The issue of the poles for lattice models

Comparison of CLE and reweighting for QCD

Dénes Sexty

Wuppertal University, Eötvös University

Lattice 2016, Southampton, 26th of July, 2016

Collaborators: Gert Aarts, Erhard Seiler, Ion-Olimpiu Stamatescu
Felipe Attanasio, Lorenzo Bongiovanni, Benjamin Jäger,
Zoltán Fodor, Sándor Katz, Csaba Török

1. poles and HDQCD
2. poles and full QCD
   [Aarts, Seiler, Sexty, Stamatescu in prep.]
3. Comparison of CLE and reweighting for full QCD
   [Fodor, Katz, Sexty, Torok (2015)]

Proof of convergence for CLE results

If there is fast decay $P(x,y) \rightarrow 0$ as $x, y \rightarrow \infty$

and a holomorphic action $S(x)$

then CLE converges to the correct result
[Aarts, Seiler, Stamatescu (2009)
Aarts, James, Seiler, Stamatescu (2011)]

Non-holomorphic action for nonzero density

$$S = S_W[U_\mu] + \ln \mathrm{Det}\, M(\mu)$$

measure has zeros $(Det\, M = 0)$
complex logarithm has a branch cut
$\longrightarrow$ meromorphic drift
[Mollgaard, Splittorff (2013), Greensite(2014)]

Incorporating poles to proof, investigations of toy models

[See Gert Aarts' talk]

Heavy Quark QCD at nonzero chemical potential (HDQCD)

Hopping parameter expansion of the fermion determinant
Spatial hoppings are dropped

$$\mathrm{Det}\, M(\mu) = \prod_x \det(1 + C\, P_x)^2 \det(1 + C'\, P_x^{-1})^2$$

$$P_x = \prod_\tau U_0(x + \tau a_0) \qquad C = [2\kappa \exp(\mu)]^{N_\tau} \qquad C' = [2\kappa \exp(-\mu)]^{N_\tau}$$

$$S = S_W[U_\mu] + \ln \mathrm{Det}\, M(\mu)$$

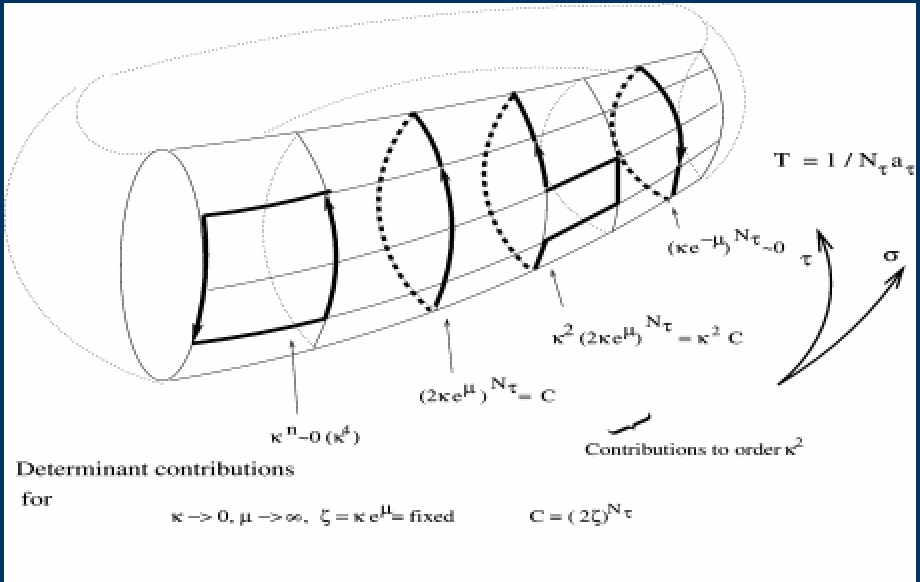
Studied with reweighting

$$R = e^{\sum_x C\, \mathrm{Tr}\, P_x + C'\, \mathrm{Tr}\, P^{-1}}$$

[De Pietri, Feo, Seiler, Stamatescu (2007)]
[Rindlischbacher, de Forcrand (2015)]

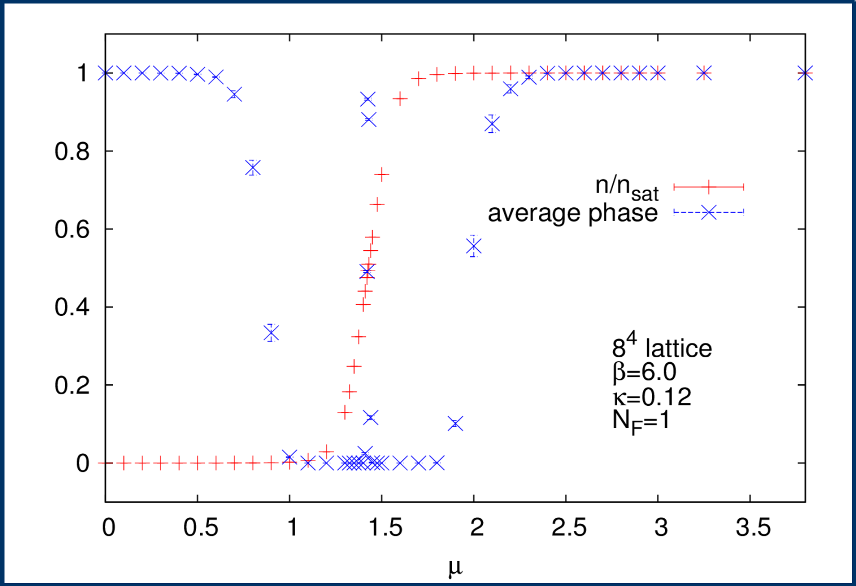
CLE study using gaugecooling

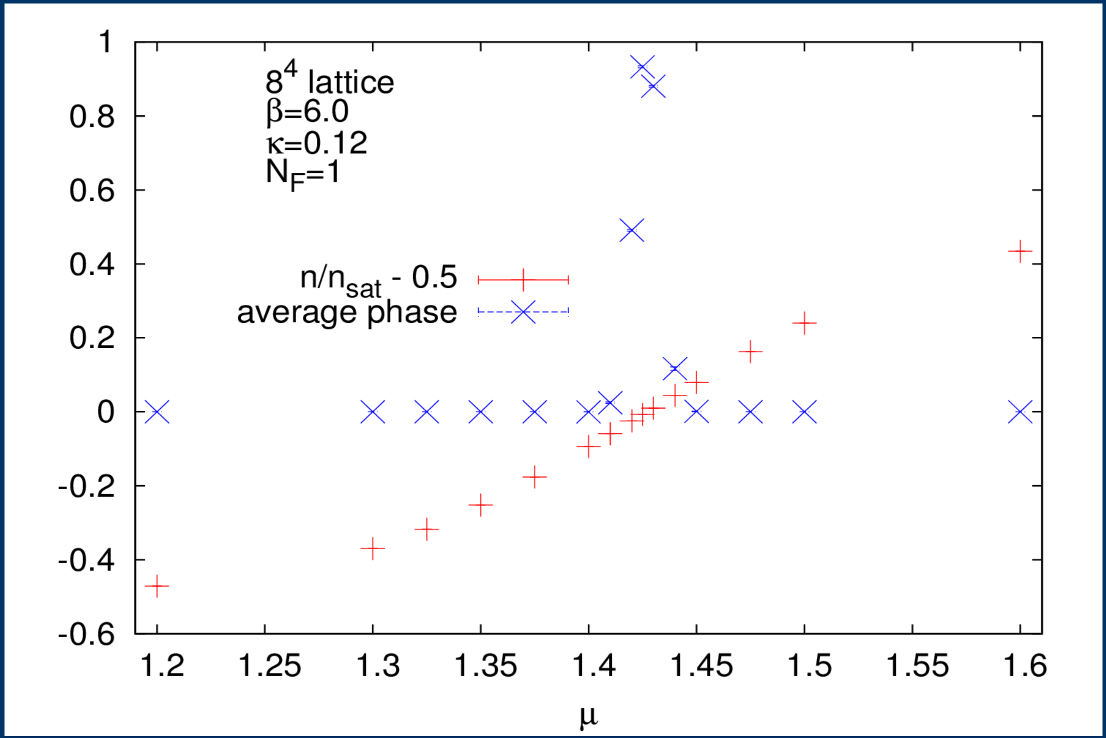
[Seiler, Sexty, Stamatescu (2012)]
[Aarts, Attanasio, Jäger, Sexty (2016)]



