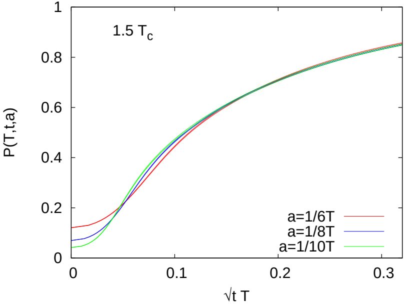
$$P(T,a) \sim e^{-c(T)/a}\, P_r(T) \qquad \to 0 \;\; \text{as } a \to 0$$

- For observable with continuum limit: need to consider "fat" Polyakov loop.
- Use of flowed Polyakov loop and route to renormalized Polyakov loop. (1512.04892)
- Other studies: Petreczky & Schadler, 1509.07874; Bazavov et al., 1603.06637
- Flow behavior of Gluon condensates.

Flowed Polyakov loop

- For Wilson flow to time t, operators constructed are smeared to a radius $\sim \sqrt{8t}$ (Luscher '06)
For $\sqrt{t} \gg a$, negligible cutoff effect





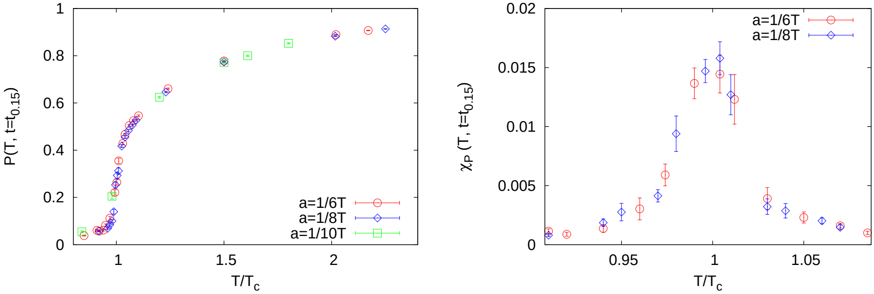
On the other hand, thermal physics requires $T \gg \frac{1}{\sqrt{t}}$

- $T \ll \frac{1}{\sqrt{t}} \lesssim \frac{1}{a}$

Polyakov loop at a fixed flow time

We can look at Polyakov loops flowed to a fixed hadronic scale.

Petreczky & Schadler, 1509.07874



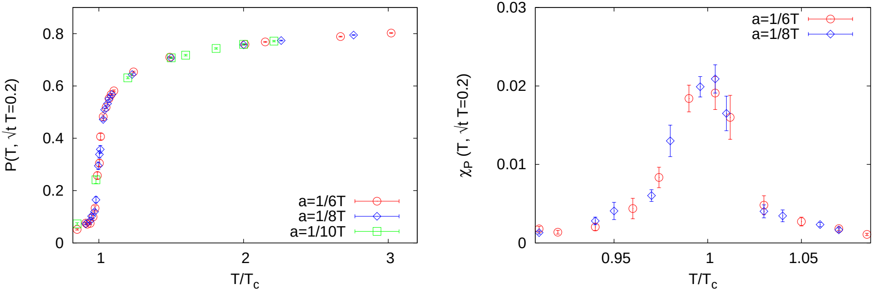
- T_c fixed by peak of (thin) P susceptibility, T/T_c using flow time.
- Difficult to cover a wide temperature range; $\sqrt{t_{0.3}}T_c \sim 0.25$

Francis et al., 1503.05652; Asakawa et al, 1503.06516

$\sqrt{t}\,T$ fixed

Alternately, fix flow time in temperature units.

