Computing the muon anomalous magnetic moment using the hybrid method with physical quark masses

Matt Spraggs

UNIVERSITY OF Southampton

RBC/UKQCD

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RBC/UKQCD Collaboration

BNL and RBRC
Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Taichi Kawanai
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

Columbia University
Ziyuan Bai
Norman Christ
Luchang Jin
Christopher Kelly
Bob Mawhinney
Greg McGlynn
David Murphy
Jiqun Tu

University of Connecticut
Tom Blum

CERN
Marina Marinkovic

Edinburgh University
Peter Boyle
Guido Cossu
Luigi Del Debbio
Richard Kenway
Julia Kettle
Ava Khamseh
Brian Pendleton
Antonin Portelli
Oliver Witzel
Azusa Yamaguchi

KEK
Julien Frison

Peking University
Xu Feng

Plymouth University
Nicolas Garron

University of Southampton
Jonathan Flynn
Vera Gülpers
James Harrison
Andreas Jüttner
Andrew Lawson
Edwin Lizarazo
Chris Sachrajda
Francesco Sanfilippo
Matt Spraggs
Tobias Tsang

York University (Toronto)
Renwick Hudspith

External
Kim Maltman
Randy Lewis

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Motivation

Magnetic moment:

$$\boldsymbol{\mu} = g\frac{e}{2m}\mathbf{S};\ \ U = -\boldsymbol{\mu}\cdot\mathbf{B};\ \ a_\mu = \frac{g_\mu - 2}{2}$$

Contributions to a_μ [PDG, 2015]

Contribution	$a_\mu \times 10^{11}$	Uncertainty
QED (5-loop)	116584718.95	0.08
Electroweak (2-loop)	153.6	1.0
LO hadronic (HVP)	6923	42
NLO hadronic	7	26
Theory Total	116591803	49.4
Experiment Total	116592091	63.3

Hadronic Light-by-Light (HLbL): [Blum et al., 2015; Jin, Tuesday, 14:20]

- 3.6σ discrepancy between theory and experiment.
- Fermilab $g-2$ and JPARC set to reduce experimental error.

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Magnetic moment:

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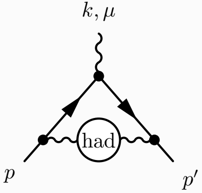
Hadronic Light-by-Light (HLbL): [Blum et al., 2015; Jin, Tuesday, 14:20]

- 3.6σ discrepancy between theory and experiment.
- Fermilab $g-2$ and JPARC set to reduce experimental error.
- Greatest uncertainty comes from (non *ab initio*) HVP.

Hadronic Vaccuum Polarization

HVP in Euclidean Space [T. Blum, 2002]



- $a_\mu^{(2)\text{had}} = 4\alpha^2 \int_0^\infty \mathrm{d}\hat{q}^2 f(\hat{q}^2)\hat{\Pi}(\hat{q}^2)$
- $\hat{\Pi}(\hat{q}^2) = \Pi(\hat{q}^2) - \Pi(0)$
- $\Pi_{\mu\nu}(\hat{q}) = (\delta_{\mu\nu}\hat{q}^2 - \hat{q}_\mu \hat{q}_\nu)\Pi(\hat{q}^2)$

Computation:

$$\Pi_{\mu\nu}(\hat{q}) = Z_V \sum_{f,x} Q_f^2 \mathrm{e}^{\mathrm{i}q\cdot x} \left\langle \mathcal{V}_\mu^f(x) V_\nu^f(0) \right\rangle ; \;\; \hat{q} = \frac{2}{a} \sin\left(\frac{aq}{2}\right)$$

Challenges:

- Integrand highly peaked near $\hat{q}^2 = m_\mu^2/4$.
- Lattice imposes lower bound on non-zero momenta ($q_\mu = \frac{2\pi n_\mu}{N_\mu}$).
- HVP cannot be directly computed at $\hat{q}^2 = 0$.

HVP Components

Wick contractions give various HVP components: $\Pi^{\ell}_{\mu\nu}(\hat{q}) + \Pi^{s}_{\mu\nu}(\hat{q}) + \cdots$
Separate continuum limits exist for each contraction
[Jüttner and Della Morte, 2010].

Complete

- Strange contribution [Blum et al., 2016].
- Disconnected contribution [Blum et al., 2015].

