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Centre vortices are the seeds of dynamical chiral symmetry breaking

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Collaborators
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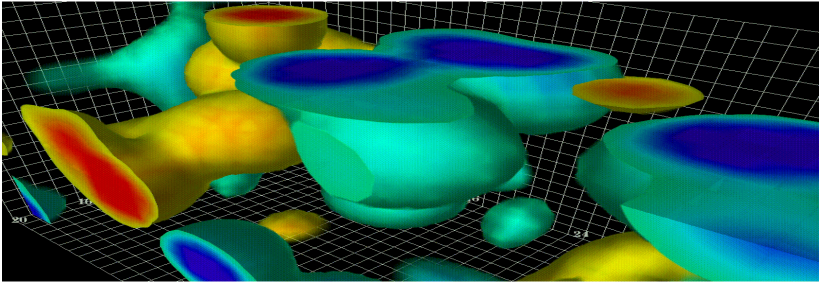
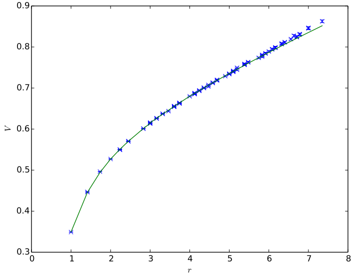
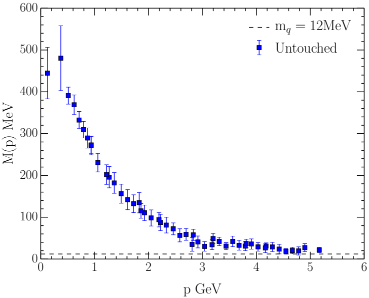


34^{th} International Symposium on Lattice Field Theory
Southampton, UK, July 24-30, 2016

QCD Key Features

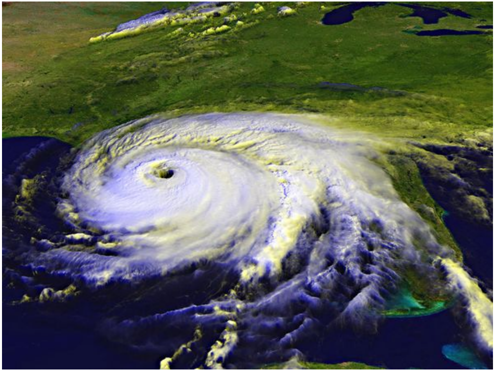


Quark Confinement

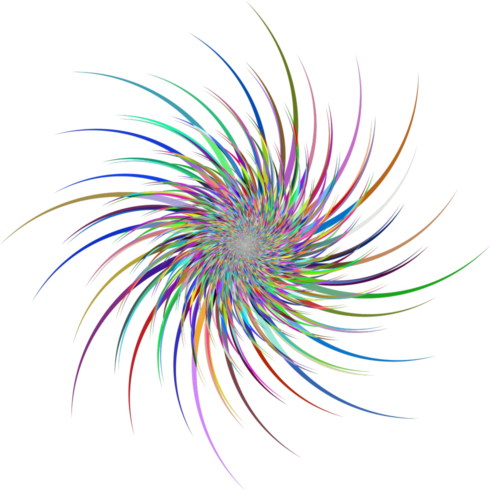


Dynamical mass generation

Emergent Phenomena



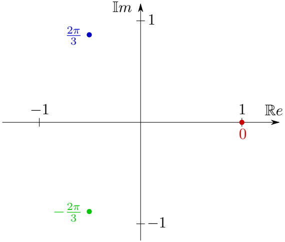
"An emergent behavior or emergent property can appear when a number of simple entities (agents) operate in an environment, forming more complex behaviors as a collective."



Centre-Vortex Model of Confinement

Centre Vortices

- Centre group of $SU(3)$ is $\{zI : z = 1, e^{\pm \frac{2\pi i}{3}}\} \simeq \mathbb{Z}_3$.



- Define a centre transformation on a subset of links by

$$U \to ZU, Z \in \mathbb{Z}_3.$$

- A surface A is pierced by a centre vortex if

$$U(\partial A) \underset{Z}{\to} zU(\partial A), \quad z \neq 1.$$

Identifying Centre Vortices

- Apply transformation $\Omega(x)$ to Maximal Centre Gauge,

$$\frac{1}{3}\mathrm{Tr}U_\mu^\Omega(x) = r_\mu(x)\exp(\frac{2\pi i}{3}\phi_\mu(x)).$$

- Project onto $\mathbb{Z}_3$ by choosing $m \in \{-1, 0, 1\}$ closest to ϕ,

$$Z_\mu(x) = \exp\left[\frac{2\pi i}{3}m_\mu(x)\right]\mathrm{I},$$

- Identify P-vortices via centre-projected plaquette,

$$P_{\mu\nu}(x) = Z_\mu(x)Z_\nu(x+\mu)Z_\mu^\dagger(x+\nu)Z_\nu^\dagger(x) \neq I.$$

Gauge Fields

$$U_\mu(x) = Z_\mu(x) \cdot R_\mu(x)$$

Gauge Fields

$$U_\mu(x) = Z_\mu(x) \cdot R_\mu(x)$$

Untouched

Gauge Fields

$$U_\mu(x) = Z_\mu(x) \cdot R_\mu(x)$$

Untouched

Vortex-only

Gauge Fields

$$U_\mu(x) = Z_\mu(x) \cdot \boldsymbol{R_\mu(x)}$$

Untouched

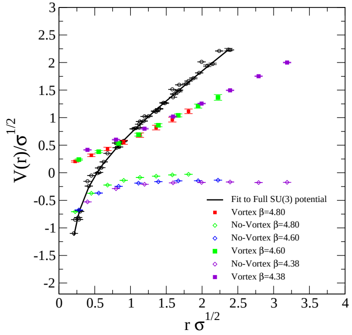
Vortex-only

Vortex-removed

Static Quark Potential

- Confinement $\rightarrow$ static quark potential is linear

Area Law: $$V(r) \sim \sigma r \leftrightarrow W(\mathcal{C}) \sim e^{-\sigma A}$$



Bowman et al, Phys. Rev. D **84***, 034501 (2011)*

MCG procedure cannot simultaneously identify all SU(3) vortex matter.

O'Cais et al, Phys. Rev. D **82***, 114512 (2010)*

Dynamical Chiral Symmetry Breaking

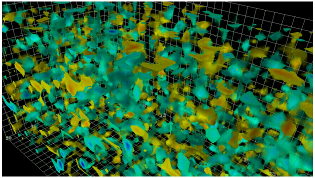
- Topology and dynamical χSB connected via:
  - Banks-Casher relation, $\langle \overline{q}q \rangle = -\pi\, \rho(0), \text{ as } m_q \to 0$
  - Atiyah-Singer index theorem, $Q = n_- - n_+$ at $m_q = 0$.
- Examine local topological charge density,

$$q(x) = \frac{g^2}{16\pi^2} \mathrm{Tr}[\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}(x) F_{\rho\sigma}(x)]$$

- Non-trivial topology ($\leftrightarrow$ instantons) characterised by non-zero topological charge,

$$Q = \sum_x q(x).$$

Topological Charge



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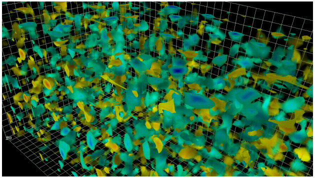
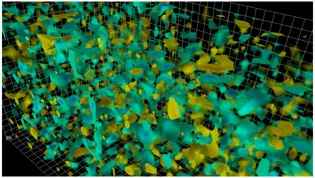


