Fermion bags, topology and index theorems

Shailesh Chandrasekharan
(Duke University)

work done in collaboration with V. Ayyar

Lattice 2016, Southampton UK

Supported by:
US Department of Energy, Nuclear Physics Division

Summary

Summary

The concepts of topology and index theorems
that arise in the context of QCD,
have analogies in simple fermion lattice field theories
with staggered fermions,
when formulated in the fermion bag approach

Summary

The concepts of topology and index theorems
that arise in the context of QCD,
have analogies in simple fermion lattice field theories
with staggered fermions,
when formulated in the fermion bag approach

This connection gives a more complete perspective
on fermion mass generation mechanisms,
including a mechanism where fermions acquire a mass
through four-fermion condensates instead of
fermion bilinear condensates.

Summary

The concepts of topology and index theorems
that arise in the context of QCD,
have analogies in simple fermion lattice field theories
with staggered fermions,
when formulated in the fermion bag approach

This connection gives a more complete perspective
on fermion mass generation mechanisms,
including a mechanism where fermions acquire a mass
through four-fermion condensates instead of
fermion bilinear condensates.

Can non-Abelian gauge theories also demonstrate
this alternate mechanism of fermion mass generation?

QCD Partition Function

QCD Partition Function

Partition function of a non-Abelian gauge theory (formal, continuum, finite volume)

QCD Partition Function

Partition function of a non-Abelian gauge theory (formal, continuum, finite volume)

$$Z = \int [dA] \, e^{-S_G(A)} \int [d\overline{\psi} \, d\psi] \, e^{-\overline{\psi} \, D(A) \, \psi}$$

background gauge field integration

weight of the background gauge field

$$D(A) = (\gamma_\mu \partial_\mu - iA_\mu)$$

anti-Hermitian operator depends on the gauge field

Fermion Bag Analogy

Fermion Bag Analogy

Partition function of staggered fermion lattice field theories
in the fermion bag approach.

Fermion Bag Analogy

Partition function of staggered fermion lattice field theories
in the fermion bag approach.

$$Z = \sum_{\mathrm{B}} \mathrm{e}^{-S(\mathrm{B})} \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-\overline{\psi}\ W(\mathrm{B})\ \psi}$$

sum over fermion bag configurations

weight of a fermion bag configuration

anti-Hermitian “fermion bag” matrix depends fermion bag configuration

Topology and Index Theorem in QCD

Topology and Index Theorem in QCD

For every gauge field configuration A we can define a topological charge: Q

Topology and Index Theorem in QCD

For every gauge field configuration A we can define a topological charge: Q

Dirac operators satisfy:

$$\gamma_5 \, D(A) = -D(A) \, \gamma_5$$

non-zero modes come in pairs $\quad D(A)\,|\lambda\rangle = i\lambda\,|\lambda\rangle \qquad D(A)\,\gamma_5\,|\lambda\rangle = -i\lambda\,\gamma_5\,|\lambda\rangle$

zero modes are eigenstates of γ_5 $\quad D(A)\,|z^{\pm}\rangle = 0, \quad \gamma_5\,|z^{\pm}\rangle = \pm\,|z^{\pm}\rangle$

$n_{\pm} =$ number of $|z^{\pm}\rangle$ modes

The index of the Dirac operator $D(A)$: $(n_+ - n_-)$

Topology and Index Theorem in QCD

For every gauge field configuration A we can define a topological charge: Q

Dirac operators satisfy:

$$\gamma_5 \; D(A) \; = -D(A) \; \gamma_5$$

non-zero modes come in pairs $\quad D(A) \, |\lambda\rangle \; = \; i\lambda \, |\lambda\rangle \qquad D(A) \, \gamma_5 \, |\lambda\rangle \; = \; -i\lambda \, \gamma_5 \, |\lambda\rangle$

zero modes are eigenstates of γ_5 $\quad D(A) \, |z^{\pm}\rangle \; = \; 0, \quad \gamma_5 \, |z^{\pm}\rangle \; = \; \pm \, |z^{\pm}\rangle$