Ongoing

- Light contribution [C. Lehner, Tuesday, 14:00].
- Charm contribution.
- Isospin breaking effects [Guelpers, Tuesday, 15:20; Harrison, Tuesday, 15:00].

HVP Computation

Strange and charm correlator computations

- Conserved current at sink.
- $\mathbb{Z}_2$ wall source - Ward identity $q^\mu \Pi_{\mu\nu} = 0$ in large hit limit.

Zero-mode Subtraction

Reduce statistical noise at low-$\hat{q}^2$ by subtracting $q_t = 0$ component:

$$\Pi_{\mu\nu}(\hat{q}) = \sum_t e^{i q_t \cdot t} C_{\mu\nu}(t) - \sum_t C_{\mu\nu}(t)$$

[Bernecker and Meyer, 2011; C. Lehner and T. Izubuchi, 2014]

Restriction to diagonal of HVP tensor (remove longitudinal part and reduce cut-off effects):

$$\Pi(\hat{q}^2) = \frac{1}{3} \sum_i \frac{\Pi_{ii}(\hat{q})}{\hat{q}^2}; \ \hat{q}_i = 0$$

Light correlator computation [C. Lehner, Tuesday, 14:00]

- Local current at sink.
- Compute $\Pi_{\mu\nu}(\hat{q})$ using Bernecker-Meyer kernel.

Simulations

Ensembles

RBC/UKQCD 2+1f domain wall fermion ensembles with physical pion masses [RBC/UKQCD, 2014]:

Parameter	48I	64I
$L^3 \times T \times L_s$	$48^3 \times 96 \times 24$	$64^3 \times 128 \times 12$
m_π	139 MeV	139 MeV
m_K	499 MeV	507 MeV
a^{-1}	1.73 GeV	2.36 GeV
$m_\pi L$	3.86	3.78

Target $m_K = 495.7$ MeV

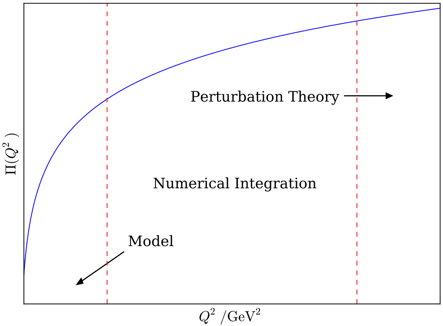
Strange Measurements

Unitary and physical/partially quenched strange masses to account for m_s mistuning.

The Hybrid Method

Motivation

- Systematic error of the parametrisation at low-$\hat{q}^2$ grows with cut.
- Perturbation theory only valid at high-$\hat{q}^2$.
- How are these reconciled? [Golterman, Maltman and Peris, 2014]



Variations

- Parametrisations
- $\hat{q}^2$ cuts
- Techniques to constrain parametrisation (fits, moments)
- Numerical integration method

Low-q^2 Parametrisations

Padé Approximants

Motivated by the spectral decomposition of the HVP [Aubin, Blum, Golterman and Peris, 2012]:

$$R_{mn}(\hat{q}^2) = \Pi_0 + \hat{q}^2 \left(\sum_{i=0}^{m-1} \frac{a_i}{b_i + \hat{q}^2} + \delta_{mn} c \right);\ n = m,\ m+1.$$

Conformal Polynomials

Map domain of analyticity onto region within unit disc. Better convergence properties [Golterman, Maltman and Peris, 2014]:

$$P_n^E(\hat{q}^2) = \Pi_0 + \sum_{k=1}^{n} p_k w^k,$$

$$w = \frac{1 - \sqrt{1+z}}{1 + \sqrt{1+z}},\ z = \frac{\hat{q}^2}{E^2}.$$

Time Moments [HPQCD, 2014]

1. Tensor decomposition:
$$\sum_t e^{iq_t t} C_{\mu\mu}(t) = \hat{q}_t^2 \Pi(\hat{q}_t^2)$$
2. Differentiate w.r.t. q_t:
$$\Delta_{q_t}^{2n} \left(\sum_t e^{iq_t t} C_{\mu\mu}(t) \right)\Bigg|_{q_t=0} = \Delta_{q_t}^{2n} \left(\hat{q}_t^2 \Pi(\hat{q}_t^2) \right)\Big|_{q_t=0}$$
3. Plug in a parametrisation for $\Pi\left(\hat{q}^2\right)$ and solve for parameters.