Critical chemical potential in HDQCD

Phase average $\langle \exp(2i\varphi) \rangle = \left\langle \frac{\mathrm{Det}\, M(\mu)}{\mathrm{Det}\, M(-\mu)} \right\rangle$





Hard sign problem $1<\mu<1.8$

Except in the middle at half filling $\mu_c = -\ln(2\kappa)$

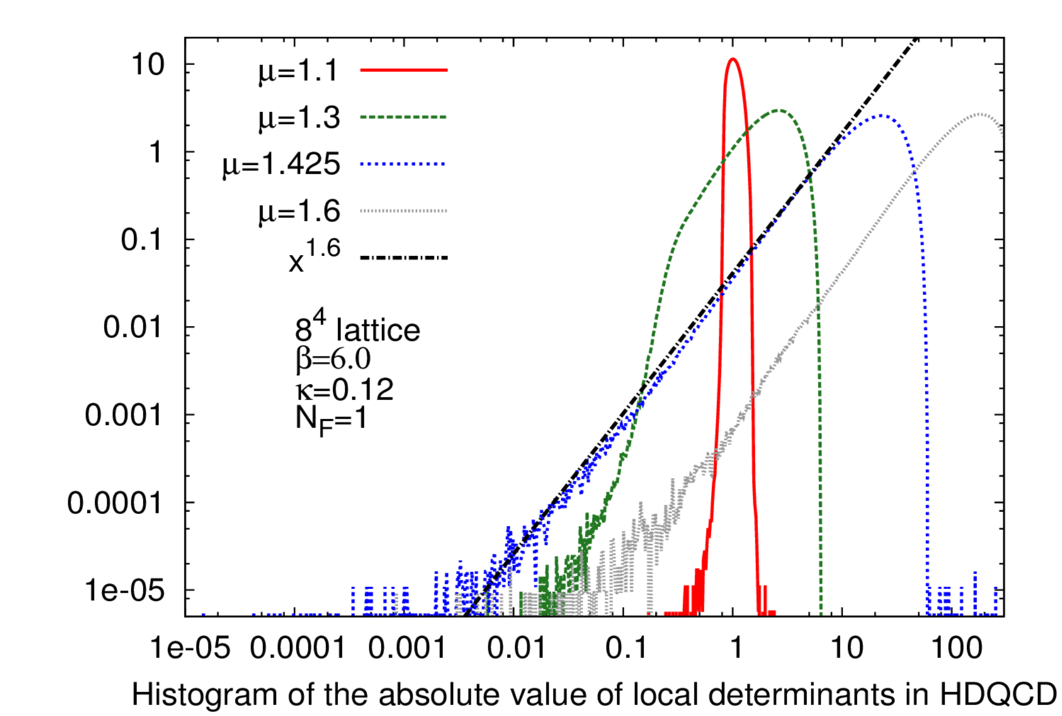
$$\mathrm{Det}\, M(\mu) = \prod_x \det(1+2\kappa e^{\mu} P_x)^2 \det(1+2\kappa e^{-\mu} P_x^{-1})^2$$

$$\det(1+CP) = 1 + C^3 + C\,\mathrm{Tr}\,P + C^2\,\mathrm{Tr}\,P^{-1}$$

At μ_c only the second factor has a(n exponentially suppressed) sign problem

Do poles play a role in HDQCD?

Distribution around the zero of the determinant



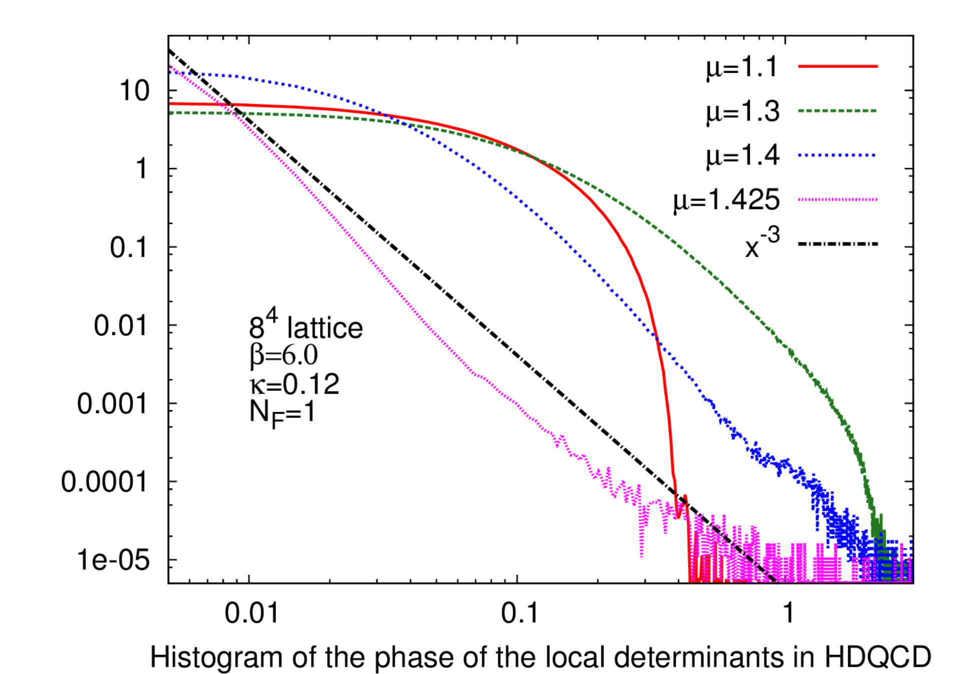
Histogram of the absolute value of local determinants in HDQCD

Only gets close to the pole around μ_c

Where it shows criticality

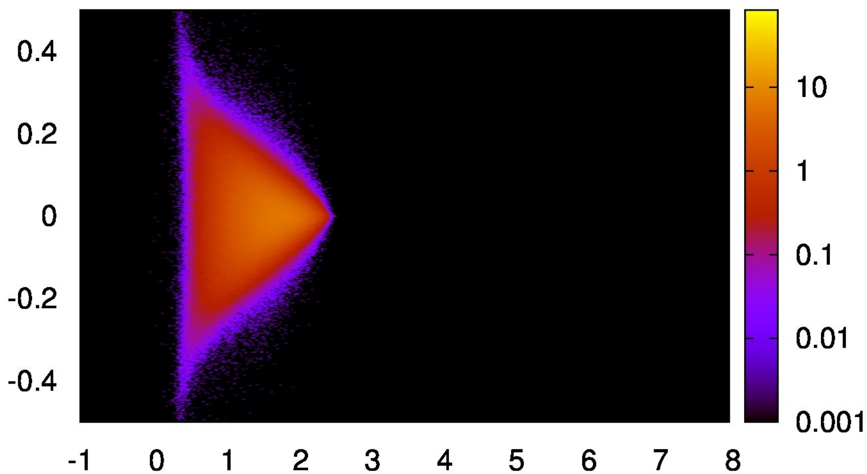
Otherwise the pole is outside of the distribution