$n_{\pm} \; =$ number of $|z^{\pm}\rangle$ modes

The index of the Dirac operator $D(A)$: $(n_+ - n_-)$

Index Theorem: $\quad Q \; = \; (n_+ - n_-)$

Topology and Index Theorem in QCD

For every gauge field configuration A we can define a topological charge: Q

Dirac operators satisfy:

$$\gamma_5 \, D(A) = -D(A) \, \gamma_5$$

non-zero modes come in pairs $\quad D(A)\,|\lambda\rangle = i\lambda\,|\lambda\rangle \qquad D(A)\,\gamma_5\,|\lambda\rangle = -i\lambda\,\gamma_5\,|\lambda\rangle$

zero modes are eigenstates of γ_5 $\quad D(A)\,|z^{\pm}\rangle = 0, \quad \gamma_5\,|z^{\pm}\rangle = \pm\,|z^{\pm}\rangle$

$n_{\pm} =$ number of $|z^{\pm}\rangle$ modes

The index of the Dirac operator $D(A)$: $(n_+ - n_-)$

Index Theorem: $Q = (n_+ - n_-)$ $\qquad$ D(A) has at least |Q| zero modes

Fermion Bag Analogy

Fermion Bag Analogy

For every fermion bag configuration B we can define a topological charge: Q

Fermion Bag Analogy

For every fermion bag configuration B we can define a topological charge: Q

Fermion bag matrix satisfies:

$$\Xi\, W(B) \;=\; -\, W(B)\, \Xi$$

non-zero modes come in pairs $\quad W(B)\,|\lambda\rangle \;=\; i\lambda\,|\lambda\rangle \qquad W(B)\,\Xi\,|\lambda\rangle \;=\; -i\lambda\,\Xi\,|\lambda\rangle$

zero modes are eigenstates of Ξ $\quad W(B)\,|z^{\pm}\rangle \;=\; 0, \quad \Xi\,|z^{\pm}\rangle \;=\; \pm\,|z^{\pm}\rangle$

$n_{\pm} \;=\;$ number of $|z^{\pm}\rangle$ modes

The index of the Dirac operator $W(B)$: $(n_{+} - n_{-})$

Fermion Bag Analogy

For every fermion bag configuration B we can define a topological charge: Q

Fermion bag matrix satisfies:

$$\Xi \; W(B) \; = \; - \; W(B) \; \Xi$$

non-zero modes come in pairs $\quad W(B) \, |\lambda\rangle \; = \; i\lambda \, |\lambda\rangle \qquad W(B) \, \Xi \, |\lambda\rangle \; = \; -i\lambda \, \Xi \, |\lambda\rangle$

zero modes are eigenstates of Ξ $\quad W(B) \, |z^{\pm}\rangle \; = \; 0, \quad \Xi \, |z^{\pm}\rangle \; = \; \pm \, |z^{\pm}\rangle$

$n_{\pm} \; = \;$ number of $\; |z^{\pm}\rangle \;$ modes

The index of the Dirac operator $W(B)$: $(n_+ - n_-)$

Index Theorem: $\quad Q \; = \; (n_+ - n_-)$

Fermion Bag Analogy

For every fermion bag configuration B we can define a topological charge: Q

Fermion bag matrix satisfies:

$$\Xi \, W(B) \;=\; - \, W(B) \, \Xi$$

non-zero modes come in pairs $\quad W(B)\,|\lambda\rangle \;=\; i\lambda\,|\lambda\rangle \qquad W(B)\,\Xi\,|\lambda\rangle \;=\; -i\lambda\,\Xi\,|\lambda\rangle$

zero modes are eigenstates of Ξ $\quad W(B)\,|z^{\pm}\rangle \;=\; 0, \quad \Xi\,|z^{\pm}\rangle \;=\; \pm\,|z^{\pm}\rangle$

$n_{\pm} \;=\;$ number of $|z^{\pm}\rangle$ modes

The index of the Dirac operator $W(B)$: $(n_{+} - n_{-})$

Index Theorem: $\quad Q \;=\; (n_{+} - n_{-})$ $\qquad$ W(B) has at least |Q| zero modes

