Supergravity from Gauge Theory

Evan Berkowitz
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1602.01473 1603.03055, EB, Masanori Hanada, Jonathan Maltz
1606.04948 1606.04951, Monte Carlo String + M-theory Collaboration (MCSMC)
EB, Enrico Rinaldi, Masanori Hanada, Pavlos Vranas, Goro Ishiki, Shinji Shimasaki, So Matsuura
+ forthcoming work

LLNL-PRES-692051
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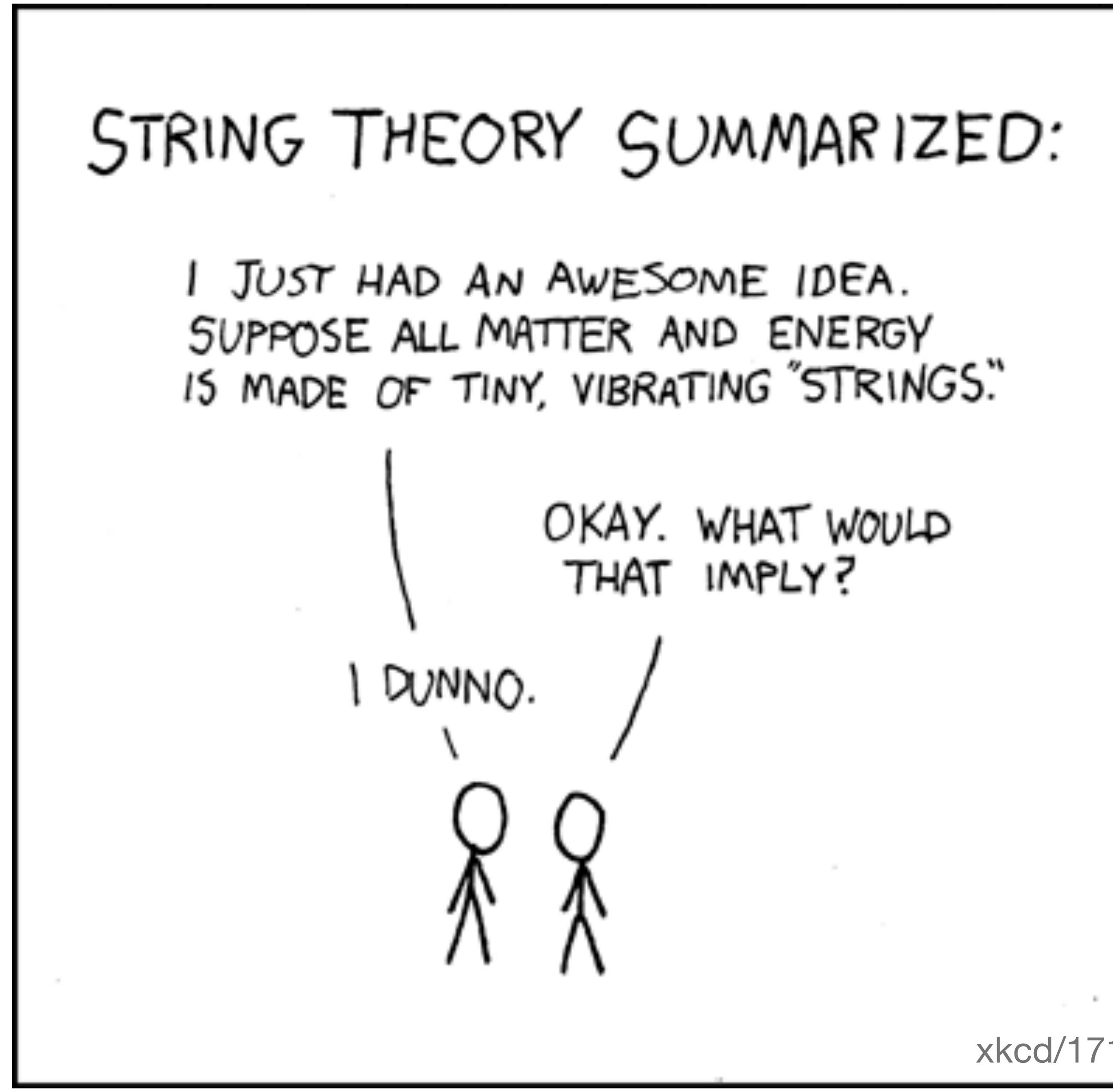
Outline

- Gauge/Gravity Duality & Motivation
- The BFSS Matrix Model
- Data Collection
- Fitting
- BFSS matches SUGRA



xkcd/171

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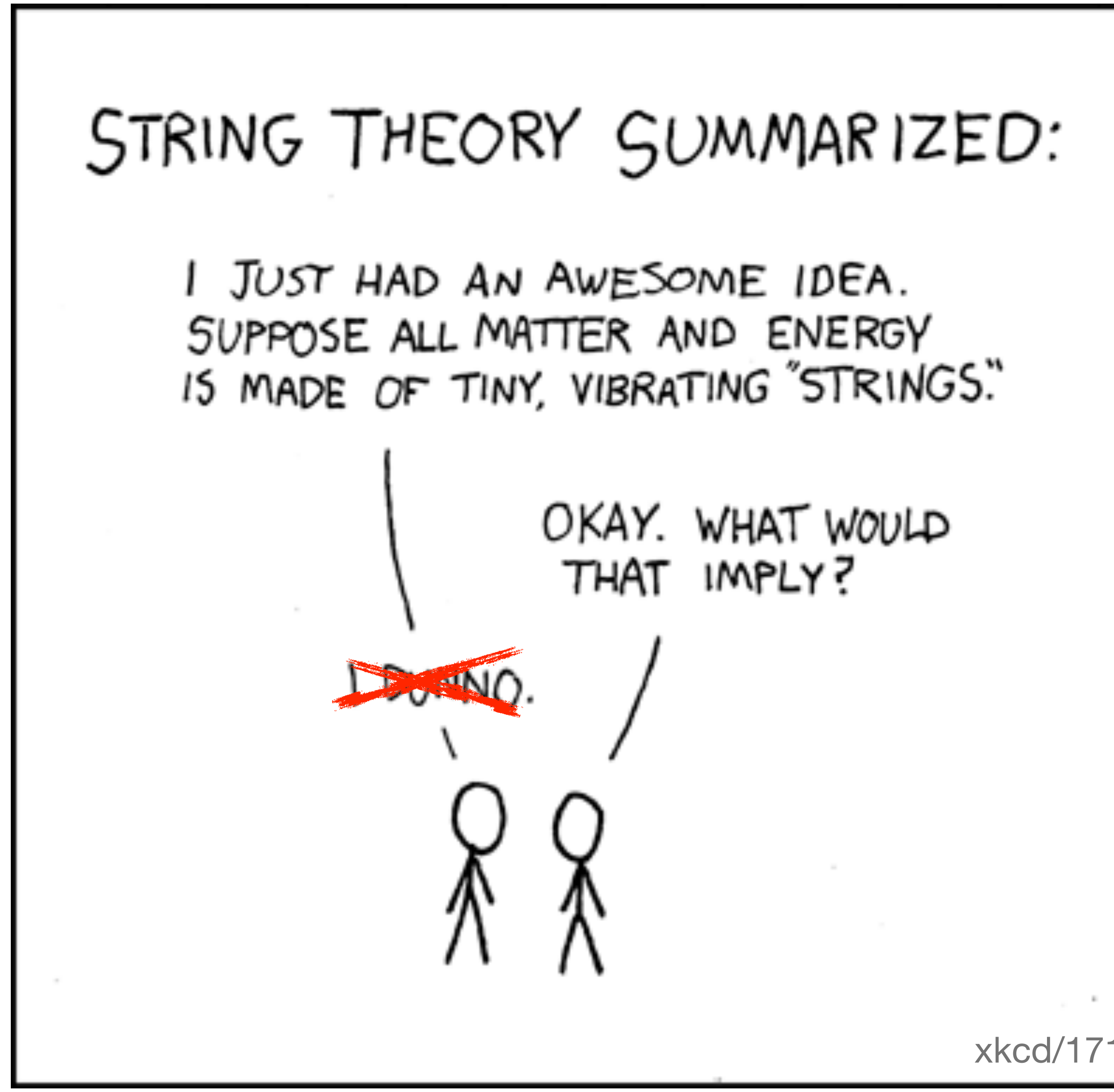

xkcd/171

Gauge / Gravity Duality Conjecture

Maldacena hep-th/9711200

Gauge / Gravity Duality Conjecture

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Quantum Gravity

Gauge Theory

Weak / Strong

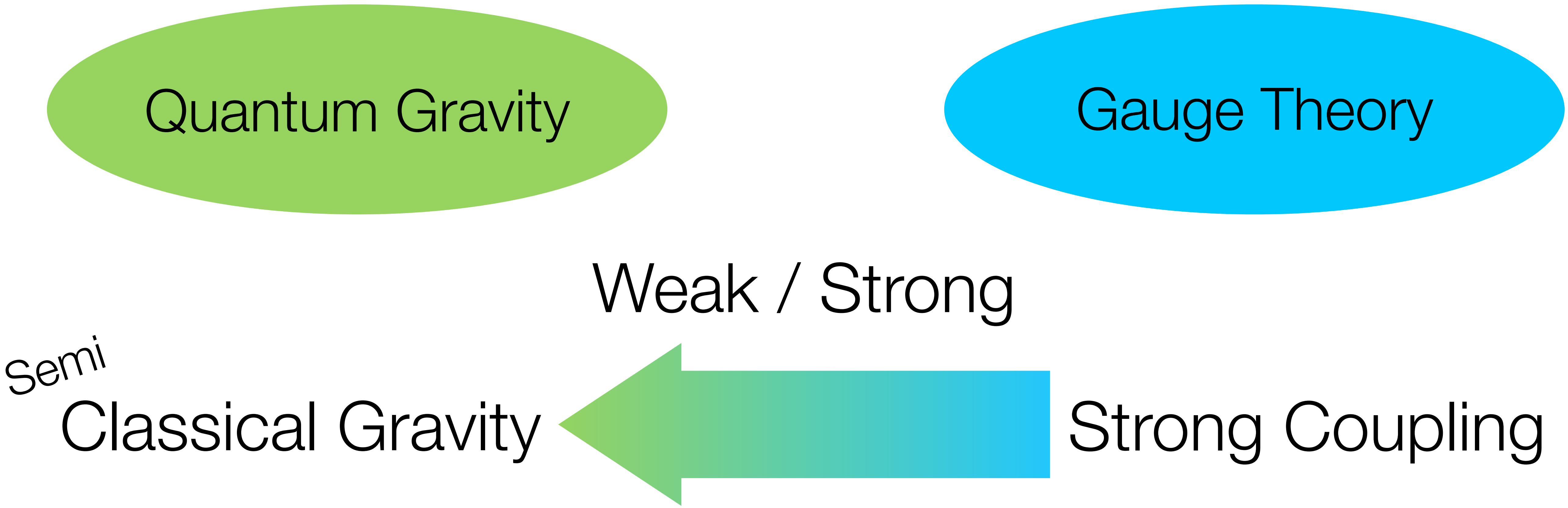
Semi Classical Gravity → Strong Coupling

$$\frac{\eta}{s} = \frac{1}{4\pi}$$ (NOT QCD!)

KSS hep-th/0405231

Gauge / Gravity Duality Conjecture

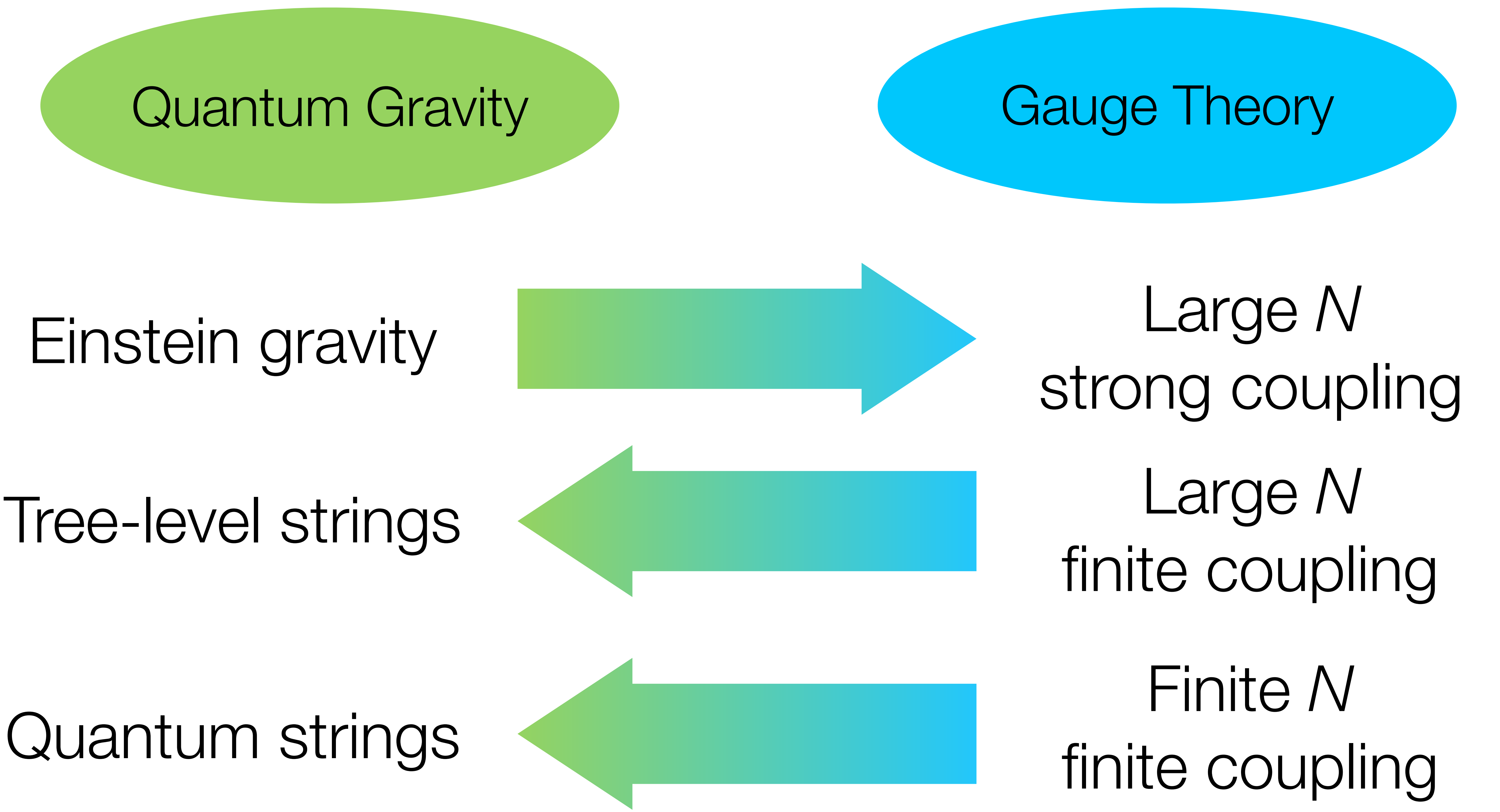
Maldacena hep-th/9711200



$$S = \frac{A}{4}$$ (After B&H)

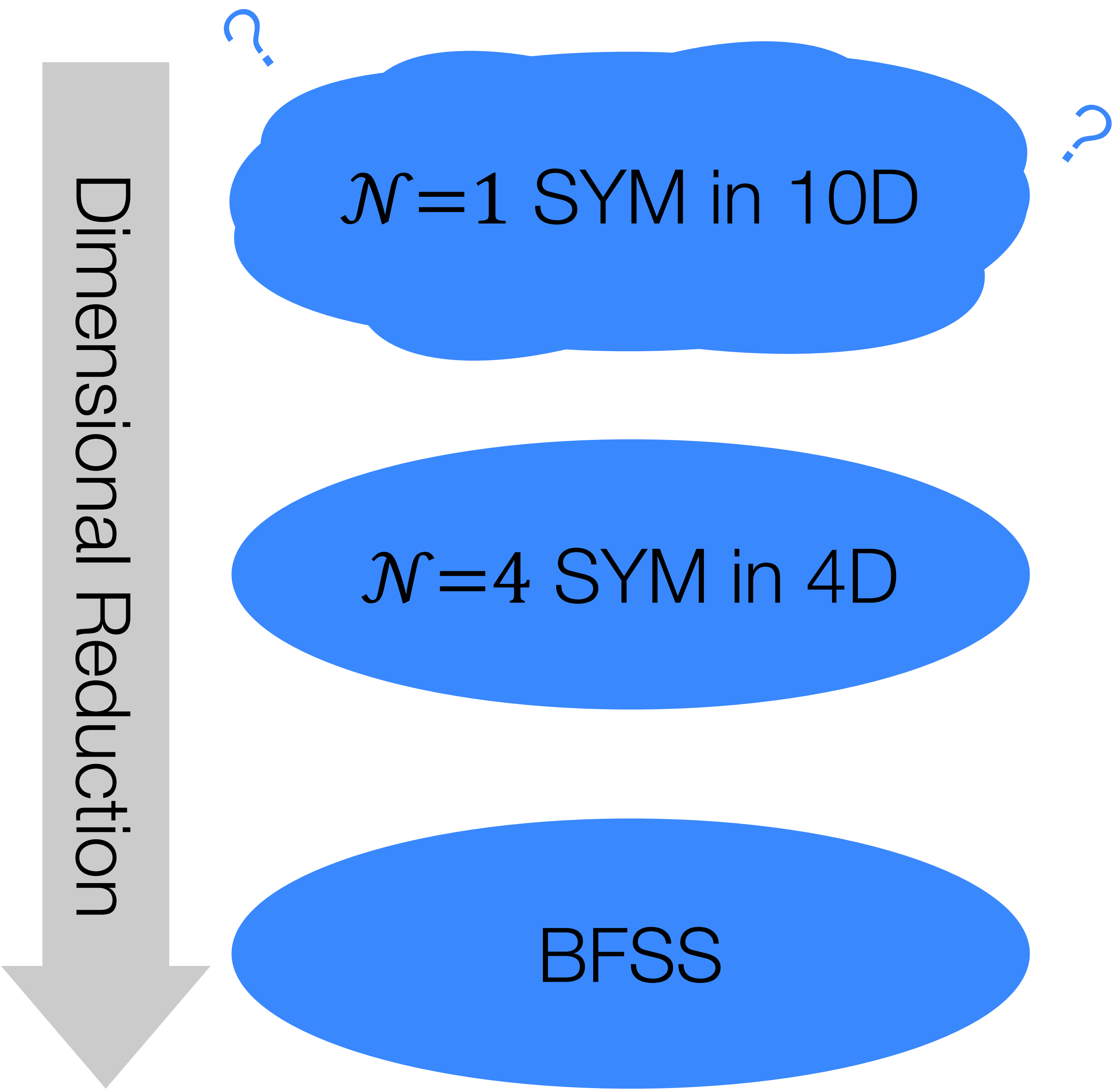
Gauge / Gravity Duality Conjecture

Maldacena hep-th/9711200



Numerical Approaches

Catterall, Kaplan, and Unsal 0903.4881



- Large number of spacetime dimensions
- Naive lattice breaks SUSY
- Numerical fine-tunings?

Kaplan, Katz and Unsal hep-lat/0206019
Kaplan and Unsal hep-lat/0503039
Schaich and Catterall 1505.03135
1508.00884

- 0+1 dimensional
- No fine-tunings
- Easy numerics

BFSS Matrix Model

Banks Fischler Shenker Susskind hep-th/9610043

$$L = \frac{1}{2g_{YM}^2}\mathrm{Tr}\left\{(D_t X_M)^2 + [X_M, X_{M'}]^2 + i\bar{\psi}^\alpha D_t \psi^\beta + \bar{\psi}^\alpha \gamma^M_{\alpha\beta}[X_M, \psi^\beta]\right\}$$

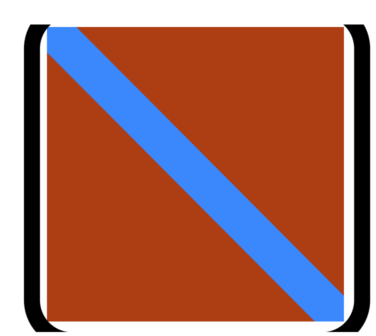
$$D_t \cdot = \partial_t \cdot - i[A_t, \cdot]$$

M: 1...9 X, ψ: bosonic, fermionic N×N matrices

α: 1...16 γ: left-handed part of 9+1D

- Obvious nonperturbative definition (discretized QM)
- Defined for all *N* and g_{YM}.
- Unitary

9 ×



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$$\lambda = g_{YM}^2 N$$

$$m_{D0} \sim g_s^{-1} \sim g_{YM}^{-2} \sim N$$

- 11D SUGRA at low temperature!
- Dimensionful coupling → easy scale setting

dimensionless temperature $\lambda^{-1/3} T$

BFSS conjecture:

maximally supersymmetric matrix quantum mechanics = M theory

BFSS Cartoon

$$L = \frac{1}{2g_{YM}^2}\mathrm{Tr}\left\{(D_t X_M)^2 + [X_M, X_{M'}]^2 + i\bar{\psi}^\alpha D_t \psi^\beta + \bar{\psi}^\alpha \gamma^M_{\alpha\beta}[X_M, \psi^\beta]\right\}$$