Worst case for poles: zero temperature lattice



Histogram of the phase of the local determinants in HDQCD

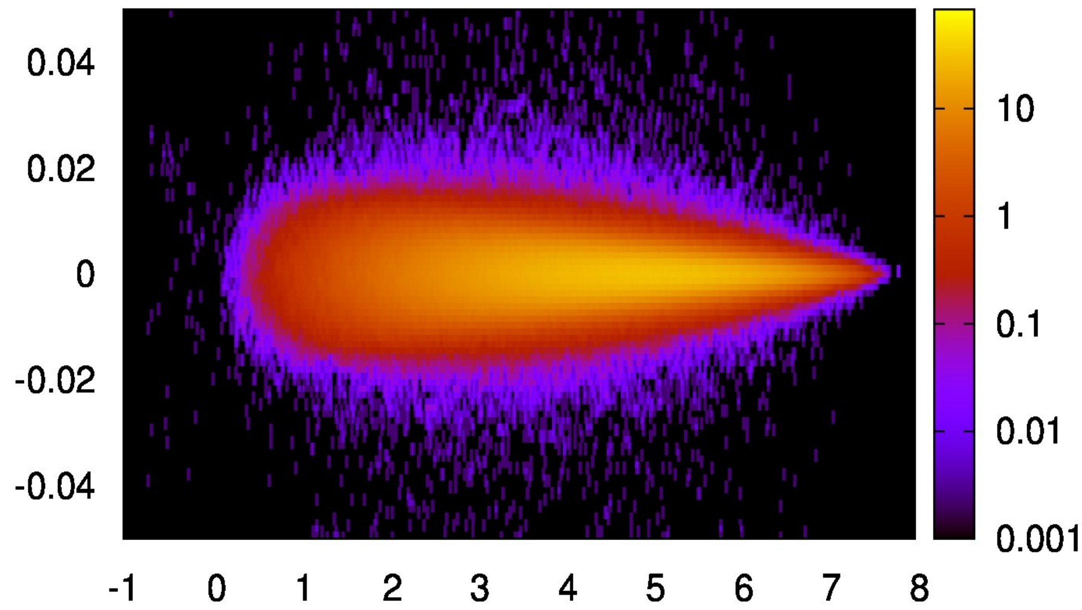
Distribution of the local determinants on the complex plane



$\mu = 1.3$

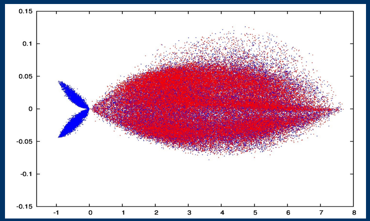
Well separated from poles

Exact results



$\mu = 1.425 = \mu_c$

Distribution close to real axis,
but “touches” pole



Very faint “whiskers”
Similar to the toy model case
Negligible contribution to averages

Conclusion for HDQCD

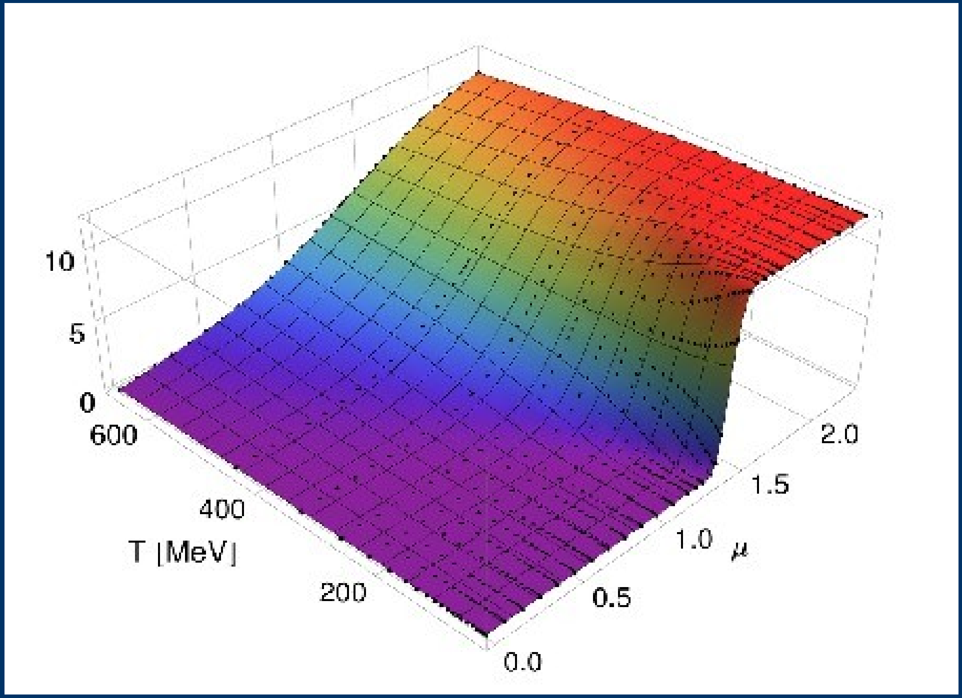
Results are unaffected by poles almost everywhere

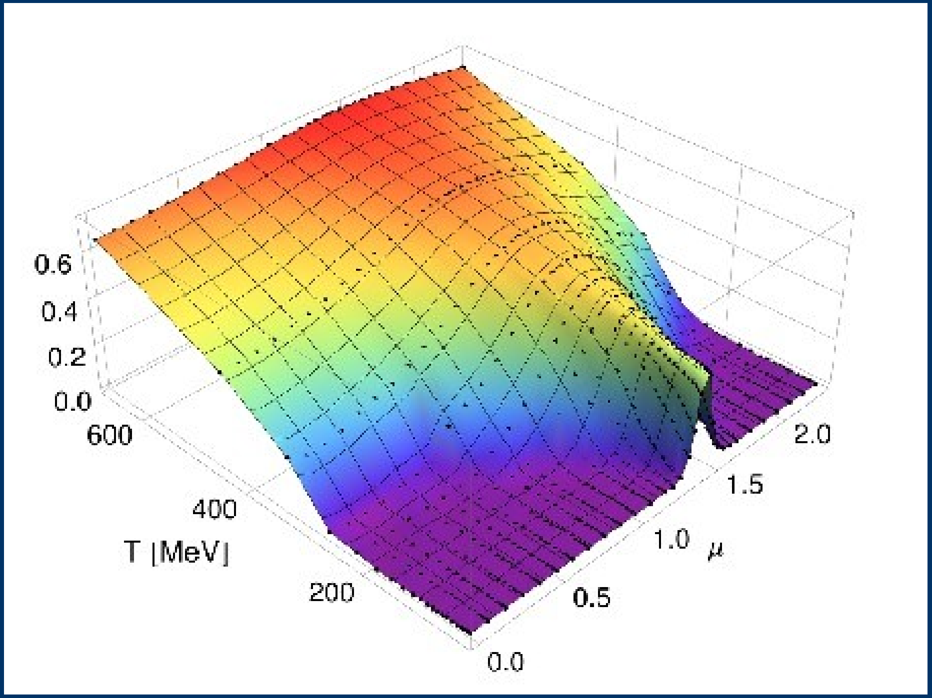
Near the critical chemical potential
we have indications that results are probably OK
affected by a negligibly small contamination

Phase diagram mapped out with complex Langevin

[Aarts, Attanasio, Jäger, Sexty arxiv:1606.05561]

[See Felipe Attanasio's talk]



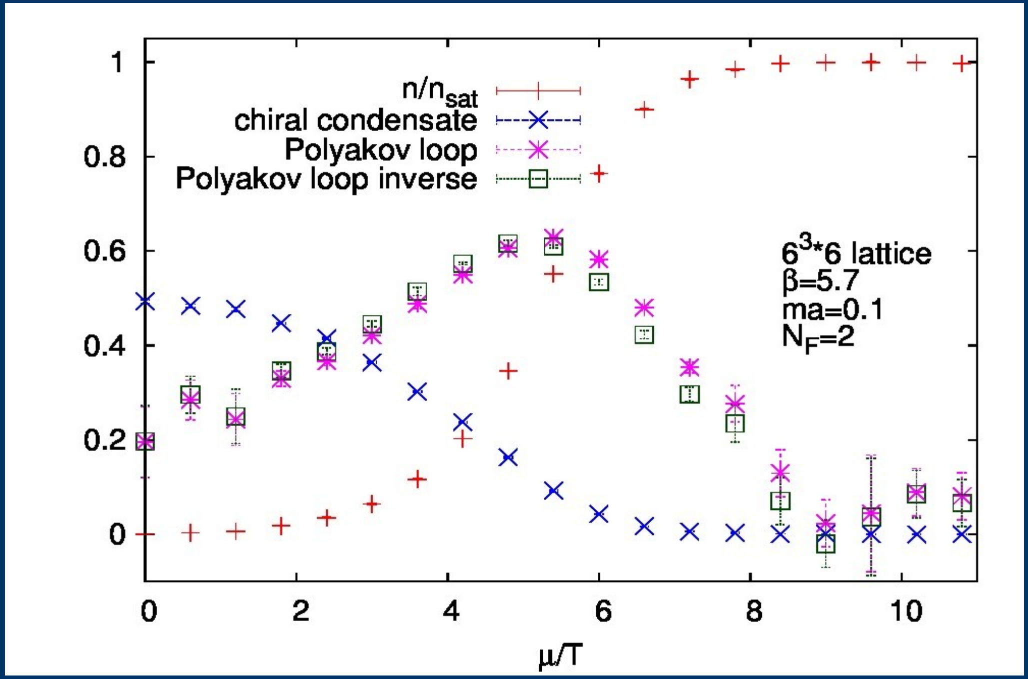


Full QCD and the issue of poles

Unimproved staggered and Wilson fermions with CLE

[Sexty (2014), Aarts, Seiler, Sexty, Stamatescu (2015)]

