Non perturbative renormalization of flavor singlet quark bilinear operators in lattice QCD

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Renormalization of singlet quark bilinear operators

The aim of the project is the investigation of the non-perturbative renormalization of flavor singlet operators.

The strange quark contribution to the spin of the nucleon is an example of computation that requires the knowledge of singlet renormalization constants:[1]

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G \qquad \Delta\Sigma = \Delta u + \Delta d + \Delta s$$

Challenges:

- Determination of disconnected contributions
- Fermion contribution to the renormalization of $\hat{T}_{\mu\nu}$
- Mixing with gluonic observables

[1] G. S. Bali *et al*, Phys. Rev. Lett. 108, 222001

The RI'-MOM scheme

The renormalization constants in the RI'-MOM scheme are defined by the renormalization condition

$$\frac{1}{12} Z_q^{-1} Z_\Gamma \mathrm{Tr}_{\mathrm{spin,color}}(\tilde{V}_\Gamma(p) \tilde{V}_\Gamma^{\mathrm{Born}}(p)^{-1}) = 1 \,,$$

where $\tilde{V}_\Gamma(p)$ is the amputated vertex function for the operator Γ at momentum p. The scale is set as $\mu^2 = p^2$.

The contraction of quark fields of the vertex function

$$V_\Gamma(p) = \sum_{x,y,z} \exp\{-ip(x-y)\} \langle \psi(x) \bar{\psi}(z) \Gamma \psi(z) \bar{\psi}(y) \rangle \,,$$

gives rise to connected

$$V_\Gamma(p) = \sum_{x,y,z} \exp\{-ip(x-y)\} \langle \gamma_5 S(z,x)^\dagger \gamma_5 \Gamma S(z,y) \rangle$$

and disconnected contributions

$$V_\Gamma(p) = -N_f \sum \exp\{-ip(x-y)\} \langle S(x,y) \mathrm{Tr}_0(\Gamma S(z,z)) \rangle \,.$$

Ensembles analized

We perform the non-perturbative computation of the renormalization constants on the $N_f = 2$ QCDSF ensembles[2]

	$\beta = 5.20$	$\beta = 5.29$	$\beta = 5.40$
$a[\mathrm{fm}]$	0.081	0.071	0.060
r_0/a	5.454(20)	7.004(54)	8.285(74)
$M_\pi[\mathrm{MeV}]$	600-300	420-150	490-260

We measure the vertex function on 100 Landau gauge fixed configurations for each ensemble. We compute $\mathrm{Tr}_0(\Gamma S(z,z))$ with 20 stochastic estimators.

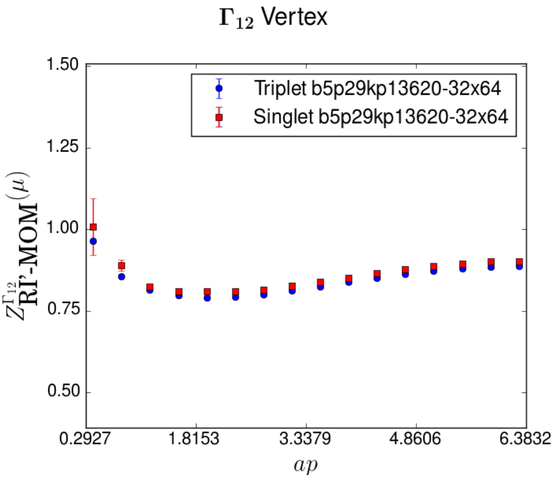
For the matching with perturbation theory and the computation of the conversion factors, we use $r_0\Lambda_{\overline{\mathrm{MS}}} = 0.789(52)$.[3]

[2] M. Göckeler *et al.* Phys. Rev. D 82, 114511; G. S. Bali *et al.* Phys. Rev. D 91, 054501; G. S. Bali *et al.* Nucl. Phys. B 866, 1-25

[3] ALPHA Collaboration, Nucl. Phys. B 865, 397-429

The tensor operator

The tensor operator $\sigma_{\mu\nu}$ has only a small difference when comparing singlet versus non singlet renormalization constants



Symmetric direction of the momenta $p_n = 2\pi \left\{ \frac{n}{N_s}, \frac{n}{N_s}, \frac{n}{N_s}, \frac{2n}{N_t} \right\}$