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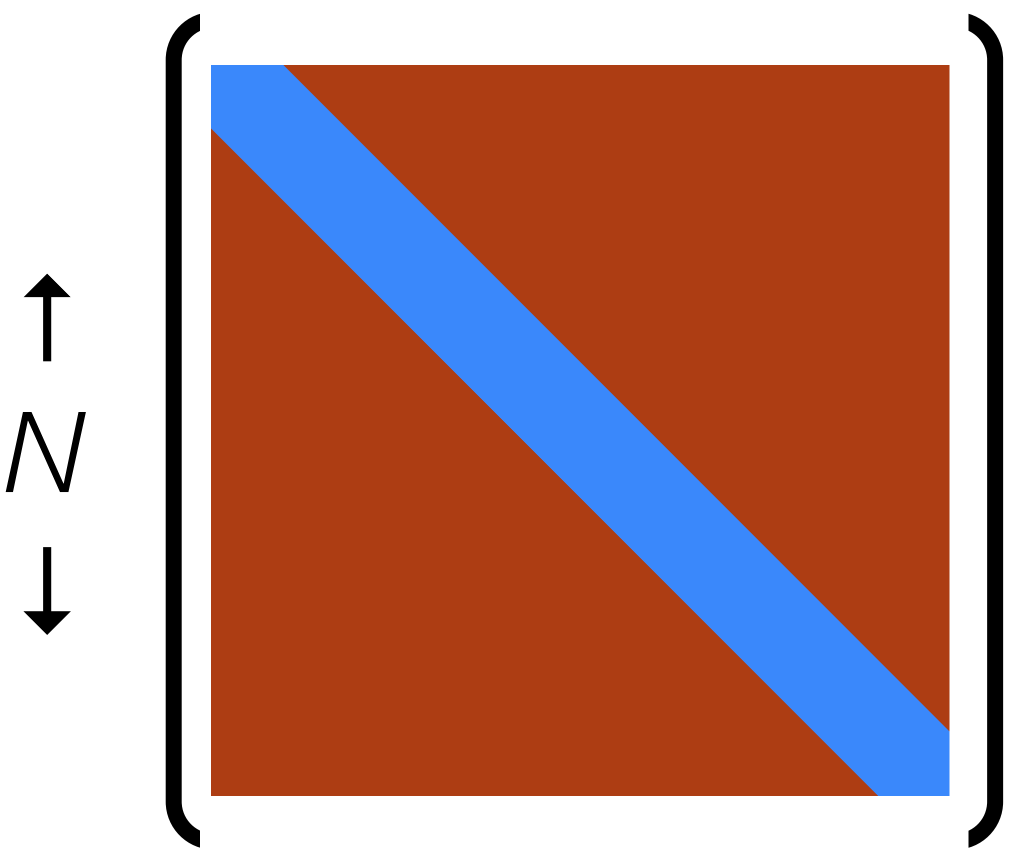
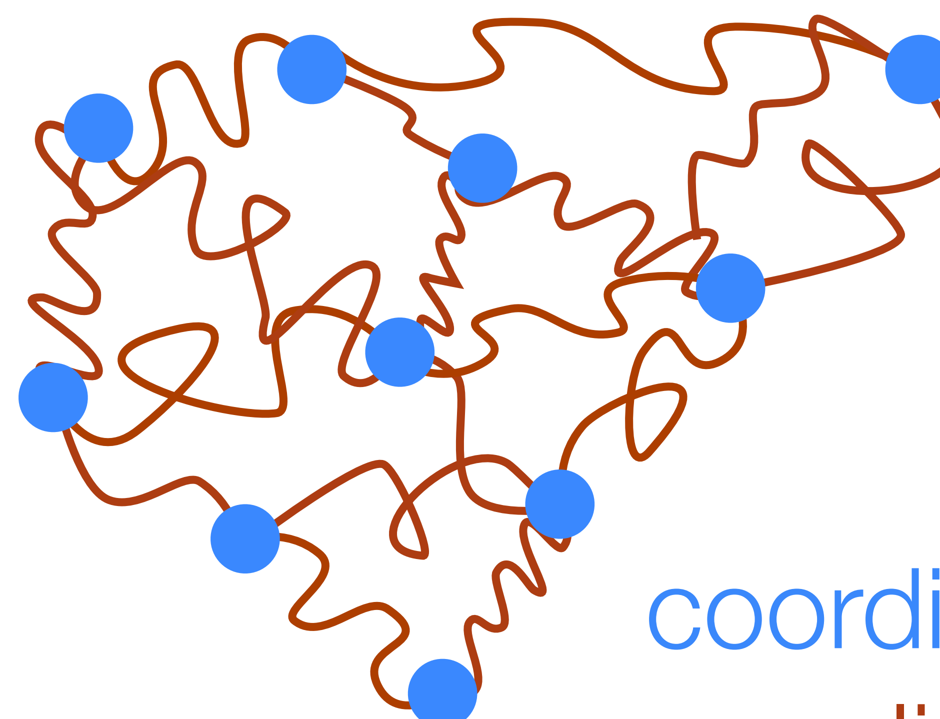




coordinates

couplings

Witten
hep-th/9510135

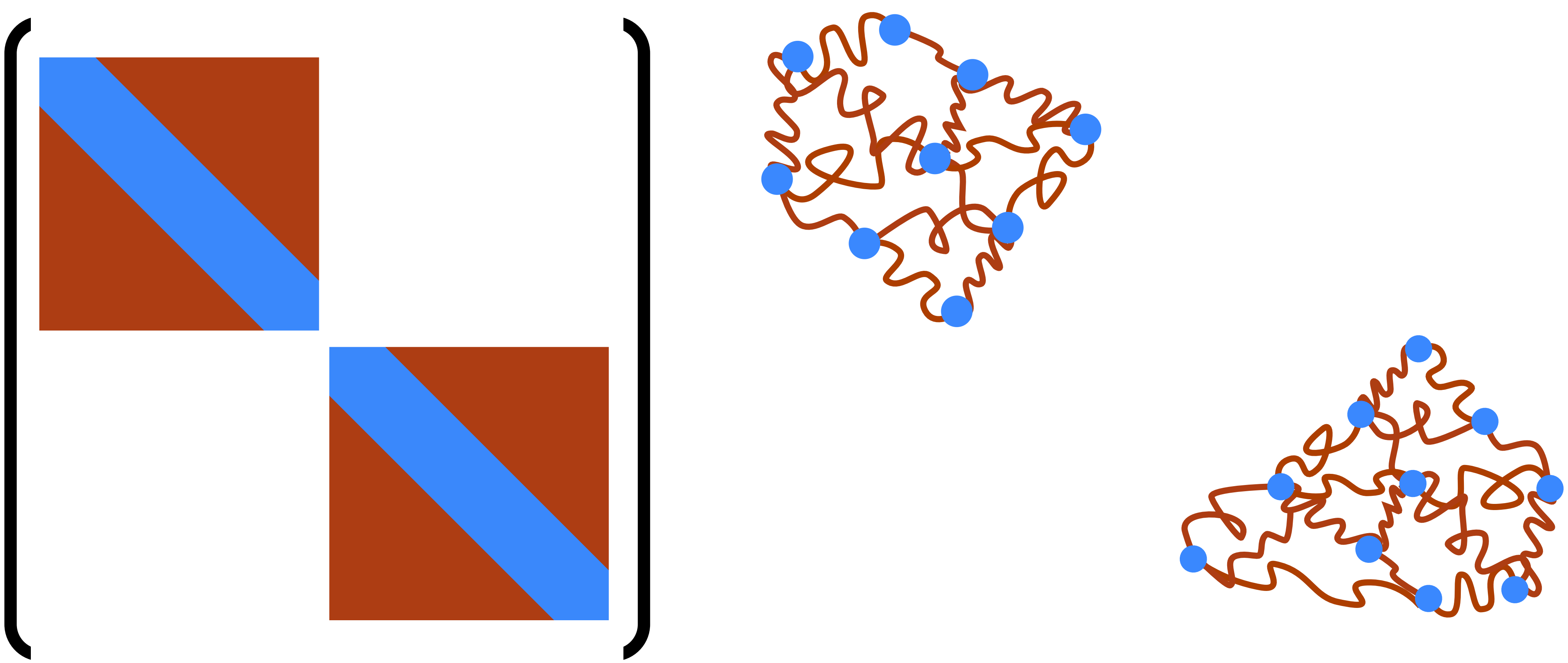
Single Black 0-Brane

BFSS Has Flat Directions

de Wit, Lüscher, Nicolai - Nucl. Phys. B 320, 135 (1989)

$$L = \frac{1}{2g_{YM}^2}\mathrm{Tr}\left\{(D_t X_M)^2 + [X_M, X_{M'}]^2 + i\bar{\psi}^\alpha D_t \psi^\beta + \bar{\psi}^\alpha \gamma^M_{\alpha\beta}[X_M, \psi^\beta]\right\}$$

Potential ~ $\mathrm{Tr}\left\{[X_M, X_{M'}]^2\right\}$



2 black branes

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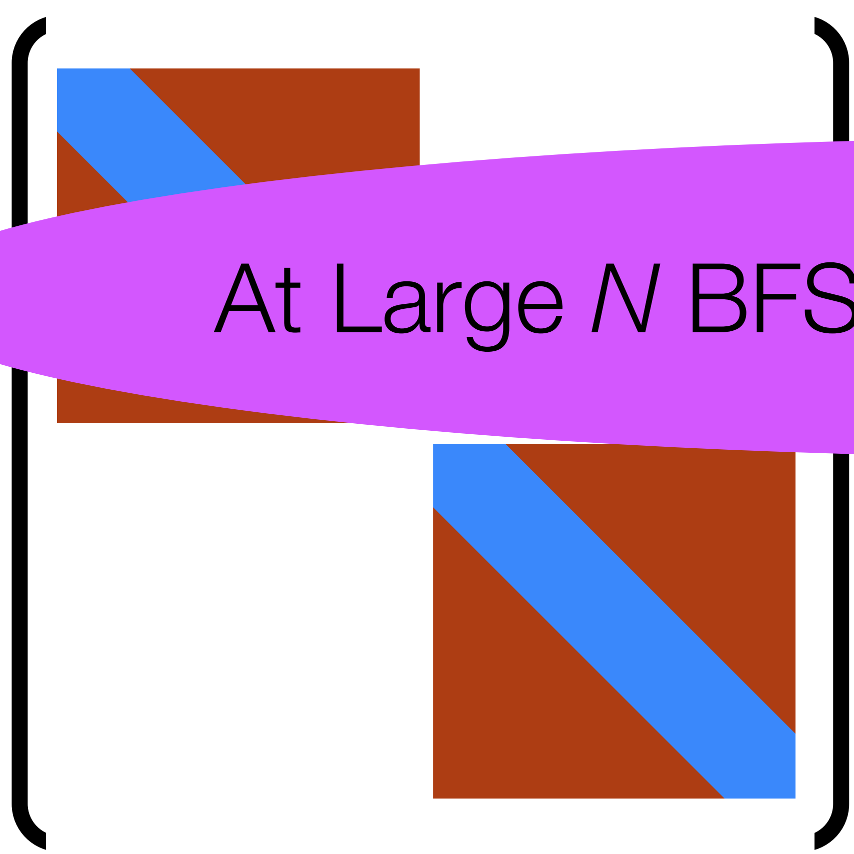
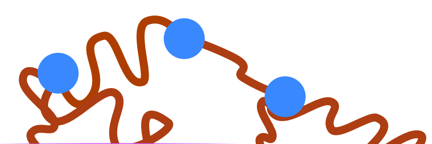
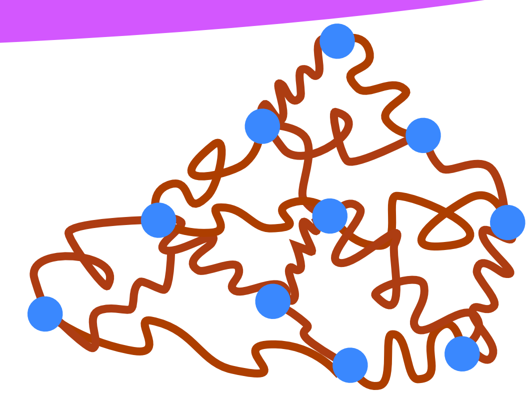




At Large *N* BFSS is a 2nd-quantized theory!



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BFSS Cartoon

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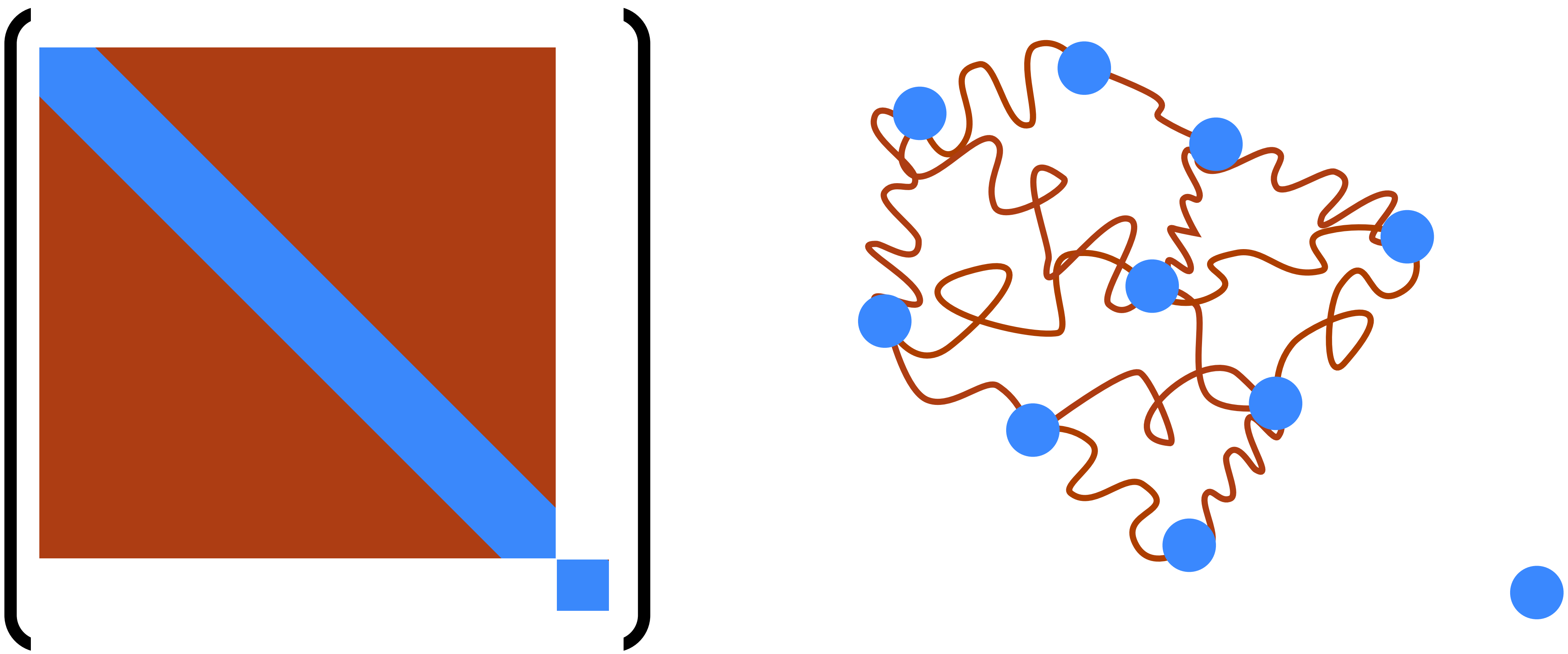

1 Black 0-Brane + 1 D0 Radiation

BFSS Cartoon

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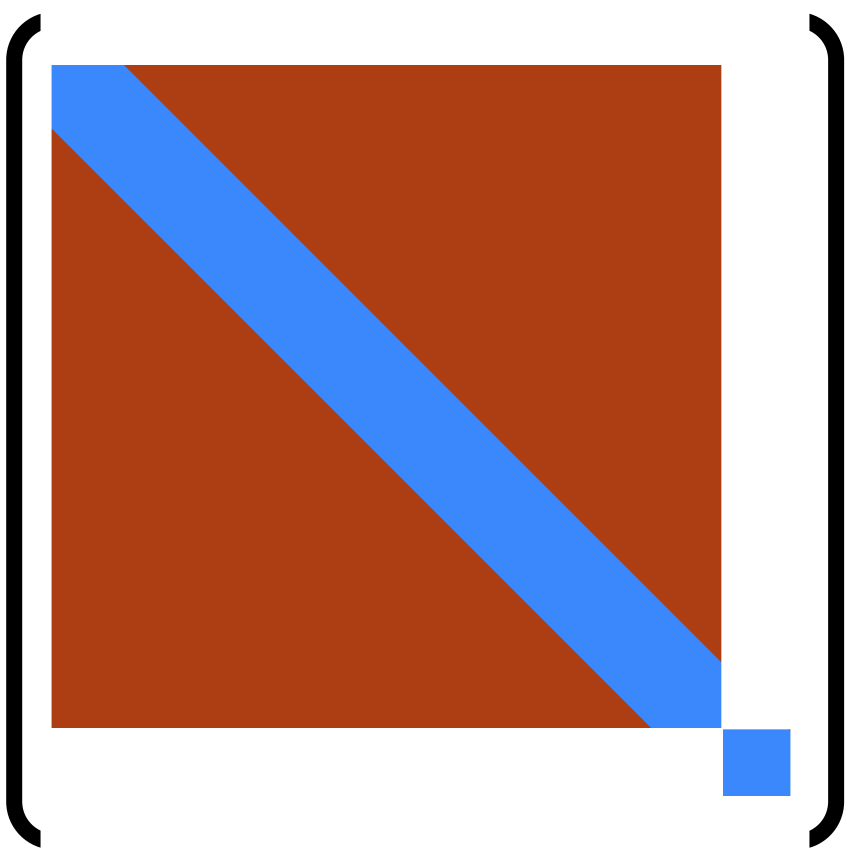
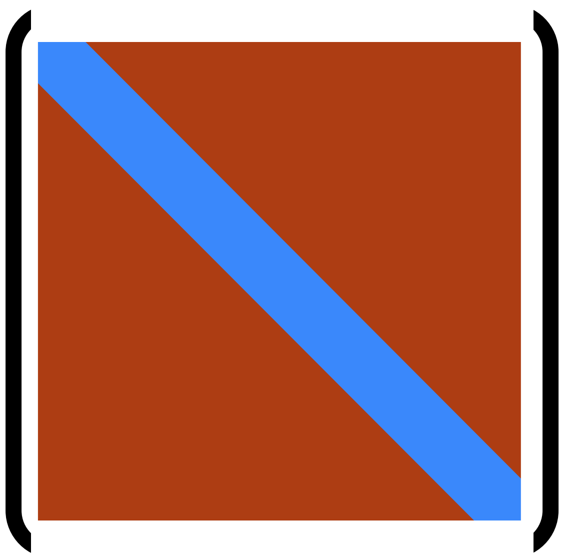


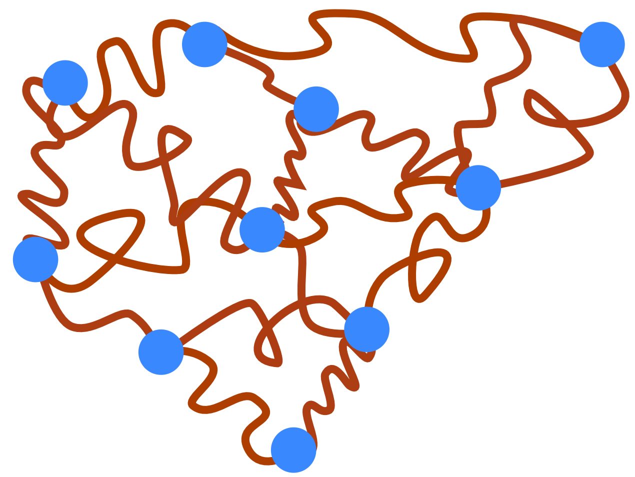
- Chaos → flat directions will ultimately be found
- Flat directions + chaos → evaporation by D0 emission has specific heat < 0! 1602.01473 1603.03055

1 Black 0-Brane + 1 D0 Radiation

Monte Carlo Considerations

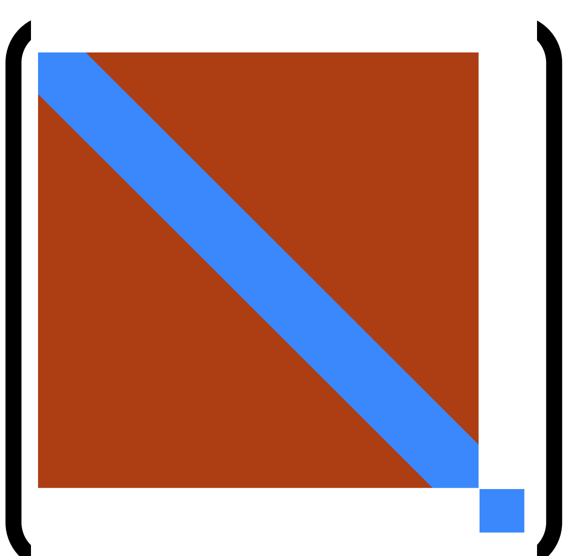
Goal: study a metastable state?

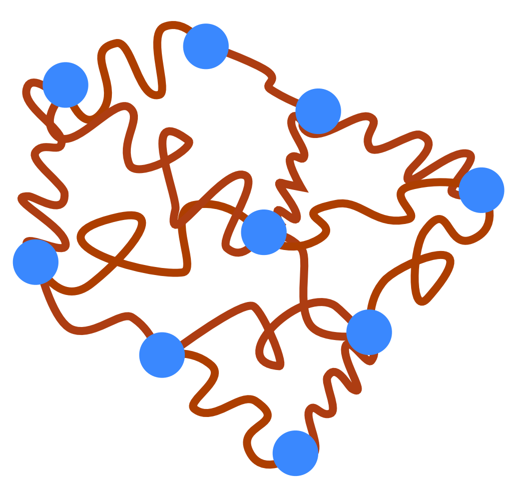




DOF ~ N^2

$t_{recurrence} \sim e^{+N^2}$





DOF ~ $(N-1)^2$

Monte Carlo Considerations

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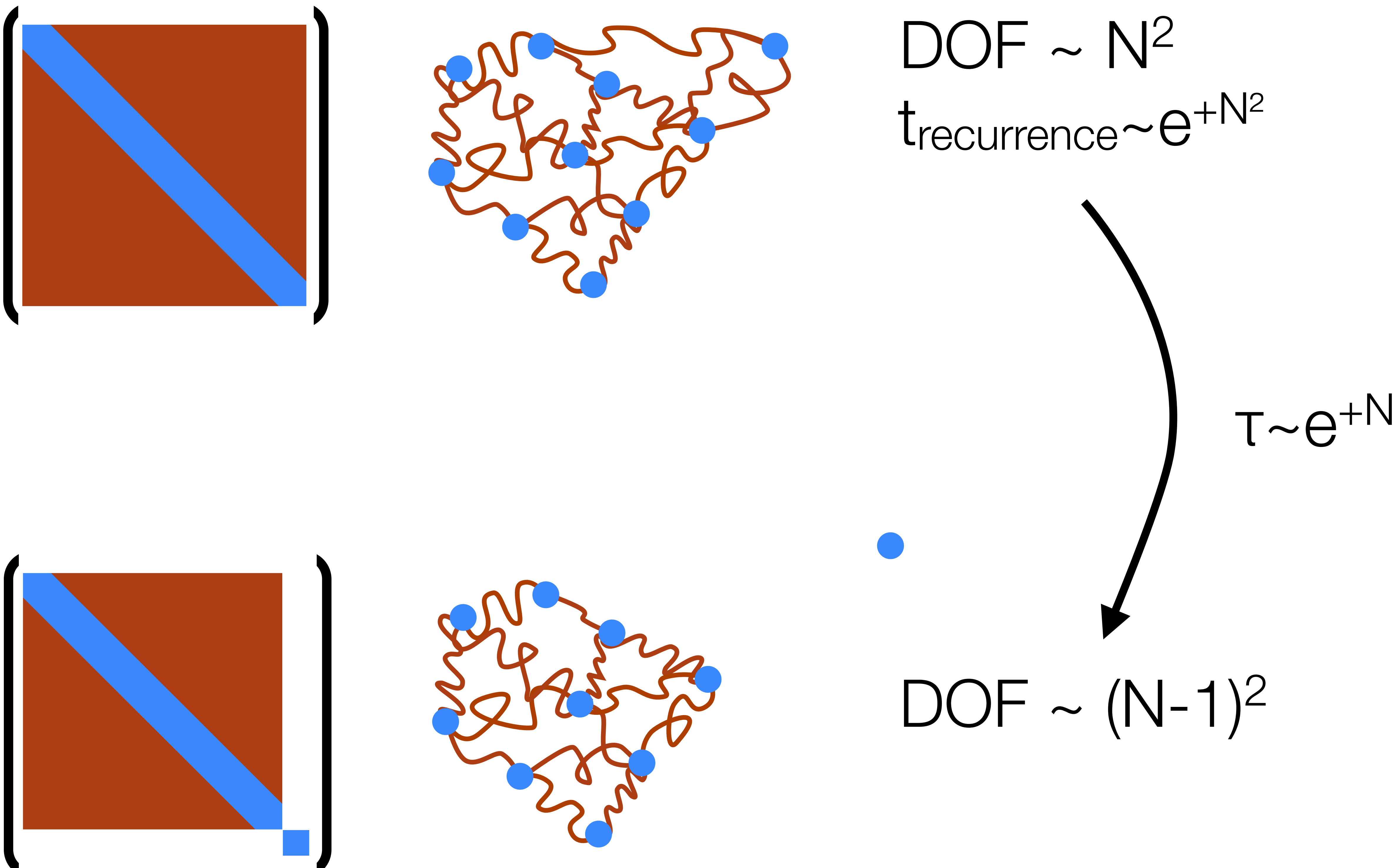