Notes

- Account for discrete momenta using finite differences.
- Expansion around $\hat{q}^2 = 0$, data here carry more weight.
- Resulting parameters independent of cut.

Continuous Momenta

- Fourier transform to arbitrary momenta [Bernecker and Meyer, 2011; Feng et al., 2013]:

$$\Pi_{\mu\nu}(\hat{q}) = \sum_t e^{iq_t \cdot t} C_{\mu\nu}(t) - \sum_t C_{\mu\nu}(t)$$

$$q_t = \frac{2\pi n_t}{T}, \; n_t \in [-T/2, T/2)$$

- Compute HVP directly at arbitrary $\hat{q}$.
- No hybrid method, parametrisation independent.
- Can show interpolation effects are $\mathcal{O}(\exp(-m_\pi T))$ [Portelli, Lattice 2015; Del Debbio and Portelli, to appear].

Overview of Strange Analyses

Hybrid Method

- Matching method: diagonal fit, time moments.
- Parametrisations:
  - Padés: $R_{1,1}$, $R_{1,2}$.
  - Conformal polynomials: P_3^E, P_4^E; $E = 500$ MeV, 600 MeV.
- Low cuts: 0.5, 0.7, 0.9 GeV^2, $\left(\frac{2\pi}{aT}\right)^2 \approx 0.013\,\mathrm{GeV}^2$.
- High cut: 5.0 GeV^2.
- Mid-$\hat{q}^2$ integration technique: trapezium rule, Simpson's rule.

Sine Cardinal Interpolation

- $\Delta n_t = 0.005$
- High cut: 5.0 GeV^2.

[Blum et al., 2016]

Extrapolations

- Strange quark mistuning: $\sim$1% on 48I, $\sim$5% on 64I.
- Partially quenched measurements.
- Continuum limit.
- Strange mass extrapolation.

Strange Ansatz

$$a_{\mu}^{(s)\mathrm{had},s}(a^2, am_s) = a_{\mu,0}^{(s)\mathrm{had},s} + \alpha a^2 + \beta \frac{am_s - am_s^{\mathrm{phys}}}{am_s^{\mathrm{phys}} + am_{\mathrm{res}}}$$

Systematic Effects

Accounted For

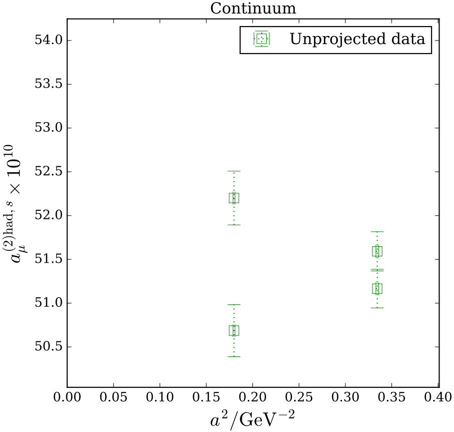
- $\hat{q}^2$ cuts.
- Low-$\hat{q}^2$ parametrisation.
- Matching method.
- Mid-$\hat{q}^2$ integration method.
- Finite volume effects negligible via G–parity.
- Non-unitarity effects negligible.
- Disconnected diagrams [Blum et al., 2015].

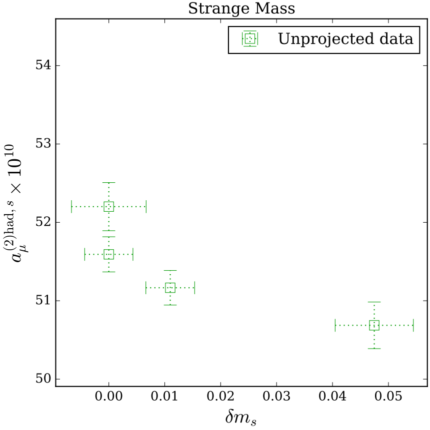
Long Term

- Charm quark in the sea.
- Isospin breaking effects, including EM effects [Guelpers, Tuesday, 15:20; Harrison, Tuesday, 15:00].