$$S_{eff} = S_g(U) - N_f \ln \det M(U)$$
$$= S_g(U) - N_f \sum_i \ln \lambda_i(U)$$



Drift term of fermions

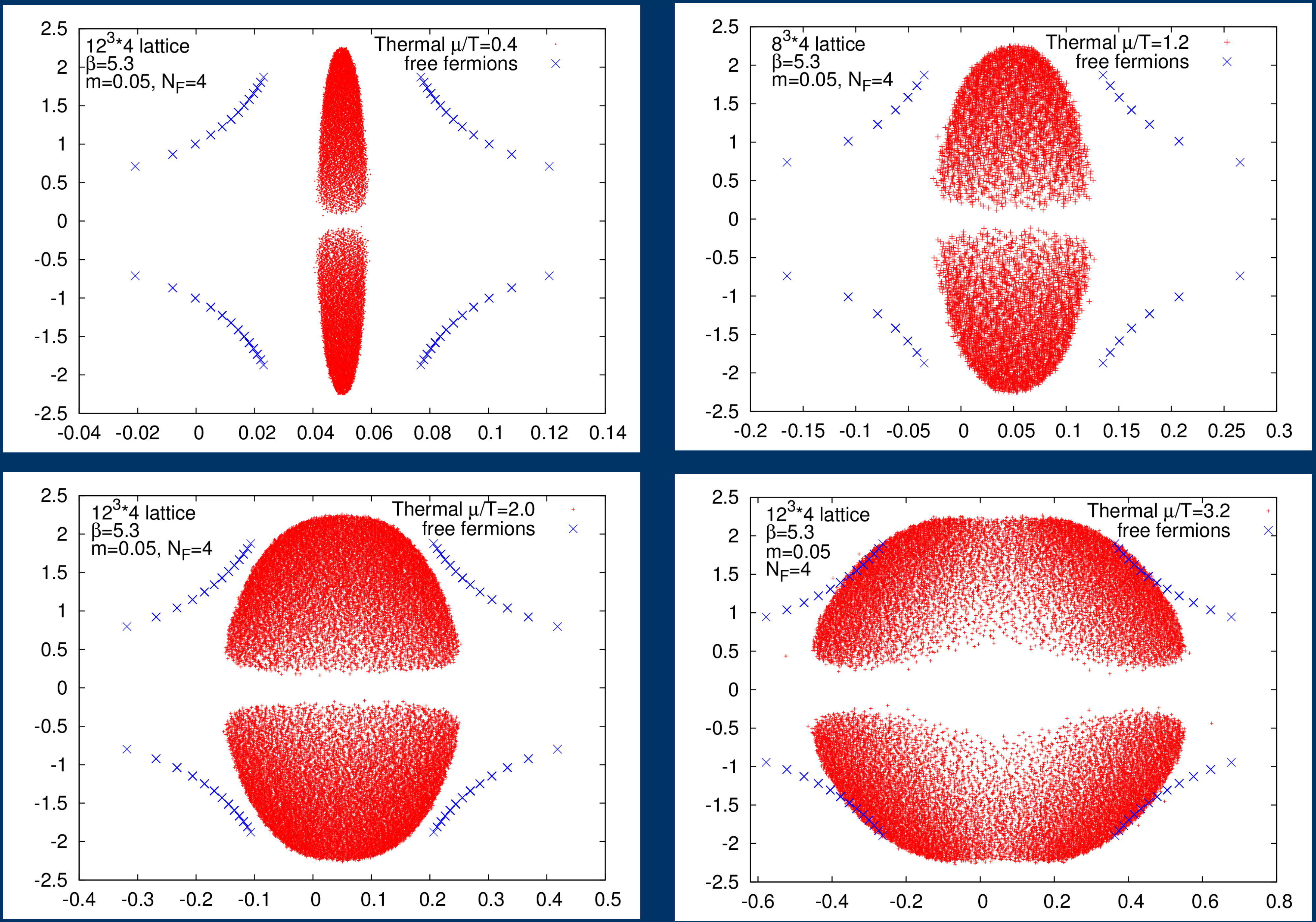
$$K_f = N_f \sum_i \frac{D\lambda_i(U)}{\lambda_i(U)}$$

