The running coupling of $N_f = 12$

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Why (still) $N_f = 12$?

Our previous results: consistent with χSB (no continuum)

Cheng Hasenfratz Liu Petropoulos Schaich 1404.0984: running coupling shows fixed point in continuum $g_*^2 = 6.2(2)$

Other inconsistencies among several works ...

How can this be?

Let's find out!

This will be a QFT talk :)

Strategy

Let's calculate the β-function in the exact same continuum scheme as 1404.0984, with fully controlled systematics and see what we get

Note: discretization slightly different

Continuum scheme

- Finite volume gradient flow $g^2 \sim \langle t^2 E(t) \rangle$
- Flow to fixed fraction of volume $c = \sqrt{8t}/L = 1/5$
- Periodic gauge field
- Anti-periodic fermion field in all directions
- Finite $s = 2$ scale change
- Massless fermions with only multiplicative renormalization

Any change in 6 properties above changes the continuum step function

Discretization

- Dynamical gauge action: tree-level Symanzik
- Gauge action along the flow: tree-level Symanzik
- E: clover
- SSC in our 1406.0827 terminology
- Stout staggered (no rooting)

Not exactly the same as 1404.0984 (shouldn't matter anyway)

Running coupling

Step scaling: $(g^2(sL) - g^2(L))/\log(s^2)$ as a function of $g^2(L)$

Usual procedure (ourselves included in previous works):

- Calculate $g^2(L/a, \beta)$ for a number of $L/a \to sL/a$ lattice volume pairs for many β values
- Interpolate in β to get any $g^2(\beta)$ for fixed L/a
- Continuum extrapolate discrete β-function in a^2/L^2
- Combine statistical and systematic errors
- Systematic errors: type of interpolations, how many lattice spacings (i.e. $L/a \to sL/a$ pairs) used

Cheng et al. 1404.0984 result

Find $g_*^2(L/a, \beta_*)$ and extrapolate these in a^2/L^2

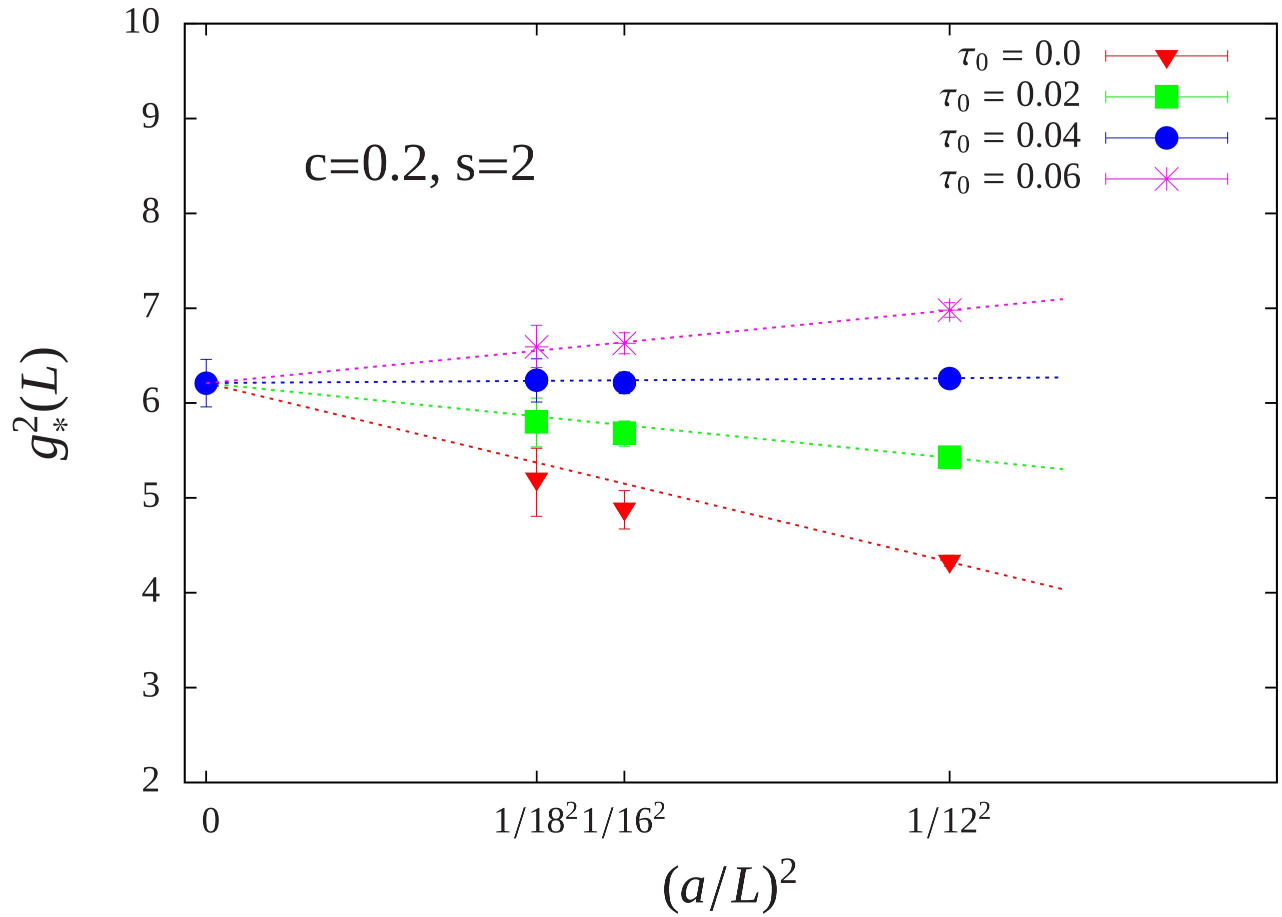
Non-perturbative improvement (different discretization of E)