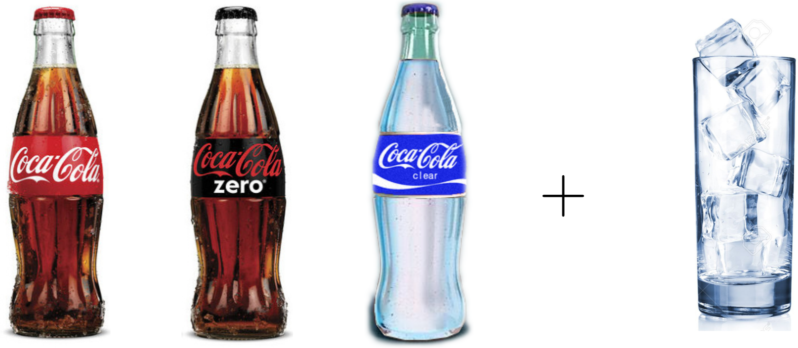
Vortex-only



Vortex-removed

Topological Charge

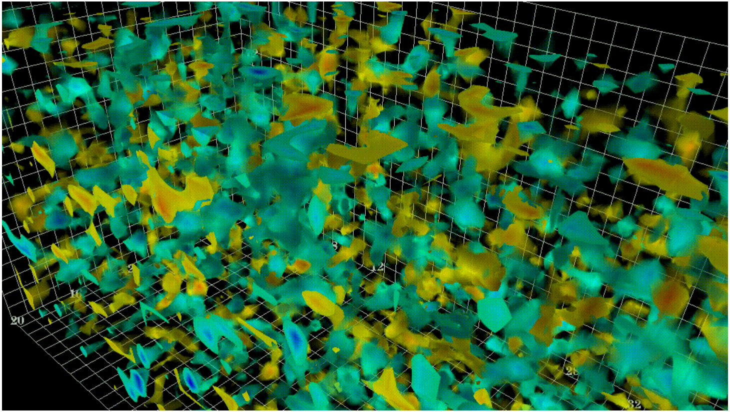
Topological Charge



Simulation Details

- Results calculated on 50 pure $SU(3)$ gauge configurations.
  - Lattice size of $20^3 \times 40$, with $a = 0.125\,\mathrm{fm}$.
  - Lušcher-Weisz mean-field improved action.
- Cooling
  - Minimise the local action.
  - Successive sweeps remove short-range noise, leave an approximation to classical solution.
  - Performed using an $\mathcal{O}(a^4)$-three-loop improved action.
- Topological charge density
  - Calculated using an $\mathcal{O}(a^4)$-five-loop improved $F_{\mu\nu}$.
  - Expect to reveal instanton-like objects.

Untouched Configurations with Cooling



Untouched Configurations with Cooling

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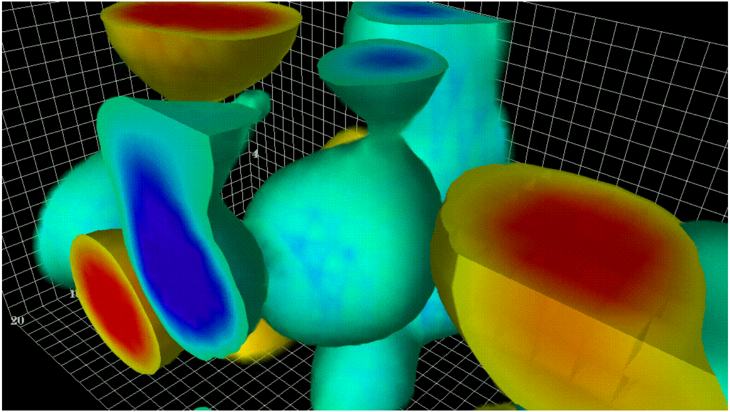
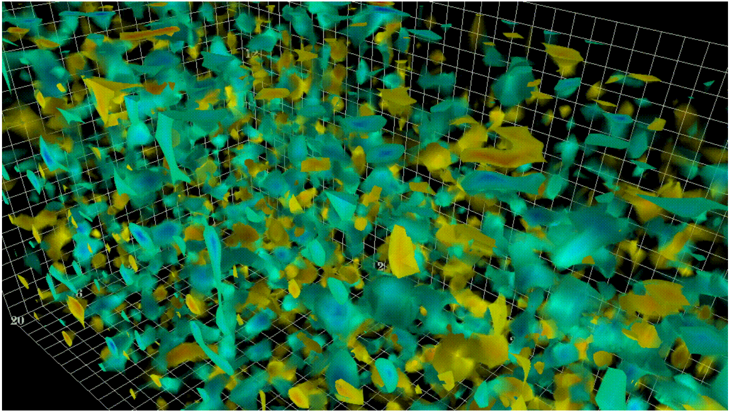


Vortex Removed Configurations with Cooling



Vortex Removed Configurations with Cooling

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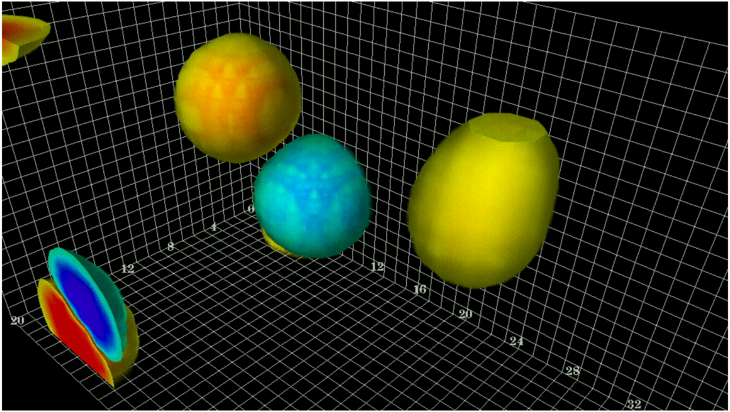
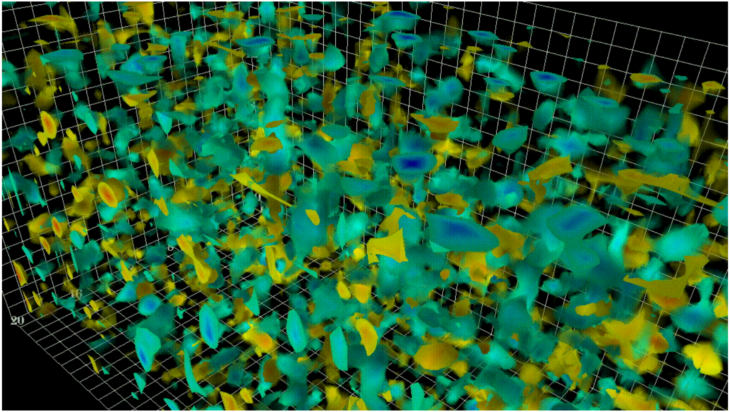
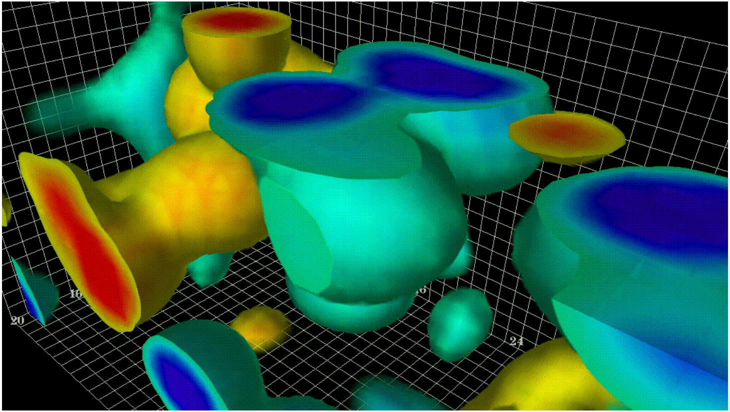