Poles can be an issue if eigenvalue density around zero is not vanishing

Total phase of the determinant is sum off all the phases

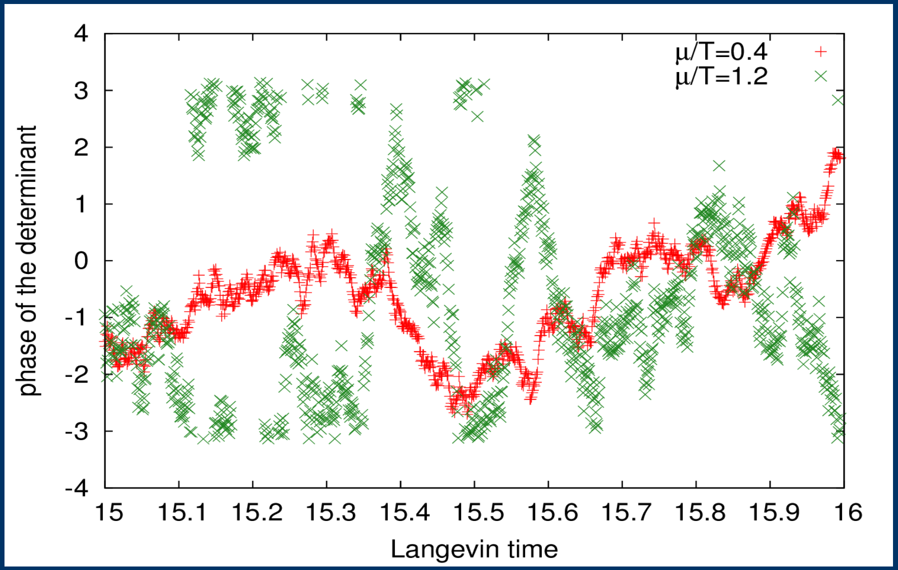
→ Sign problem can still be hard

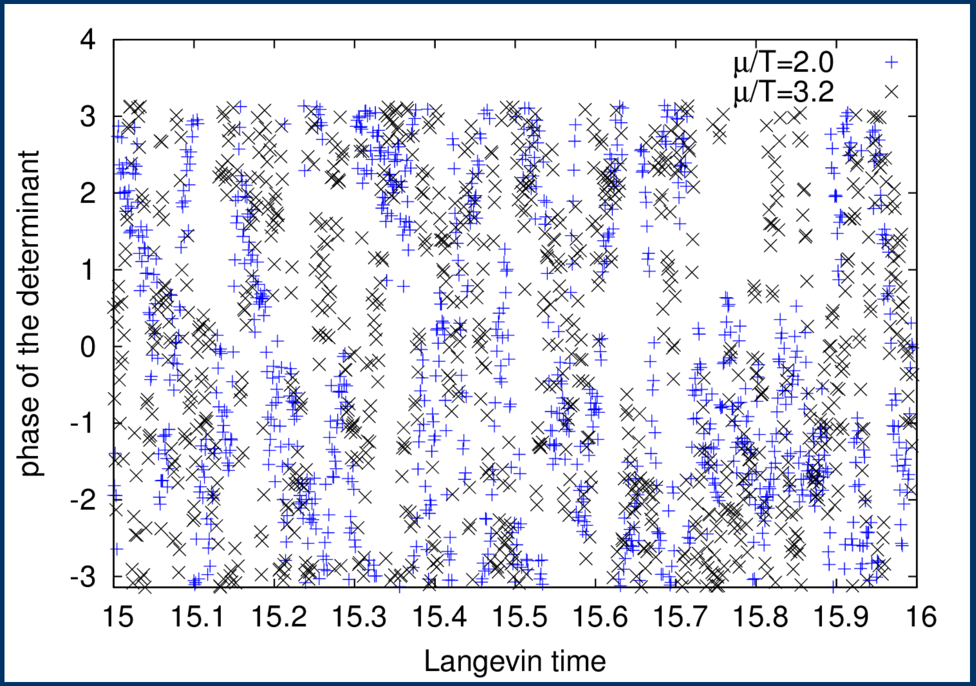
Spectrum of the Dirac operator above the deconfinement transition



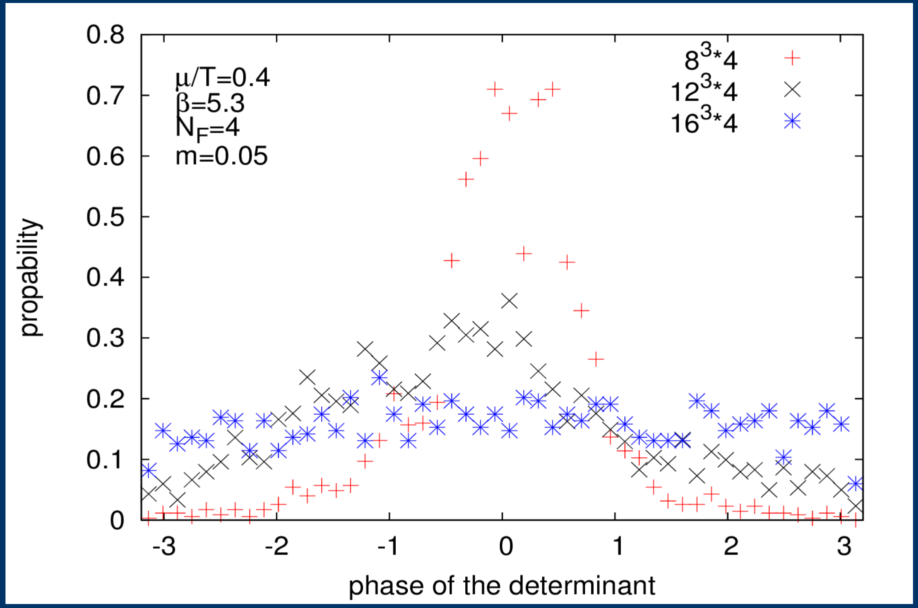
The phase of the determinant

Langevin time evolution





Histogram



Conclusions for full QCD

At high temperatures
eigenvalue density is zero at the origin
Even tough the sign problem can be hard

At low temperatures
Non-zero eigenvalue density is expected
(Banks-Casher relation)
Can we deal with it?

Reweighting

$$\langle F \rangle_\mu = \frac{\int DU\, e^{-S_E} \det M(\mu)\, F}{\int DU\, e^{-S_E} \det M(\mu)} = \frac{\int DU\, e^{-S_E} R \dfrac{\det M(\mu)}{R} F}{\int DU\, e^{-S_E} R \dfrac{\det M(\mu)}{R}}$$

$$= \frac{\langle F \det M(\mu)/R \rangle_R}{\langle \det M(\mu)/R \rangle_R}$$

$R = \det M(\mu=0),\ |\det M(\mu)|$, etc.

$$\left\langle \frac{\det M(\mu)}{R} \right\rangle_R = \frac{Z(\mu)}{Z_R} = \exp\left(-\frac{V}{T} \Delta f(\mu, T)\right)$$

$\Delta f(\mu, T)$ =free energy difference

Exponentially small as the volume increases $\langle F \rangle_\mu \rightarrow 0/0$

Reweighting works for large temperatures and small volumes

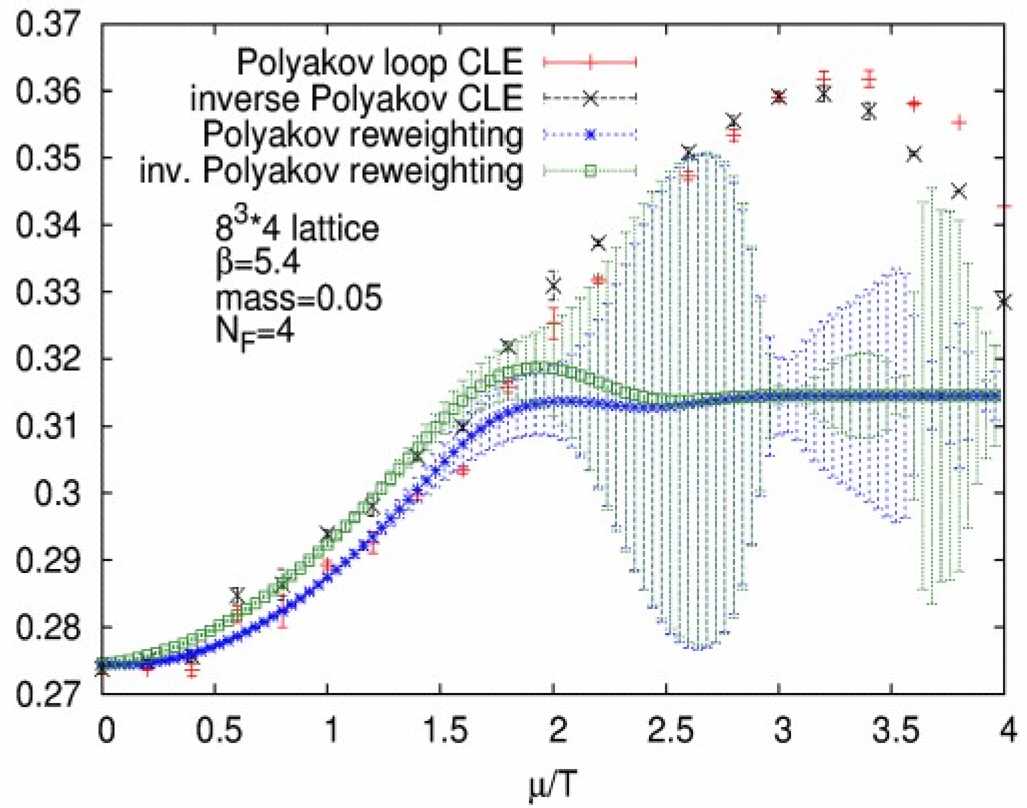
Sign problem gets hard at $\mu/T \approx 1$