$$g^2(t) \sim \langle t^2 E(t+t_0)\rangle = \frac{t^2}{(t+t_0)^2}\langle (t+t_0)^2 E(t+t_0)\rangle$$

t_0 kept fixed in lattice units

Expand in t_0, from $O(t_0)$ term clearly: continuum unchanged, slope can change

Cheng et al. 1404.0984 result



Pink, green, red: why $O(a^2/L^2)$?

Running coupling, our work

Eliminate systematic error from interpolations

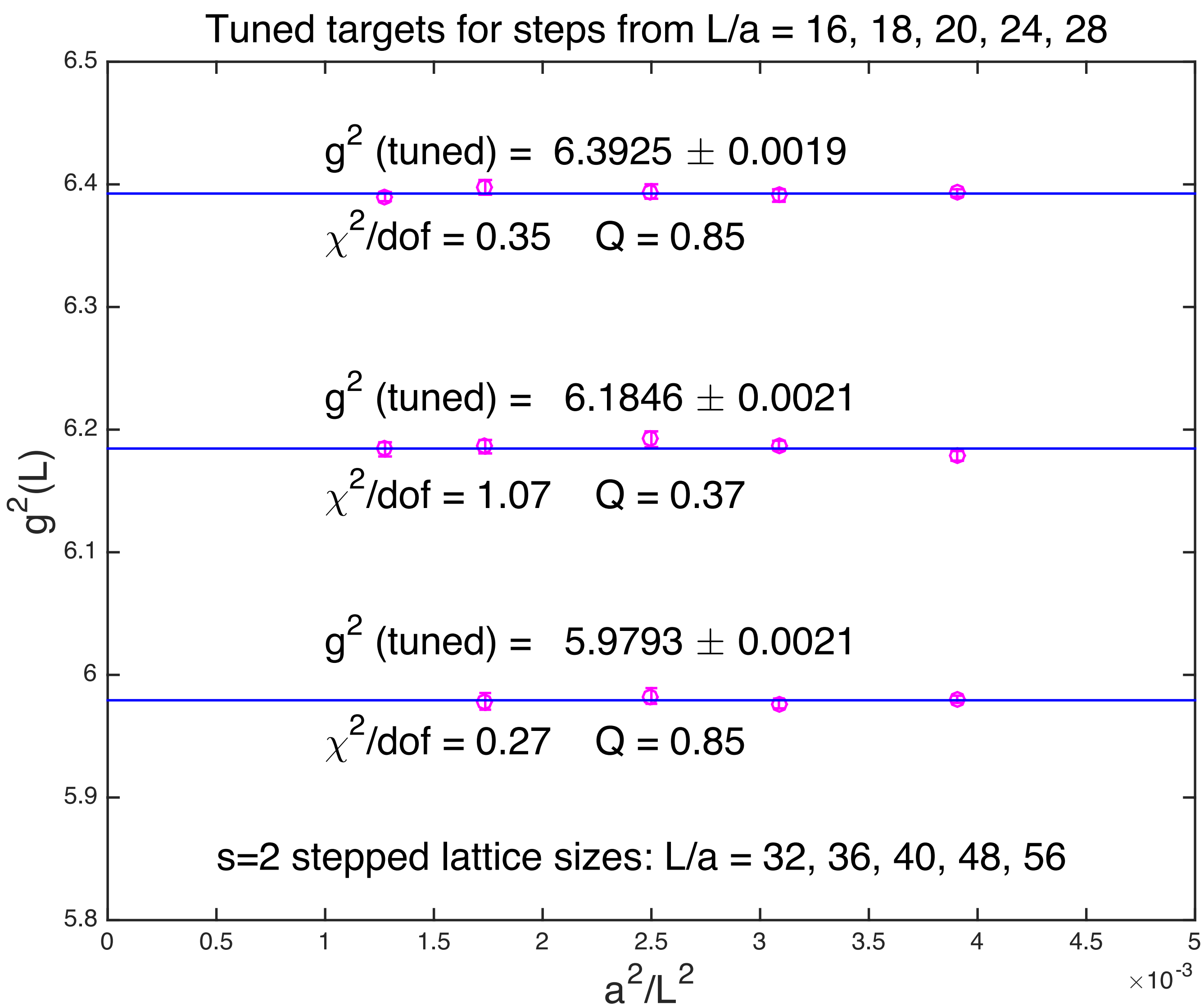
$\longrightarrow$ really tune to fix targets $g^2(L)$: very costly

