Monte Carlo simulations of $O(N)\phi_3^4$ and ϕ_2^4

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Outline

- MC simulations on ϕ^4_2-theory:
  - starting point
  - simulation strategy
  - (future) results

- $O(N) - \phi^4$-model:
  - extension to $O(N) - \phi^4_d$, with $d = 2, 3, 4$
  - future prospectives

- Conclusions

ϕ^4 theory

P. Bosetti, B. De Palma, M. Guagnelli , *"Monte Carlo determination of the critical coupling in ϕ_2^4 theory"* (2015, **Phys. Rev. D92, 034509**).

$$\mathcal{L}_E = \frac{1}{2}\left(\partial_\nu \phi\right)^2 + \frac{1}{2}\mu_0^2\phi^2 + \frac{g}{4}\phi^4, \qquad [g] = [\mu_0^2] \quad \rightarrow \quad f_0 = \frac{g}{\mu^2}$$

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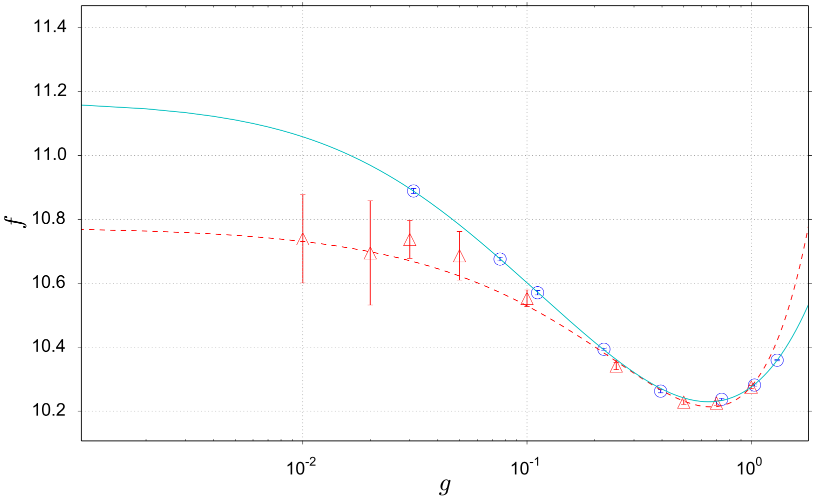
Method	f_0	Authors, year
DLCQ	5.52	Harindranath, Vary – 1988
QSE diag.	10	Lee, Lee, Salwen – 2000,
DMRG	$9.9816(16)$	Sugihara – 2004,
Monte Carlo cluster	$10.8^{0.1}_{0.05}$	Schaich, Loinaz – 2009,
Monte Carlo SLAC der.	$10.92(13)$	Wozar, Wipf – 2012,
Uniform Matrix p. s.	$11.064(20)$	Milsted, Haegeman, Osborne – 2013,
Ren. Hamiltonian	$11.88(56)$	Rychkov, Vitale – 2015,
Resummation	$11.00(4)$	Pelissetto, Vicari – 2015
Monte Carlo worm	$11.15(6)(3)$	**Here we are**–2015

Final results for $f(g)$ in logarithmic scale



[1] Triangular points are results from **D. Schaich, W. Loinaz**, Phys. Rev. D79 (2009)

Our strategy for the computation of f_0

We consider the lattice action

$$\mathcal{S}_E = -\beta \sum_x \sum_\nu \varphi_x \varphi_{x+\hat{\nu}} + \sum_x \left[\varphi_x^2 + \lambda(\varphi_x^2 - 1)^2\right] = \mathcal{S}_I + \mathcal{S}_{Site},$$

$$\phi = \sqrt{\beta}\varphi, \qquad \mu_0^2 = 2\frac{1-2\lambda}{\beta} - 4, \qquad g = \frac{4\lambda}{\beta^2}.$$

In this representation we can perform the strong coupling expansion

$$Z(x_1, \cdots, x_n) = \sum_{\{k\}} w(k) \prod_x c(d(x)),$$

$$w(k) = \prod_l \frac{\beta^{k(l)}}{k(l)!} \qquad c(\lambda, d(x)) = \int_{-\infty}^{+\infty} d\varphi(x) \mathrm{e}^{-\varphi(x)^2 - \lambda[\varphi(x)^2 - 1]^2} \varphi(x)^{d(x)}$$

In this way we pass from site-located fields to link fields.
The worm algorithm [2] allows to sample these configurations by local moves

[2] **Korzec, Vierhaus, Wolff**, Computer Physics Communications 182 (2011)