Asakawa et al. (2013)

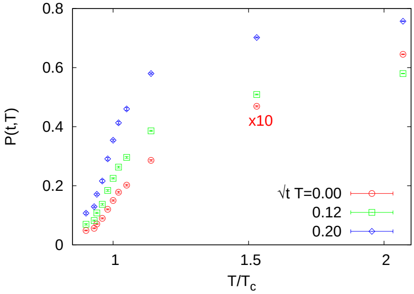


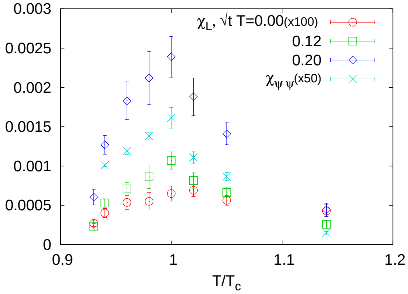
- Temperatures covering 0.9-3 T_c for SU(3).
- Center symmetry dictates susceptibility always peaks at T_c.

$N_f = 2$

N_f=2 staggered, $32^3 x8$ lattices, $m_\pi \sim 220$ MeV.

With R. Gavai





- Broad peak for thin Polyakov loop, sharpens with flow.
- Peak stays fixed after $\sqrt{t} > a$, agrees with $\chi^{\text{dis}}_{\bar{\psi}\psi}$ peak (T_c).
- Shift of P susceptibility peak with flow time found in Bazavov et al., 1603.06637.

Mass dependence?

Renormalized Polyakov loop from flow

- Renormalization of Polyakov loop well-studied. For lattice spacing a multiplicative renormalization as

$$P_r(T) \;=\; Z(a)^{1/aT}\; P(T,a)$$

Polyakov '77

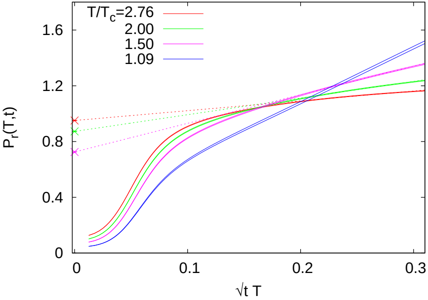
- Various well-motivated prescriptions for non-perturbative renormalization of $P_r(T)$.
- Here, using Wilson flow, we will explore the efficacy of a perturbative renormalization.
- $$\begin{aligned} P_r(T) &= \lim_{t\to 0} \exp\left(\frac{R(g^2(t))}{\sqrt{t}T}\right) P(T,t) \\ R(g^2(t)) &= \frac{\sqrt{\pi}}{6\sqrt{2}\pi^2}\, g^2(t) \;+\mathcal{O}(g^4) \end{aligned}$$

Datta et al, 1512.04892

Renormalizing perturbatively

$$\exp\left(\frac{R(g^2(t))}{\sqrt{t}\,T}\right) P(T,t) \;=\; P_r(T) + \mathcal{O}(t)$$

For small enough t can attempt linear extrapolation to $t \to 0$.

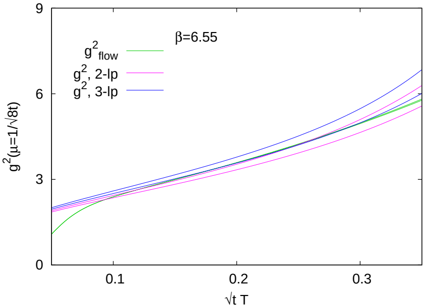


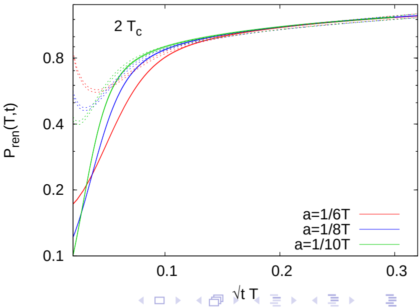
- This follows small-t expansion strategy of Suzuki, 1304.0533, used by Asakawa et al, 1312.7492 for equation of state (Also Kanaya, wednesday)

A note on the coupling

To calculate the coupling, we can use $\frac{T_c}{\Lambda_{\overline{MS}}^{N_f=0}} = 1.20 \pm 0.02$ (Datta & Gupta, 0909.5591). Other determinations: 1.24 ± 0.10 (Francis et al., 1503.05652) and 1.18 ± 0.01 (Asakawa et al., 1503.06516). Alternately, we can extract it from