Vortex Only Configurations with Cooling

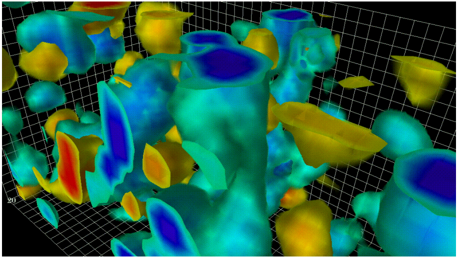


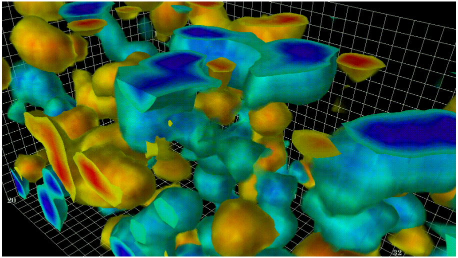
Vortex Only Configurations with Cooling

Vortex Only Configurations with Cooling

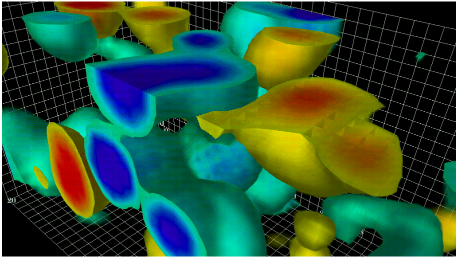


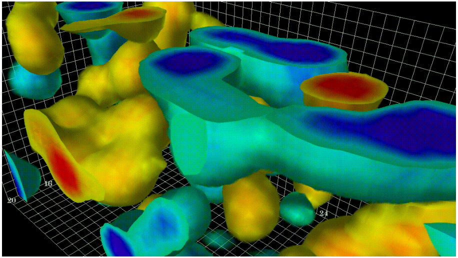
UT and VO comparison (10 sweeps)



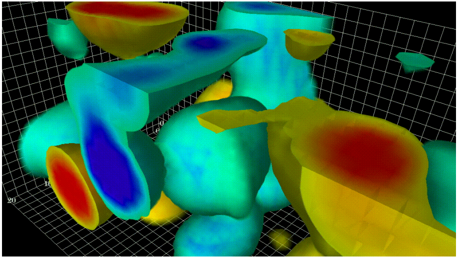


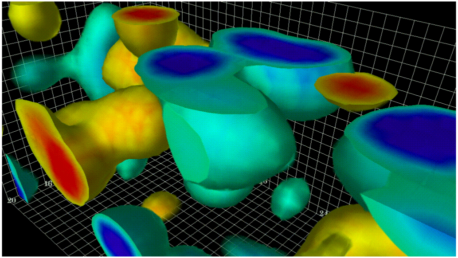
UT and VO comparison (40 sweeps)





UT and VO comparison (80 sweeps)





Centre Vortices and Instantons

- Topological charge density:
  - Centre vortices have a connection to instanton degrees of freedom.
  - Vortex removal destabilizes instanton-like objects under cooling.
  - Vortex-only background creates instanton-like objects under cooling.
- Centre vortices are the seeds of instantons!
  - Cooling turns thin vortices into thick vortices.
- How similar are the vortex only and untouched backgrounds on the ensemble level?

Instantons

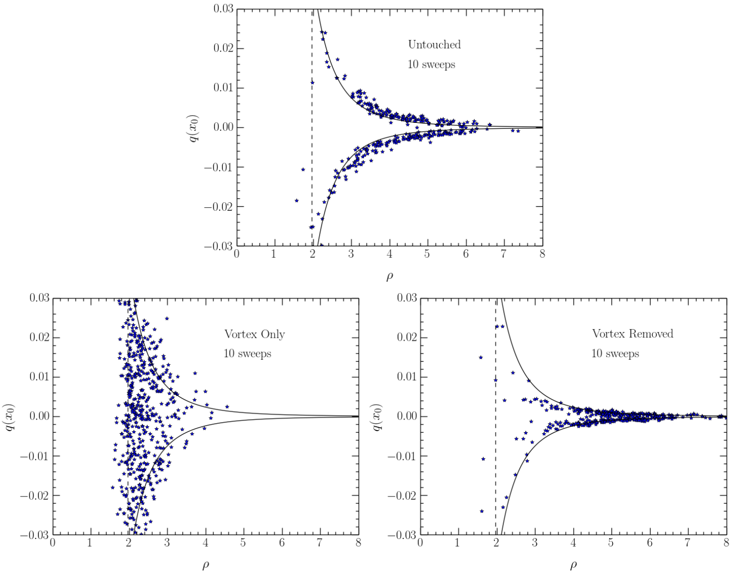
- We directly examine the instanton degrees of freedom on our ensembles
- Scan each configuration for local maxima of the action, fit the instanton solution around them