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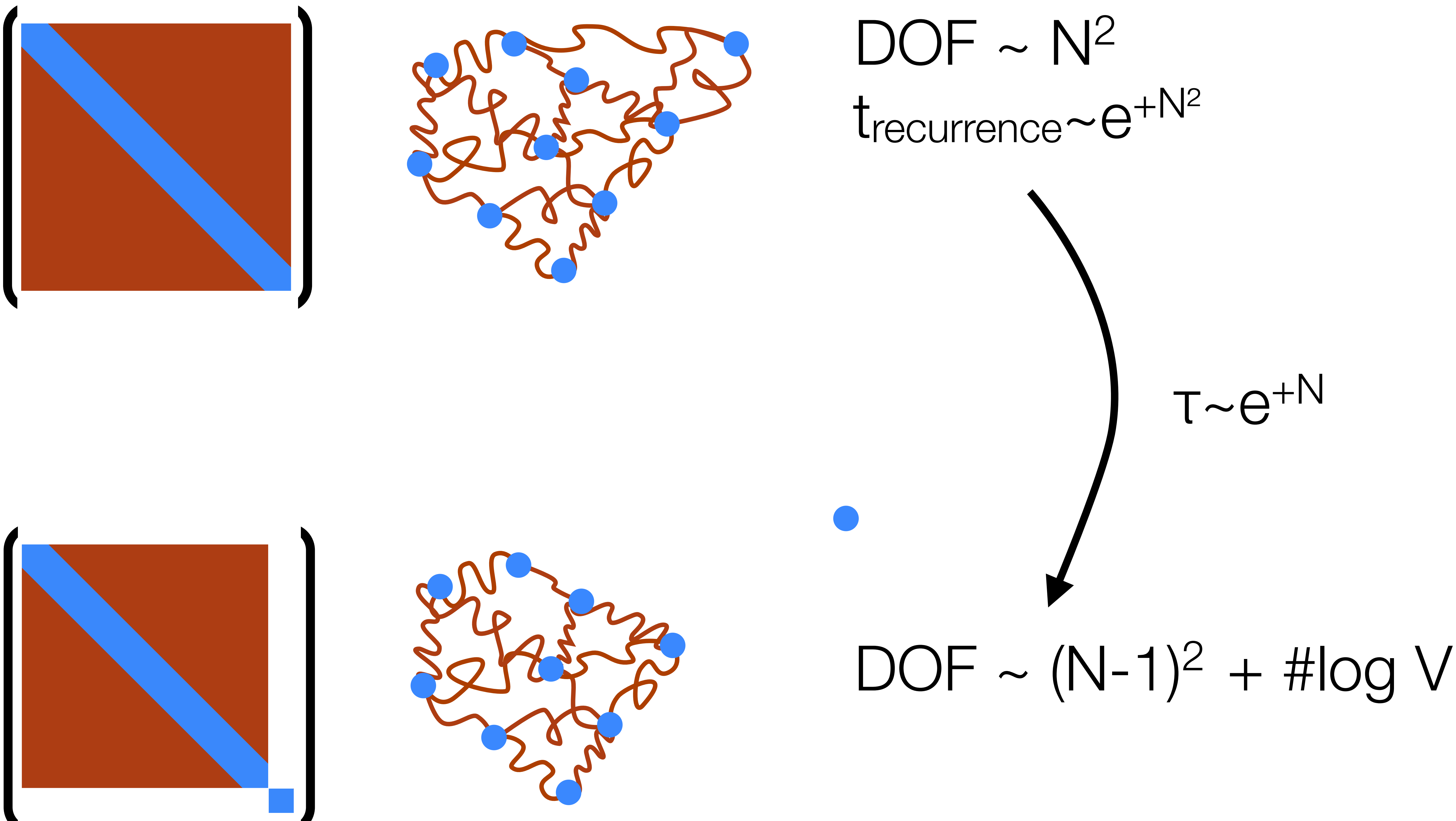
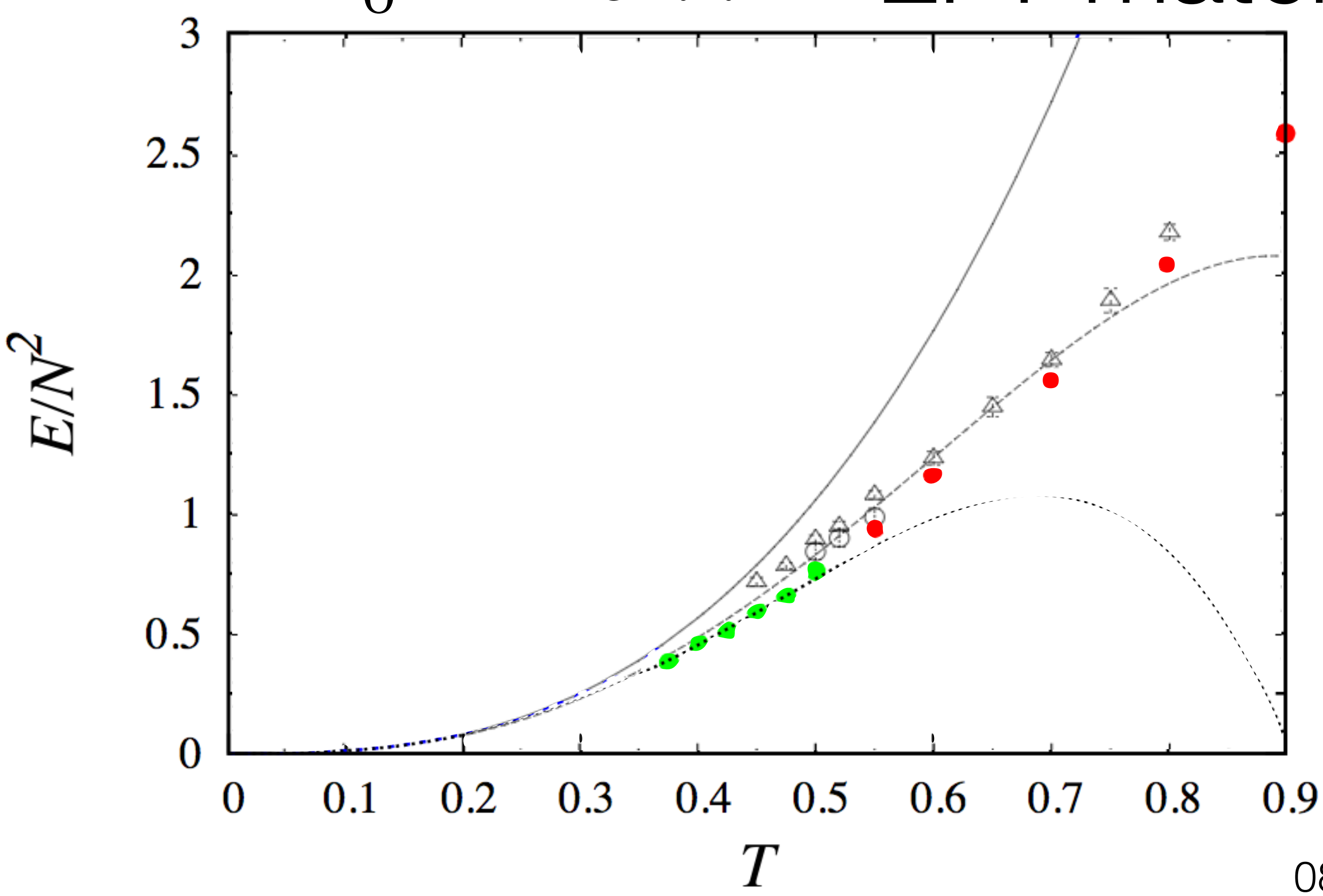


Aim: Test Duality Nonperturbatively: Finite T BH Internal Energy in SUGRA $\stackrel{?}{=}$ BFSS

$$E/N^2 = \frac{\left(a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8} + \cdots\right)}{N^0} + \frac{\left(b_0 T^{0.4} + b_1 T^{2.2} + \cdots\right)}{N^2} + \mathcal{O}\left(\frac{1}{N^4}\right)$$

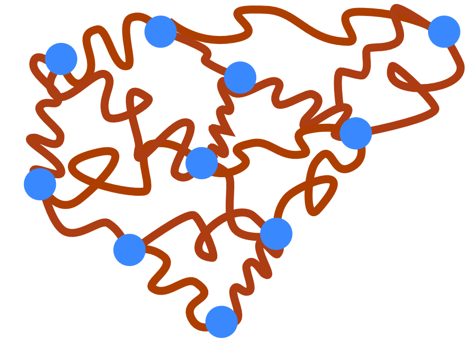
Known: $a_0 = \quad 7.41$ classical SUGRA

$b_0 = -5.77$ EFT matching



- No large N
- No continuum
- a_0 fixed by hand
- NLO Disagreement

0811.3102 Hanada-Hyakutake-Nishimura-Takeuchi
1503.08499 Kadoh-Kamata



Black Hole Internal Energy

MCSMC - arXiv:1606.04948 arXiv:1606.04951

IIA String theory

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Fast Scrambling arXiv:1512.00019 Gur-Ari, Hanada, Shenker $t_{\text{scramble}} \sim \log N$

SUGRA arXiv:0707.4454 Anagnostopoulos et al.
arxiv:0803.4273 Catterall+Wiseman
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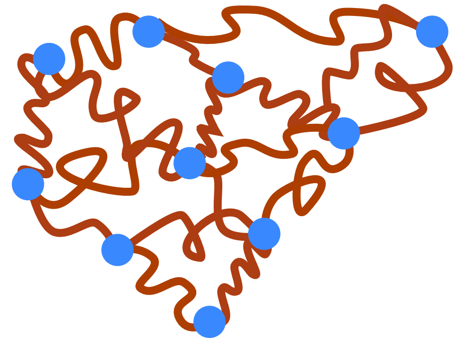
arXiv:1506.01366 Filev, O'Connor

Finite N arXiv:0811.3102 Hanada, Hyakutake, Nishimura, Takeuchi
arXiv:1311.5603 arXiv:1603.00538 Hanada, Hyakutake, Ishiki, Nishimura
arXiv:1606.04948 arXiv:1606.04951 MCSMC

Polyakov loop arXiv:0811.2081 Hanada, Miwa, Nishimura, Takeuchi

2-point functions arXiv:1108.5153 Hanada, Nishimura, Sekino, Yoneya, 2009, 2011



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Example MC History T=0.5 N=24 L=32

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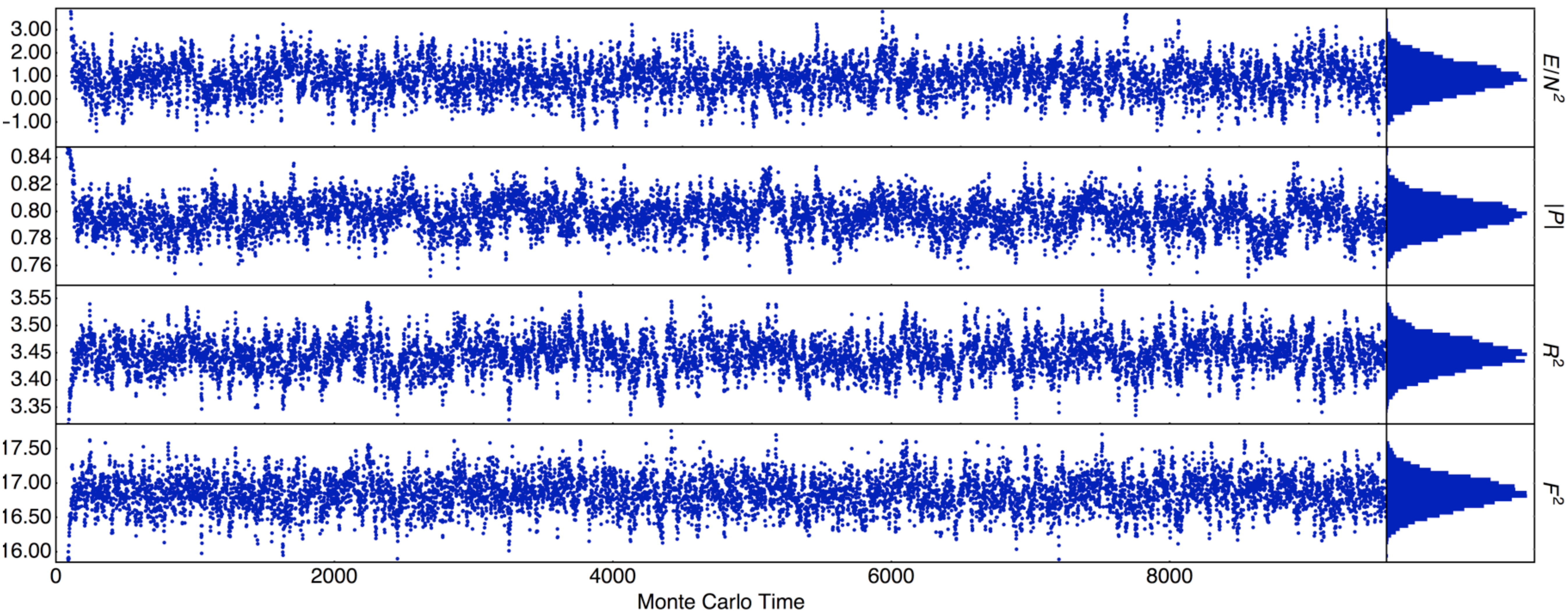
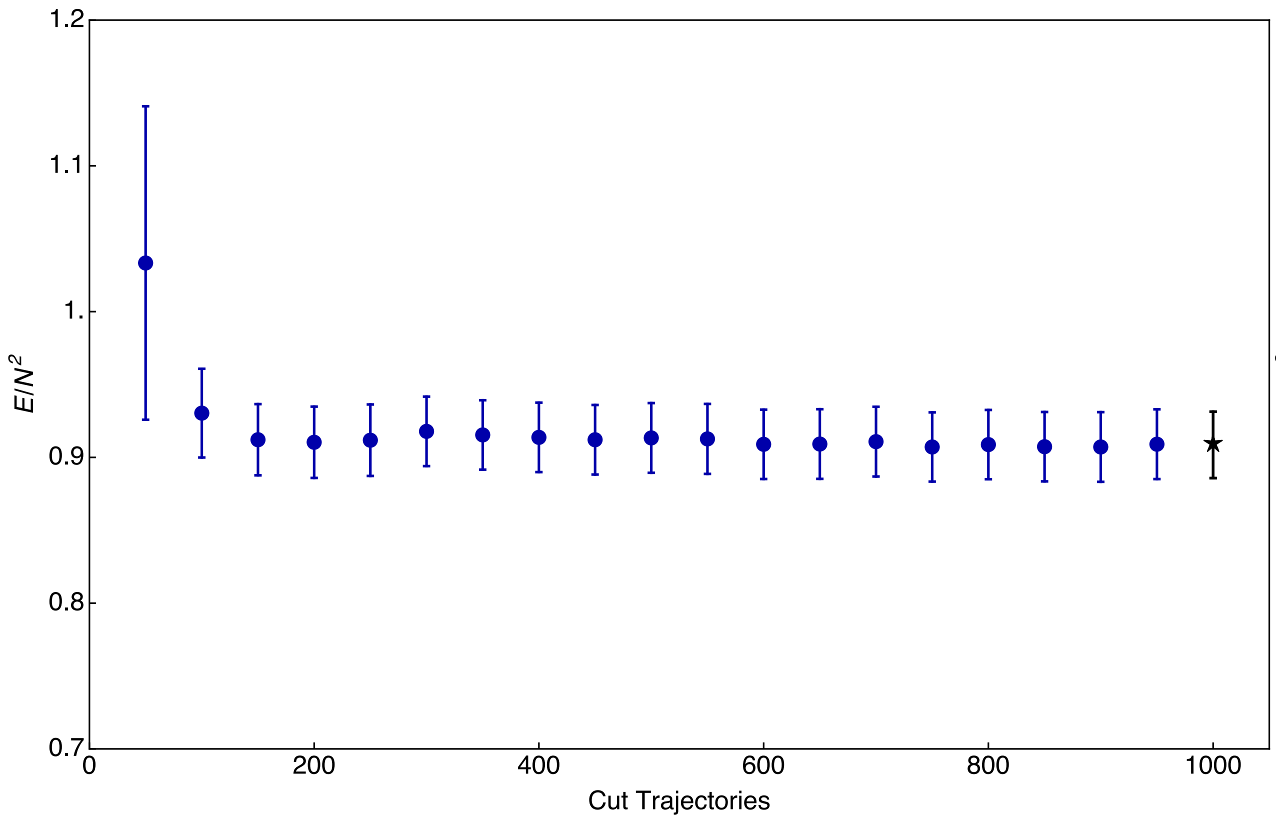


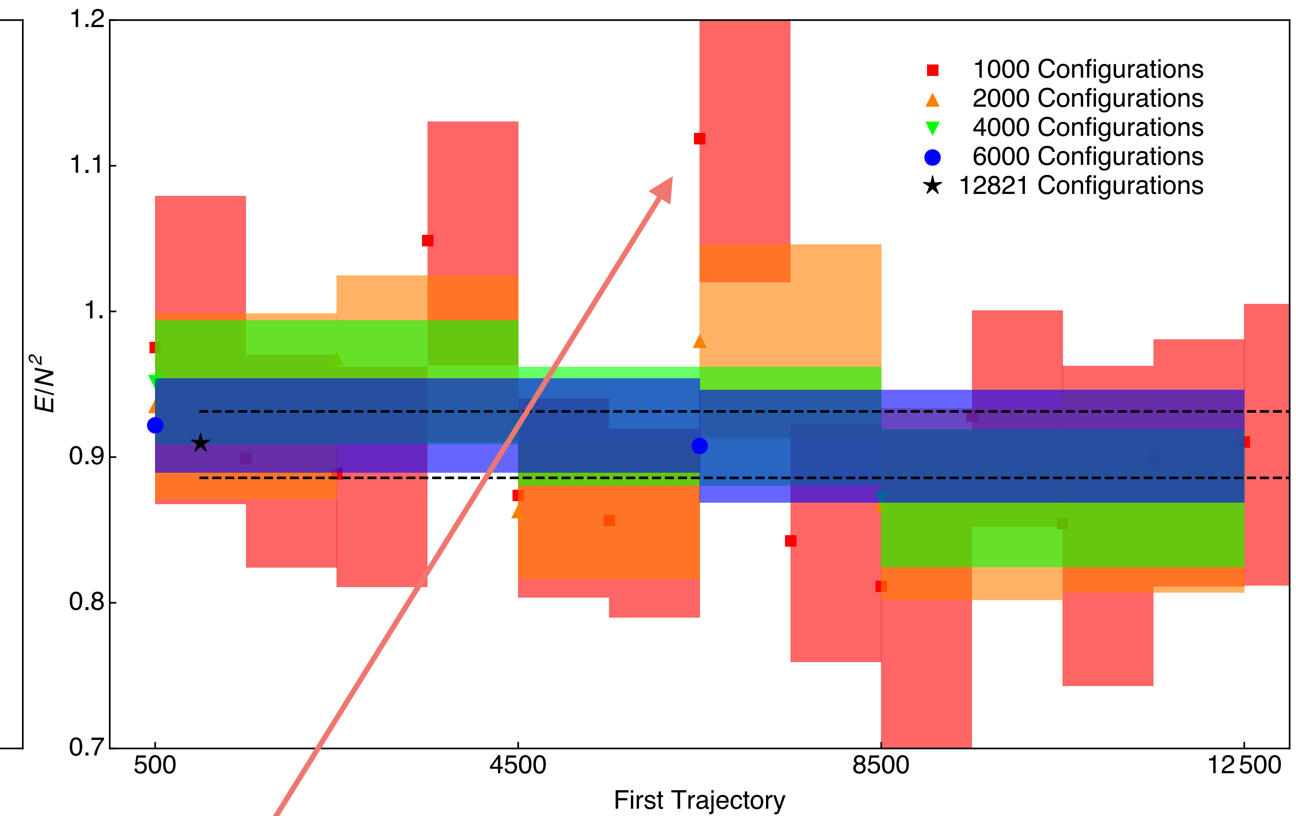
Required Statistics

T=0.5 N=16 L=32

MCSMC - arXiv:1606.04948 arXiv:1606.04951



Thermalization cut

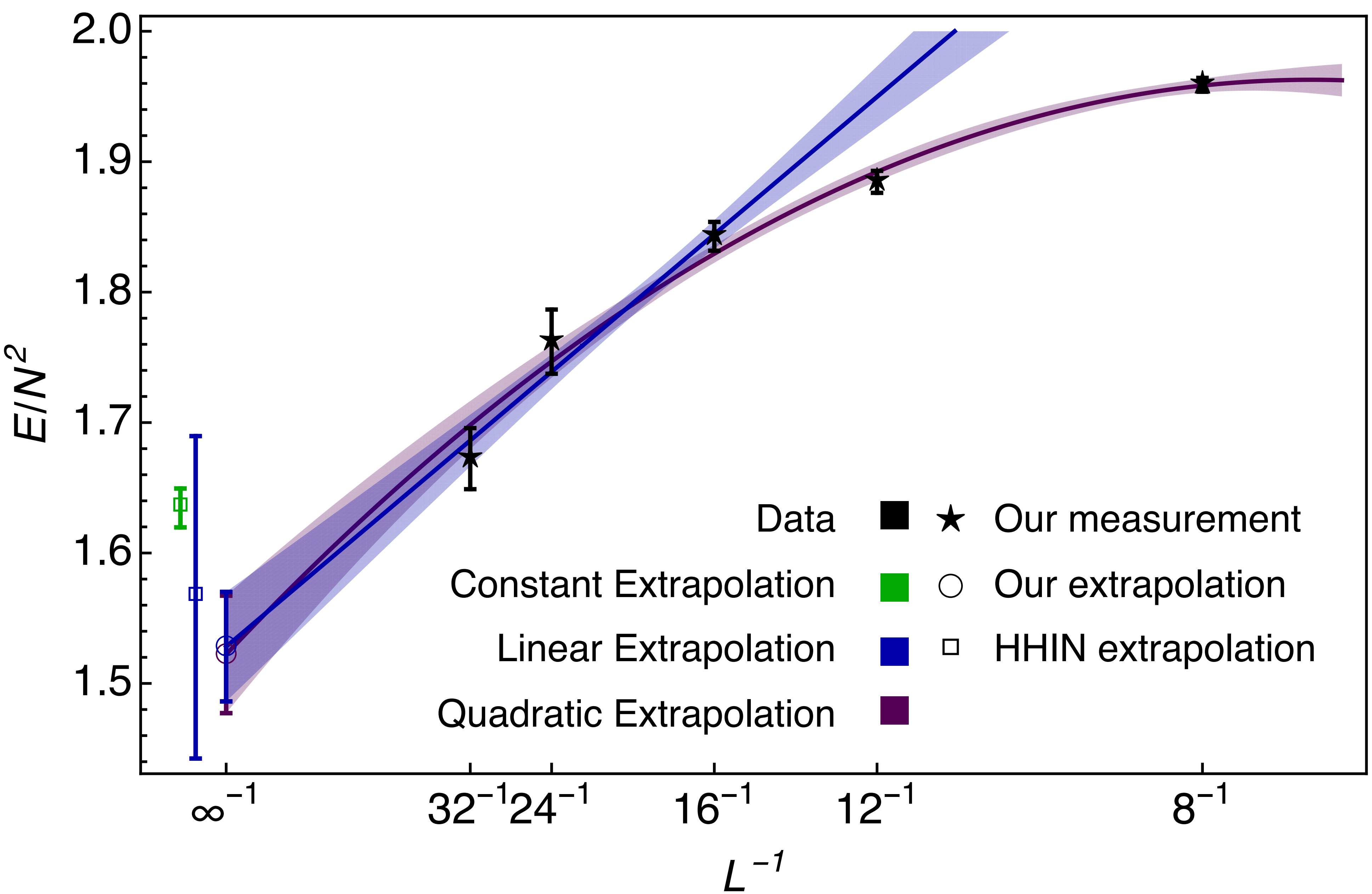


Statistical stability

long-lived fluctuations

Continuum Extrapolations

MCSMC - arXiv:1606.04948 arXiv:1606.04951



Fixed N T=0.7 N=16

$$\frac{E}{N^2} = e_0 + \frac{e_1}{L} + \frac{e_2}{L^2} + \mathcal{O}\left(L^{-3}\right)$$