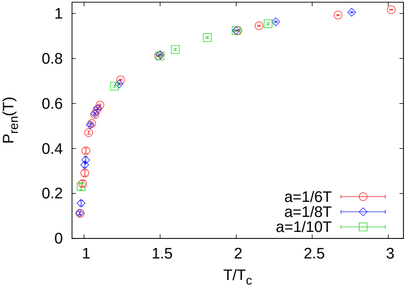
$$\langle \mathcal{E}(t) \rangle = \frac{3}{16\pi^2} \left(g^2_{\overline{MS}}(\mu = 1/\sqrt{8t}) + 1.0978 g^4 + .. \right)$$





The matching program

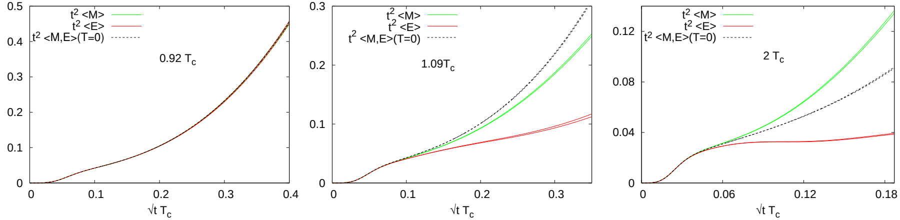
- R function of t only $\Longrightarrow$ run this program at a high T, use calculated R for renormalization at lower temperatures.
- Use $P_r(3T_c) = 1.017(1)$ to calculate $R(t) \Longrightarrow P_r$ at lower temperature. Run iteratively to stay in allowed t range.



This follows Gupta & Kaczmarek, PR D 77 (2007) 034503, except our renormalization factor depends on t and not a, so no need to do sets of simulations to match a.

Electric and magnetic condensate

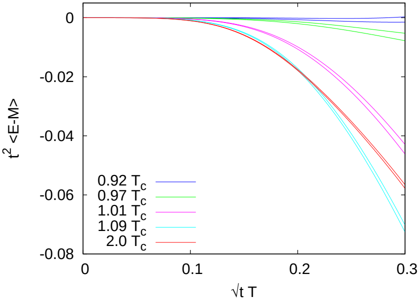
Flow behavior of $E = \mathrm{Tr}\, G_{0i} G_{0i}$ and $M = \frac{1}{2} \mathrm{Tr}\, G_{ij} G_{ij}$

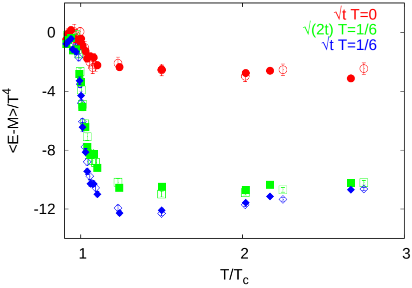


On crossing T_c the flow behavior of the electric and magnetic parts of the condensate change dramatically.

A LO calculation captures this behavior, though effect stronger in interacting theory. (1512.04892)

Electric and magnetic condensate

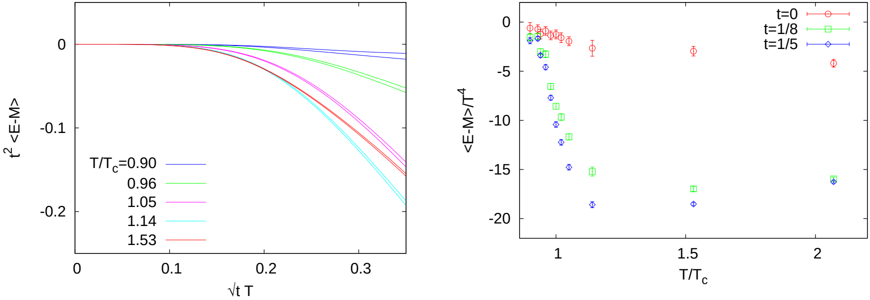




For $t \gg a^2$, the quantity $\frac{\langle E-M\rangle(T,t)}{T^4} \propto \frac{s}{T^3}$
Flow behavior can be related to entropy release at deconfinement.