Example of a Fermion Bag Approach

Example of a Fermion Bag Approach

Consider free massive staggered fermions:

$$S = \frac{1}{2}\sum_{x,\alpha}\eta_{x,\alpha}\Big(\overline{\psi}_x\psi_{x+\alpha} - \overline{\psi}_{x+\alpha}\psi_x\Big) \ + \ m\sum_{x}\overline{\psi}_x\psi_x$$

$$\sum_{x,y}\overline{\psi}_x D_{xy}\psi_y$$

Example of a Fermion Bag Approach

Consider free massive staggered fermions:

$$S = \frac{1}{2}\sum_{x,\alpha} \eta_{x,\alpha}\left(\overline{\psi}_x \psi_{x+\alpha} - \overline{\psi}_{x+\alpha}\psi_x\right) \; + \; m \sum_x \overline{\psi}_x \psi_x$$

$$\sum_{x,y} \overline{\psi}_x D_{xy} \psi_y$$

D is an anti-Hermitian matrix of the form

$$D \; = \; \begin{array}{cc} \begin{array}{cc} \text{even} & \text{odd} \end{array} & \\ \begin{pmatrix} 0 & C \\ -C^T & 0 \end{pmatrix} & \begin{array}{c} \text{even} \\ \text{odd} \end{array} \end{array}$$

Example of a Fermion Bag Approach

Consider free massive staggered fermions:

$$S = \frac{1}{2}\sum_{x,\alpha} \eta_{x,\alpha}\left(\overline{\psi}_x \psi_{x+\alpha} - \overline{\psi}_{x+\alpha}\psi_x\right) \; + \; m \sum_x \overline{\psi}_x \psi_x$$

$$\sum_{x,y} \overline{\psi}_x D_{xy} \psi_y$$

D is an anti-Hermitian matrix of the form

$$D = \begin{array}{cc} \begin{array}{cc} \text{even} & \text{odd} \end{array} & \\ \begin{pmatrix} 0 & C \\ -C^T & 0 \end{pmatrix} & \begin{array}{l} \text{even} \\ \text{odd} \end{array} \end{array} \qquad \Xi = \begin{array}{cc} \begin{array}{cc} \text{even} & \text{odd} \end{array} & \\ \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} & \begin{array}{l} \text{even} \\ \text{odd} \end{array} \end{array}$$

Example of a Fermion Bag Approach

Consider free massive staggered fermions:

$$S = \frac{1}{2}\sum_{x,\alpha} \eta_{x,\alpha}\left(\overline{\psi}_x \psi_{x+\alpha} - \overline{\psi}_{x+\alpha}\psi_x\right) \; + \; m \sum_x \overline{\psi}_x \psi_x$$

$$\sum_{x,y} \overline{\psi}_x D_{xy} \psi_y$$

D is an anti-Hermitian matrix of the form

$$D = \begin{array}{cc} \begin{array}{cc} \text{even} & \text{odd} \end{array} & \\ \left(\begin{array}{cc} 0 & C \\ -C^T & 0 \end{array}\right) & \begin{array}{c} \text{even} \\ \text{odd} \end{array} \end{array} \qquad \Xi = \begin{array}{cc} \begin{array}{cc} \text{even} & \text{odd} \end{array} & \\ \left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array}\right) & \begin{array}{c} \text{even} \\ \text{odd} \end{array} \end{array}$$

$$D\Xi = -\Xi D$$

Partition function: $$Z = \int [d\overline{\psi}\; d\psi]\; \mathrm{e}^{-S}$$

Partition function: $$Z = \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-S}$$

Femion Bag Approach:

$$Z = \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-\overline{\psi} D \psi}\ \mathrm{e}^{-m \sum_x \overline{\psi}_x \psi_x}$$

$$= \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-\overline{\psi} D \psi} \prod_x \left(1 - m\, \overline{\psi}_x \psi_x\right)$$

$$= \sum_B m^k \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-\overline{\psi}\ W(B)\ \psi}$$

Partition function: $Z = \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-S}$

Femion Bag Approach:

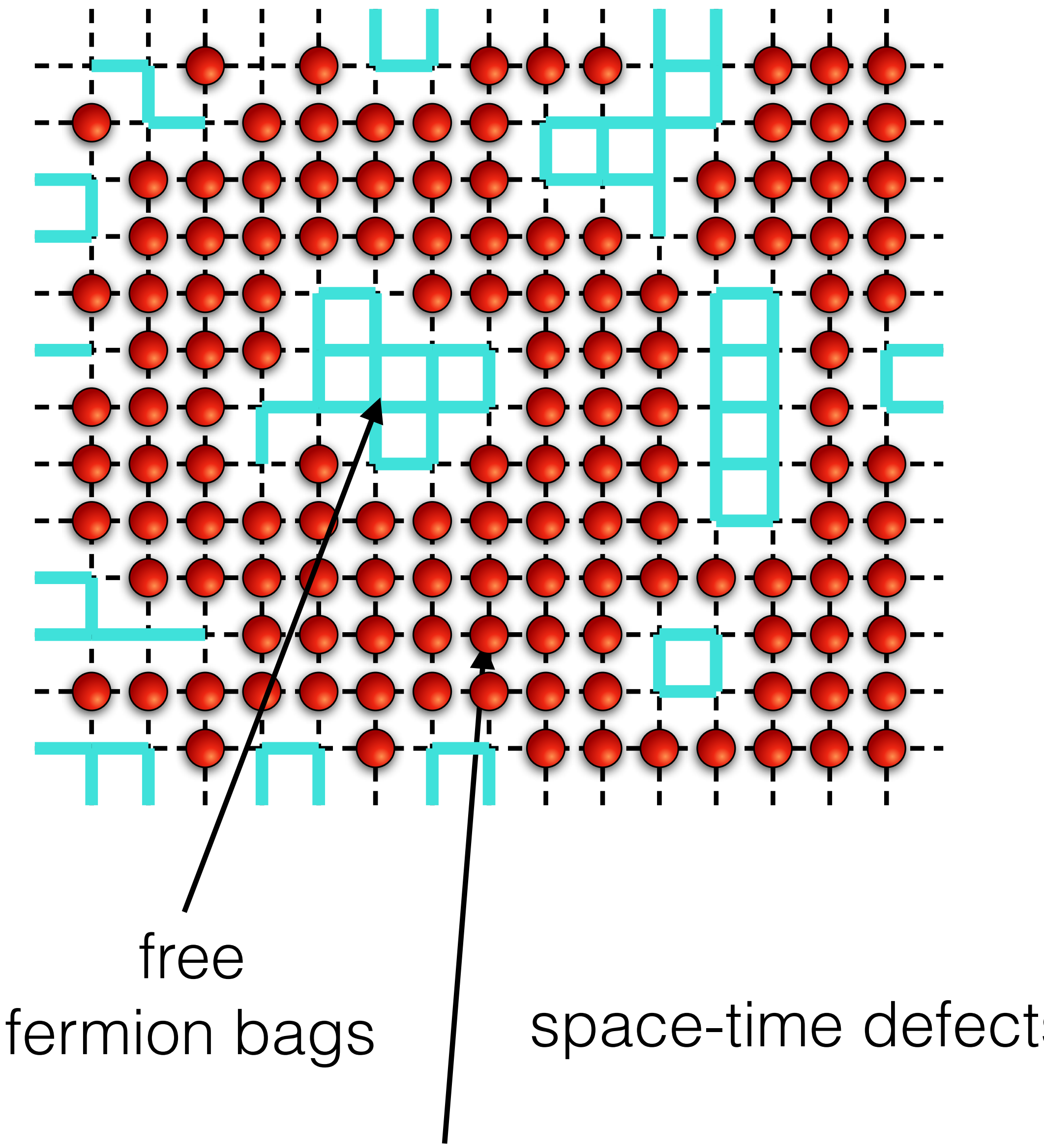
$$Z = \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-\overline{\psi}D\psi}\ \mathrm{e}^{-m\sum_x \overline{\psi}_x\psi_x}$$

$$= \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-\overline{\psi}D\psi} \prod_x \left(1 - m\ \overline{\psi}_x\psi_x\right)$$

$$= \sum_B m^k \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-\overline{\psi}\ W(B)\ \psi}$$

B = B1 + B2 + …

configuration B



Partition function: $$Z = \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-S}$$

Femion Bag Approach:

$$Z = \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-\overline{\psi} D \psi}\ \mathrm{e}^{-m \sum_x \overline{\psi}_x \psi_x}$$

$$= \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-\overline{\psi} D \psi} \prod_x \left(1 - m\, \overline{\psi}_x \psi_x\right)$$