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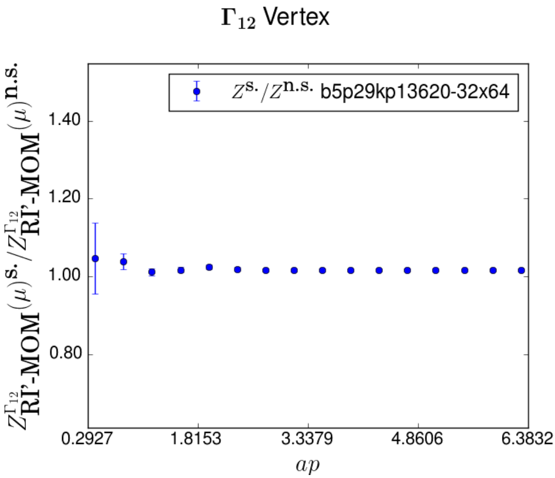
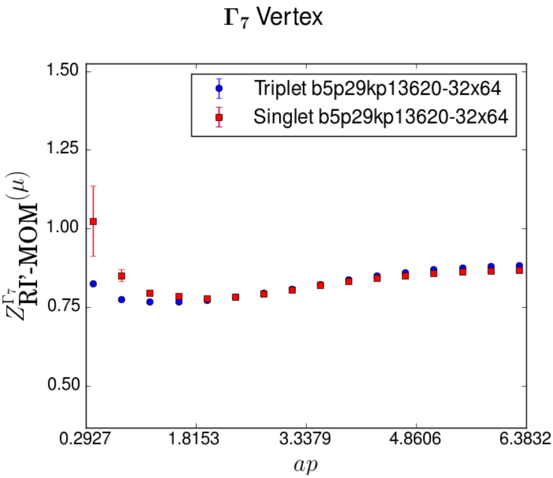


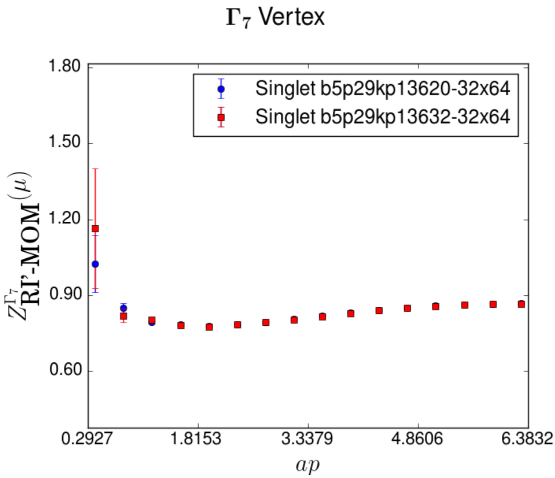
The axial vector operator

The axial vector operator $\gamma_5\gamma_\mu$ has only a small difference when comparing singlet versus non singlet renormalization constants:



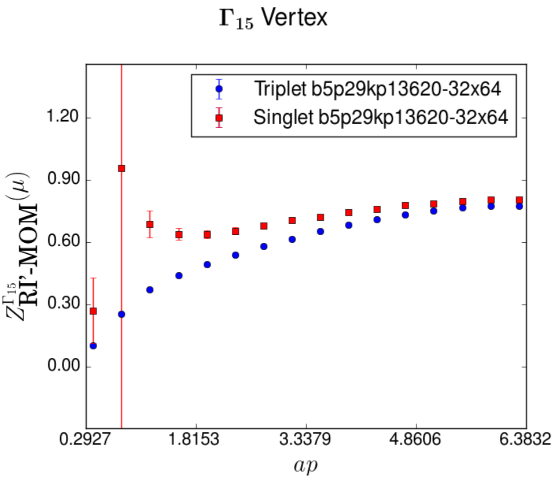
The axial vector operator

The axial vector operator $\gamma_5\gamma_\mu$ has only a very mild dependence on the pion mass.



The pseudoscalar operator

The singlet pseudoscalar operator γ_5, unlike its triplet counterpart, is not expected to develop a pole for $m_q \to 0$. Large difference for small momenta:



Calculation of RGI renormalization constants

The calculation of the renormalization group invariant Z_Γ^{RGI} (independent from scheme and scale) is performed in two steps.