Continuum Extrapolations

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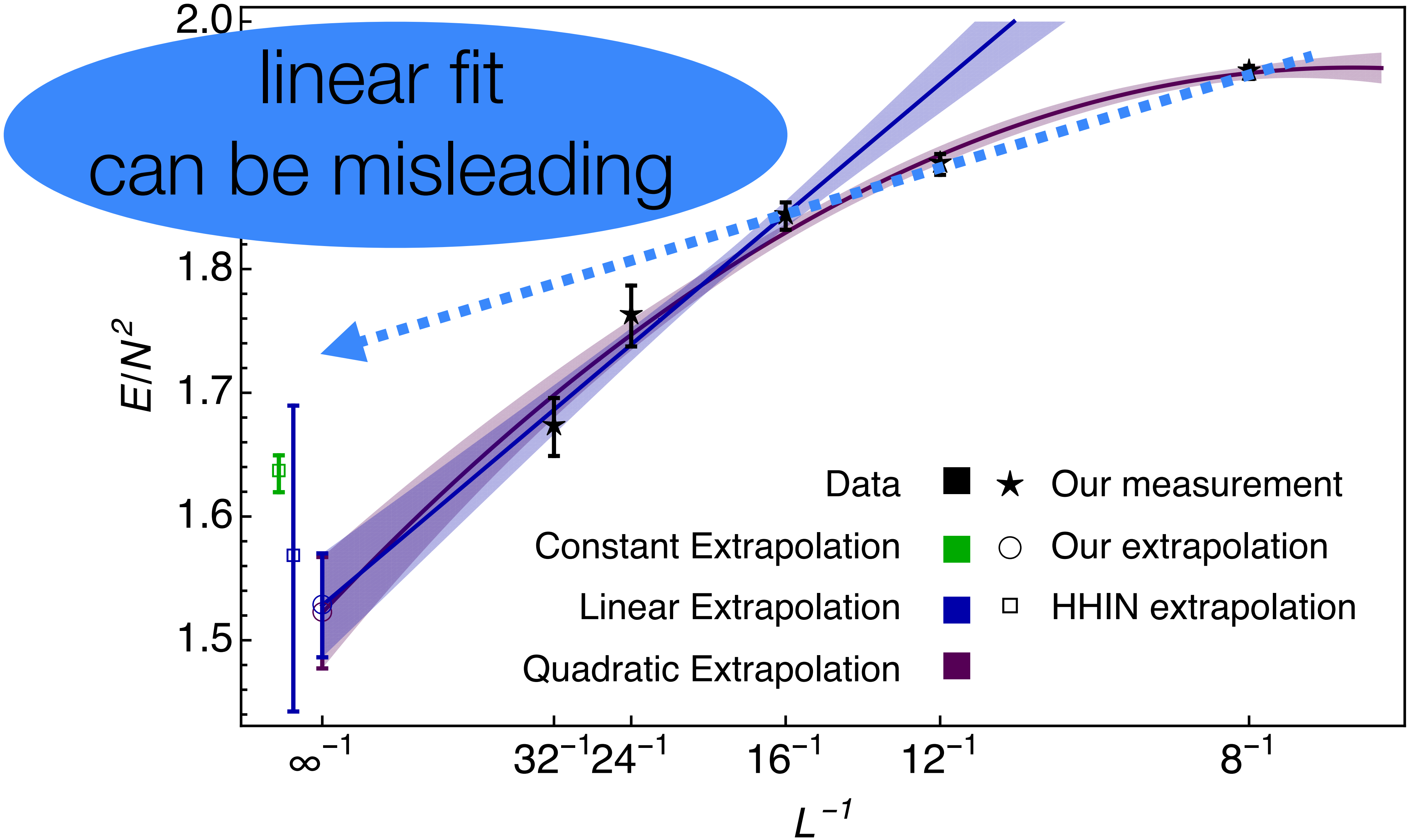

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Simultaneous Extrapolation T=0.5

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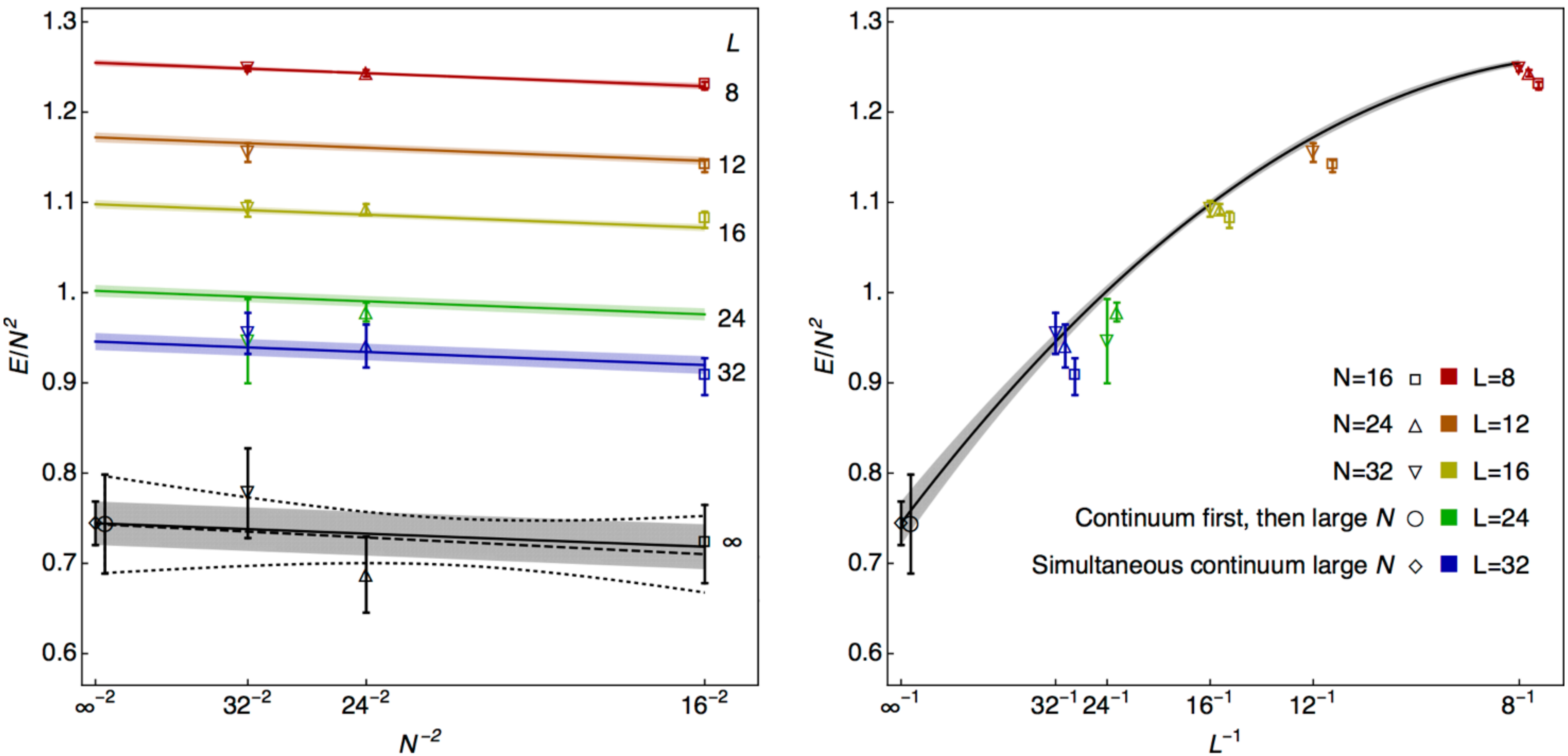

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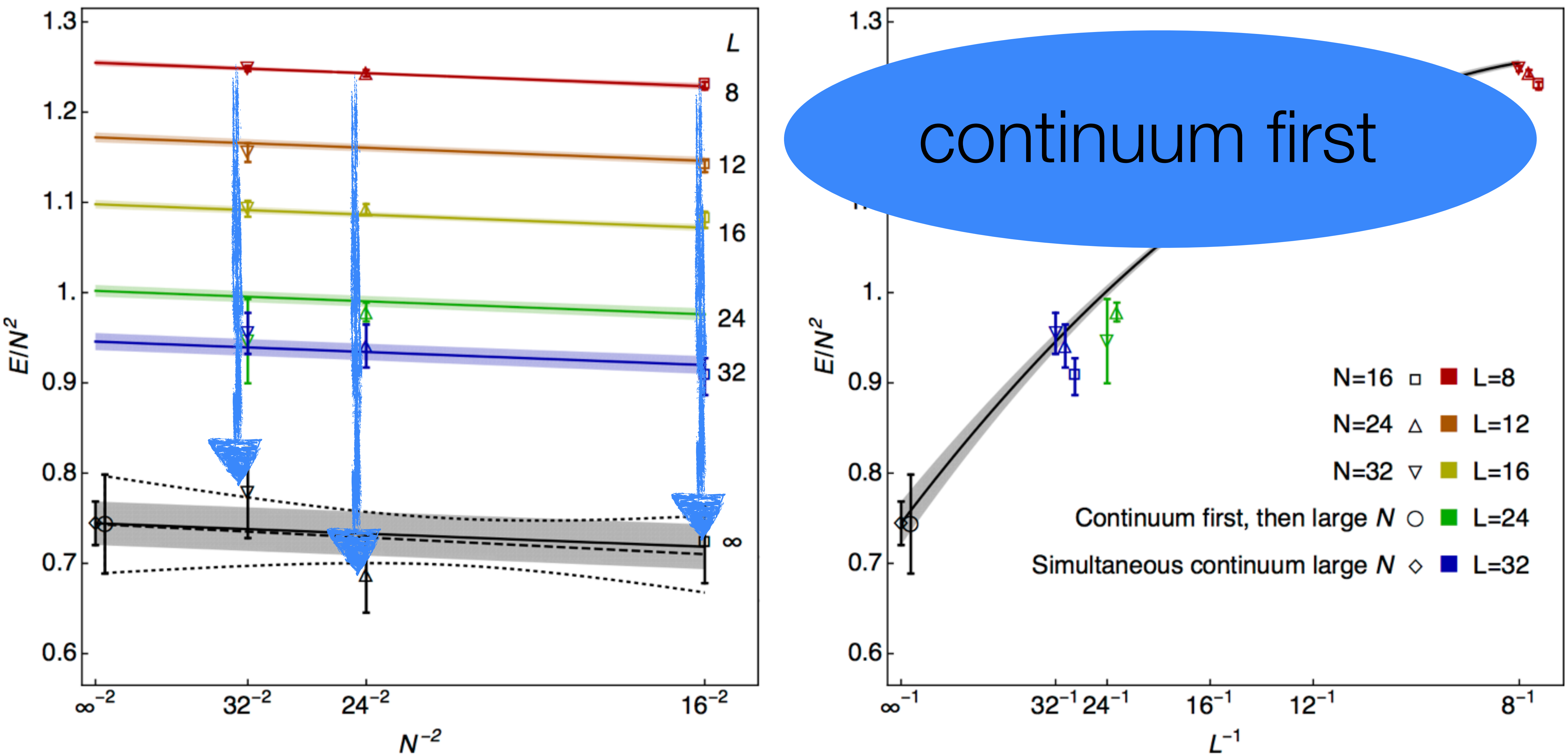

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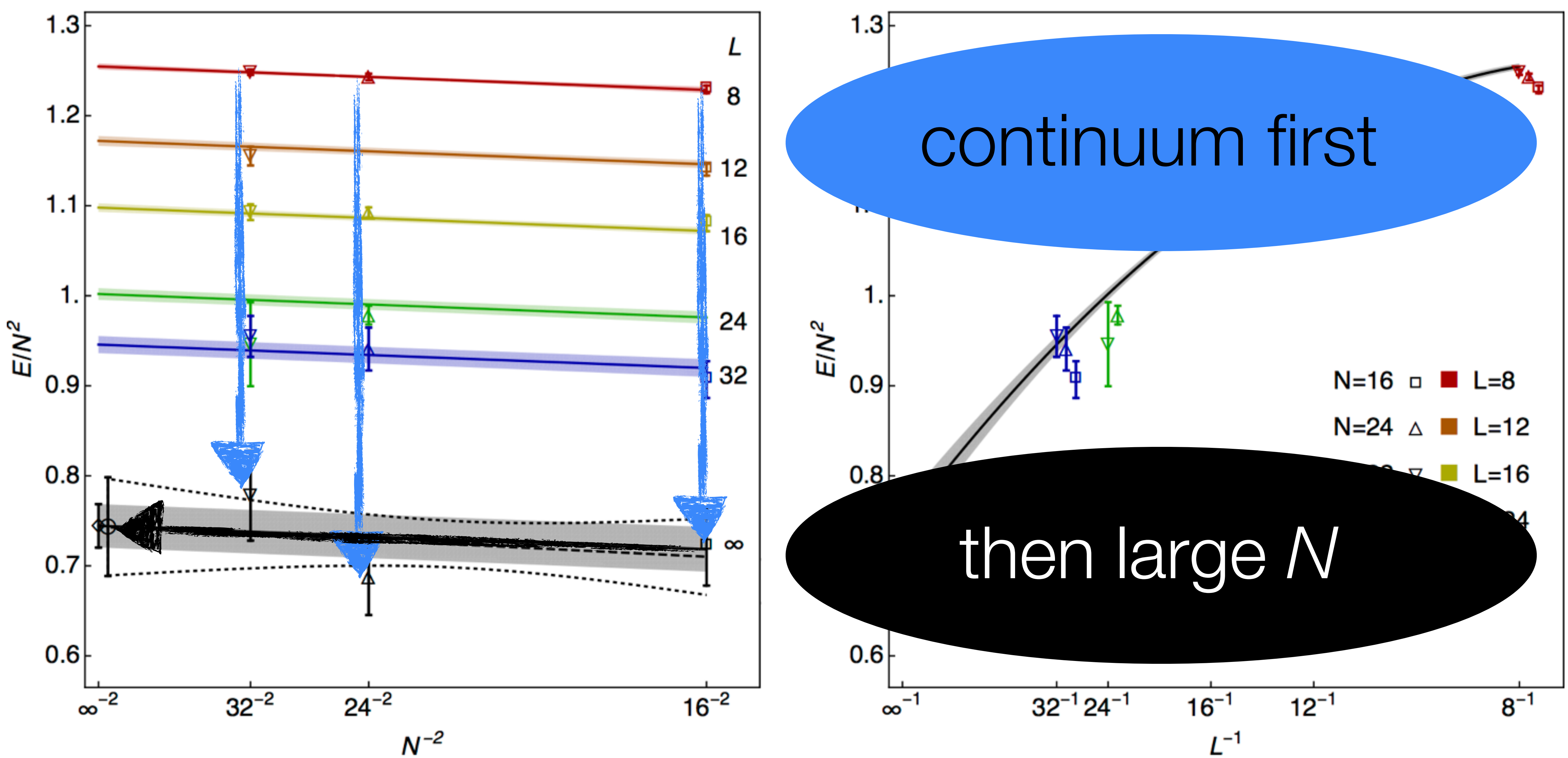

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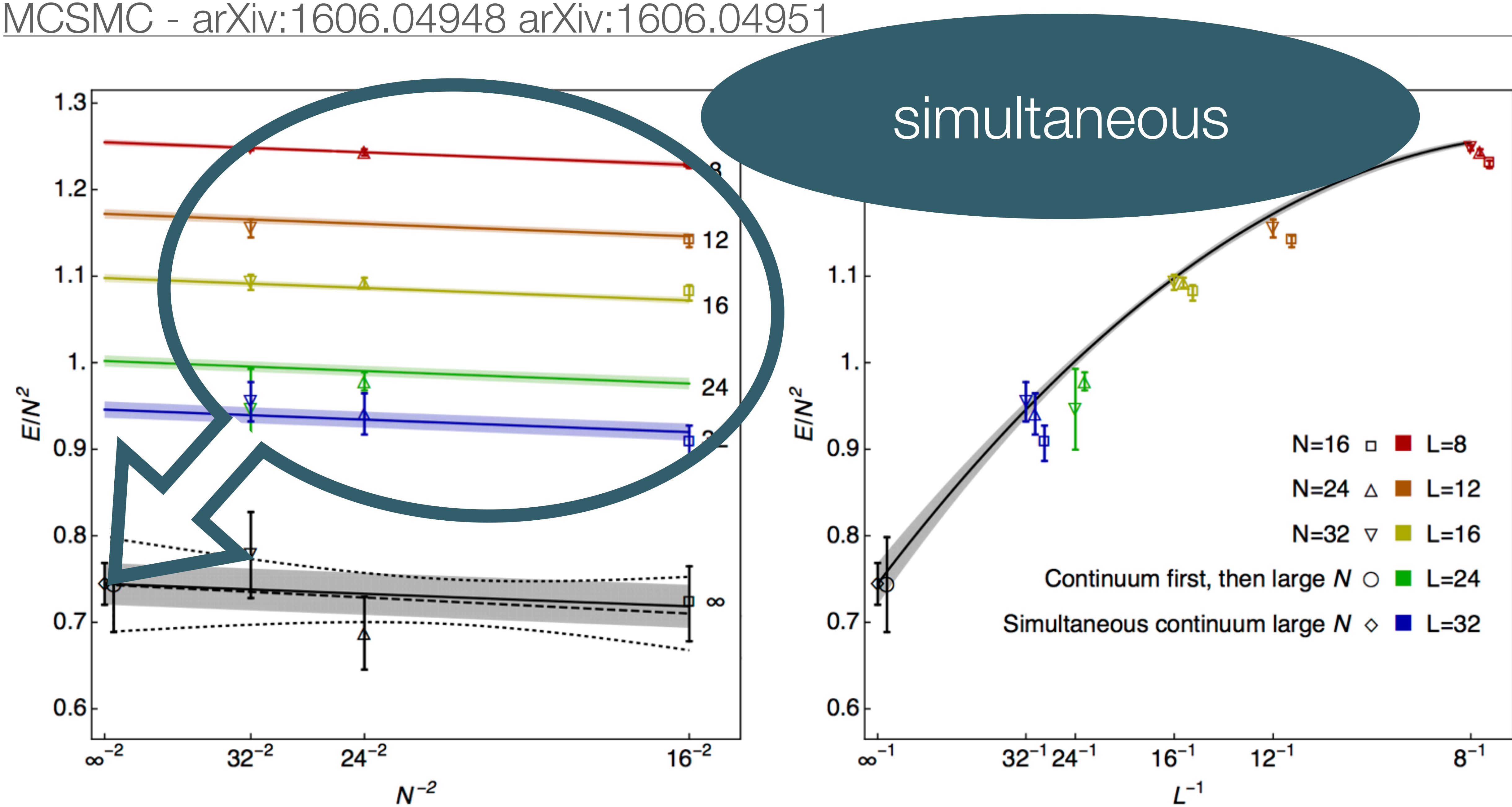

Simultaneous

e_{00} e_{01}, e_{02}, e_{10}

$$\frac{E}{N^2} = \sum_{ij \geq 0} \frac{e_{ij}}{N^{2i} L^j}$$

Simultaneous Extrapolation

T=0.5

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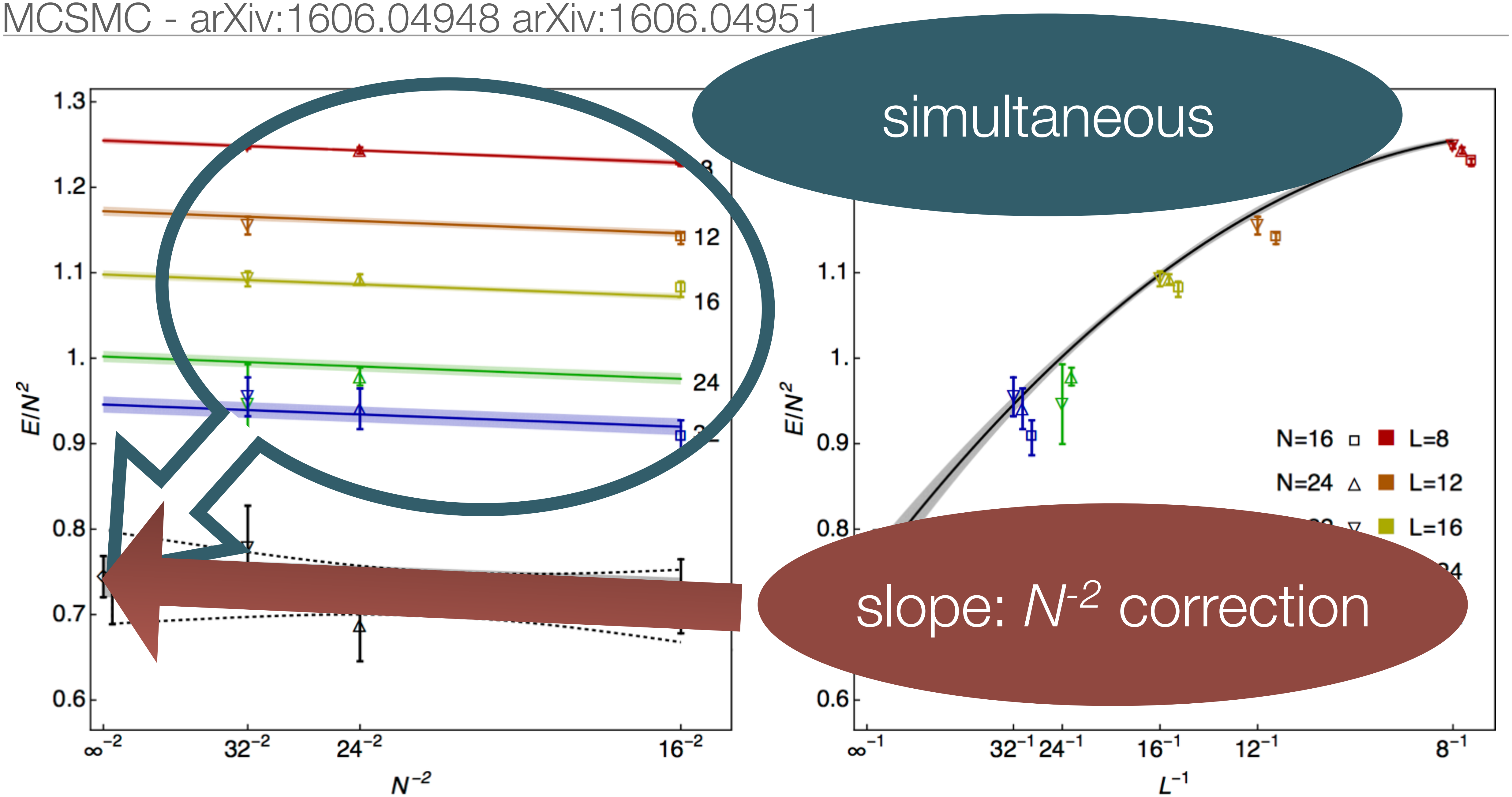