$$S_0(x) = \xi \frac{6}{\pi^2} \frac{\rho^4}{((x - x_0)^2 + \rho^2)^4}$$

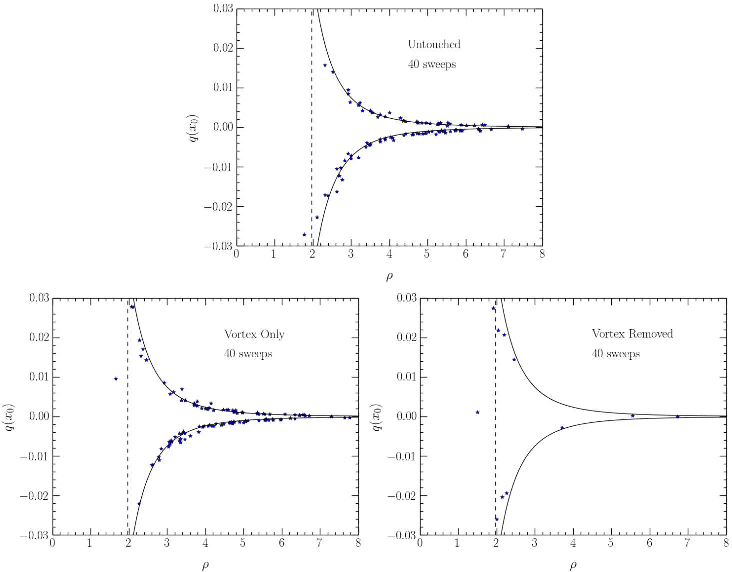
- Compare to theoretical relationship,

$$q(x_0) = Q \frac{6}{\pi^2 \rho^4}$$

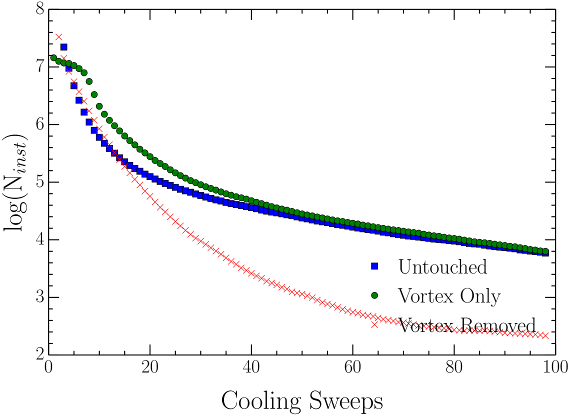
Instanton ρ vs $q(x_0)$



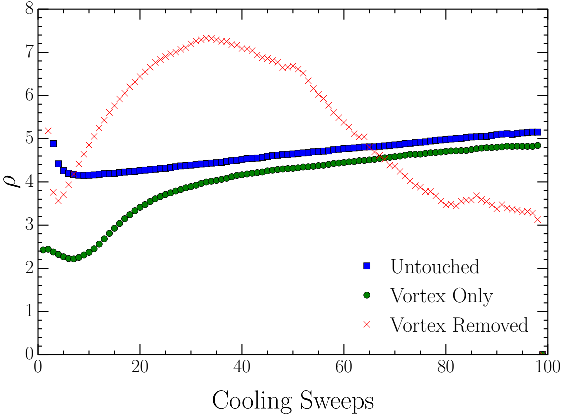
Instanton ρ vs $q(x_0)$



Number of Instantons



Instanton Radius

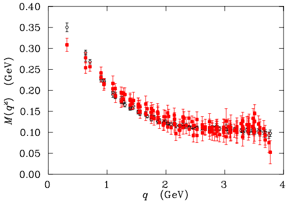


Landau Gauge Quark Propagator

- IR Enhancement of $M(p)$ $\to$ dynamical mass generation

$$S(p) = \frac{Z(p)}{i\not{q} + M(p)}.$$

- Previous results with an ASQTAD action



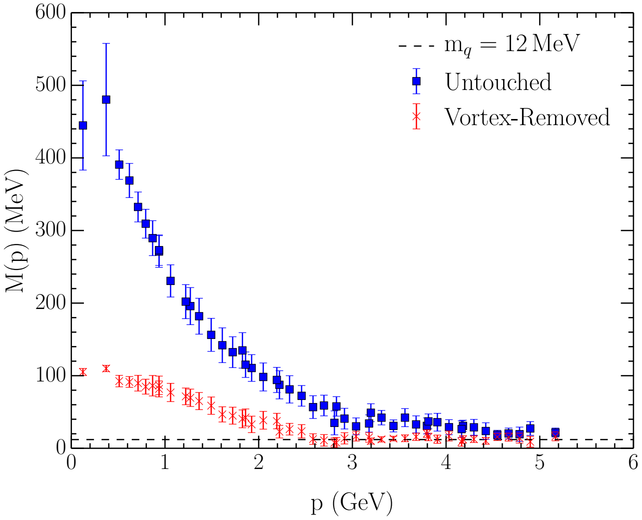
Performed with $m_0 a = 0.048$, $a = 0.122$ on a $16^3 \times 32$ lattice

From Bowman et al, Phys. Rev. D **84**, *034501 (2011)*

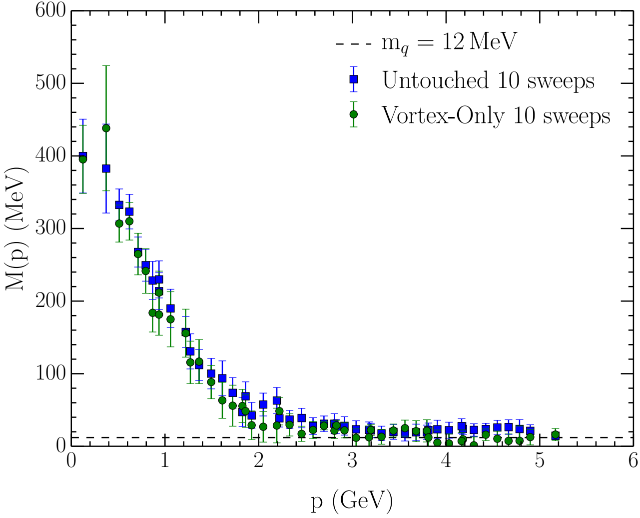
Overlap Quark Propagator

- Previous studies:
  - Fermion action explicitly breaks chiral symmetry.
  - $\Rightarrow$ Use overlap fermions instead.
- Vortex only configurations:
  - Consist only of centre elements $\Rightarrow$ very rough.
  - Overlap operator has smoothness condition!
  - $\Rightarrow$ 10 sweeps of cooling on vortex only configurations
- Overlap mass parameter of $\mu = 0.004 \rightarrow m_q = 12$ MeV.

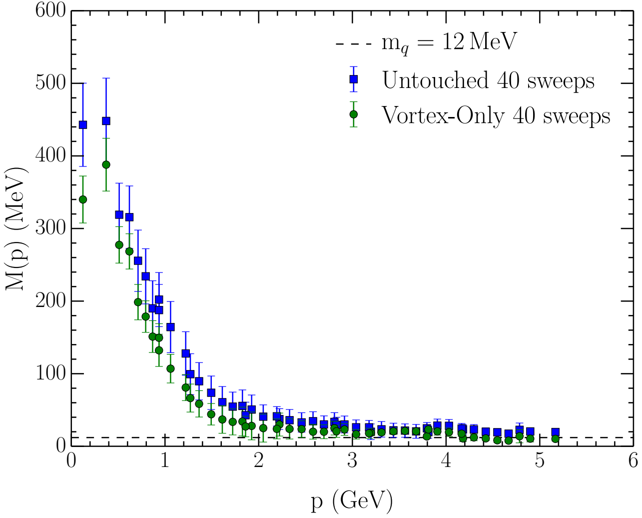
VR Mass function



VO Mass function, 10 sweeps



VO Mass function, 40 sweeps



The story so far...

- Chirally sensitive overlap action 'sees' changes to gauge field:
  - Vortex removal destabilizes instanton-like objects
  - Vortex only background contains seeds of instanton-like objects
- Using the overlap operator, we have shown for the first time a clear link between DχSB and centre vortices in $\mathrm{SU}(3)$
  - Removing centre vortices destroys dynamical mass generation
  - Dynamical mass generation completely reproduced from centre vortices alone
- What about the hadron spectrum?

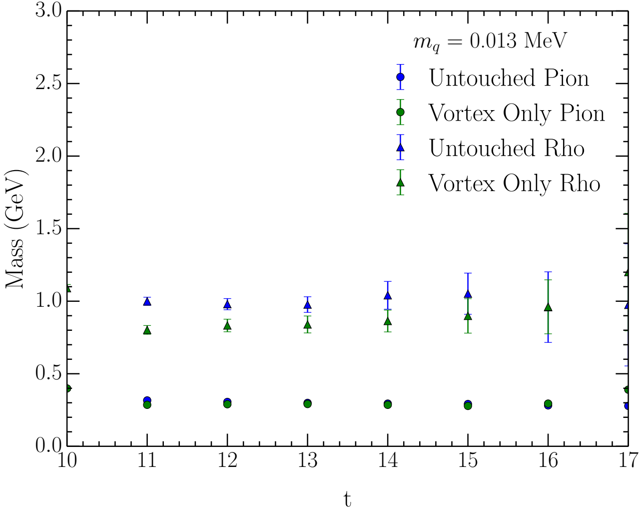
Hadron spectrum

Meson	I, J^{PC}	Operator
π	$1, 0^{+-}$	$\bar{q}\,\gamma_5\,\frac{\tau^a}{2}\,q$
ρ	$1, 1^{--}$	$\bar{q}\,\gamma_i\,\frac{\tau^a}{2}\,q$
a_0	$1, 0^{++}$	$\bar{q}\,\frac{\tau^a}{2}\,q$
a_1	$1, 1^{++}$	$\bar{q}\,\gamma_i\,\gamma_5\,\frac{\tau^a}{2}\,q$