1. Conversion to the $\overline{\mathrm{MS}}$ scheme: requires the computation in perturbation theory of $Z_{\mathrm{RI'\text{-}MOM}}^{\overline{\mathrm{MS}}}(\mu)$.
2. Absorb the dependence of $Z_{\overline{\mathrm{MS}}}(\mu)$ on μ using $\Delta Z_{\overline{\mathrm{MS}}}(\mu)$

$$\Delta Z_{\overline{\mathrm{MS}}}(\mu) = \left(2\beta_0 \frac{g^{\overline{\mathrm{MS}}}(\mu)^2}{16\pi^2}\right)^{-\frac{\gamma_0}{2\beta_0}} \exp\left\{\int_0^{g^{\overline{\mathrm{MS}}}(\mu)} \left(\frac{\gamma^{\overline{\mathrm{MS}}}(g')}{\beta^{\overline{\mathrm{MS}}}(g')} + \frac{\gamma_0}{\beta_0 g'}\right) \mathrm{d}\, g'\right\}.$$

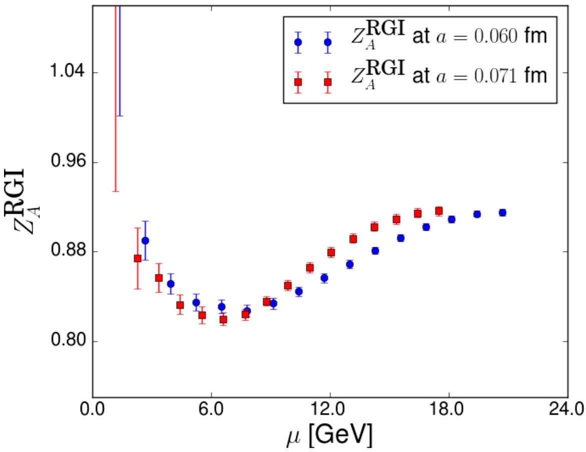
Requires the computation of scale dependence of the operator Γ in the $\overline{\mathrm{MS}}$ scheme defined by $\gamma^{\overline{\mathrm{MS}}} = -\mu \frac{d}{d\mu} \log Z^{\overline{\mathrm{MS}}}(\mu)$.

Finally, Z_Γ^{RGI} is left only with a lattice spacing dependence

$$Z_\Gamma^{\mathrm{RGI}}(a) = \Delta Z_{\overline{\mathrm{MS}}}(\mu) Z_{\mathrm{RI'\text{-}MOM}}^{\overline{\mathrm{MS}}}(\mu, a) Z^{\mathrm{RI'\text{-}MOM}}(\mu, a)\,. \qquad (1)$$

Calculation of RGI renormalization constants

The renormalization group invariant constant Z_A^{RGI} has still a remaining μ dependence at large μ due to "lattice artefacts".

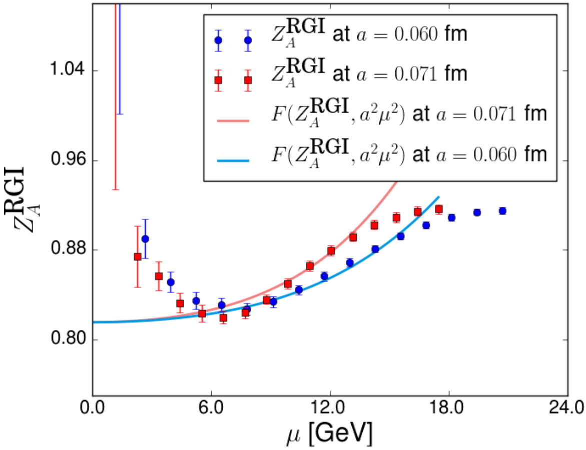


"Window problem": $\Lambda_{\mathrm{QCD}} \ll \mu \ll \frac{1}{a}$

Calculation of RGI renormalization constants

Solution: combined fit of *all* data following the ansatz

$$F(Z_A^{\mathrm{RGI}}, a^2\mu^2) = Z_A^{\mathrm{RGI}} + c_1 a^2\mu^2 + c_2(a^2\mu^2)^2 + \ldots$$