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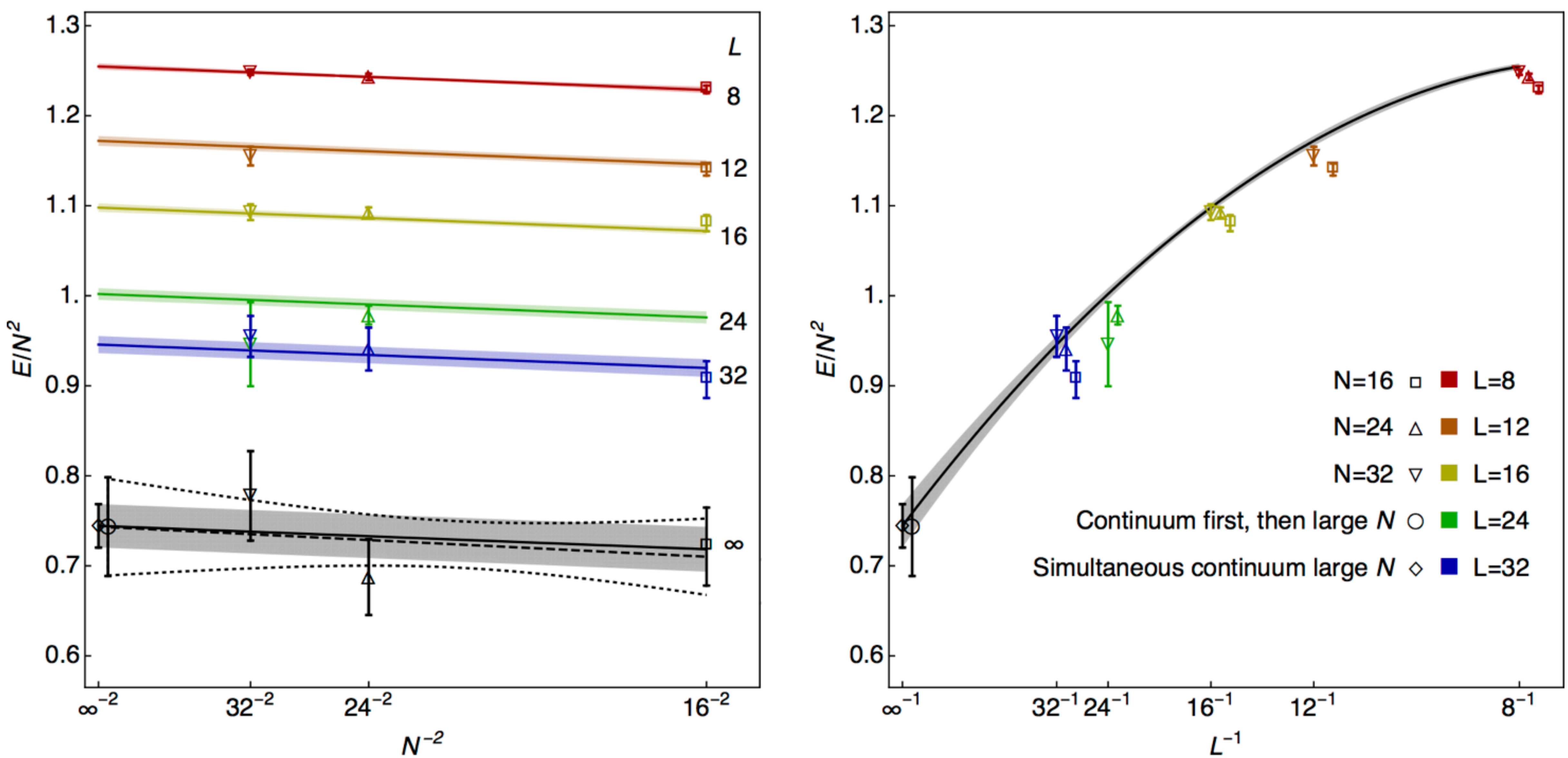

Simultaneous

$e_{00}, e_{01}, e_{02}, e_{10}$

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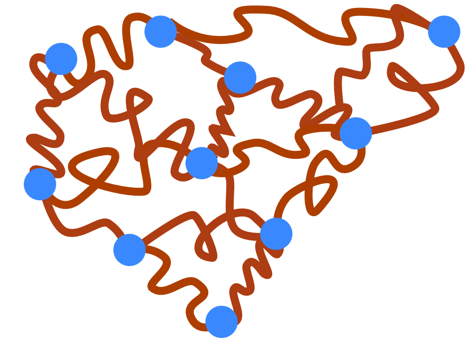

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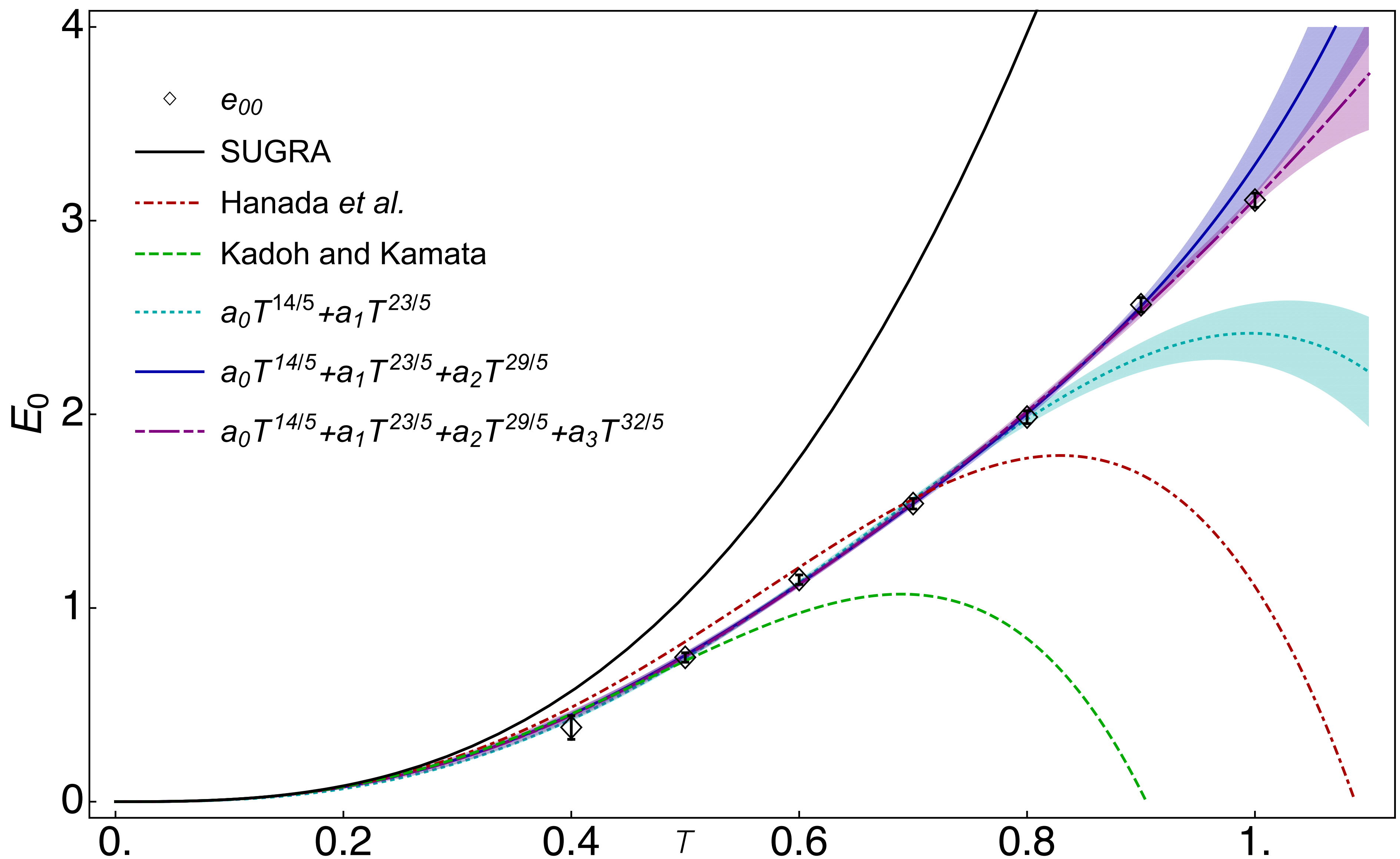
Stringy Prediction
$$a_0 = \quad 7.41$$
$$b_0 = -5.77$$



χ^2=0.76
DOF=3
a_0=7.4±0.5
a_1=-9.7±2.2
a_2=5.6±1.8

Strong coupling

Weak coupling

Black Hole Internal Energy

MCSMC - arXiv:1606.04948 arXiv:1606.04951

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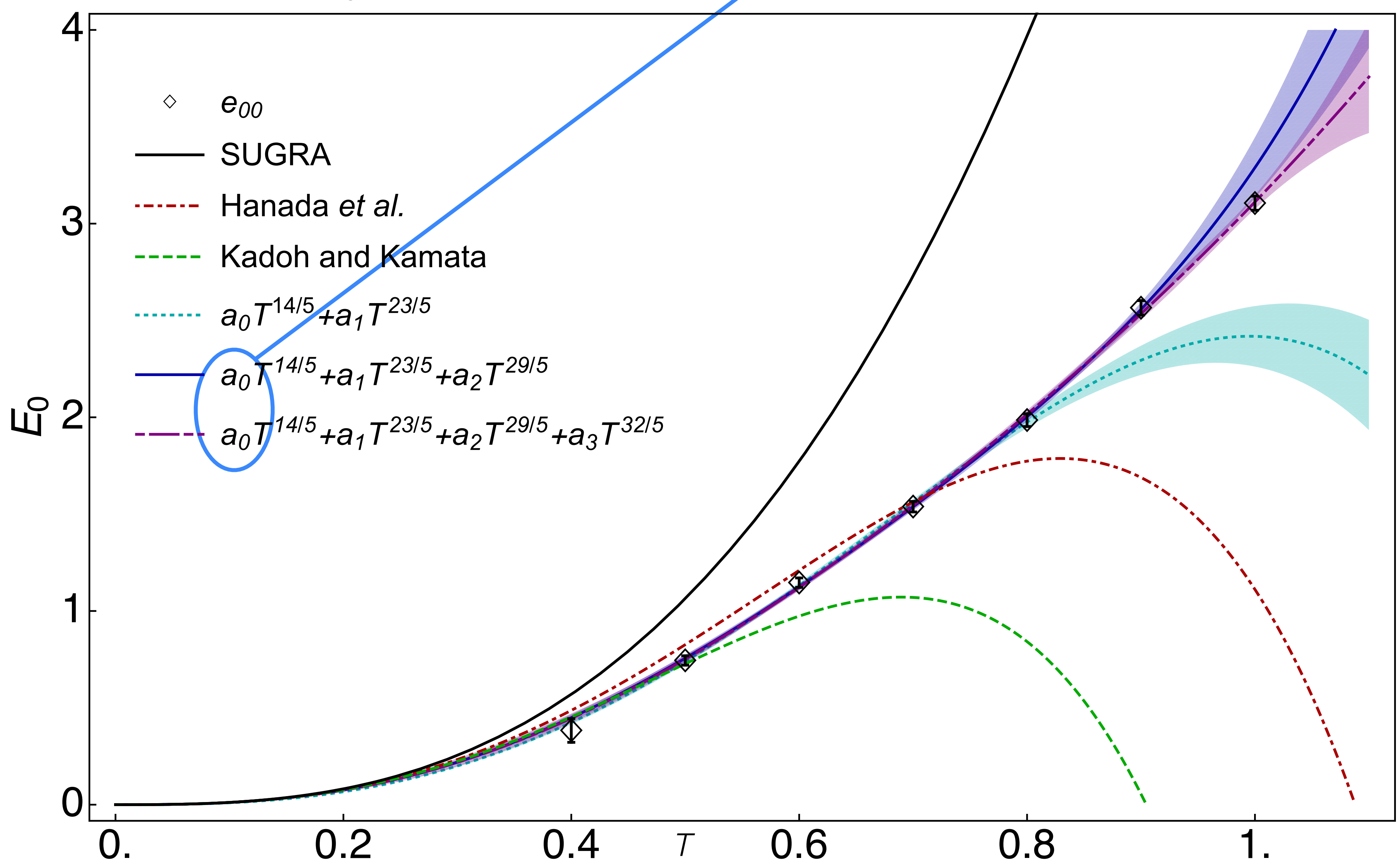
!



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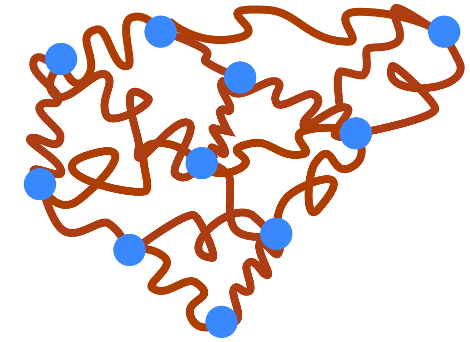

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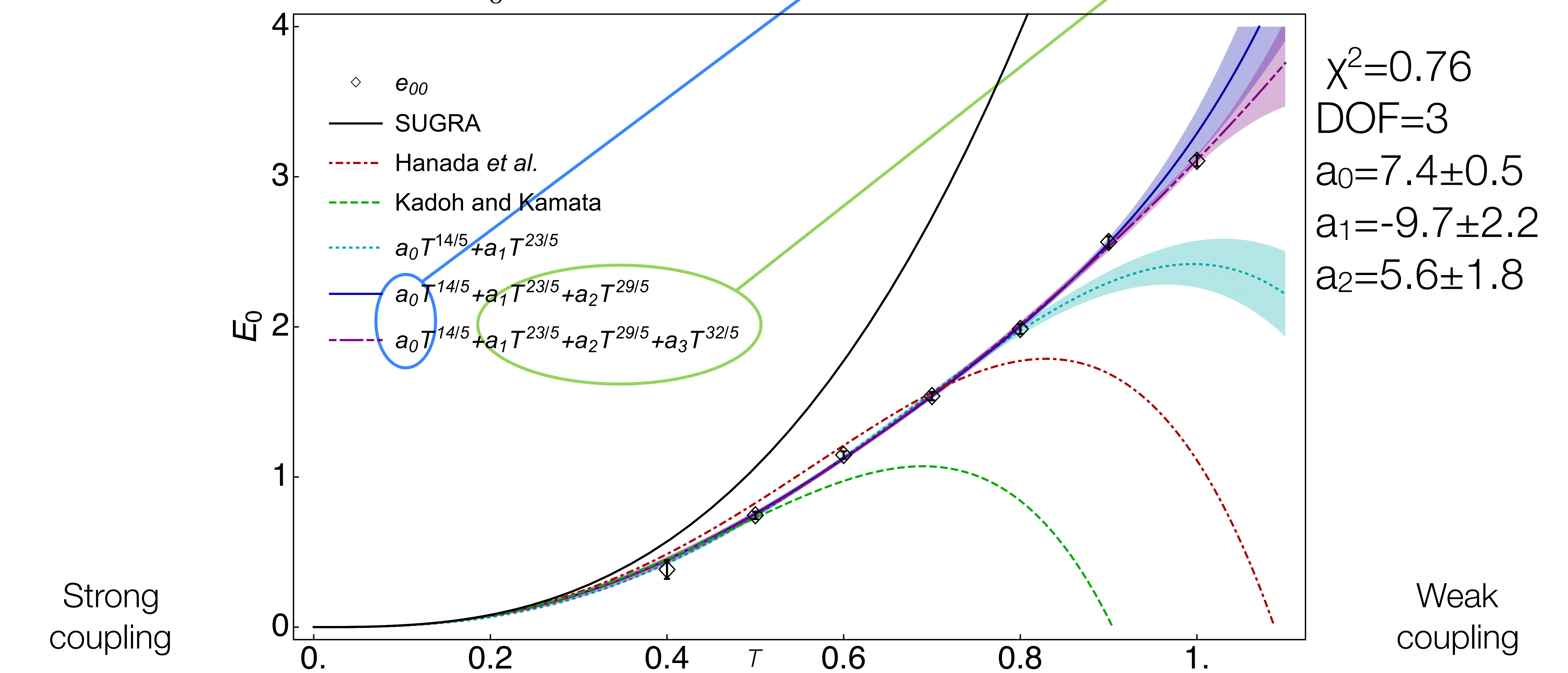
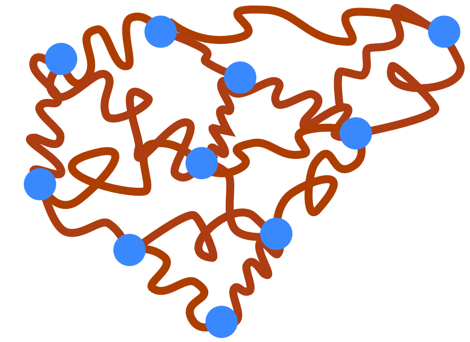


χ^2=0.76
DOF=3
a_0=7.4±0.5
a_1=-9.7±2.2
a_2=5.6±1.8



Black Hole Internal Energy

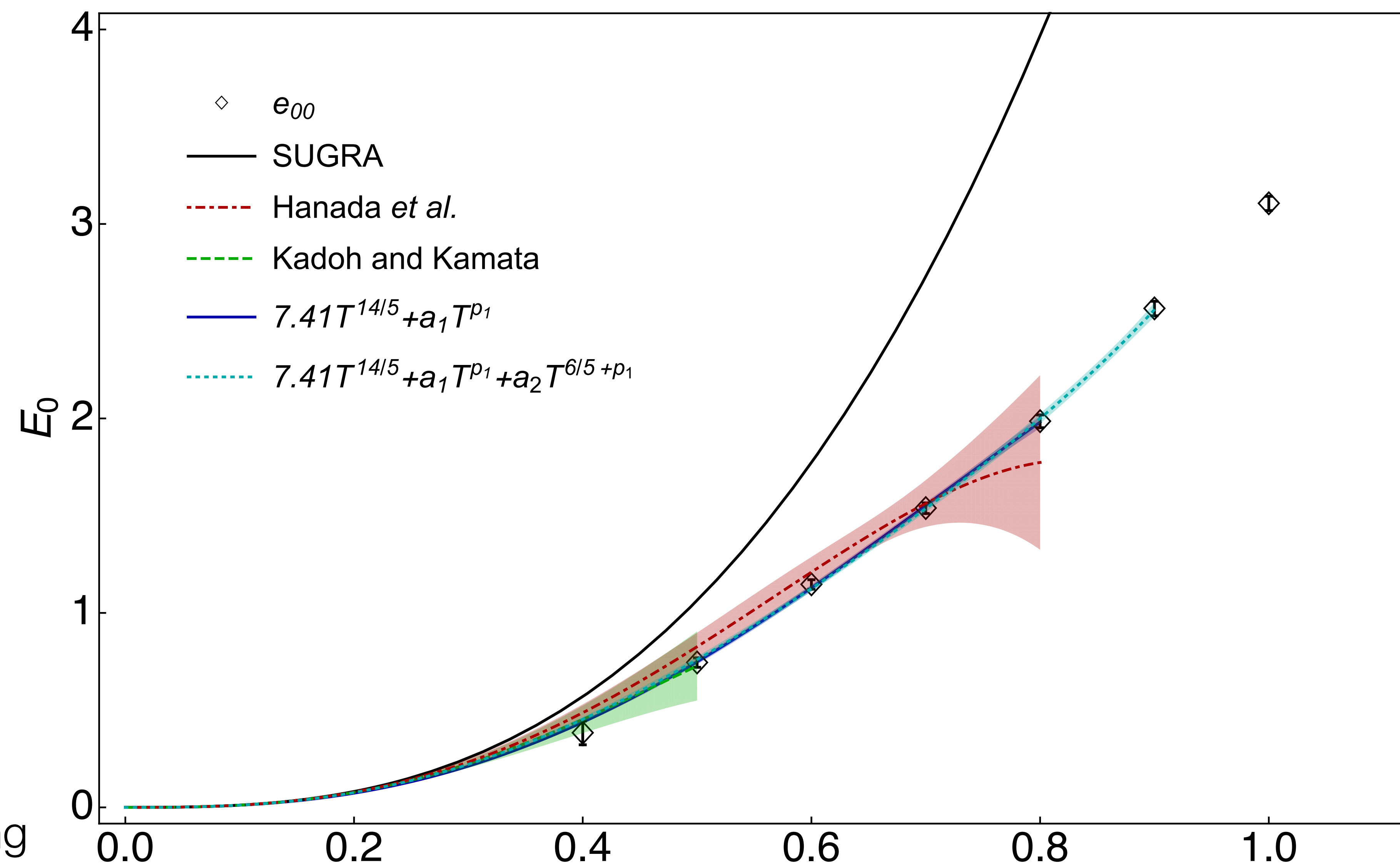
MCSMC - arXiv:1606.04948 arXiv:1606.04951

IIA String theory

$$E/N^2 = \frac{(a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8} + \cdots)}{N^0} + \frac{(b_0 T^{0.4} + b_1 T^{2.2} + \cdots)}{N^2} + \mathcal{O}\left(\frac{1}{N^4}\right)$$