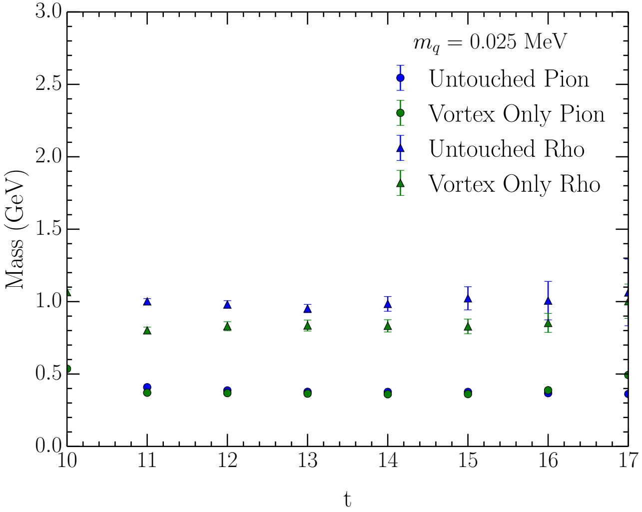
Baryon	I, J^P	Operator
N	$\frac{1}{2}, \frac{1}{2}^+$	$[u^T\,C\,\gamma_5\,d\,]\,u$
Δ	$\frac{3}{2}, \frac{3}{2}^+$	$[u^T\,C\,\gamma_i\,u\,]\,u$

- Overlap fermion action.
- 100 sweeps of Gaussian smearing on source.
- 10 sweeps of cooling on VO ensemble.