Target $g^2(L) \sim 6.0,\ \ 6.2,\ \ 6.4$ with volume pairs

$$16 \rightarrow 32,\ \ 18 \rightarrow 36,\ \ 20 \rightarrow 40,\ \ 24 \rightarrow 48,\ \ 28 \rightarrow 56$$

(for $g^2(L) \sim 6.0$ no $28 \rightarrow 56$)

Tuning to fixed $g^2(L)$ for smaller volumes L/a

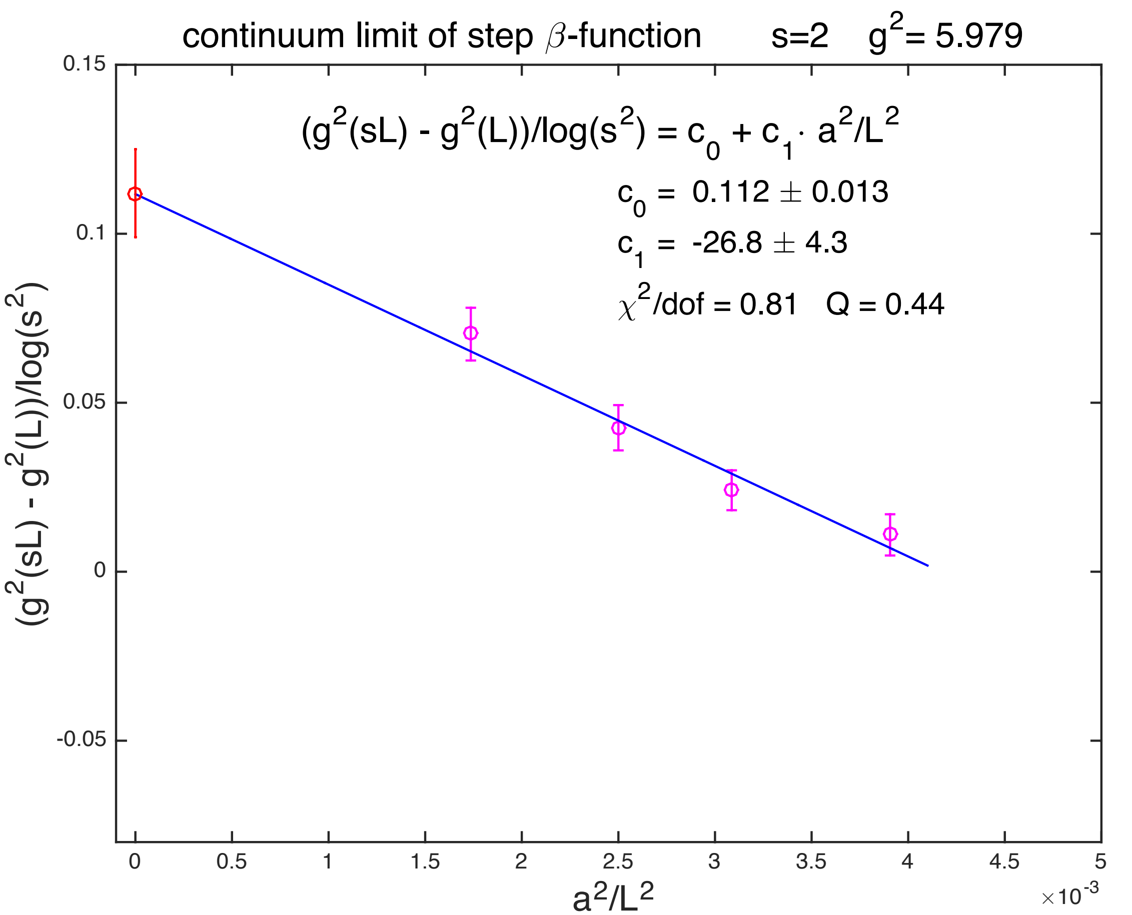


Step scaling to larger volumes sL/a

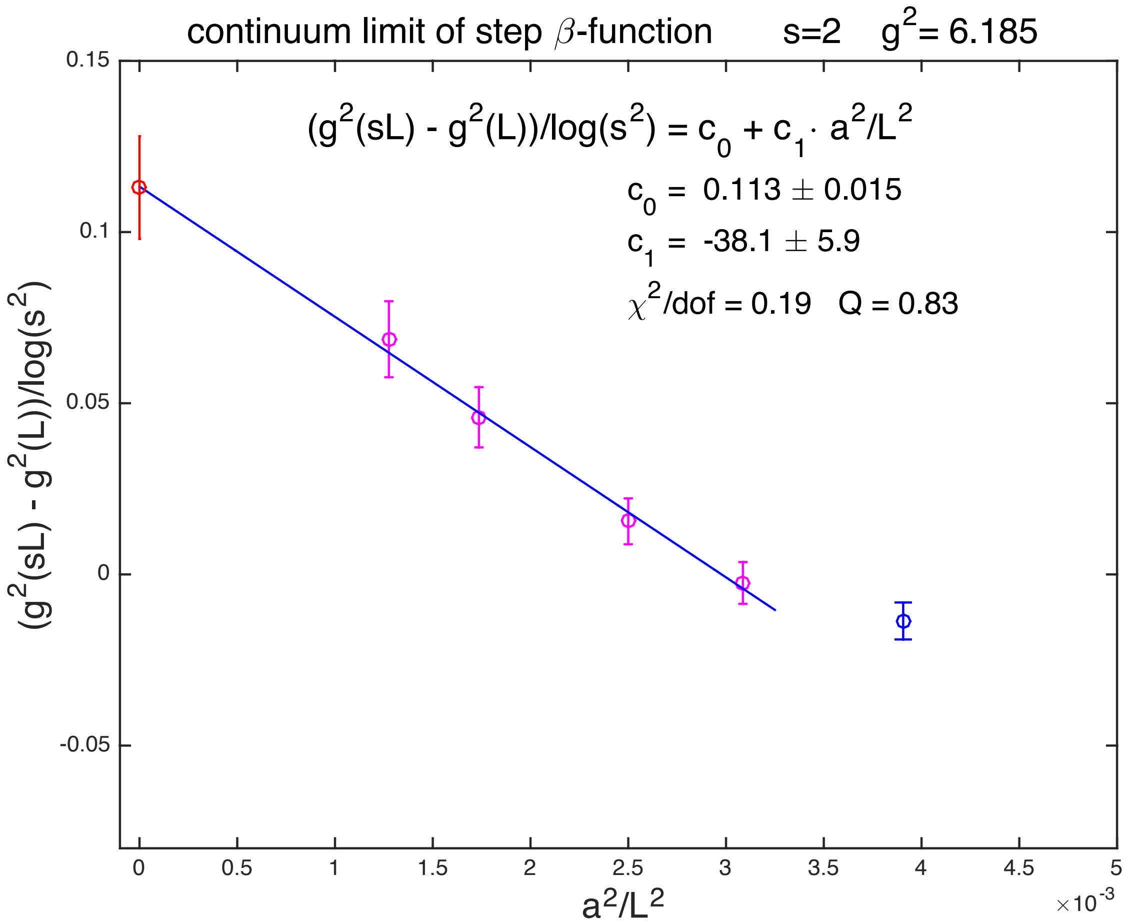
Larger volume runs with tuned bare coupling