Stringy Prediction

$$a_0 = 7.41$$
$$b_0 = -5.77$$



$p_1=4.6\pm0.3$

$a_1=-10.2\pm2.4$

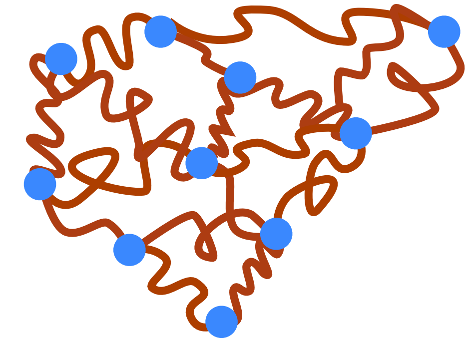
$a_2=6.2\pm2.6$

$\chi^2=2.6$

DOF=3

Strong coupling

Weak coupling



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$p_1 = 4.6 \pm 0.3$

Stringy Prediction

$$a_0 = 7.41$$

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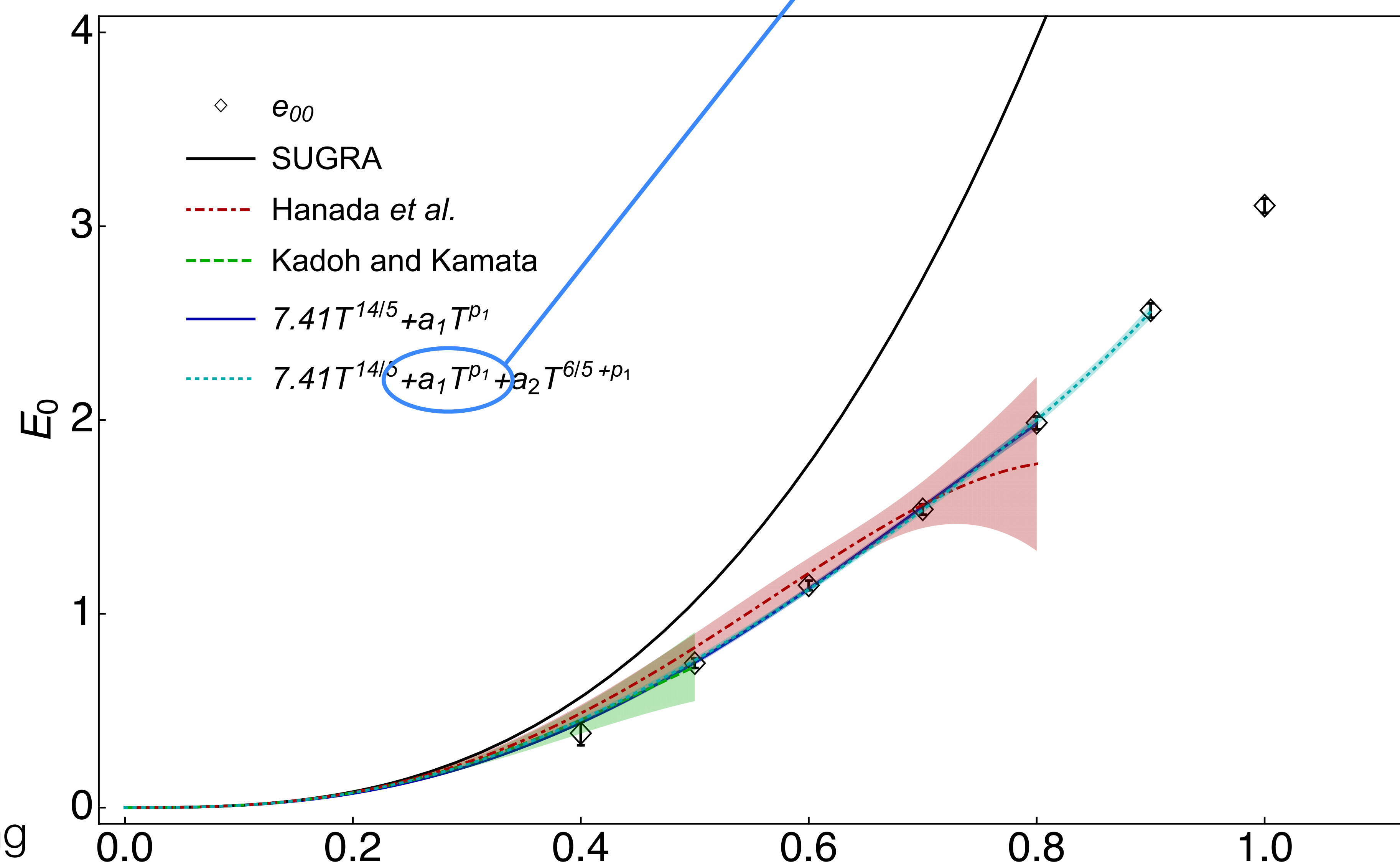

$p_1 = 4.6 \pm 0.3$

$a_1 = -10.2 \pm 2.4$

$a_2 = 6.2 \pm 2.6$

$\chi^2 = 2.6$

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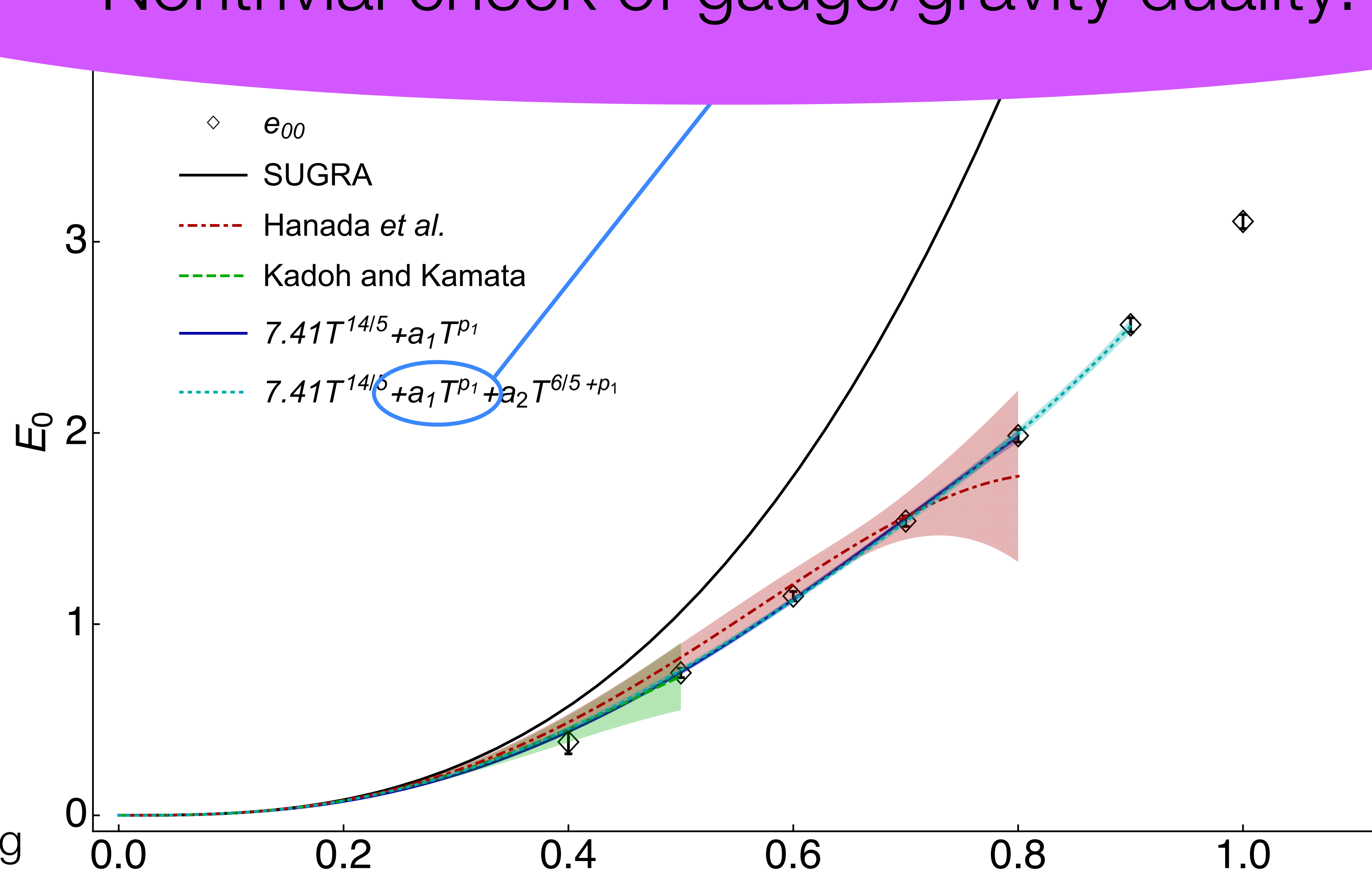
Stringy ...

Nontrivial check of gauge/gravity duality!



Strong coupling

Weak coupling

$p_1=4.6\pm0.3$

$a_1=-10.2\pm2.4$

$a_2=6.2\pm2.6$

$\chi^2=2.6$

DOF=3

$1/N^2$ Correction

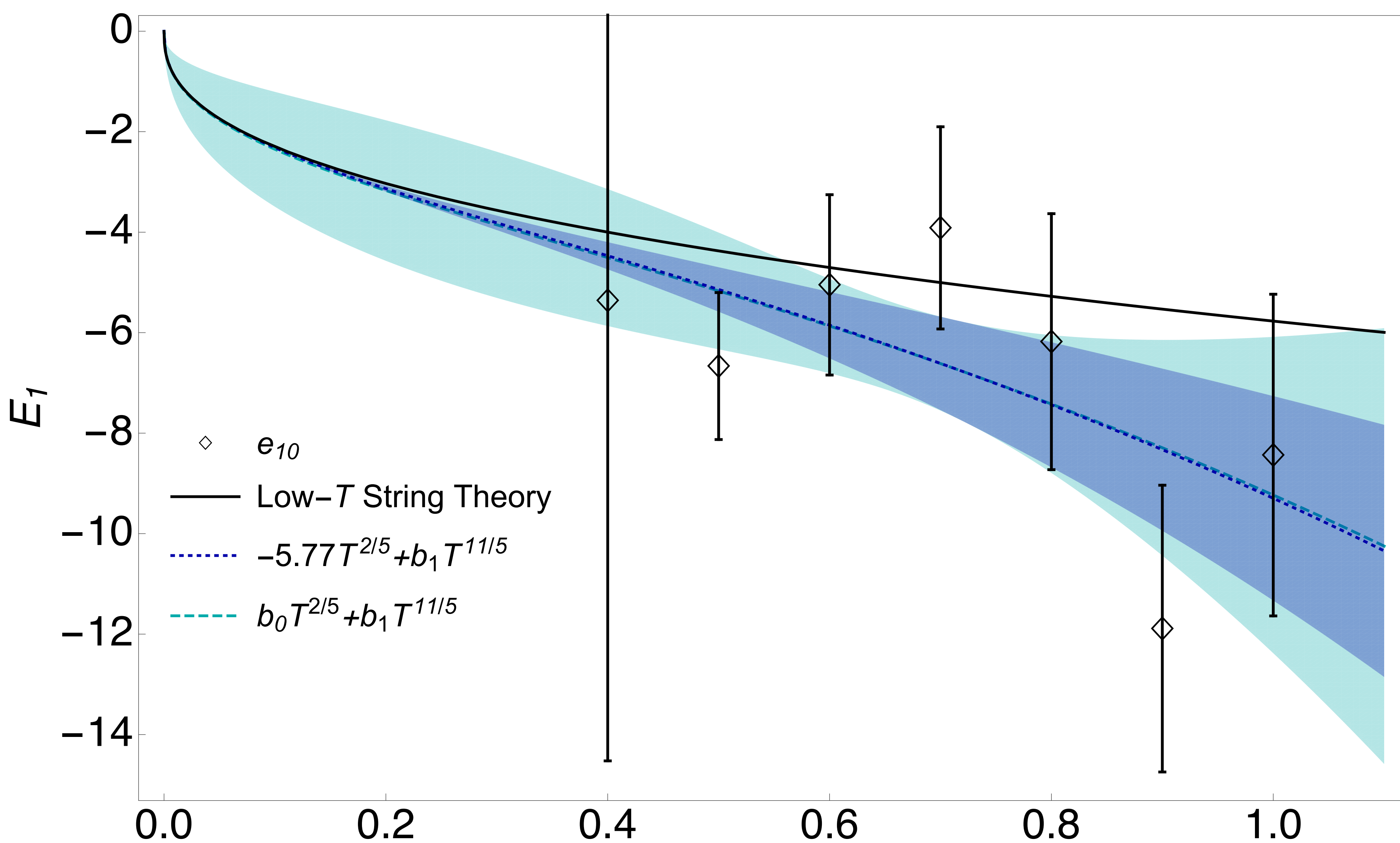
MCSMC - arXiv:1606.04948 arXiv:1606.04951

IIA String theory

$$E/N^2 = \frac{(a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8} + \cdots)}{N^0} + \frac{(b_0 T^{0.4} + b_1 T^{2.2} + \cdots)}{N^2} + \mathcal{O}\left(\frac{1}{N^4}\right)$$

Stringy Prediction

$$a_0 = 7.41$$
$$b_0 = -5.77$$

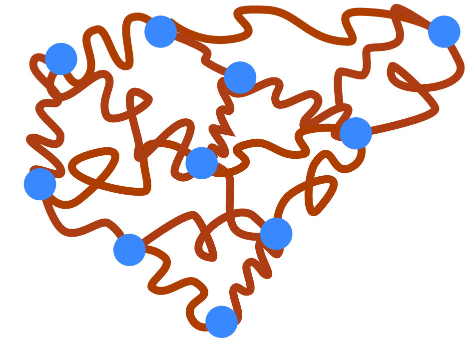


$b_0 = -5.8 \pm 3.0$

$b_1 = -3.4 \pm 5.7$

$\chi^2 = 4.8$

DOF=4



$1/N^2$ Correction

MCSMC - arXiv:1606.04948 arXiv:1606.04951

IIA String theory

$$E/N^2 = \frac{(a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8} + \cdots)}{N^0} + \frac{(b_0 T^{0.4} + b_1 T^{2.2} + \cdots)}{N^2} + \mathcal{O}\left(\frac{1}{N^4}\right)$$

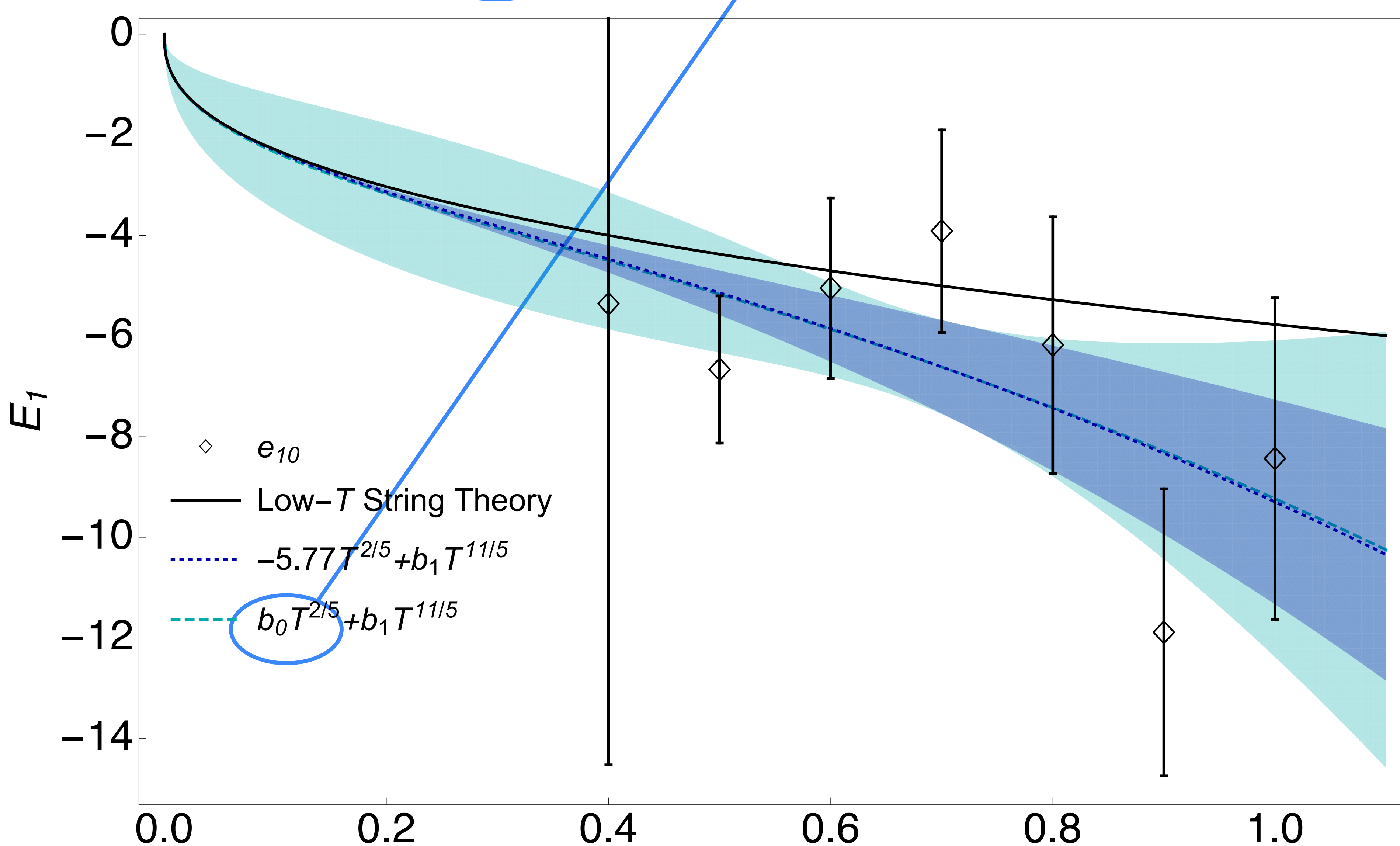
Finite N
quantum strings

Stringy Prediction

$$a_0 = 7.41$$

$$b_0 = -5.77$$

!



$b_0 = -5.8 \pm 3.0$
$b_1 = -3.4 \pm 5.7$
$\chi^2 = 4.8$
DOF=4

Summary

Gravity → Gauge Theory

Summary

Gravity ⟵ Gauge Theory

Summary

- Nontrivial checks of gauge/gravity duality
- 0+1D BFSS Matrix Model reproduces 11D SUGRA
- BFSS ~ SUGRA + predictions about (quantum!) stringy corrections

Future

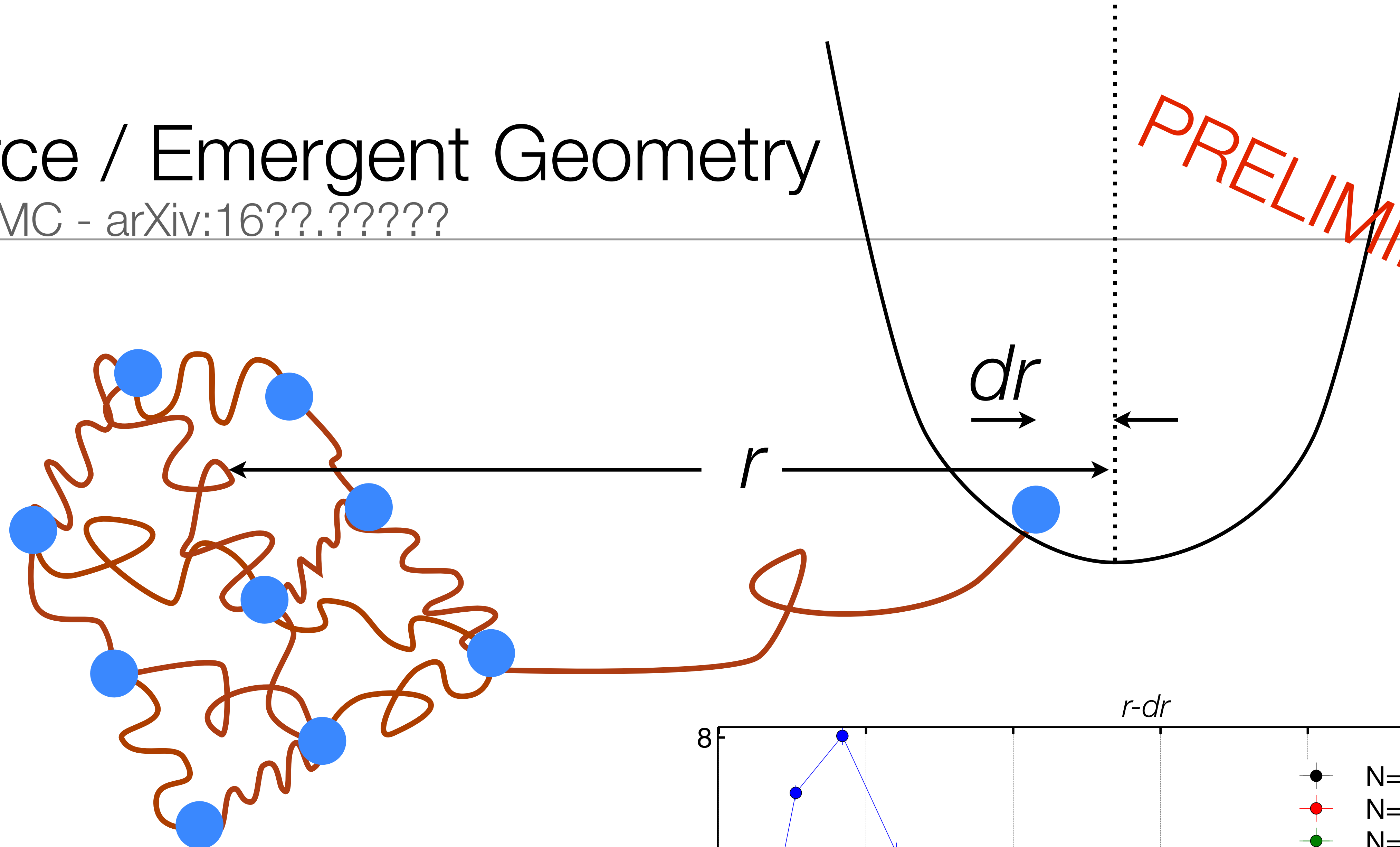
- Can we verify emergent spacetime far from the eigenvalue bunch?
- Can we push towards the M-theory region, see ~massless Hawking radiation?
- How does entanglement enter this story?

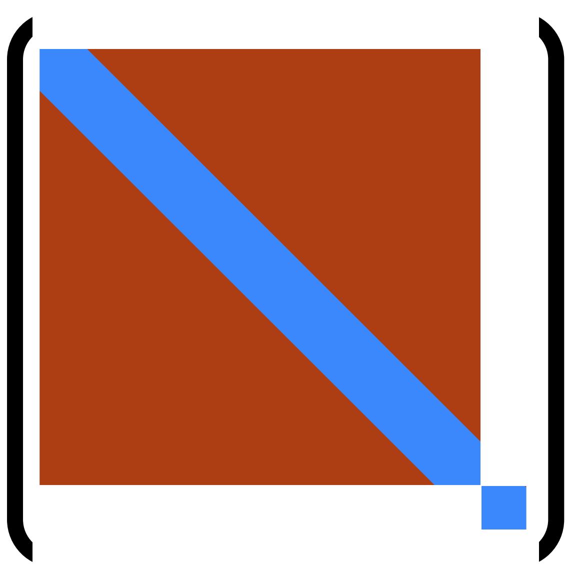
Backup Slides

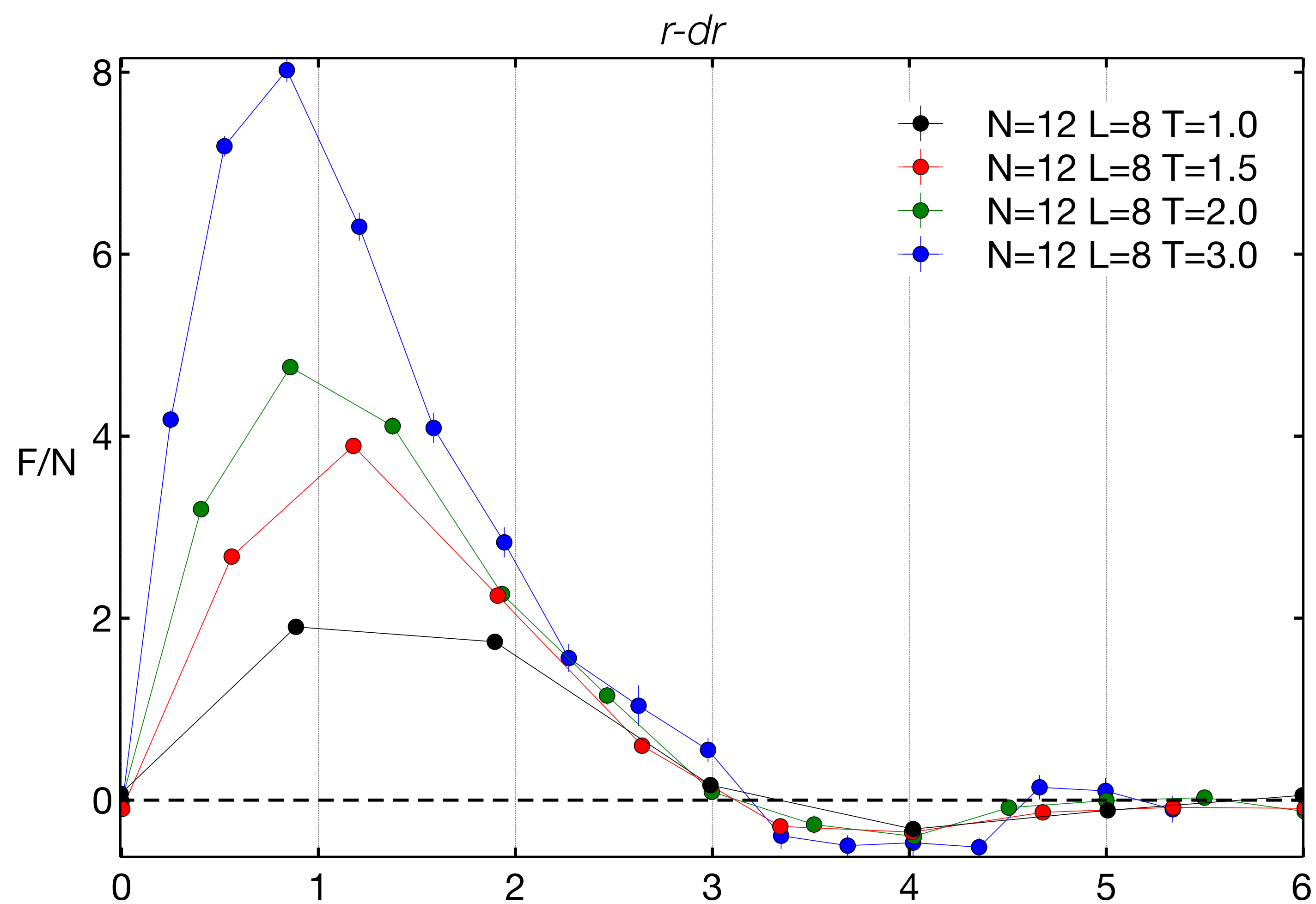
Force / Emergent Geometry

MCSMC - arXiv:16??.?????