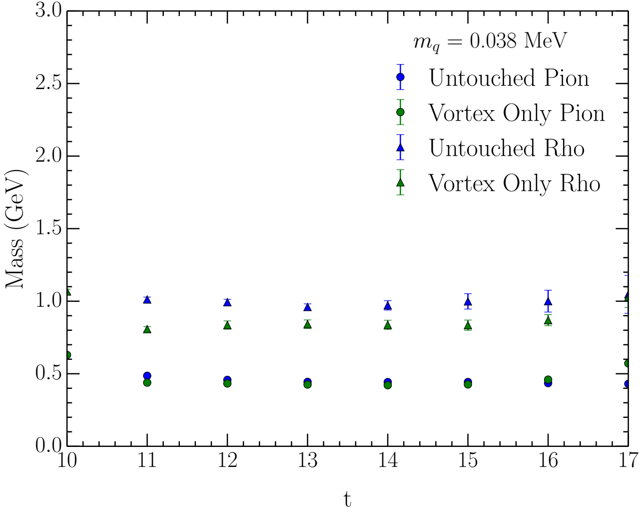
Vortex-only spectrum: π, ρ



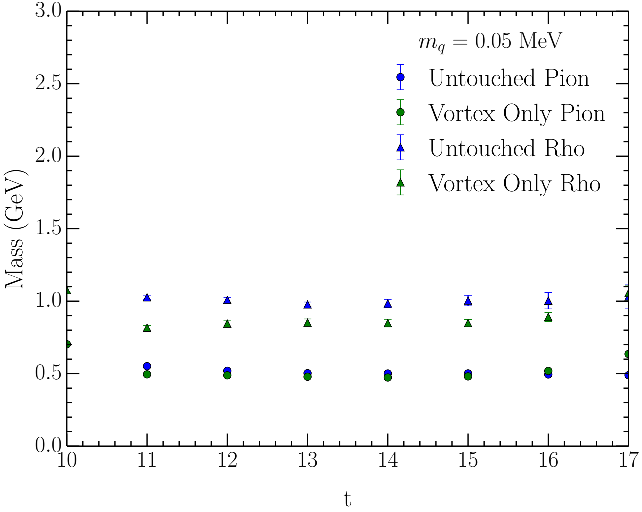
Vortex-only spectrum: π, ρ



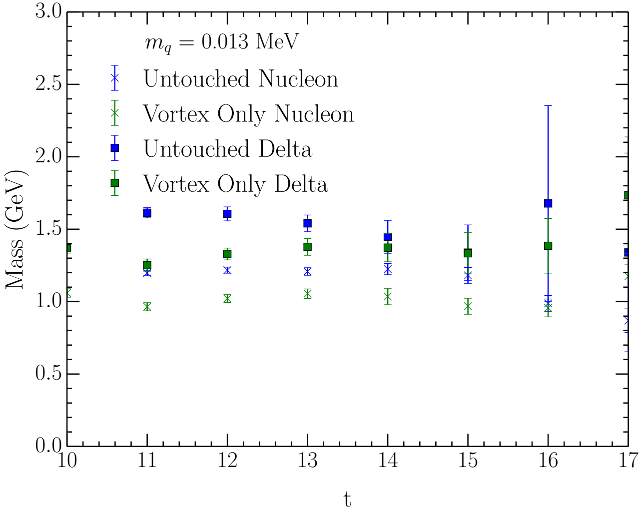
Vortex-only spectrum: π, ρ



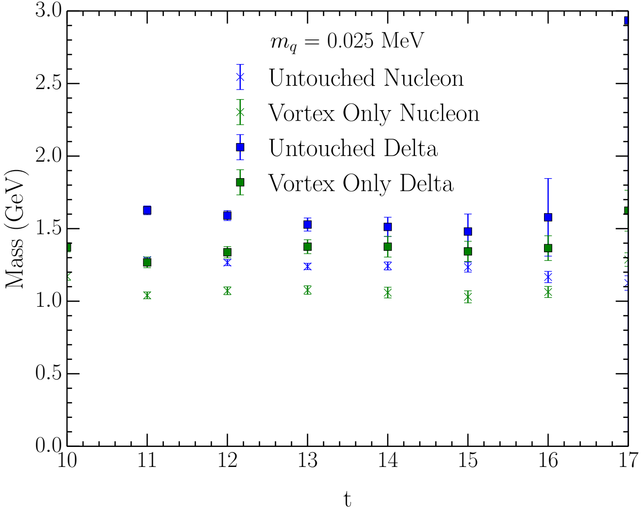
Vortex-only spectrum: π, ρ



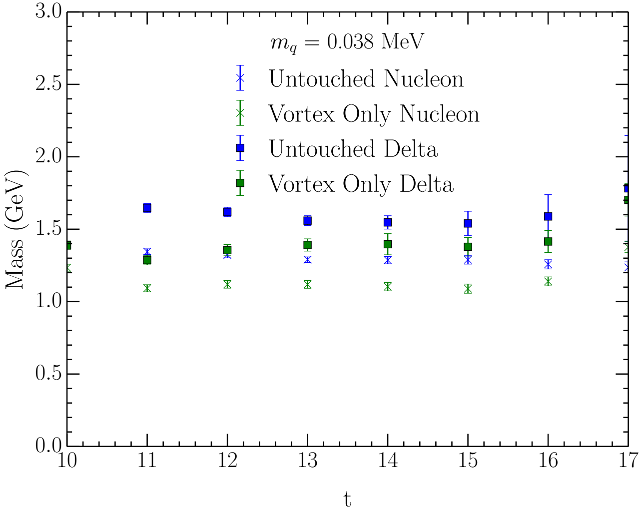
Vortex-only spectrum: N, Δ



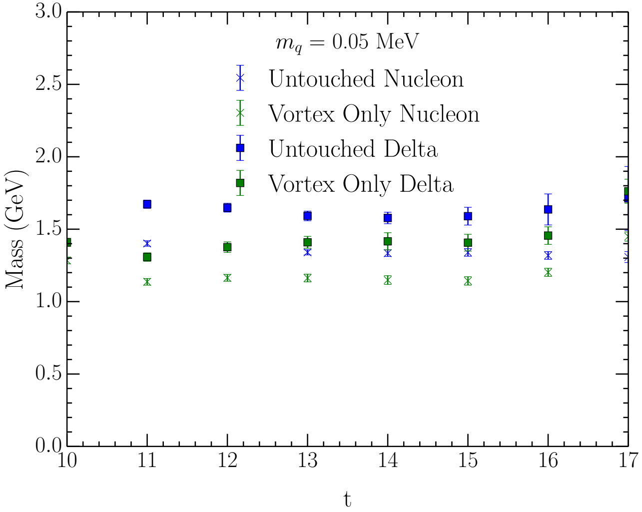
Vortex-only spectrum: N, Δ



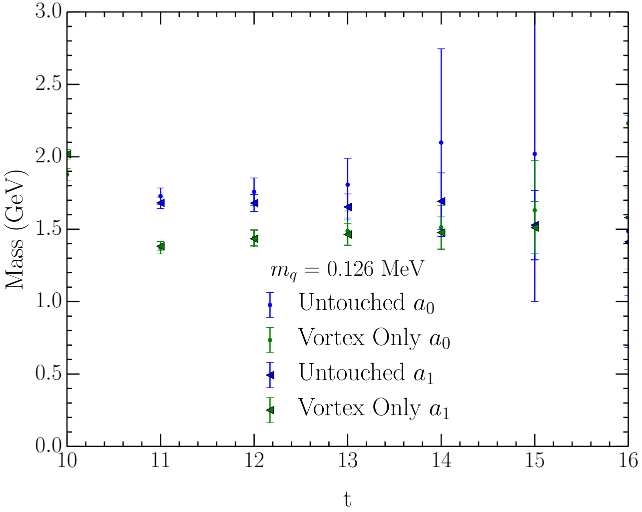
Vortex-only spectrum: N, Δ



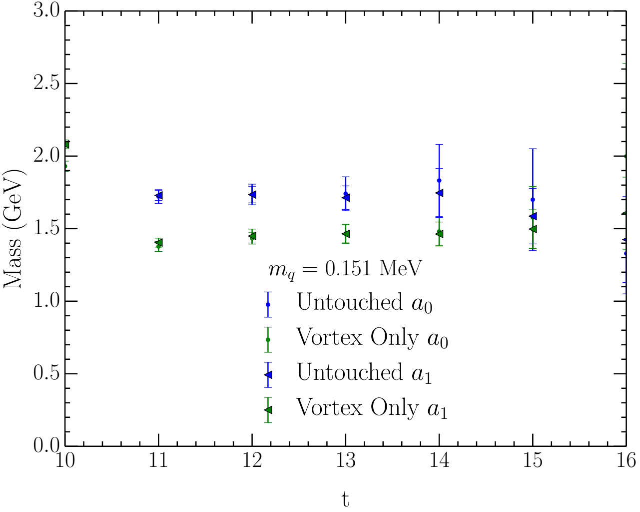
Vortex-only spectrum: N, Δ



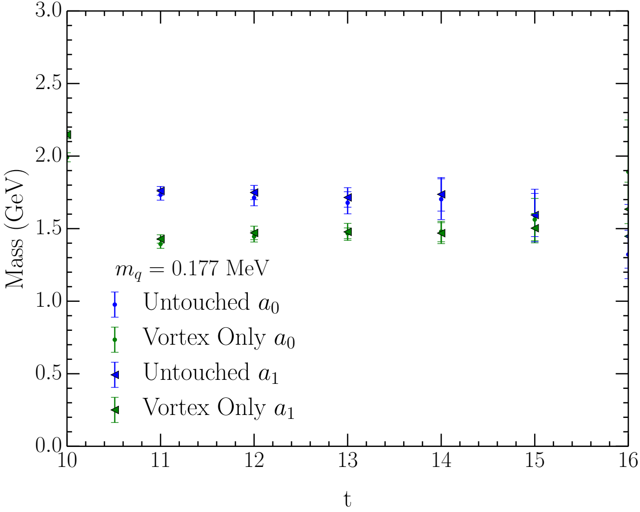
Vortex-only spectrum: a_0, a_1



Vortex-only spectrum: a_0, a_1



Vortex-only spectrum: a_0, a_1



Restoration of Chiral Symmetry

- If vortices are responsible for $D\chi SB$, then their removal should restore chiral symmetry

$$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R} \times \mathrm{U}(1)_\mathrm{A}$$

- Expect baryon currents related by chiral transformations to become degenerate

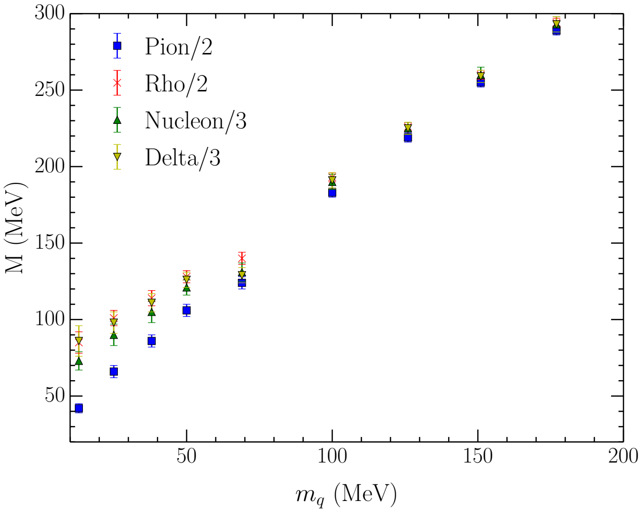
$$\begin{array}{ccc} \pi & \xleftrightarrow{\mathrm{U}(1)_\mathrm{A}} & a_0 \\ \rho & \xleftrightarrow{\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}} & a_1 \\ N & \xleftrightarrow{\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}} & \Delta \end{array}$$

- $\mathrm{U}(1)_\mathrm{A}$ is related to the axial anomaly and may be restored separately to $\mathrm{SU}(2)_\mathrm{L} \times \mathrm{SU}(2)_\mathrm{R}$

Restoration of Chiral Symmetry

- Chiral symmetry restoration only reasonable at light quark masses, explicitly broken at heavy mass.
- At small quark masses, we should see a *chiral regime* with corresponding hadron degeneracies.
  - No chiral transformation $\pi \leftrightarrow \rho \Rightarrow$ expect different masses.
  - No $D\chi SB \Rightarrow \pi$ is no longer a pseudo-Goldstone boson.
- At large quark masses, we should see a *constituent regime* of weakly-interacting constituent quarks.
  - Baryons $m_N \simeq m_\Delta \simeq 3m_Q$, mesons $m_\pi \simeq m_\rho \simeq 2m_Q$.
- N, Δ should be degenerate at all quark masses;
  - Via $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry at low mass,
  - Both composed of 3 dressed quarks at high mass.

VR constituent quark mass



VR Meson Spectrum: Chiral Regime

- Restoration of $\mathrm{U}(1)_A$ symmetry $\rightarrow$ degeneracy of π and ground state a_0.
- Restoration of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry $\rightarrow$ degeneracy of ρ and ground state a_1.