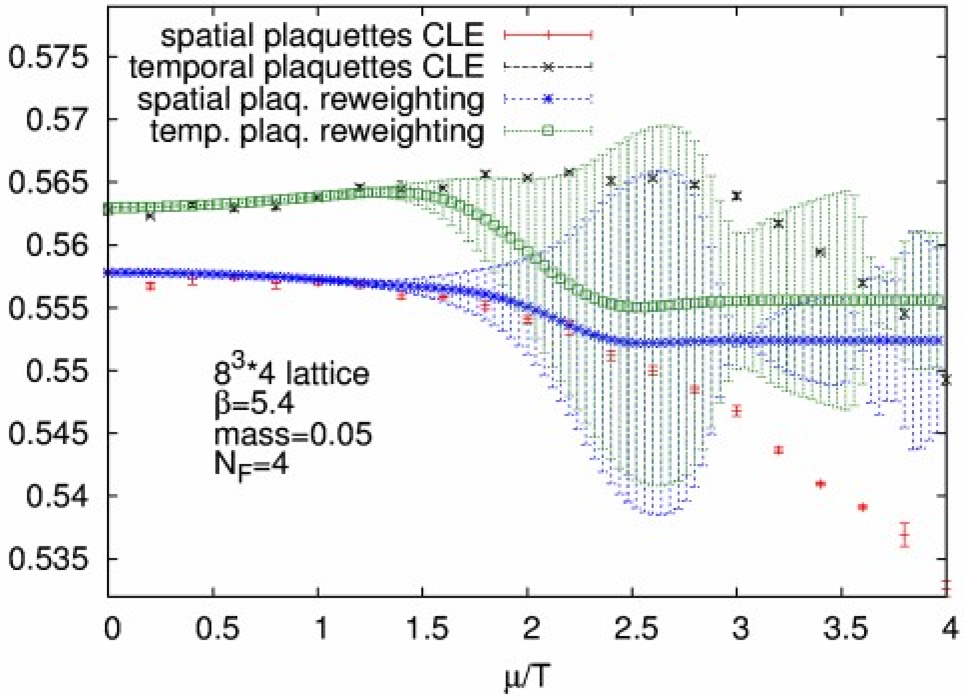
Comparison with reweighting for full QCD

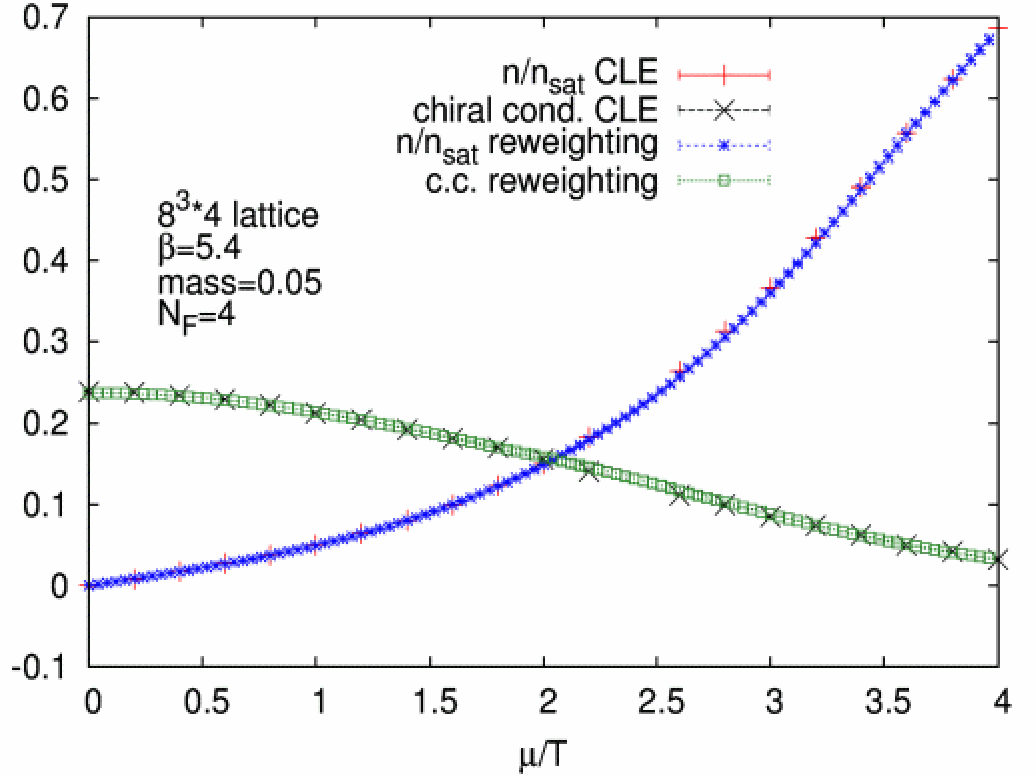
[Fodor, Katz, Sexty, Török 2015]