Renormalization group invariants renomalization constants

After performing the subtraction of the lattice artefacts, we can finally extract the RGI renormalization constants

Singlet $Z^{\rm RGI}$

$$
\begin{aligned}
Z_A^{\rm RGI,\,S} &= 0.815(\ 7)\left(^{30}_{-1}\right)\\
Z_P^{\rm RGI,\,S} &= 0.430(10)\left(^{10}_{-20}\right)\\
Z_T^{\rm RGI,\,S} &= 0.896(12)\left(^{10}_{-4}\right)\\
Z_S^{\rm RGI,\,S} &= 0.29(1)\left(^{3}_{-2}\right)
\end{aligned}
$$

Triplet $Z^{\rm RGI}$

$$
\begin{aligned}
Z_A^{\rm RGI,\,N.S.} &= 0.7647(14)\\
Z_P^{\rm RGI,\,N.S.} &= 0.3544(61)\\
Z_T^{\rm RGI,\,N.S.} &= 0.9137(48)\\
Z_S^{\rm RGI,\,N.S.} &= 0.4585(61)
\end{aligned}
$$

PRELIMINARY RESULTS!!! Consistent picture with Chambers *et al.* Phys. Lett. B740 (2015) 30-35 for Z_A and Z_S.

Consistency check for the scalar singlet RC

The renormalized quark mass can be defined from the vector Ward identities

$$am^R_{\mathrm{VWI}}(\mu) = \frac{1}{Z_S^{\mathrm{s.}}(\mu)}\left(\frac{1}{2\kappa} - \frac{1}{2\kappa_c}\right),$$

or from the axial Ward identities (PCAC mass)

$$am^R_{\mathrm{PCAC}}(\mu) = \frac{Z_A^{\mathrm{n.s.}}(\mu)}{Z_P^{\mathrm{n.s.}}(\mu)} m^{\mathrm{bare}}_{\mathrm{PCAC}}.$$

Therefore we must have

$$\frac{1}{Z_S^{\mathrm{s.}}} = \frac{Z_P^{\mathrm{n.s.}}(\mu)}{Z_A^{\mathrm{n.s.}}(\mu)},$$

or equivalently, multiplying both members by $Z_S^{\mathrm{n.s.}}$ [4]

$$r_m = \frac{Z_S^{\mathrm{n.s.}}}{Z_S^{\mathrm{s.}}} = \frac{Z_S^{\mathrm{n.s.}}(\mu) Z_A^{\mathrm{n.s.}}(\mu)}{Z_P^{\mathrm{n.s.}}(\mu)}.$$

[4] G. S. Bali *et al.*, Phys. Rev. D 93, 094504 (2016)

Consistency check for the scalar singlet renormalization constant

From the formulas above, we have

$$
\begin{aligned}
r_m(\beta = 5.29) &= \frac{Z_A^{\rm n.s.}(\mu)}{Z_S^{\rm n.s.} Z_P^{\rm n.s.}(\mu)} = 1.314(20) \\
r_m(\beta = 5.29) &= \frac{Z_S^{\rm n.s.}(\mu)}{Z_S^{\rm s.}(\mu)} = 1.54(9) \\
r_m(\beta = 5.40) &= \frac{Z_A^{\rm n.s.}(\mu)}{Z_S^{\rm n.s.} Z_P^{\rm n.s.}(\mu)} = 1.205(14) \\
r_m(\beta = 5.40) &= \frac{Z_S^{\rm n.s.}(\mu)}{Z_S^{\rm s.}(\mu)} = 1.31(6)
\end{aligned}
$$

In the continuum $r_m = 1$; $r_m \neq 1$ on the lattice for Wilson fermions. The non-perturbative determination of singlet renormalization constants is consistent within 2.5σ.

Summary and outlook

Conclusions:

- The computation of singlet renormalization constants seems feasible at the cost of $O(20)$ times more computational resources
- Final error on Z^{RGI} of the order of $3 - 10$ %

Further measurements are required to extrapolate to the chiral limit.

Outlook:

- Perform the computation on the $N_f = 3$ CLS ensembles (smaller lattice spacings $\rightarrow$ milder window problem)
- Consider the renormalization of operators with derivatives

Thanks to all the RQCD collaboration for the help!