L/a	$6/g_0^2$	g^2	$6/g_0^2$	g^2	$6/g_0^2$	g^2
16	3.1519	5.9801(29)	3.0830	6.1786(39)	3.0110	6.3930(30)
32	3.1519	5.9952(79)	3.0830	6.1597(64)	3.0110	6.3233(74)
18	3.1510	5.9767(40)	3.0785	6.1871(37)	3.0055	6.3909(51)
36	3.1510	6.0101(71)	3.0785	6.1840(81)	3.0055	6.3446(64)
20	3.1499	5.9828(64)	3.0704	6.1922(64)	2.9896	6.3942(59)
40	3.1499	6.0419(73)	3.0704	6.2137(67)	2.9896	6.4000(67)
24	3.1480	5.9784(68)	3.0680	6.1861(55)	2.9800	6.3976(60)
48	3.1480	6.0758(84)	3.0680	6.2497(109)	2.9800	6.4404(122)
28			3.0698	6.1839(58)	2.9819	6.3900(37)
56			3.0698	6.2792(142)	2.9819	6.4610(124)

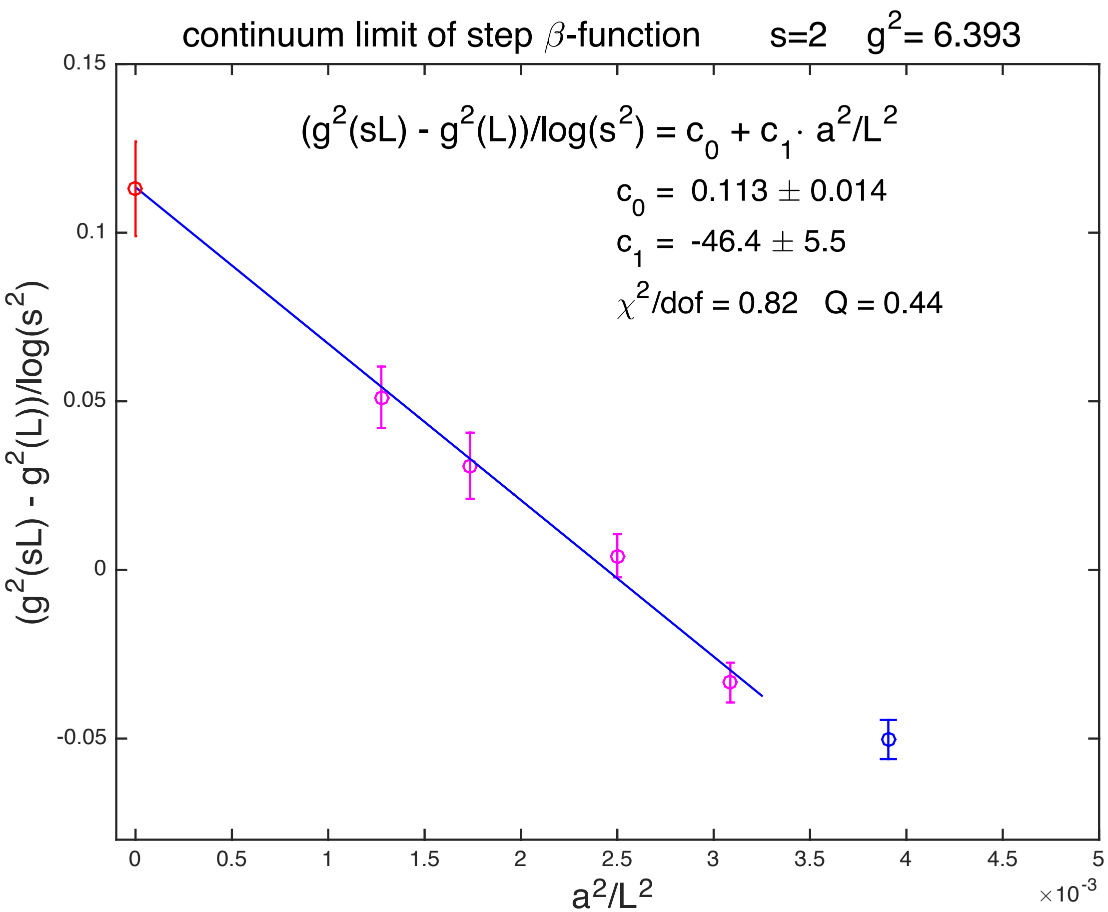
Continuum extrapolation, $g^2 = 5.979$



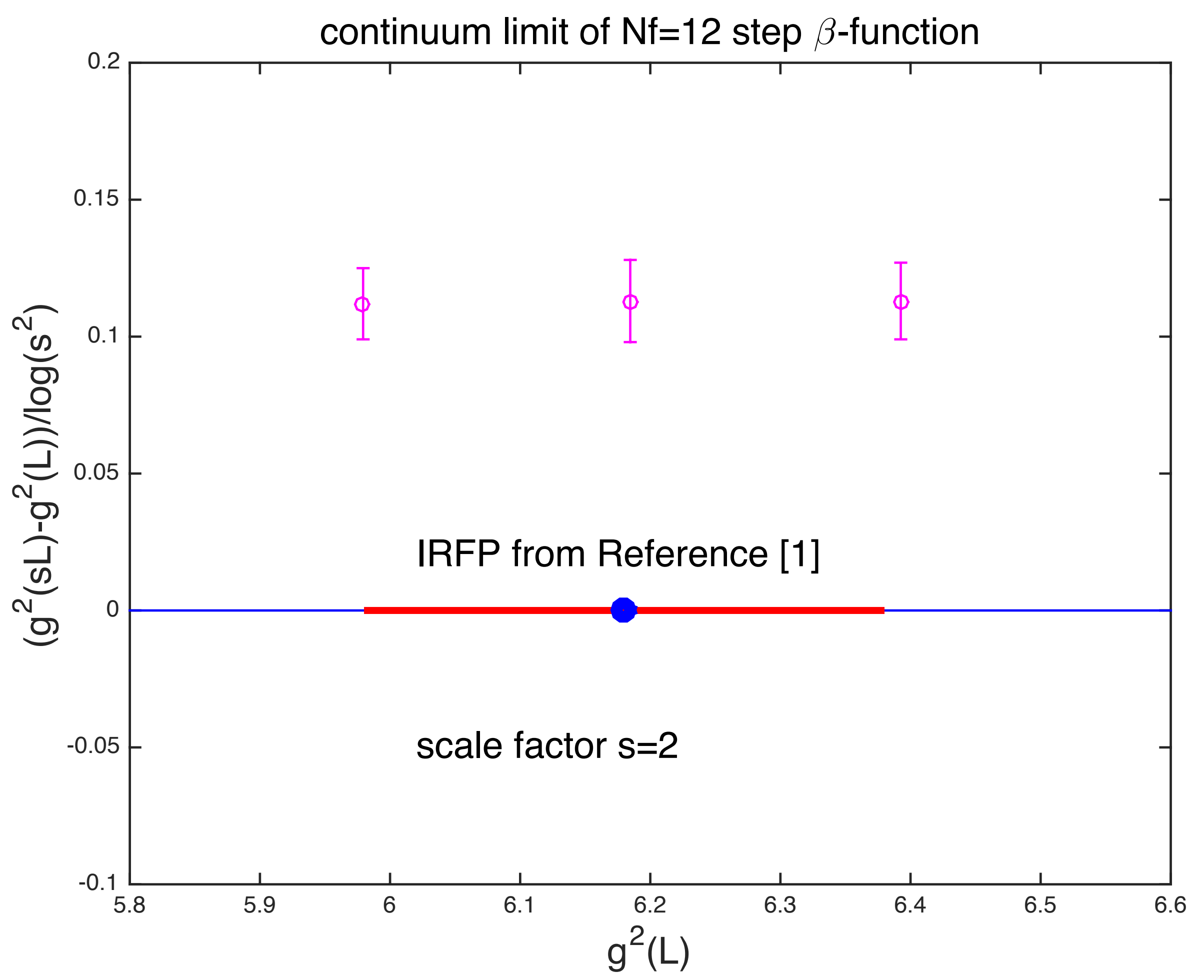
Continuum extrapolation, $g^2 = 6.185$



Continuum extrapolation, $g^2 = 6.393$



Continuum β-function



How is this possible?

Our best guess: Cheng et al. 1404.0984 too small lattice volumes

$12 \to 24, \;\; 16 \to 32, \;\; 18 \to 36$

Whereas we have

$$16 \to 32, \;\; 18 \to 36, \;\; 20 \to 40, \;\; 24 \to 48, \;\; 28 \to 56$$

Hence our best guess: underestimated systematic uncertainty

Thank you for your attention!