Our strategy for the computation of f_0

- For each lattice size, at fixed λ we search for β such that

Condition of constant physics

$$mL = L/\xi = \text{const} = z_0$$

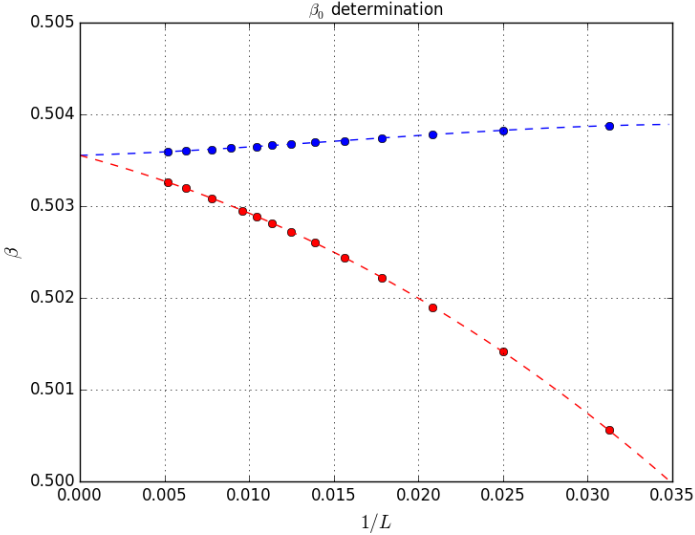
Currently we are simulating $\lambda = 0.001$ and L/a since to 320.

- Finally we extrapolate β_c at the infinite volume limit.

new! We simulate two set of date at $\lambda = 0.001$ with $z_0 = 1$ and $z_0 = 4$ and then combine the results for a better estimation of β_c with small lattice sizes

- Finally with (β_c, λ) we compute f_0, after the mass renormalization.

Infinite volume limit of β_c



Worm algorithm: loop algorithm for O(N) theories

U. Wolff, *"Simulating the All-Order Strong Coupling Expansion III: O(N) sigma loop models"*(2010)

$$Z(u,v) = \int \left[\prod_z d\mu[\sigma(z)]\right] e^{\beta \sum_{\langle xy \rangle} \sigma(x)\cdot\sigma(y)} \sigma(u)\cdot\sigma(v)$$

where

$$\int d\mu[\sigma] f(\sigma) = K_N \int d^N\sigma \delta(\sigma^2 - 1) f(\sigma)$$

In order to obtain the loop representation we need

$$G_N(J) \equiv \int d\mu[\sigma] e^{J\cdot\sigma} = \sum_{n=0}^{\infty} c[n;N](J\cdot J)^n = \sum_{n=0}^{\infty} \frac{\Gamma(N/2)}{2^{2n} n! \Gamma(N/2+n)} (J\cdot J)^n$$

In the case of ϕ^4 model with $O(N)$ symmetry the partition function is

$$Z(u,v) = \int \left[\prod_z d\mu[\phi(z)]\right] e^{\beta \sum_{\langle xy \rangle} \phi(x)\cdot\phi(y)} \phi(u)\cdot\phi(v)$$

where

$$\int d\mu[\phi] f(\phi) = K_N \int d^N\phi e^{-\phi\cdot\phi - \lambda(\phi\cdot\phi-1)^2} f(\phi)$$

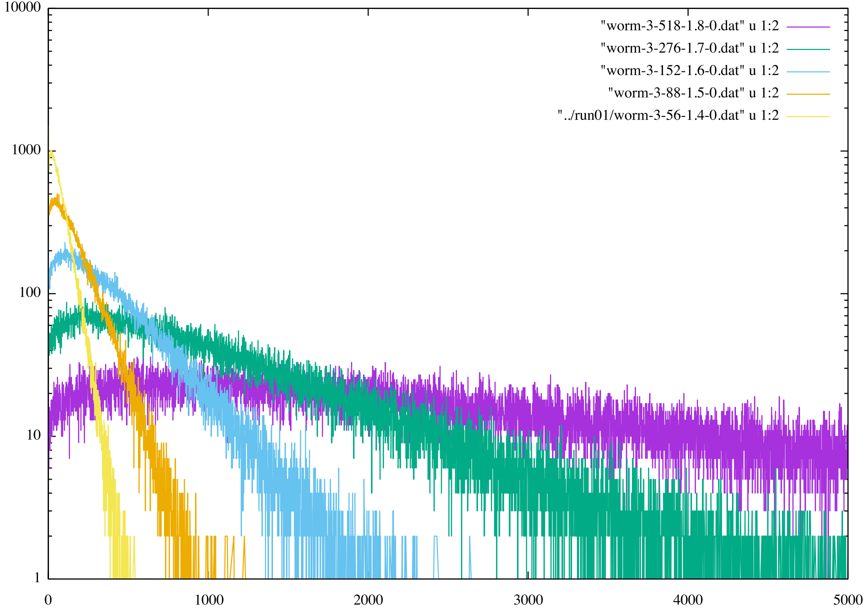
where

$$K_N^{-1} = \int d\omega \int_0^\infty d\rho \rho^{N-1} e^{-\rho^2-\lambda(\rho^2-1)^2} = \Omega_N \gamma(N-1)$$

The calculation of $G_N(J)$ proceeds as before, but with different expansion coefficients:

$$c[n;N] = \frac{\gamma(N+n-1)\Gamma(N/2)}{\gamma(N-1)2^{2n} n! \Gamma(N/2+n)}$$

Distribution of the lenght of the worms



Summary

- The goal of MC simulations on ϕ^4_2-theory is to reach a better estimate of f_0 at lowest g in order to discern which is the non-perturbative behaviour of f_0.
  Simulations are running and we are waiting for the results.