Strange Result: Extrapolations

Moments, $P_3^{0.6\mathrm{GeV}}$, low cut = 0.7 GeV^2, high cut = 5.0 GeV^2





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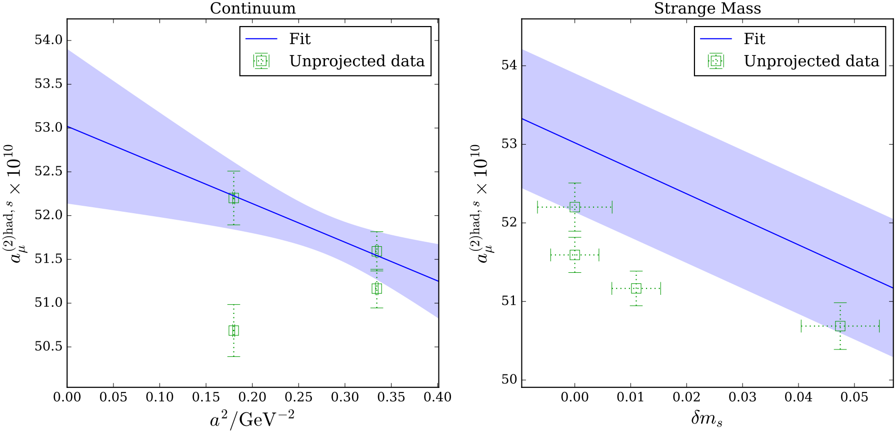

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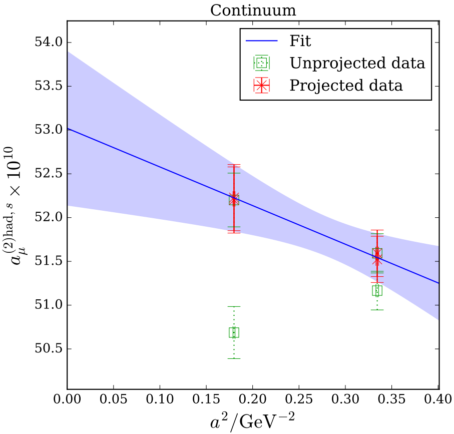
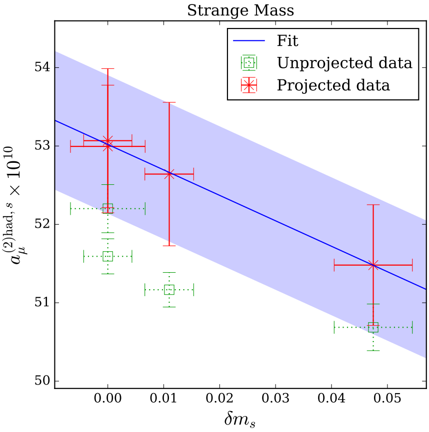
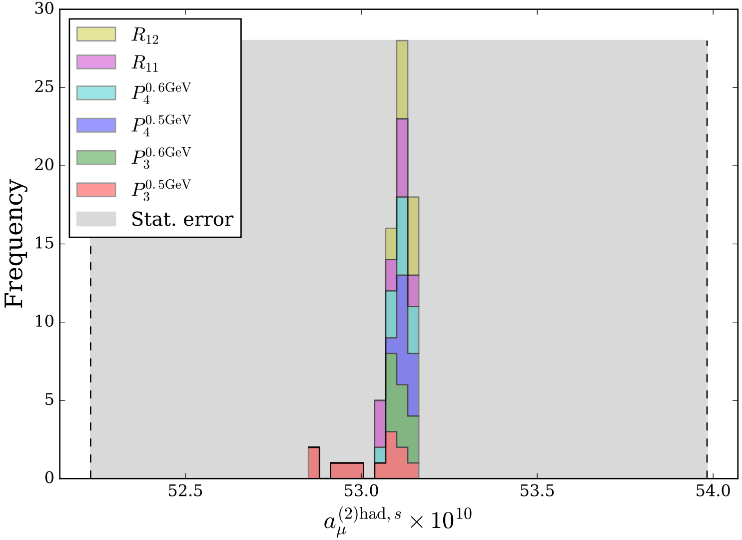
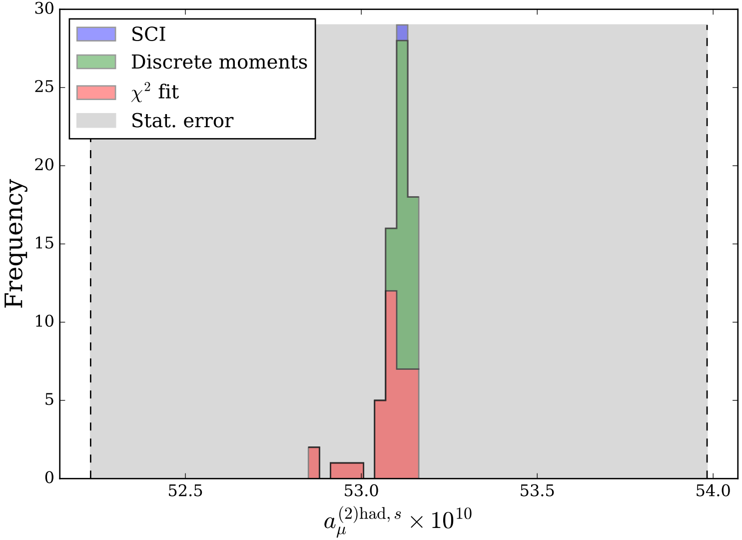