PRELIMINARY

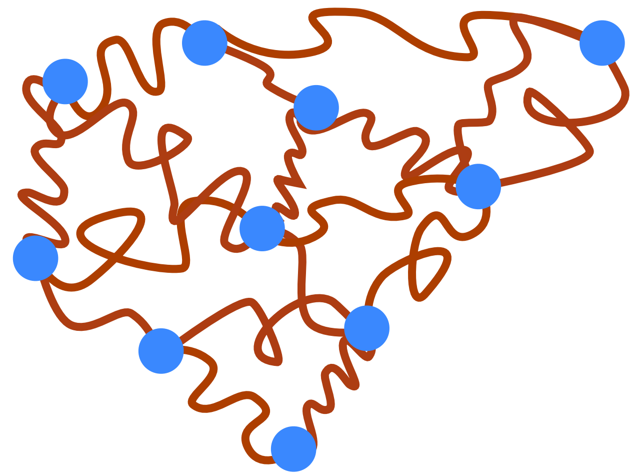






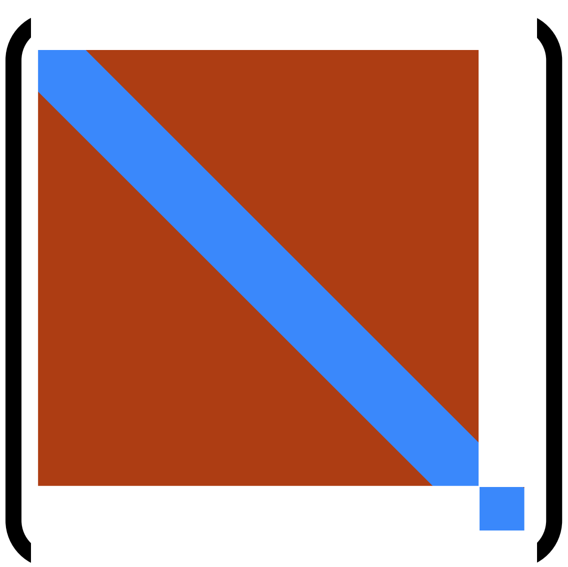
Genericly

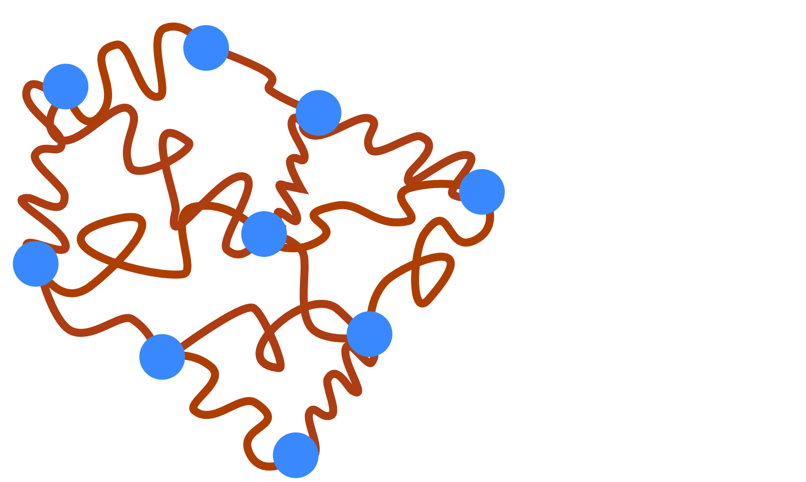
Berkowitz, Hanada, Maltz - arXiv:1602.01473