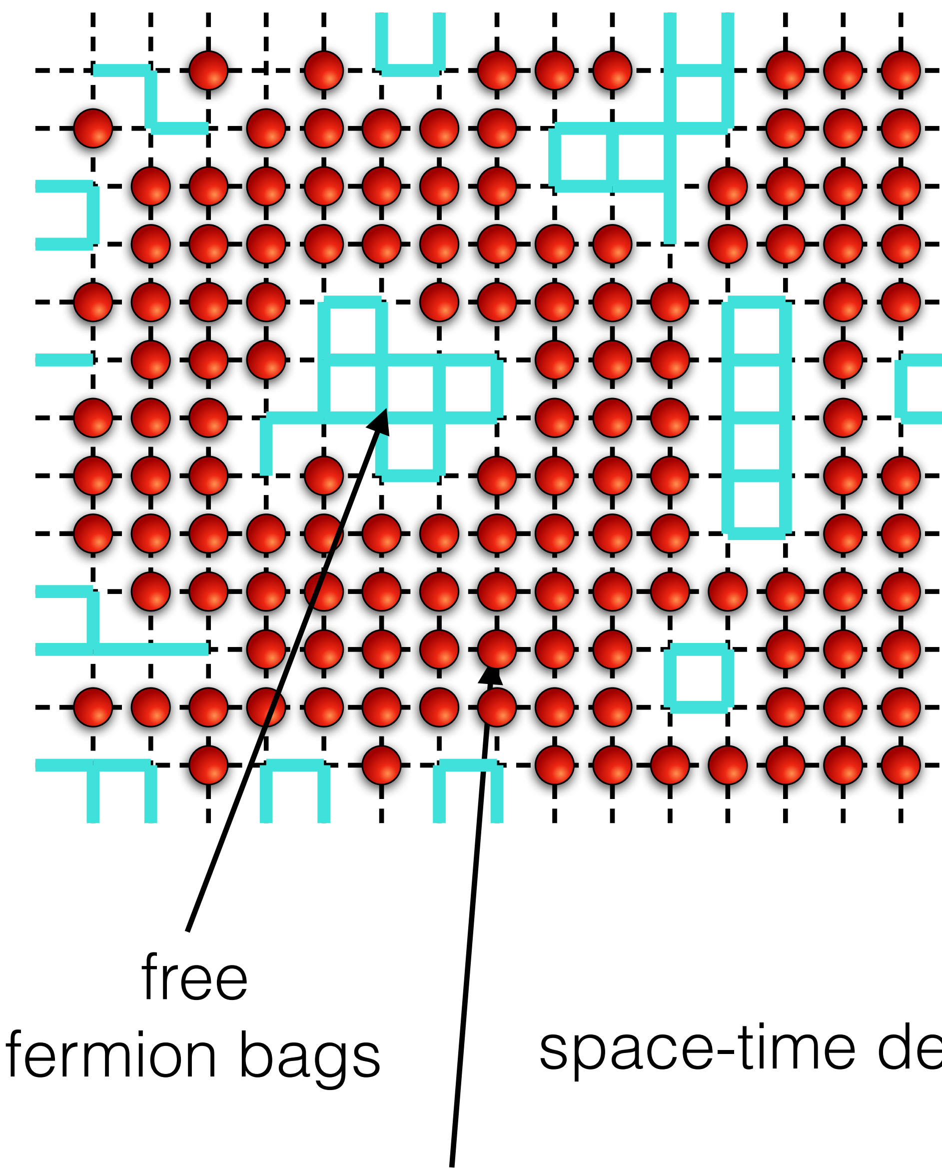
$$= \sum_B m^k \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-\overline{\psi}\ W(B)\ \psi}$$