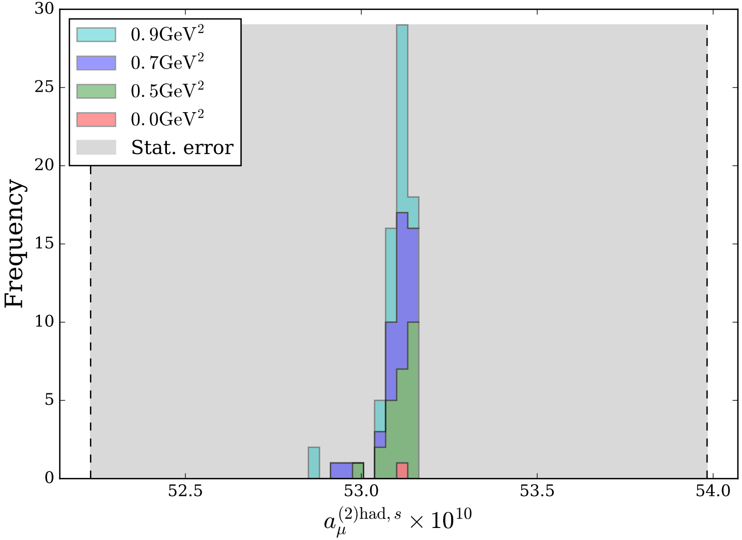
Strange Result: Parametrisations



Strange Result: Matching Method



Strange Result: Low Cuts



Overview of Light Analyses

Hybrid Method

- Matching method: diagonal fit.
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  - Conformal polynomials: $P_2^{2m_\pi}$, $P_3^{2m_\pi}$, $P_4^{2m_\pi}$.
- Low cuts: 0.5, 0.7, 0.9 GeV^2, $\left(\frac{2\pi}{aT}\right)^2 \approx 0.013\,\mathrm{GeV}^2$.
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Sine Cardinal Interpolation