Reweighting from ensemble at

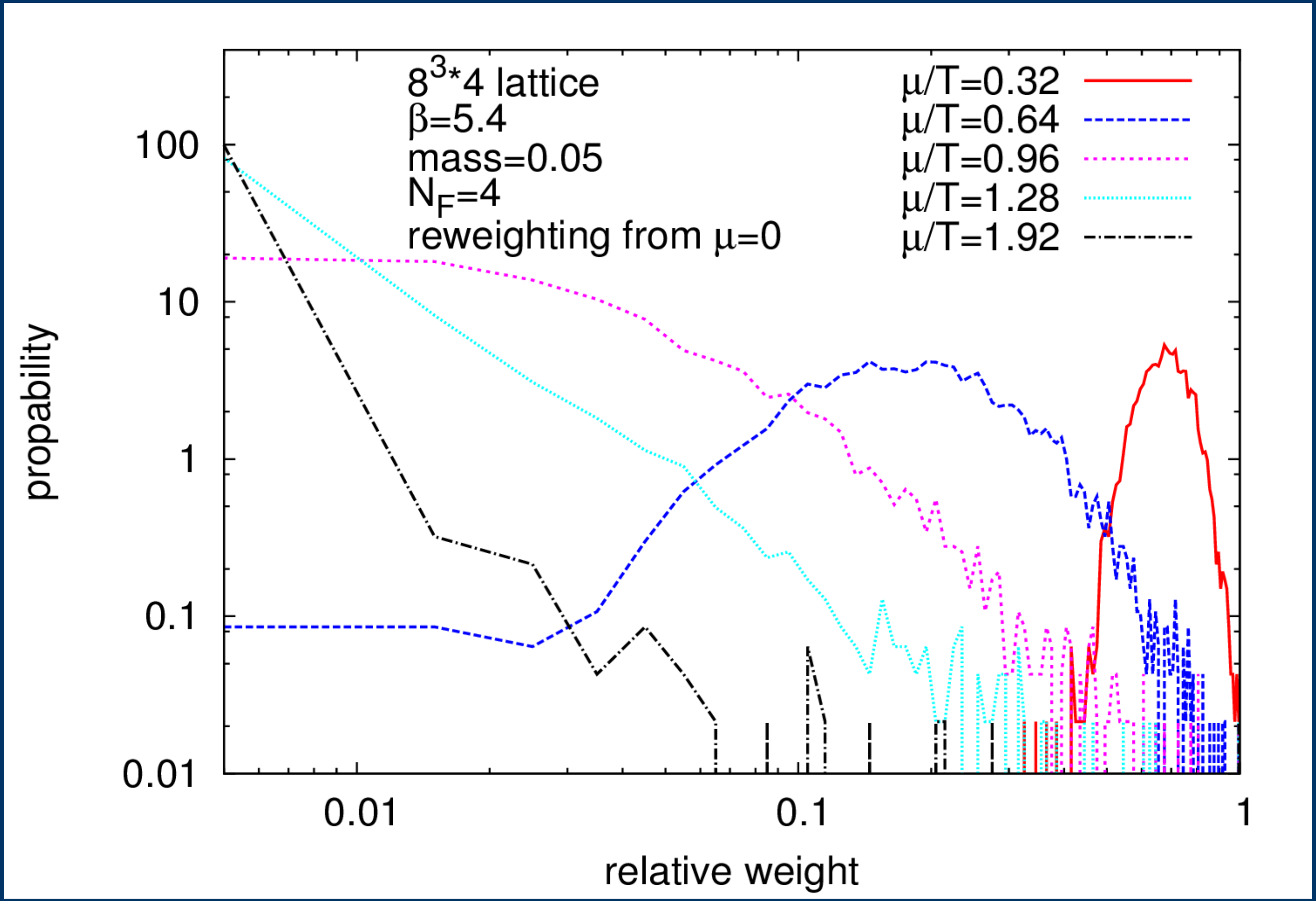
$$R = \mathrm{Det}\, M(\mu=0)$$







Overlap problem

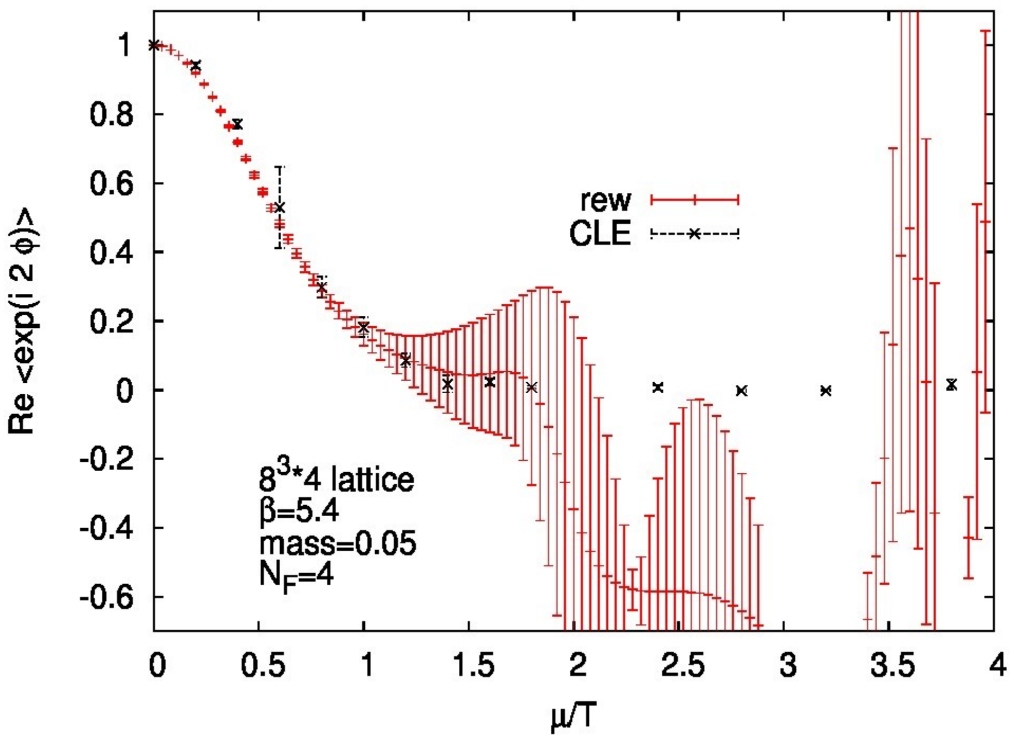


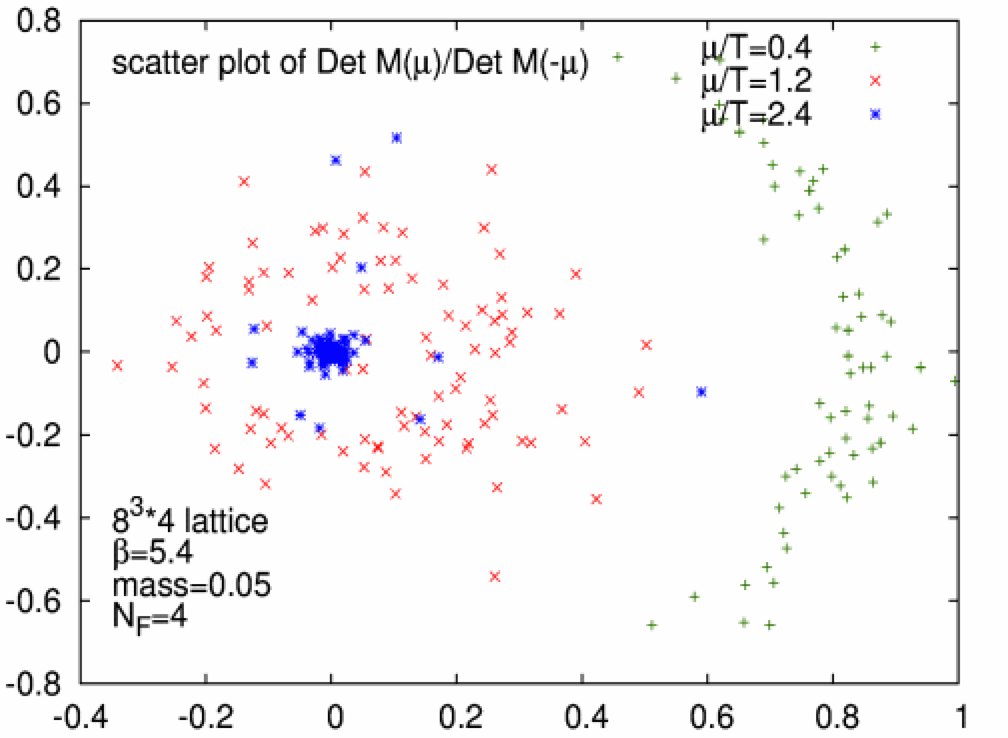
Histogram of weights
Relative to the largest weight in ensemble

Average becomes dominated by very few configurations

Sign problem

Sign problem gets hard around $\mu/T \approx 1 - 1.5$





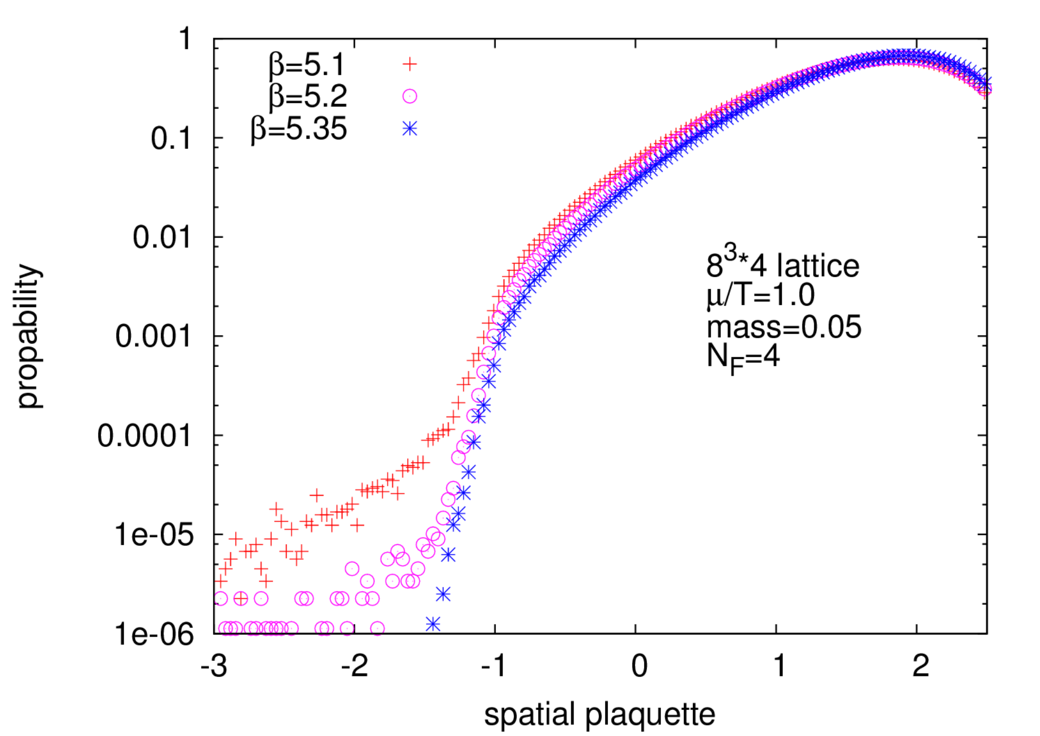
$$\langle \exp(2 i \varphi) \rangle = \left\langle \frac{\det M(\mu)}{\det M(-\mu)} \right\rangle$$