B = B1 + B2 + …

Defining $S(B) = -k\ \log(m)$

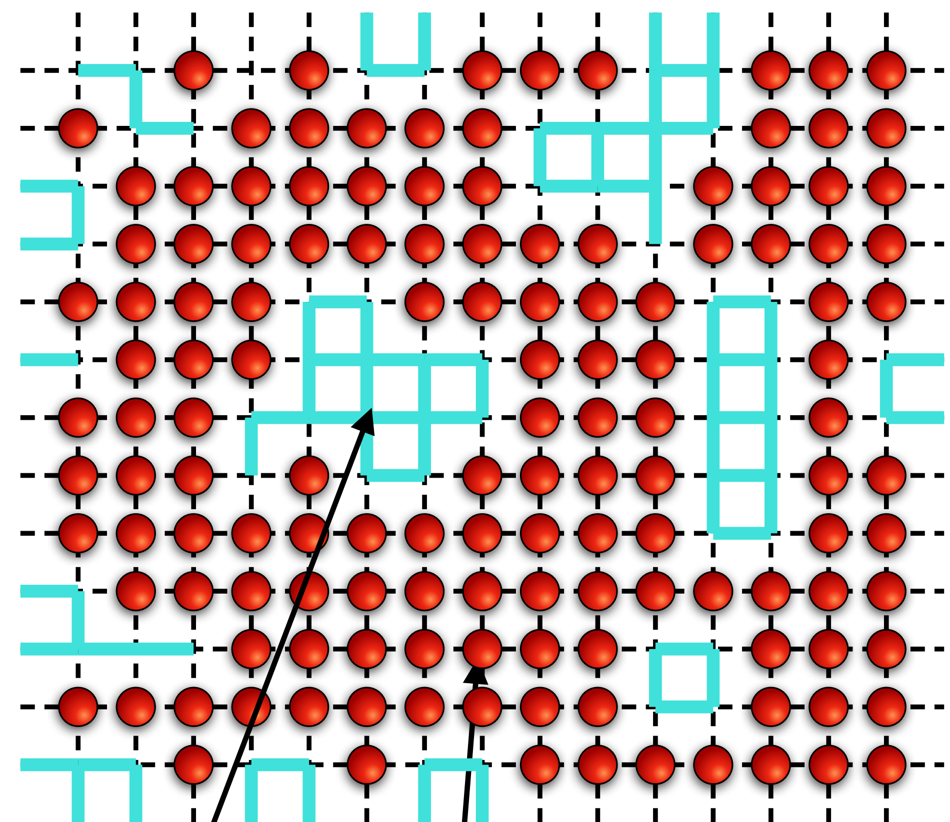
$$Z = \sum_B \exp(-S(B)) \int [d\overline{\psi}\ d\psi]\ \mathrm{e}^{-\overline{\psi}\ W(B)\ \psi}$$

configuration B



Partition function: $$Z = \int [d\overline{\psi}\; d\psi]\; \mathrm{e}^{-S}$$

configuration B



free fermion bags

space-time defects

k = total number of defects

Femion Bag Approach:

$$Z = \int [d\overline{\psi}\; d\psi]\; \mathrm{e}^{-\overline{\psi} D \psi}\; \mathrm{e}^{-m \sum_x \overline{\psi}_x \psi_x}$$

$$= \int [d\overline{\psi}\; d\psi]\; \mathrm{e}^{-\overline{\psi} D \psi} \prod_x \left(1 - m\, \overline{\psi}_x \psi_x\right)$$

$$= \sum_B m^k \int [d\overline{\psi}\; d\psi]\; \mathrm{e}^{-\overline{\psi}\; W(B)\; \psi}$$

B = B1 + B2 + ...

Defining $$S(B) = -k\ \log(m)$$

$$Z = \sum_B \exp(-S(B)) \int [d\overline{\psi}\; d\psi]\; \mathrm{e}^{-\overline{\psi}\; W(B)\; \psi}$$

$$W(B) = \begin{array}{cc} \begin{array}{cc} \text{even} & \text{odd} \end{array} & \\ \begin{pmatrix} 0 & C(B) \\ -C(B)^T & 0 \end{pmatrix} & \begin{array}{c} \text{even} \\ \text{odd} \end{array} \end{array}$$

anti-Hermitian

Topology and index theorem for a fermion bag

Topology and index theorem for a fermion bag

Topological charge of a fermion bag $Q \ = \ n_{even} - n_{odd}$

Topology and index theorem for a fermion bag

Topological charge of a fermion bag $Q = n_{even} - n_{odd}$

Fermion bag Dirac operator

$$W(B) = \begin{array}{cc} & \begin{array}{cc} \text{even} & \text{odd} \end{array} \\ & \left(\begin{array}{cc} 0 & C(B) \\ -C(B)^T & 0 \end{array} \right) \begin{array}{c} \text{even} \\ \text{odd} \end{array} \end{array}$$