2-flavor

Similar behavior seen in the 2-flavor theory, though onset is less abrupt.



Gluon condensate

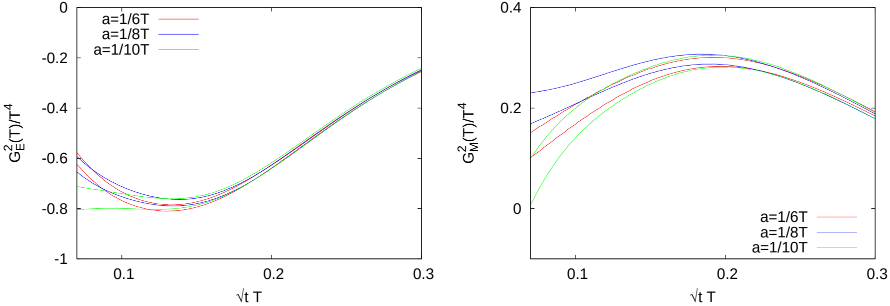
$\langle E(T,t)\rangle$ and $\langle M(T,t)\rangle$ related to electric and magnetic gluon condensates.

Remove the $1/t^2$ divergence by subtracting the $T=0$ part. Then the electric and magnetic parts of the gluon condensate can be obtained from the flowed quantities as

$$\begin{aligned} \bar{G}^2_{E,M}(T) &= \lim_{t\to 0} R(t)\cdot\{\bar{E}(T),\bar{M}(T)\} \\ R(t) &= \frac{1}{\pi^2}\left(1-2b_0 s_2 g^2(t)+\mathcal{O}(g^4)\right) \end{aligned}$$

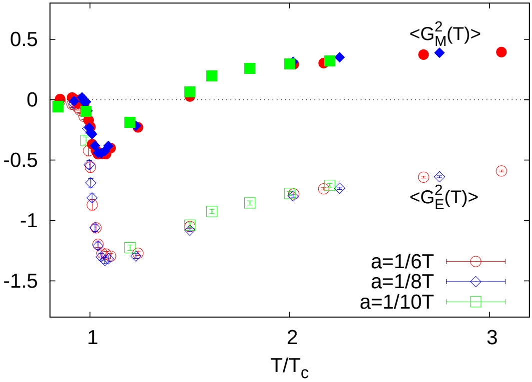
H. Suzuki, 1304.0533

Condensate vs. t



On our data, could not do reliable extrapolation even at 2 T_c. To get a qualitative feeling, we show these quantities at the smallest t value before cutoff effects set in.

Results



Qualitative features in good agreement with earlier calculation in Boyd et al., Nucl.Phys.B469('94)419.

At high temperatures approaches the LO result

$$\langle G_E^2 \rangle_T \;=\; -\langle G_M^2 \rangle_T + \mathcal{O}(g^4).$$

Summary

- Wilson flow provides a powerful tool to study the deconfinement transition.
- Can use flow to create continuum-defined Polyakov loop-like operator.
- Provides a convenient way of renormalizing the Polyakov loop.
- Together with $\langle\bar{\psi}\psi\rangle_r$, provide complete set of observables to study the transition.
- The flow behavior of electric and magnetic parts of the gluon condensate are very different.
- In principle condensates at small flow can be used to calculate renormalized condensates (Suzuki 1304.0533). But with leading order renormalization coefficients, difficult to do $t \to 0$ extrapolation even with N_t=10 lattices. Larger N_t will help.

EoS results: Kanaya, wednesday; FlowQCD, new results.