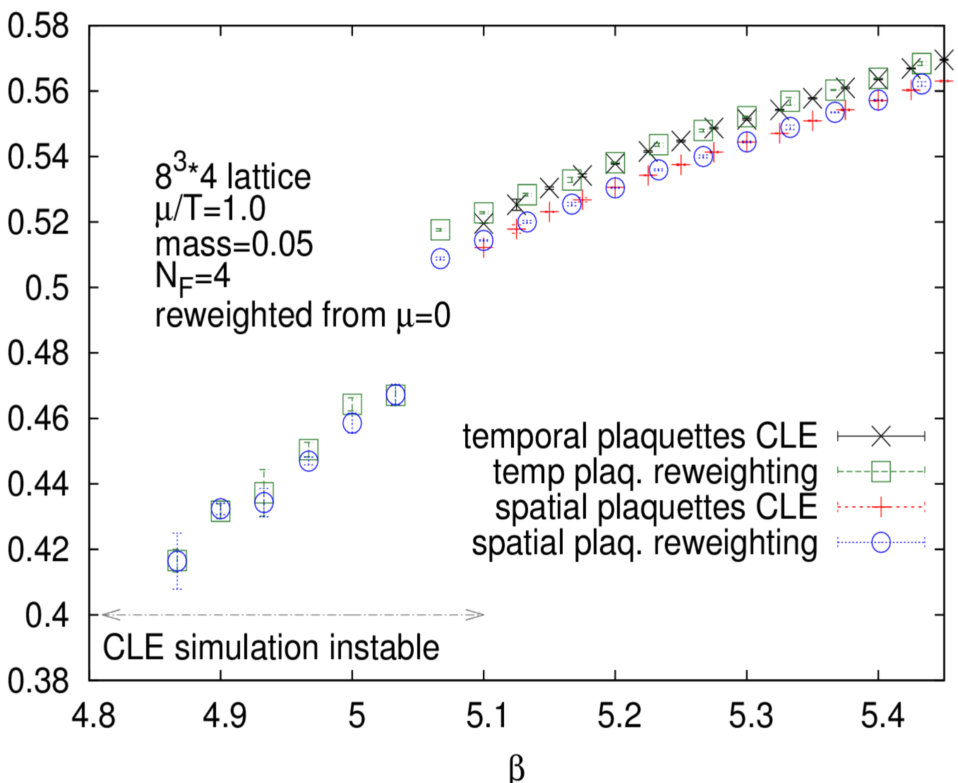
Comparisons as a function of beta

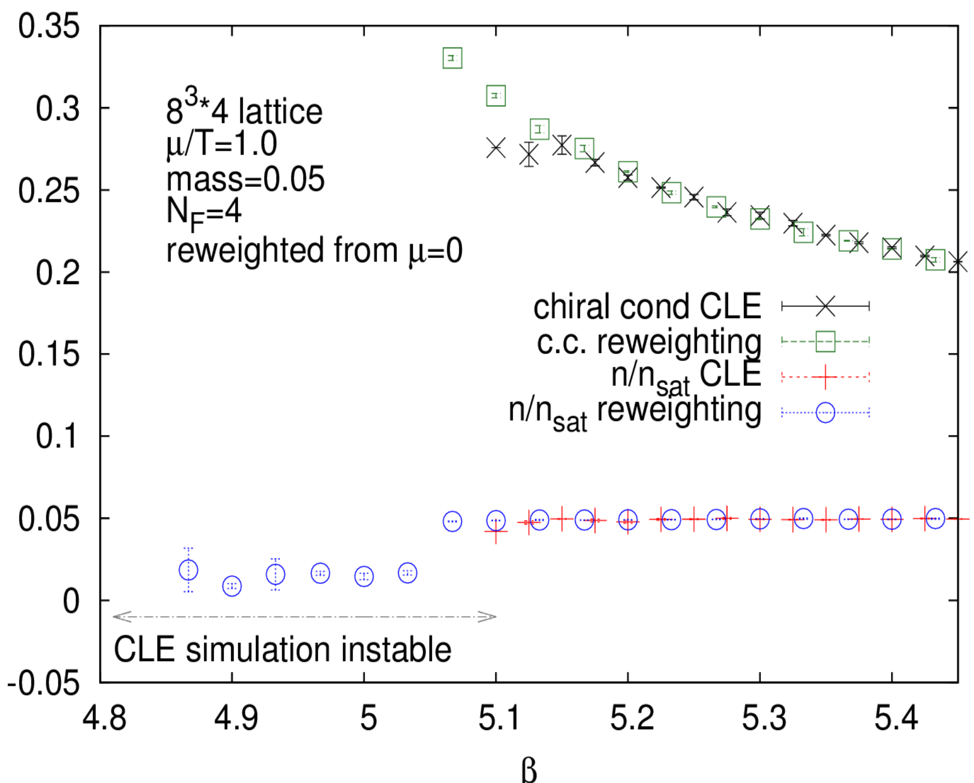
Similarly to HDQCD
Cooling breaks down at small beta

at $N_T = 4$ breakdown at $\beta = 5.1 - 5.2$

At larger N_T?

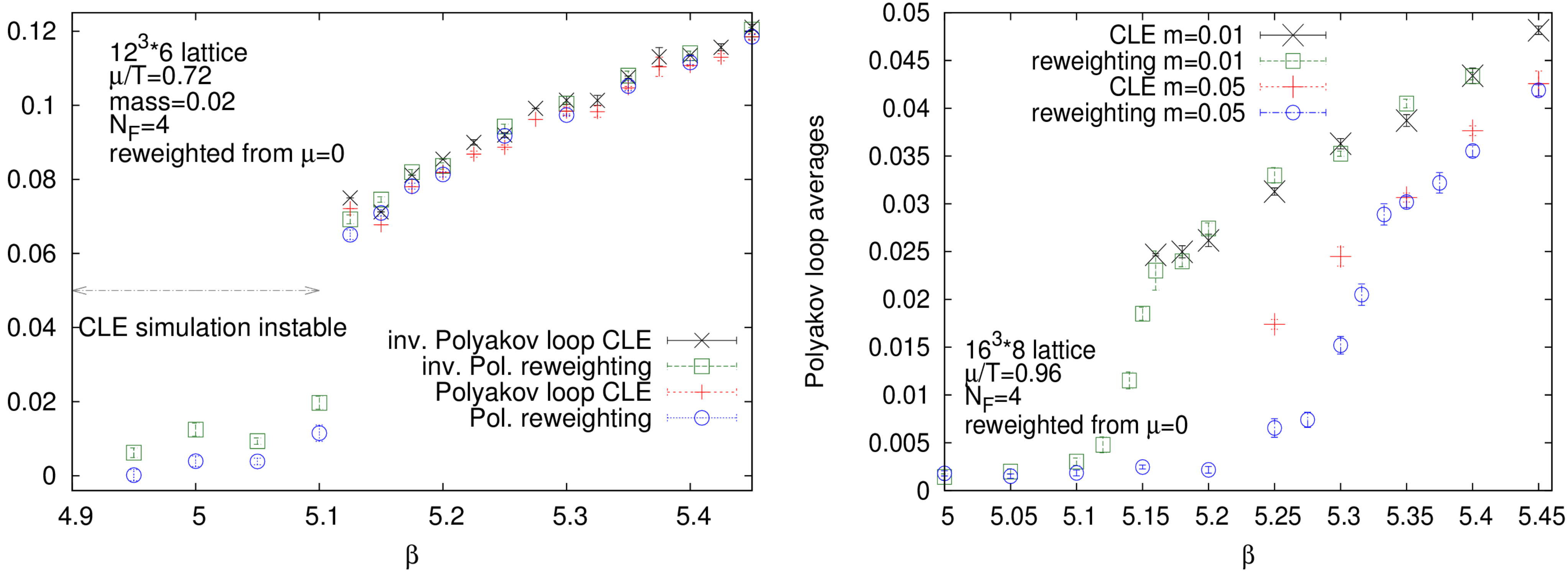






Comparisons as a function of beta

$N_T = 6$ $\qquad$ $N_T = 8$



Breakdown prevents simulations in the confined phase

for staggered fermions with $N_T = 4,6,8$

Two ensembles: $m_\pi \approx 4.8\, T_c$

$m_\pi \approx 2.3\, T_c$

Conclusions

Zeroes of the measure can affect validity of CLE
if prob. density around them is non-vanising

In HDQCD poles only have a negligible effect around
critical chemical potential, otherwise exact
In full QCD high temperature simulations are OK
Low temperatures?

Comparison of reweighting with CLE
they agree where both works
Reliability can be judged independent of the other method
Low temperature phase not yet reached with $N_T=8$