Topology and index theorem for a fermion bag

Topological charge of a fermion bag $Q = n_{even} - n_{odd}$

Fermion bag Dirac operator

$$W(B) = \begin{array}{cc|c} \text{even} & \text{odd} & \\ \end{array} \begin{array}{c} \\ \end{array}$$

$$\begin{array}{c@{}c} & \begin{array}{cc} \text{even} & \text{odd} \end{array} \\ W(B) = & \begin{pmatrix} 0 & C(B) \\ -C(B)^T & 0 \end{pmatrix} \end{array} \begin{array}{l} \text{even} \\ \text{odd} \end{array}$$

define

$$\begin{array}{c@{}c} & \begin{array}{cc} \text{even} & \text{odd} \end{array} \\ \Xi_B = & \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{array} \begin{array}{l} \text{even} \\ \text{odd} \end{array}$$

Topology and index theorem for a fermion bag

Topological charge of a fermion bag $Q = n_{even} - n_{odd}$

Fermion bag Dirac operator

$$W(B) = \begin{array}{cc} \text{even} & \text{odd} \end{array}$$

$$W(B) = \begin{array}{c} \\ \left(\begin{array}{cc} 0 & C(B) \\ -C(B)^T & 0 \end{array} \right) \end{array} \begin{array}{l} \text{even} \\ \text{odd} \end{array}$$

define

$$\Xi_B = \left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) \begin{array}{l} \text{even} \\ \text{odd} \end{array}$$

Let $n_\pm$ be the number of zero modes of $W(B)$ that are also eigenvalues of Ξ_B with eigenvalues ± 1.

Topology and index theorem for a fermion bag

Topological charge of a fermion bag $\quad Q \;=\; n_{even} - n_{odd}$

Fermion bag Dirac operator

$$W(B) \;=\; \begin{array}{cc} \text{even} & \text{odd} \\ \left(\begin{array}{c} 0 \\ -C(B)^T \end{array}\right. & \left.\begin{array}{c} C(B) \\ 0 \end{array}\right) \end{array} \begin{array}{c} \\ \text{even} \\ \text{odd} \end{array}$$

define

$$\Xi_B \;=\; \begin{array}{cc} \text{even} & \text{odd} \\ \left(\begin{array}{c} 1 \\ 0 \end{array}\right. & \left.\begin{array}{c} 0 \\ -1 \end{array}\right) \end{array} \begin{array}{c} \\ \text{even} \\ \text{odd} \end{array}$$

Let $n_\pm$ be the number of zero modes of $W(B)$ that are also eigenvalues of Ξ_B with eigenvalues ± 1.

Index theorem: $\quad Q \;=\; n_+ - n_-$

Topology and index theorem for a fermion bag

Topological charge of a fermion bag $\quad Q \;=\; n_{even} - n_{odd}$

Fermion bag Dirac operator

$$W(B) \;=\; \begin{array}{cc} \text{even} & \text{odd} \\ \left(\begin{array}{c} 0 \\ -C(B)^T \end{array}\right. & \left.\begin{array}{c} C(B) \\ 0 \end{array}\right) \end{array} \begin{array}{c} \text{even} \\ \text{odd} \end{array}$$

define

$$\Xi_B \;=\; \begin{array}{cc} \text{even} & \text{odd} \\ \left(\begin{array}{c} 1 \\ 0 \end{array}\right. & \left.\begin{array}{c} 0 \\ -1 \end{array}\right) \end{array} \begin{array}{c} \text{even} \\ \text{odd} \end{array}$$

Let $n_{\pm}$ be the number of zero modes of $W(B)$

that are also eigenvalues of Ξ_B with eigenvalues ± 1.

Index theorem: $\quad Q \;=\; n_+ - n_-$

$W(B)$ has at least $|Q|$ zero modes.

N_f = 1 QCD Chiral Condensate

N_f = 1 QCD Chiral Condensate

Partition function gets contribution only from Q = 0 sector.

$$Z = \int [dA_{Q=0}] \, \mathrm{e}^{-S_G(A_{Q=0})} \, \mathrm{Det}(D(A_{Q=0}))$$

N_f = 1 QCD Chiral Condensate

Partition function gets contribution only from Q = 0 sector.

$$Z \; = \int [dA_{Q=0}] \; \mathrm{e}^{-S_G(A_{Q=0})} \; \mathrm{Det}(D(A_{Q=0}))$$

Chiral Condensate gets contribution from the sector with topological charge one.