- $O(N)$ sigma model algorithm is extended to $O(N) - phi^4_d$ model, whit $d = 2, 3, 4$ and tested. What is missing is a deeper analysis of the features of the algorithm and the theory.

Bibliography

- **Schaich D. and Loinaz W.**, *"An Improved lattice measurement of the critical coupling in ϕ^4 theory"*, Phys.Rev. D79.056008 (2009), arXiv: hep-lat 0902.0045 (MC Cluster)
- **Harindranath, A. and Vary, J. P.**, *"Stability of the vacuum in scalar field models in 1+1 dimensions"*, PhysRevD37 (1988) (DLCQ)
- **Dean Lee and Nathan Salwen and Daniel Lee**, *"The diagonalization of quantum field Hamiltonians"*, Phys. Lett. B (2001) (QSE diag.)
- **Sugihara, Takanori**, *"Density matrix renormalization group in a two-dimensional lambda phi4 Hamiltonian lattice model"*, arXiv:hep-lat/0403008 (2004) (DMRG)
- **Milsted, A. and Haegeman, J. and Osborne, T. J.**, *"Matrix product states and variational methods applied to critical quantum field theory"*, Phys. Rev. D88 (2013) (Uniform Matrix p. s.)
- **Pelissetto A., Vicari, E.**, *"Critical mass renormalization in renormalized ? 4 theories in two and three dimensions"*, Phys. Rev. D91 (2015) (Resummation)
- **Rychkov, S. and Vitale, L. G.**, *"Hamiltonian truncation study of the φ^4 theory in two dimensions"*, Phys. Lett. B751 (2015)
- **Wozar, C. and Wipf, A.**, *"Supersymmetry Breaking in Low Dimensional Models"*, Annals Phys. 327 (2012)

Guess function for f_0

Our fit $f(g)$ over the entire range at our disposal

$$f(g) = \frac{a_0 + a_1 g + a_2 g^2 + a_3 g^3 + a_4 g^4}{1 + b_1 g + b_2 g^2 + b_3 g^3}.$$

Loinaz and Schaich guess function for fitting data

$$f(g) = \frac{g}{\mu^2} = c_0 + c_1 g + c_2 g \log g.$$

Renormalization condition

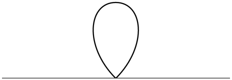


Figure : One–loop self–energy in ϕ^4

$$A(\mu_0^2) = \frac{1}{N^2}\sum_{k_1=0}^{N-1}\sum_{k_2=0}^{N-1}\frac{1}{4\left(\sin^2\frac{\pi k_1}{N}+\sin^2\frac{\pi k_2}{N}\right)+\mu_0^2},$$

$$\mu^2 = \mu_0^2 + 3gA(\mu^2).$$

backup

We write the theory on the lattice

$$\mathcal{S}_E = \sum_x \left\{ -\sum_\nu \phi_x \phi_{x+\hat{\nu}} + \frac{1}{2}\left(\mu_0^2 + 4\right)\phi_x^2 + \frac{g}{4}\phi_x^4 \right\},$$

If we switch to the following parametrization

$$\phi = \sqrt{\beta}\varphi, \qquad \mu_0^2 = 2\frac{1-2\lambda}{\beta} - 4, \qquad g = \frac{4\lambda}{\beta^2}.$$

we obtain the new action dependent on (β, λ)

$$\mathcal{S}_E = -\beta \sum_x \sum_\nu \varphi_x \varphi_{x+\hat{\nu}} + \sum_x \left[\varphi_x^2 + \lambda(\varphi_x^2 - 1)^2\right] = \mathcal{S}_I + \mathcal{S}_{Site},$$

$$(g, \mu_0^2) \qquad \rightarrow \qquad (\lambda, \beta)$$

results

λ	β_c	μ^2	g/μ^2
1.000000	0.680601(11)	0.649451(67)	13.2962(18)
0.750000	0.689117(13)	0.509730(59)	12.3935(19)
0.500000	0.686938(10)	0.367173(31)	11.5431(13)
0.380000	0.678405(11)	0.296195(32)	11.1503(15)
0.250000	0.6586276(98)	0.214762(27)	10.7340(17)
0.200000	0.6462478(78)	0.181077(21)	10.5786(15)
0.125000	0.6190716(52)	0.125924(15)	10.3605(15)
0.094000	0.6030936(89)	0.100518(23)	10.2843(26)
0.062500	0.5820989(60)	0.072073(15)	10.2370(23)
0.030000	0.5516594(71)	0.038407(17)	10.2666(48)
0.015625	0.5326936(27)	0.0211916(63)	10.3935(32)
0.007500	0.5187729(29)	0.0105457(67)	10.5704(68)
0.005000	0.5136251(17)	0.0071014(38)	10.6757(57)
0.002000	0.5064230(16)	0.0028637(35)	10.8925(132)