VR Spectrum: the a_0

Chiral regime:

- Restoration of $\mathrm{U}(1)_A$ symmetry $\rightarrow$ degeneracy of π and ground state a_0.

Constituent regime:

- Can't form the quantum numbers of the a_0 with quarks at rest.
- The a_0 mass should be the lower of two possibilities:
  - π-η state with mass $2\, m_\pi$, or
  - Two dressed quark masses with the lowest non-trivial momentum to provide overlap with an $l = 1$ orbital angular momentum state.

VR Spectrum: the a_0

- Define ratio

$$R_1 = \frac{m_{a_0}}{2\, m_\pi}$$

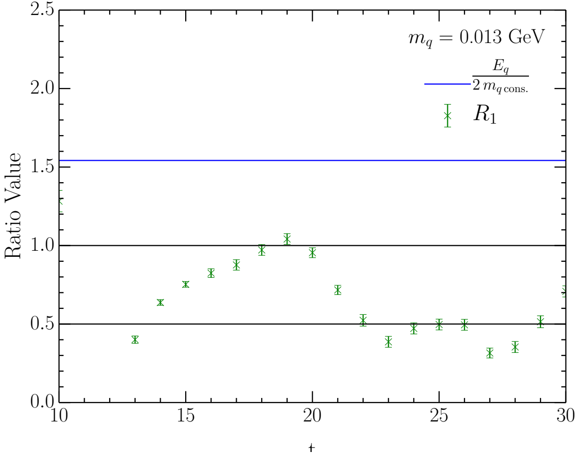
- Chiral regime $m_{a_0} = m_\pi$,

$$R_1 \to \frac{1}{2}$$

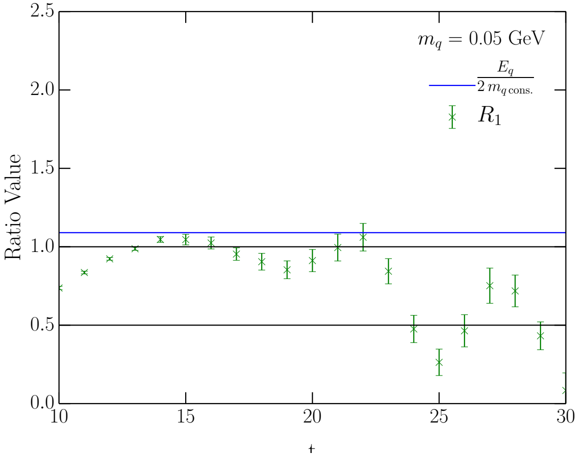
- Constituent regime:

$$R_1 \to \begin{cases} 1 & (\pi - \eta \text{ state}) \\ \frac{2\, E_q}{4\, m_{q\,\text{cons.}}} & \text{(two-quark state)} \end{cases}$$

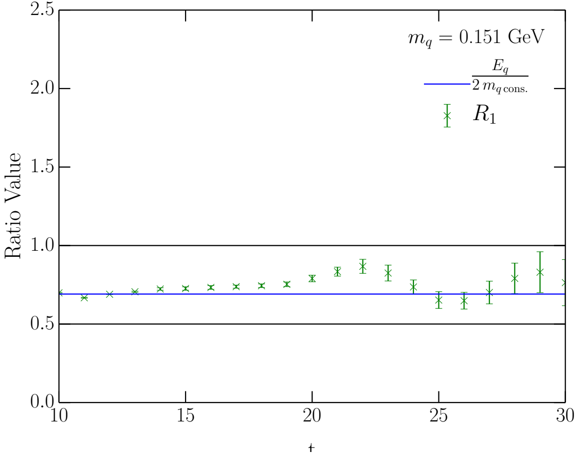
a_0, Chiral Regime



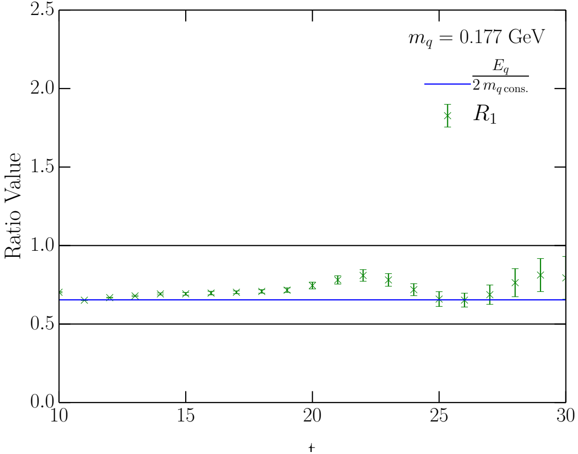
a_0, Chiral Regime



a_0, Constituent Regime



a_0, Constituent Regime



VR Spectrum: the a_1

Chiral regime:

- Restoration of $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry $\rightarrow$ degeneracy of ρ and ground state a_1.

Constituent regime:

- The a_1 should be the lower of two possibilies:
  - A ρ-η state, or
  - Two dressed quark masses with the lowest non-trivial momentum.

VR Spectrum: the a_1

- Define ratio

$$R_2 = \frac{m_{a_1}}{2\, m_\rho}$$

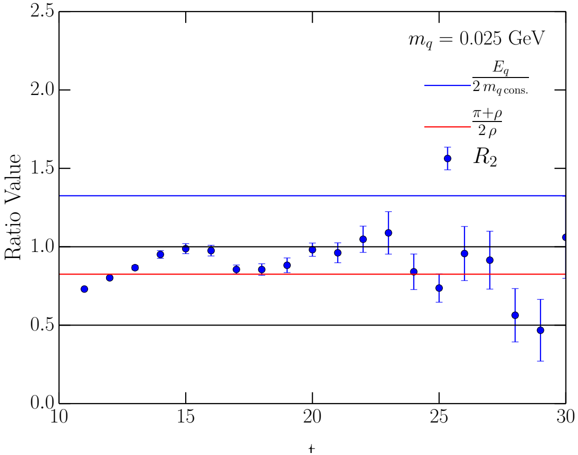
- Chiral regime $m_{a_1} = m_\rho$,

$$R_2 \to \frac{1}{2}$$

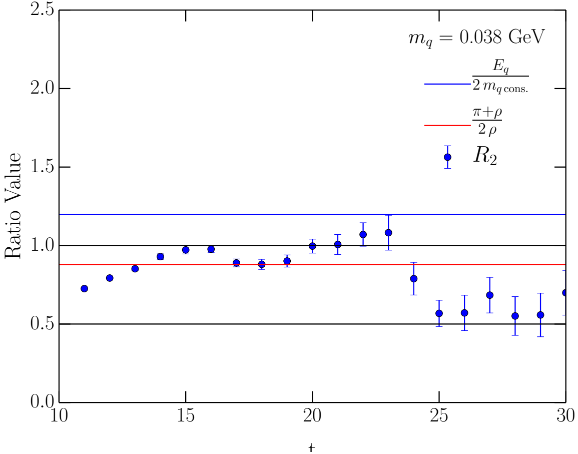
- Constituent regime $(m_\rho = m_\pi)$:

$$R_2 \to \begin{cases} 1 & (\rho - \eta \text{ state}) \\ \frac{2\, E_q}{4\, m_{q\,\text{cons.}}} & \text{(two-quark state)} \end{cases}$$