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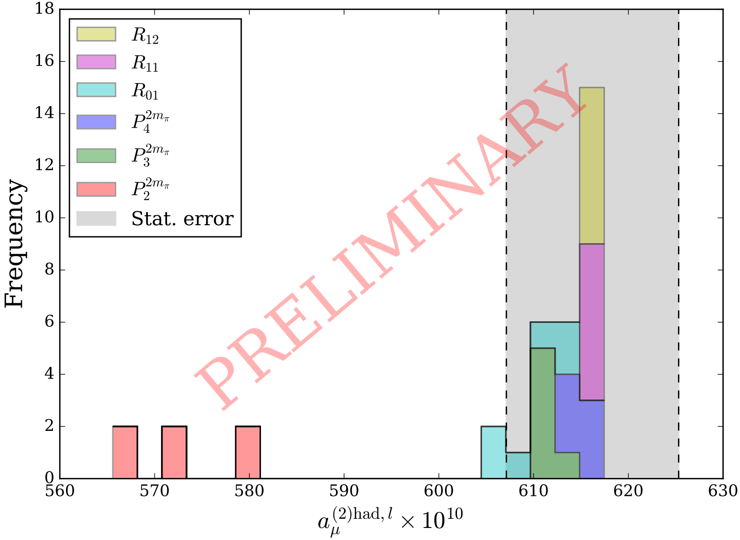
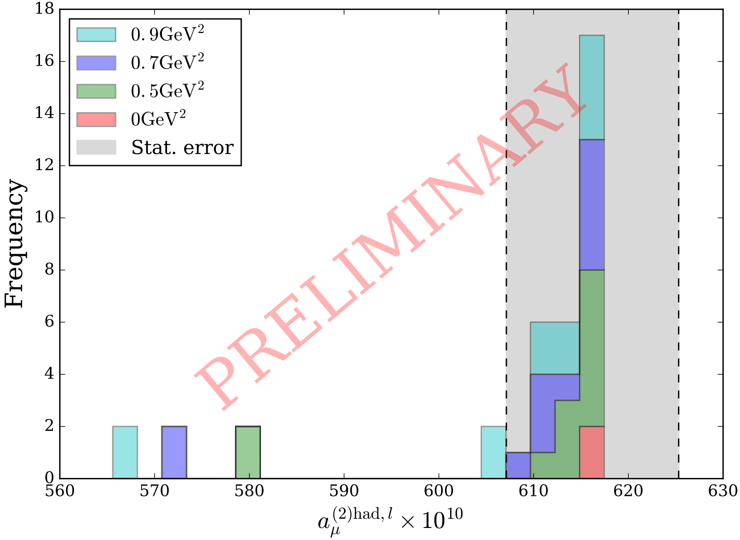
See also: [C. Lehner, Tuesday, 14:00]

Light Results: Parametrisations



Light Results: Low Cuts



Charm Contribution

HVP measurements computed on seven dynamical ensembles.

- Same computation of $\Pi_{\mu\nu}$ as strange contribution.
- Select single analysis technique motivated by strange analysis.

$L^3 \times T$	a^{-1}/GeV	m_π/MeV	Number of am_h used
$48^3 \times 96$	1.73	139	3
$24^3 \times 64$	1.78	340	3
$24^3 \times 64$	1.78	430	3
$64^3 \times 128$	2.36	139	4
$32^3 \times 64$	2.38	300	4
$32^3 \times 64$	2.38	360	4
$48^3 \times 96$	2.77	230	5

Global fit of charm data ongoing [Tsang, Friday, 14:20].

Summary

Strange Contribution [Blum et al., 2016]

- $a_\mu^{(2)\mathrm{had},s}$ computed using Möbius domain wall fermions with 2+1f.
- Extensive systematic study of analysis techniques.
- Final value of $a_\mu^{(2)\mathrm{had},s}$ largely insensitive to analysis method.
- Results consistent with other studies (HPQCD).

$$a_\mu^{(2)\mathrm{had},s} = 53.1(9)\left(^{+1}_{-3}\right) \times 10^{-10}$$

Light Contribution

- Investigation into analysis techniques ongoing.

Outlook

- Cut-off dependence of light contribution.
- Charm contribution.

Ensembles

RBC/UKQCD ensembles using 2+1f domain wall fermions with a physical pion mass.

Parameter	48I	64I
$L^3 \times T \times L_s$	$48^3 \times 96 \times 24$	$64^3 \times 128 \times 12$
am_l	0.00078	0.000678
am_s	0.0362	0.02661
am_s^{phys}	0.03580(16)	0.02539(17)
a^{-1} / GeV	1.730(4)	2.359(7)
L / fm	5.476(12)	5.354(16)
$m_\pi L$	3.863(6)	3.778(8)
am_{res}	0.0006012	0.0003116

Strange quark mistuning: partially quenched runs

Analysis Strategy: χ^2 Fit

χ^2 Fit

- Standard χ^2 minimization - covariance approximated by diagonal.

$$\chi^2 = \sum_{\hat{q}^2} \left(\frac{\Pi\left(\hat{q}^2\right) - f\left(\hat{q}^2\right)}{\delta\Pi\left(\hat{q}^2\right)} \right)^2$$

- Fits can be unstable - Z2 wall reduces d.o.f.
- Fit biased towards large $\hat{q}^2$.
- Parameters dependent on cut.