$DOF \sim N^2$

$t_{recurrence} \sim e^{+N^2}$

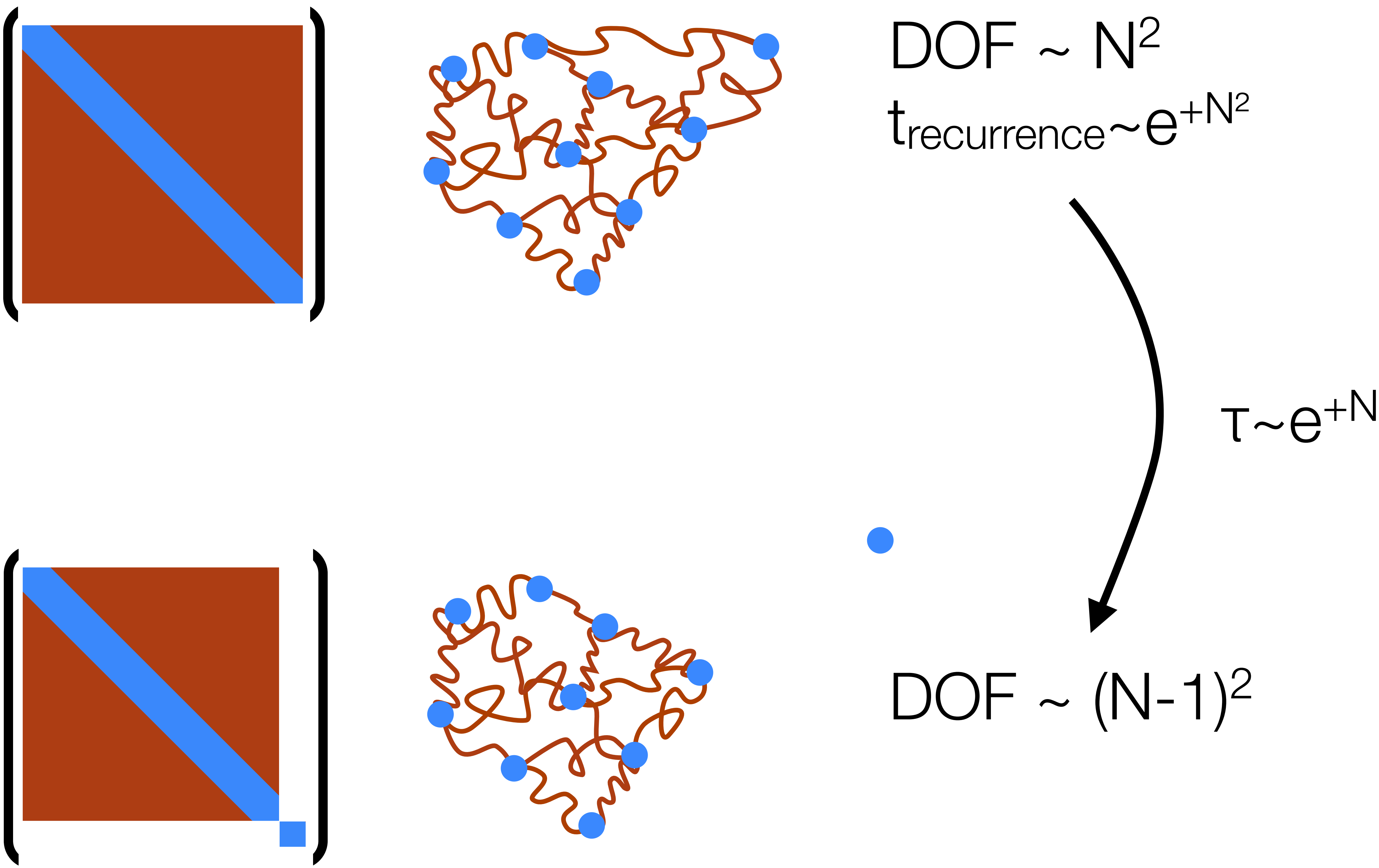




$DOF \sim (N-1)^2$

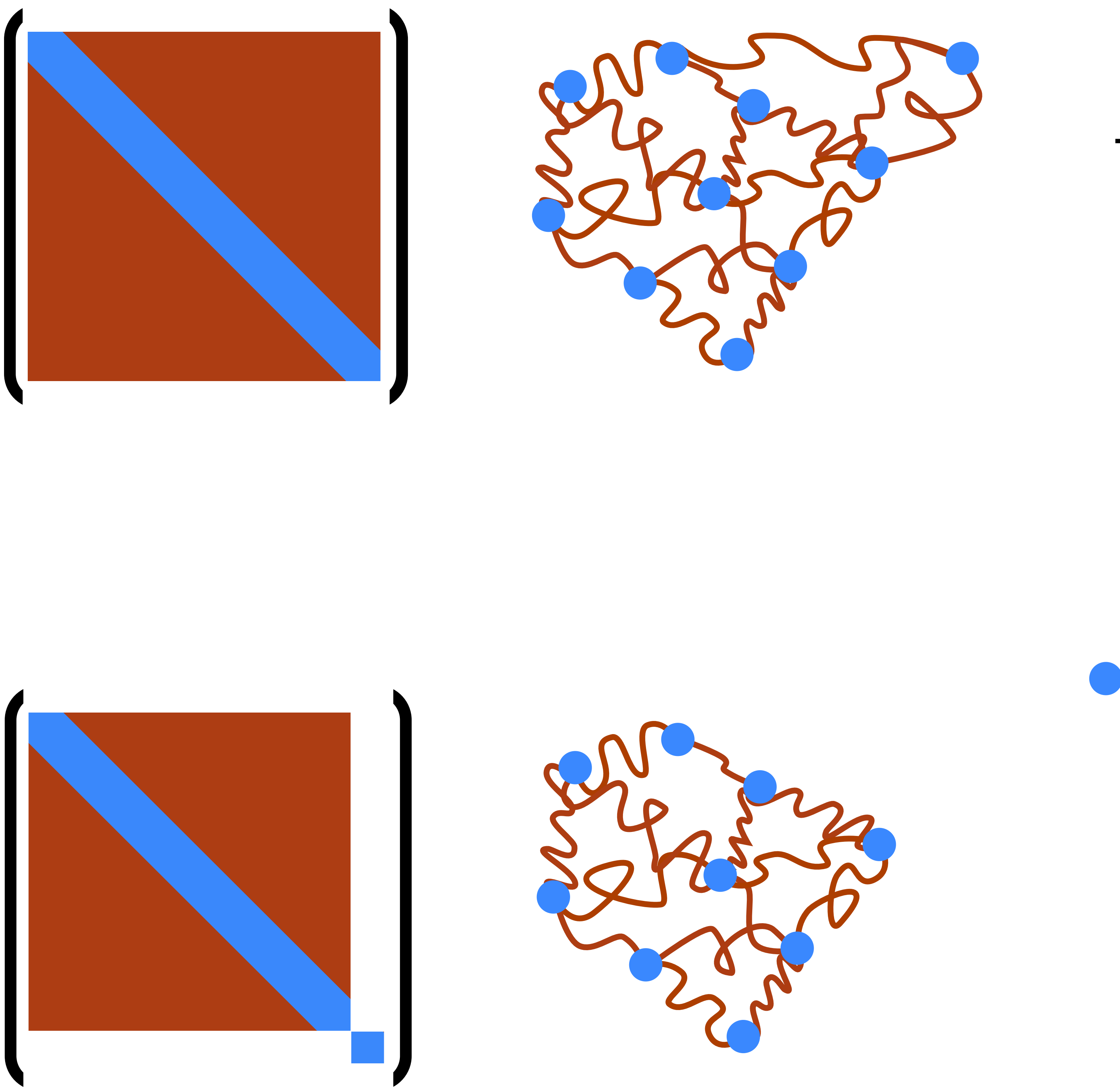
Generically

Berkowitz, Hanada, Maltz - arXiv:1602.01473



Generically

Berkowitz, Hanada, Maltz - arXiv:1602.01473



$\mathrm{DOF} \sim N^2$

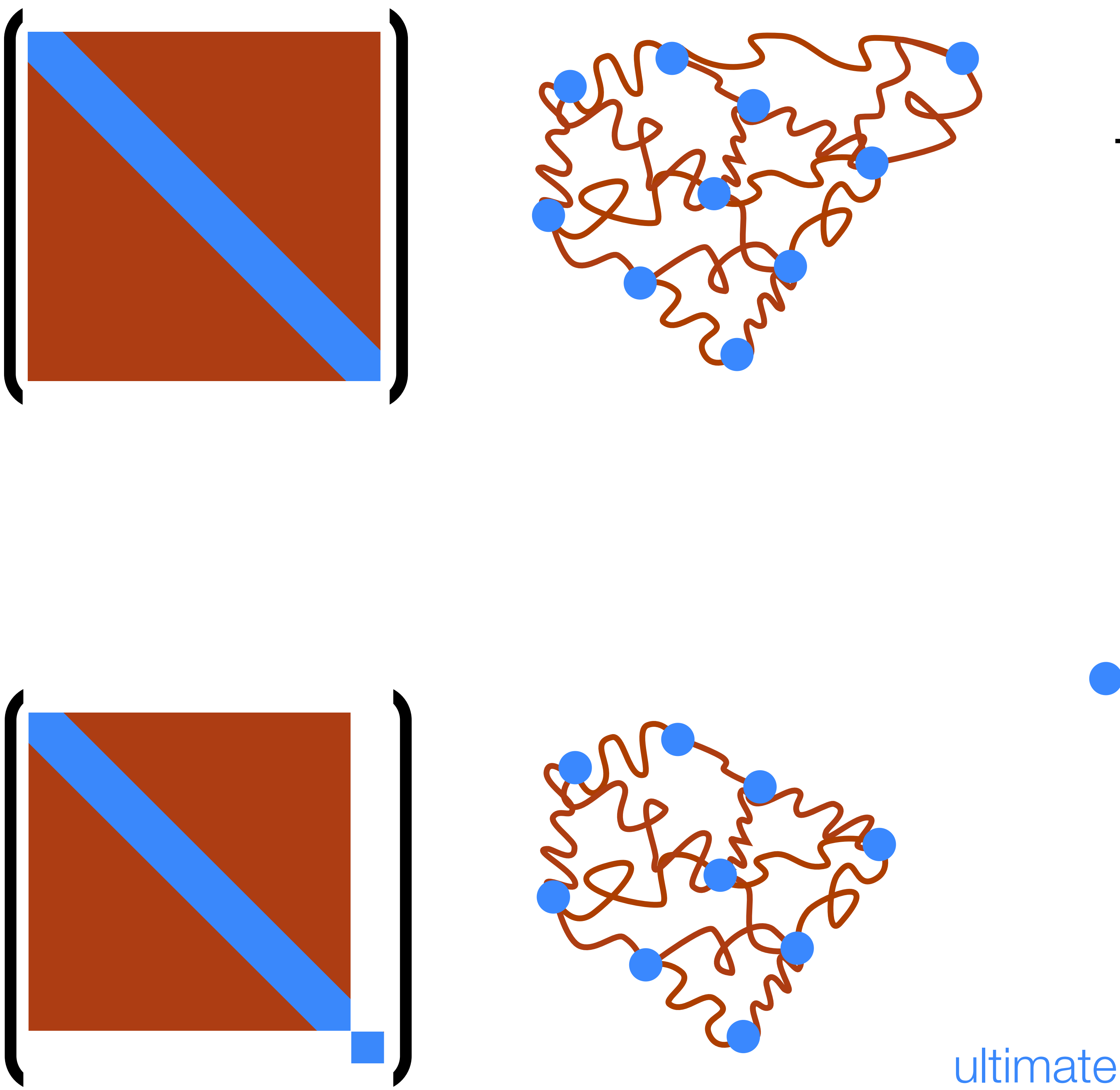
$t_{recurrence} \sim e^{+N^2}$

$\tau \sim e^{+N}$

$\mathrm{DOF} \sim (N-1)^2 + \#\log V$

Generically

Berkowitz, Hanada, Maltz - arXiv:1602.01473



$\mathrm{DOF} \sim N^2$

$t_{\mathrm{recurrence}} \sim e^{+N^2}$

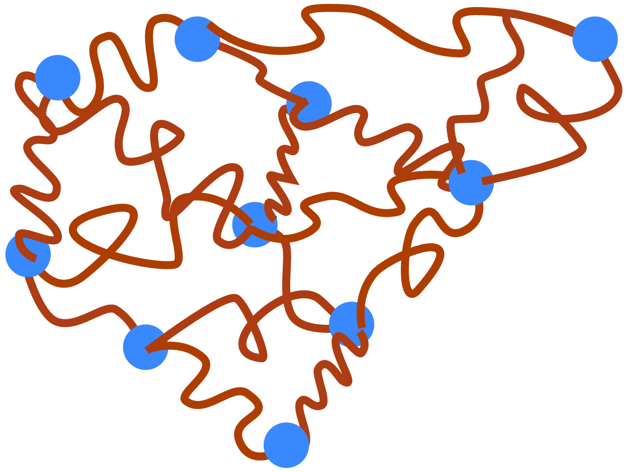
$\tau \sim e^{+N}$

$\mathrm{DOF} \sim (N-1)^2 + \#\log V$

ultimately, D0 gas will be realized

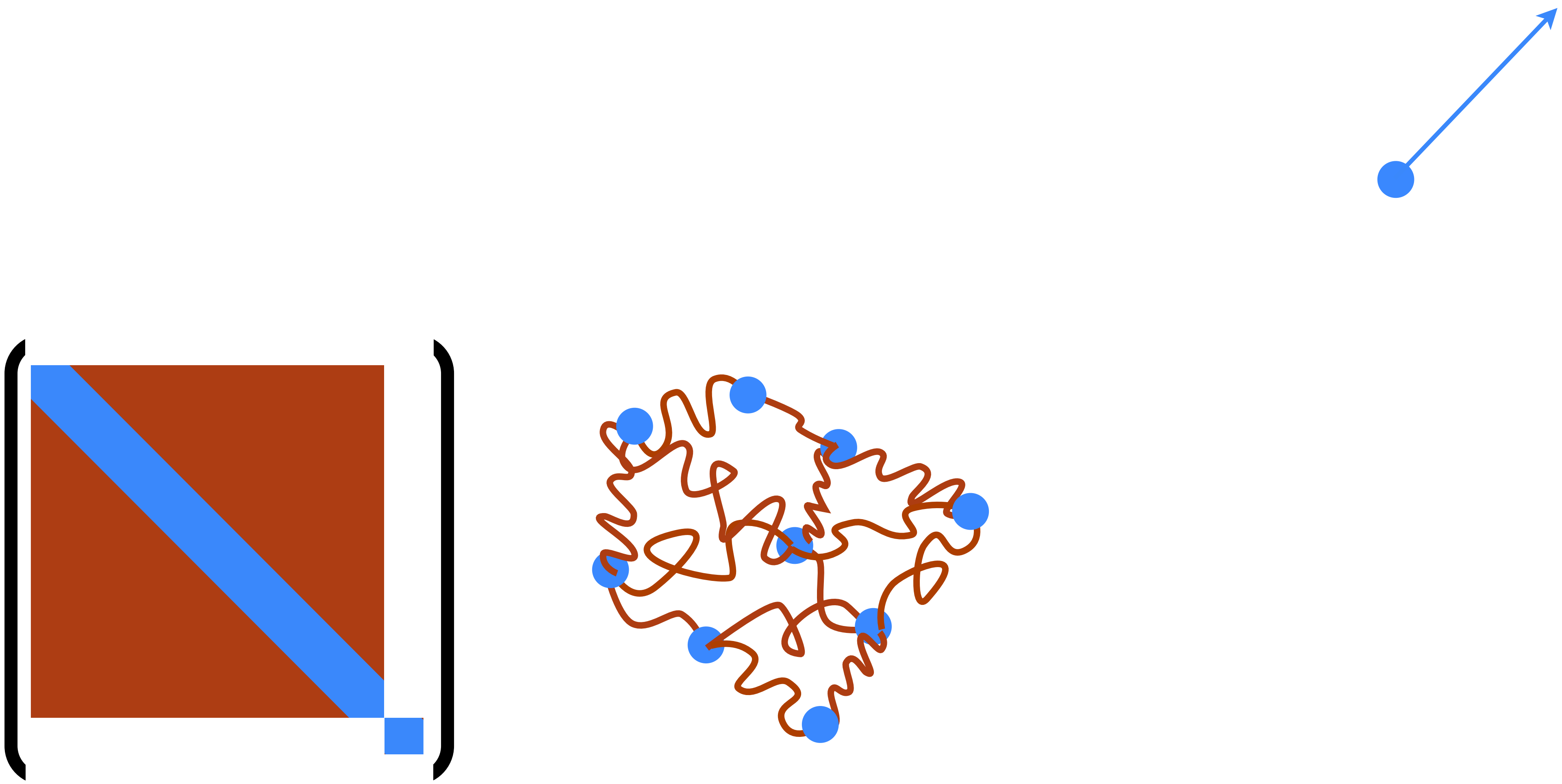
D0 branes → BH → gas of D0 branes

Berkowitz, Hanada, Maltz - arXiv:1602.01473



D0 branes → BH → gas of D0 branes

Berkowitz, Hanada, Maltz - arXiv:1602.01473



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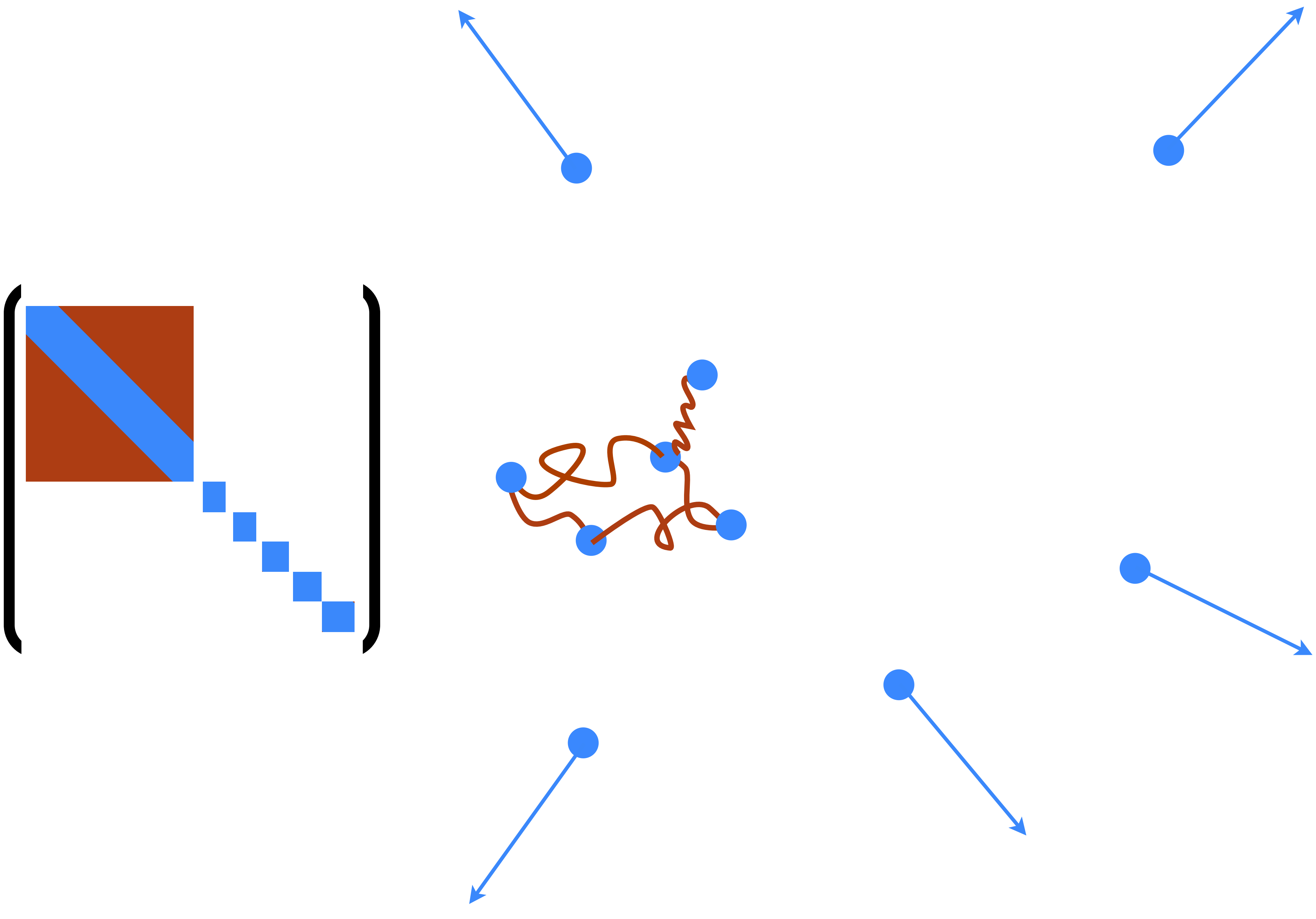
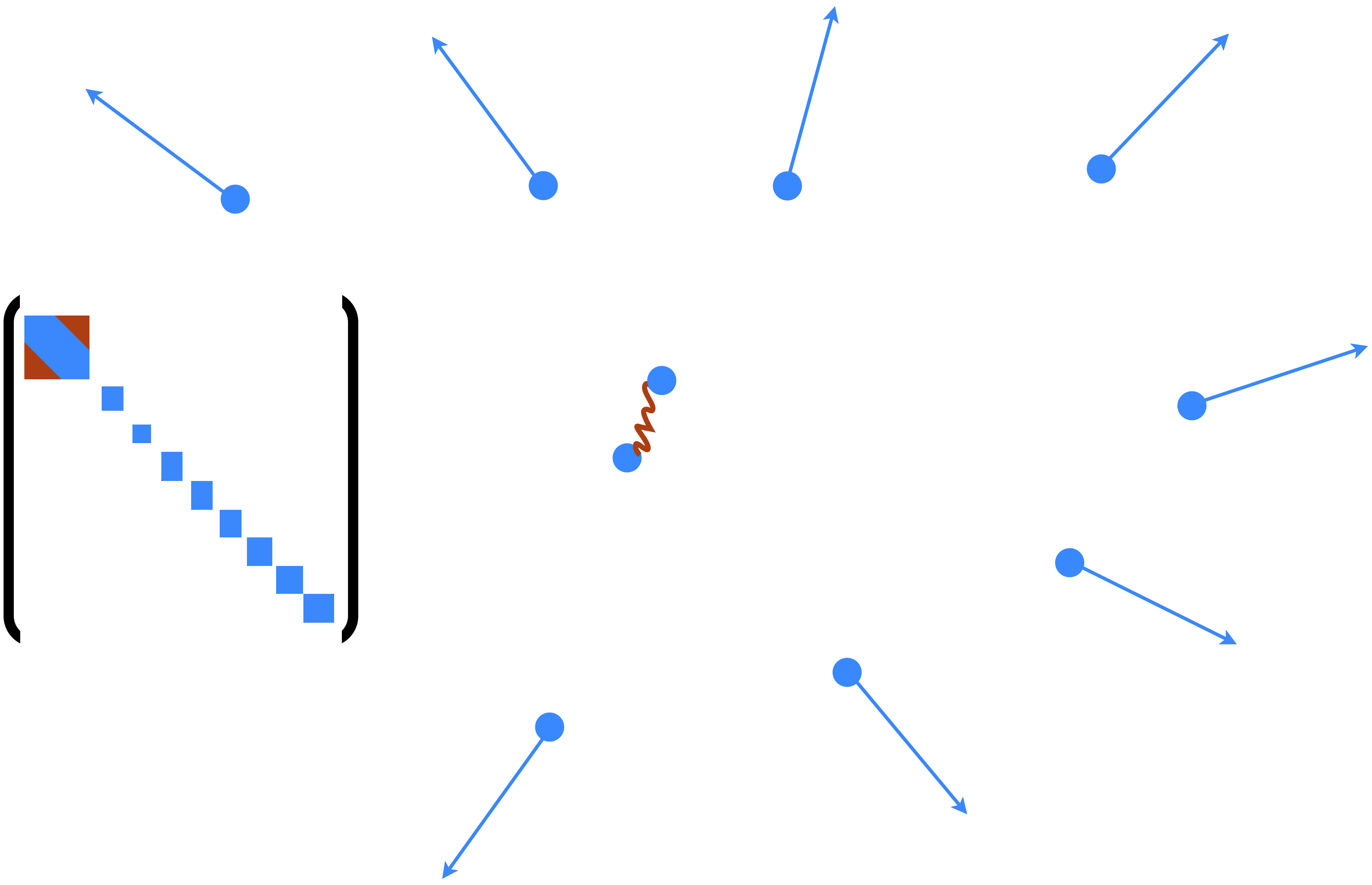


D0 branes → BH → gas of D0 branes

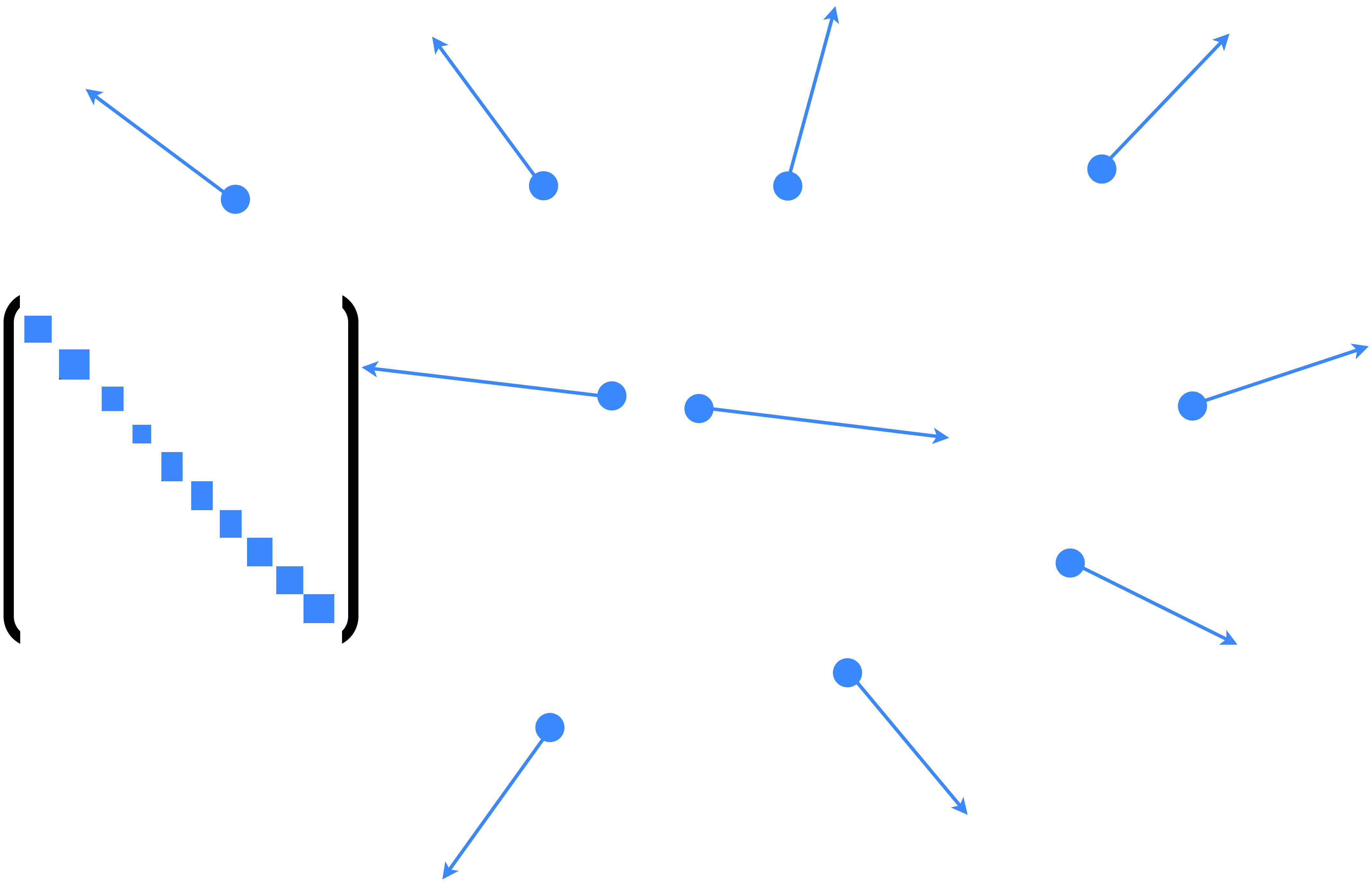
Berkowitz, Hanada, Maltz - arXiv:1602.01473



D0 branes → BH → gas of D0 branes

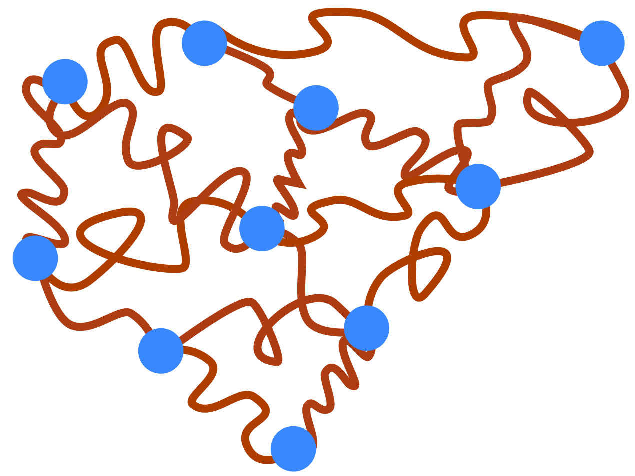
Berkowitz, Hanada, Maltz - arXiv:1602.01473

Complete evaporation takes $e^N + e^{N-1} + e^{N-2} + \ldots \sim e^N$



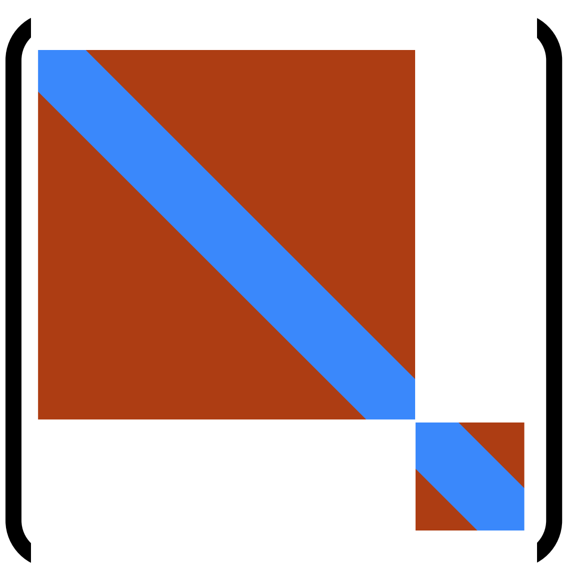
Emission of “clumps” is suppressed

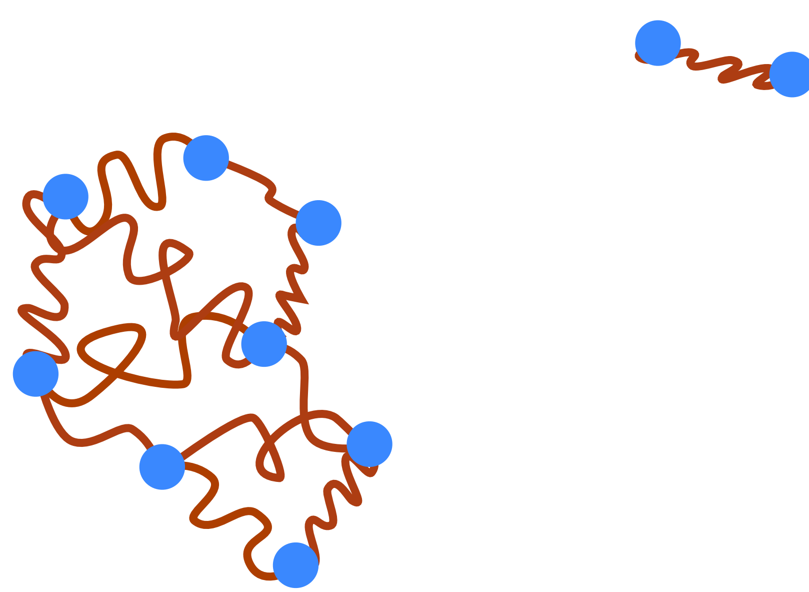
Berkowitz, Hanada, Maltz - arXiv:1602.01473



DOF ~ N^2

$t_{recurrence} \sim e^{+N^2}$

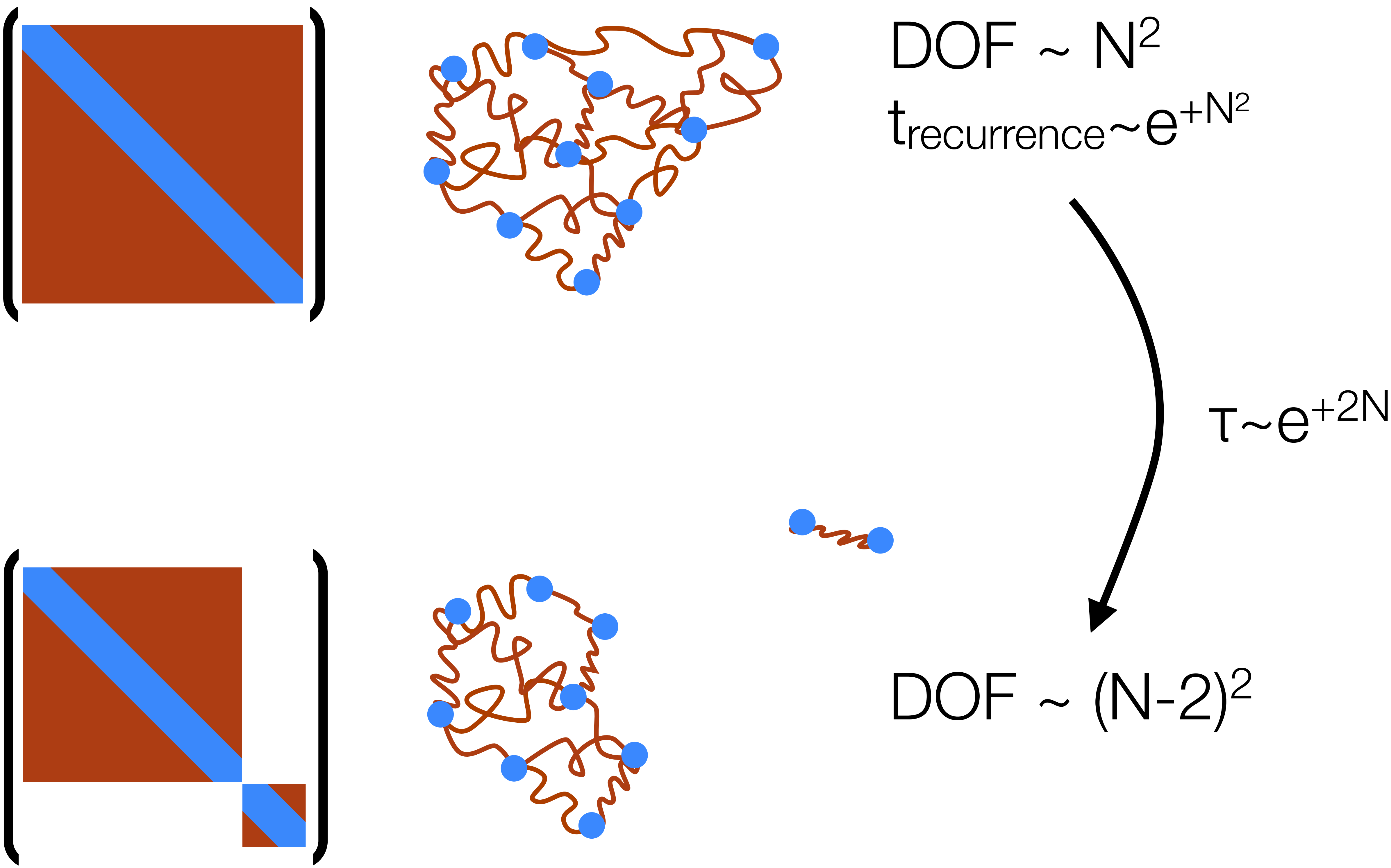




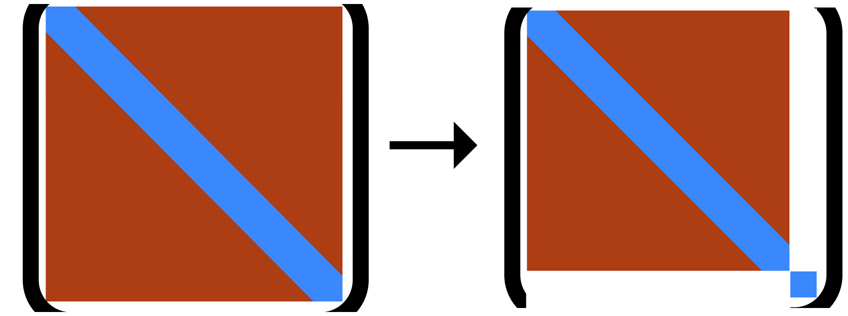
DOF ~ $(N-2)^2$

Emission of “clumps” is suppressed

Berkowitz, Hanada, Maltz - arXiv:1602.01473



Negative Specific Heat



Berkowitz, Hanada, Maltz - arXiv:1602.01473

Classically

$$E \sim cN^2T$$

(Virial: c=6)

$$cN^2T = c(N-1)^2T' + E_{D0}$$

$$T' = \left(\frac{N}{N-1}\right)^2 T - \frac{E_{D0}}{cN^2}$$
$$\approx \left(1 + \frac{2}{N}\right) T$$

Quantum Mechanically

$$E = a_0 \lambda^{-3/5} T^{14/5} N^2$$

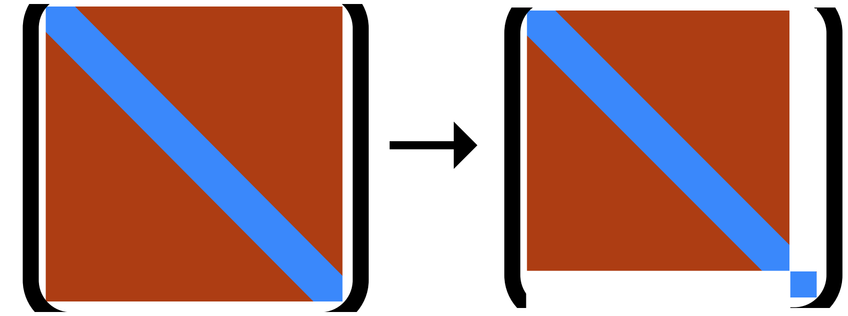
(SUGRA: a_0=7.41)

$$\lambda' = \frac{N-1}{N}\lambda$$

$$T'^{14/5} = \frac{\lambda^{-3/5}N^2}{\lambda'^{-3/5}(N-1)^2}T^{14/5} - \frac{E_{D0}}{a_0\lambda'(N-1)^2}$$
$$T' \approx \left(1 + \frac{1}{2N}\right) T$$

$T' > T$!

Negative Specific Heat



Berkowitz, Hanada, Maltz - arXiv:1602.01473

$$T' > T\,!$$

- Generic for large-N matrix models
- Valid classically and quantum-mechanically
- Evaporation process is unitary, and meaningful $\tau < t_{recurrence}$
- No remnant
- As time goes on T goes up, dynamics becomes BFSS classical → real time?

Kolmogorov-Sinai Entropy

Berkowitz, Hanada, Maltz - arXiv:1602.01473

Evaporation:

High T

$$\lambda_{\max} \sim T^{1/4} \qquad KS \propto N^2 T^{1/4} \to (N-1)^2 (T+\Delta T)^{1/4}$$

$$\Delta T \approx \frac{2}{N} T \qquad \to \left(N^2 - \frac{3N}{2}\right) T^{1/4}$$

Low T

$$\lambda_{\max} \sim T \qquad KS \propto N^2 T \to (N-1)^2 (T+\Delta T)$$

$$\Delta T \approx \frac{1}{2N} T \qquad \to \left(N^2 - \frac{3N}{2}\right) T$$

?

Lyapunov exponent grows but KS entropy falls!

Mergers: KS grows dramatically

bigger black holes are faster scramblers

Spectrum is thermal

Berkowitz, Hanada, Maltz - arXiv:1603.03055

$$W = \sum_{E_{\rm BH}+E_{\rm rad}=E} W_{\rm BH}(E_{\rm BH}) \cdot W_{\rm rad}(E_{\rm rad}), \tag{3}$$

where $W_{\rm BH} = e^{S_{\rm BH}}$, can be evaluated as

$$W = \sum_{E_{\rm rad}} \left(W_{\rm rad}(E_{\rm rad}) \cdot e^{S_{\rm BH}(E-E_{\rm rad})} \right) \simeq W_{\rm BH}(E) \sum_{E_{\rm rad}} \left(W_{\rm rad}(E_{\rm rad}) \cdot e^{-\frac{E_{\rm rad}}{T_{\rm BH}(E)}} \right), \tag{4}$$

where $T_{\rm BH} \equiv \left(\frac{dS_{\rm BH}}{dE} \right)^{-1}$. Here, corrections of order $1/N$, which contain information about quantum gravity effects, have been ignored. In principle we can calculate such corrections by fully solving the matrix model.