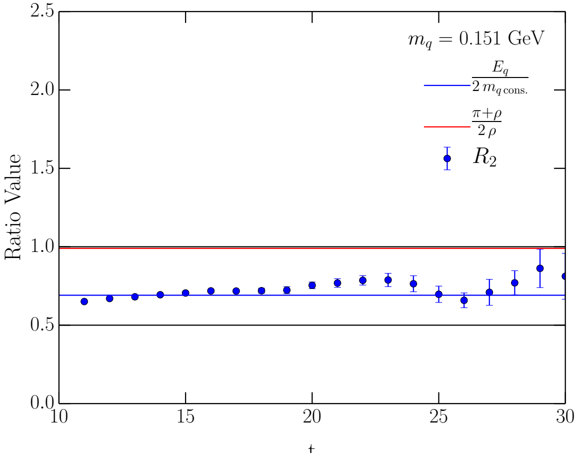
a_1, Chiral Regime



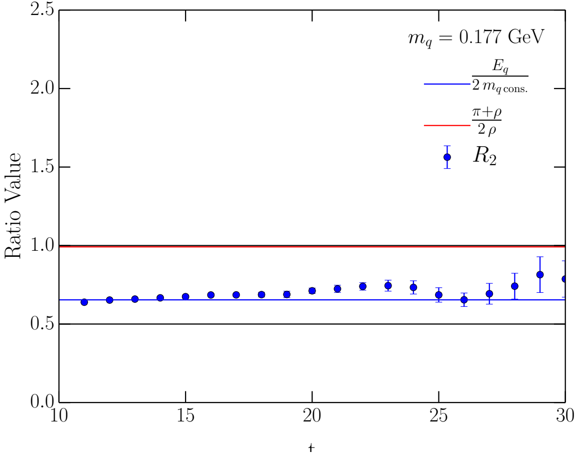
a_1, Chiral Regime



a_1, Constituent Regime



a_1, Constituent Regime



Summary (1)

- Static quark potential;
  - Vortex removal removes linear potential.
  - Vortex-only configs recreate $\sim 2/3$ of string tension.
  - Identification procedure imperfect.
- Connection to instanton degrees of freedom
  - Vortex removal destabilizes instantons under cooling.
  - Vortex-only creates instantons under cooling.
  - No one-to-one correspondence between objects on vortex only and untouched configurations.
  - Identical structure across ensembles
  - Vortices contain the 'seed' of instantons.

Summary (2)

- Overlap quark propagator;
  - Loss of dynamical mass generation with vortex removal.
  - Dynamical mass generation recreated by vortex-only background.
- Hadron spectrum;
  - Chiral symmetry restoration in light quark regime.
  - Weakly interacting theory of constituent quarks at heavy quark mass.
  - Pion is no longer a Goldstone boson.

Conclusion

Conclusion

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Conclusion

+

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Conclusion

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$=$

Centre vortices contain all information necessary to reproduce DχSB in $\mathrm{SU}(3)$ gauge theory.

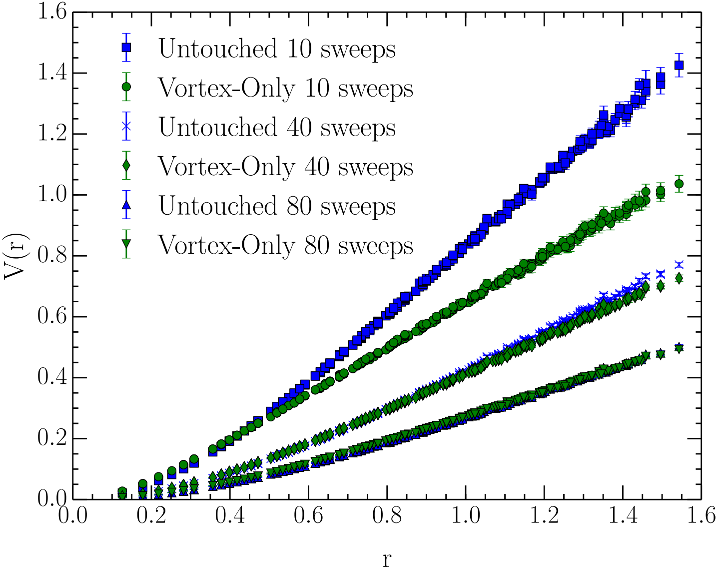
Extra Slides

Vortex-removed spectrum

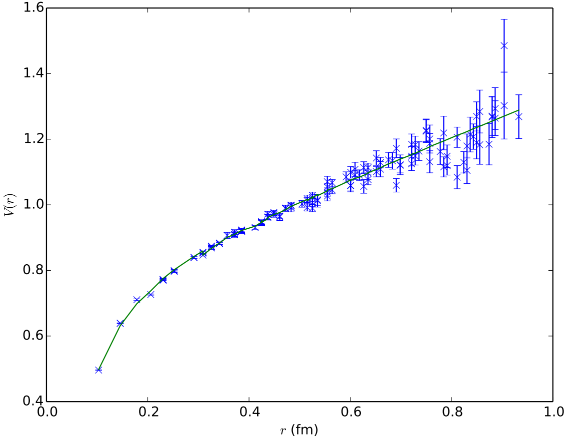
Table : Fitted masses of the pion, rho, nucleon, and Δ as a function of the bare quark mass, m_q.

m_q (MeV)	m_π (MeV)	m_ρ (MeV)	m_N (MeV)	m_Δ (MeV)
13	$85(3)$	$171(7)$	$219(6)$	$260(10)$
25	$132(4)$	$203(5)$	$272(7)$	$295(7)$
38	$173(4)$	$228(5)$	$316(7)$	$334(6)$
50	$213(4)$	$257(4)$	$365(5)$	$378(5)$
100	$366(3)$	$386(3)$	$572(5)$	$575(5)$
126	$439(3)$	$453(3)$	$676(4)$	$676(4)$
151	$510(3)$	$521(3)$	$780(4)$	$779(4)$
177	$578(3)$	$588(3)$	$881(4)$	$880(5)$

Static quark potential with cooling

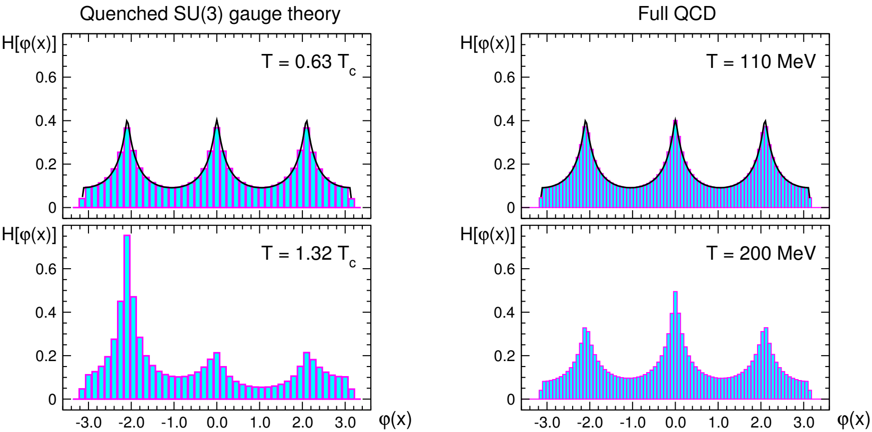


Static quark potential in full QCD



PACS-CS ensemble, $m_\pi \simeq 297$ MeV

Polyakov phase in full QCD



Danzer, Gattringer, Borsanyi and Fodor, PoS LATT2010 (176), arXiv:1010.5073