$$\langle\overline{\psi}\psi\rangle = \frac{1}{Z} \int [dA \; d\overline{\psi} \; d\psi] \; \mathrm{e}^{-S} \left\{ \frac{1}{V} \int d^4x \; \overline{\psi}(x)\psi(x) \right\}$$

$$\langle\overline{\psi}\psi\rangle \; = \; \frac{1}{ZV} \int [dA_{Q=1}] \; \mathrm{e}^{-S_G(A_{Q=1})} \left(\prod_{\lambda} \lambda(A_{Q=1}) \right)$$

$N_f = 1$ QCD Chiral Condensate

Partition function gets contribution only from Q = 0 sector.

$$Z = \int [dA_{Q=0}]\ \mathrm{e}^{-S_G(A_{Q=0})}\ \mathrm{Det}(D(A_{Q=0}))$$

Chiral Condensate gets contribution from the sector with topological charge one.

$$\langle\overline{\psi}\psi\rangle = \frac{1}{Z}\int [dA\ d\overline{\psi}\ d\psi]\ \mathrm{e}^{-S}\left\{\frac{1}{V}\int d^4x\ \overline{\psi}(x)\psi(x)\right\}$$

$$\langle\overline{\psi}\psi\rangle = \frac{1}{ZV}\int [dA_{Q=1}]\ \mathrm{e}^{-S_G(A_{Q=1})}\left(\prod_{\lambda}\lambda(A_{Q=1})\right)$$

Anomalous chiral symmetry breaking!

Fermion Bag Analogy

Fermion Bag Analogy

$$S = \frac{1}{2}\sum_{x,\alpha}\eta_{x,\alpha}\Big(\overline{\psi}_x\psi_{x+\alpha} - \overline{\psi}_{x+\alpha}\psi_x\Big) \;+\; m\sum_x \overline{\psi}_x\psi_x$$

Partition function gets contribution from Q = 0 sectors.

$$Z = \sum_{B_{Q=0}} \mathrm{e}^{k\,\log(m)}\ \mathrm{Det}(W(B_{Q=0}))$$

Chiral condensate get contribution from Q=1 sectors!

$$\langle\overline{\psi}\psi\rangle = \frac{1}{ZV}\sum_{B_{Q=1}} \mathrm{e}^{k\,\log(m)}\ \Big(\prod_{\lambda}\ \lambda(B_{Q=1})\Big)$$

Fermion Bag Analogy

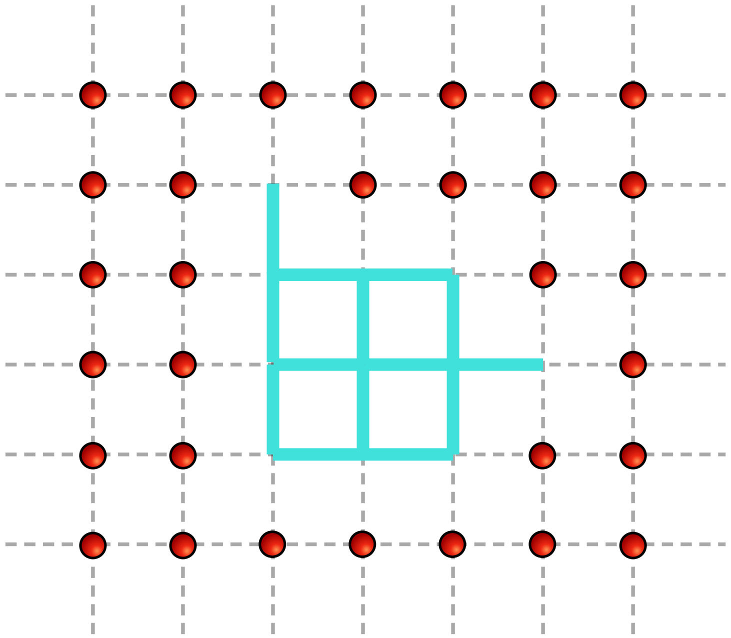
$$S = \frac{1}{2}\sum_{x,\alpha}\eta_{x,\alpha}\left(\overline{\psi}_x\psi_{x+\alpha} - \overline{\psi}_{x+\alpha}\psi_x\right) \; + \; m\sum_x \overline{\psi}_x\psi_x$$

Partition function gets contribution from Q = 0 sectors.

$$Z = \sum_{B_{Q=0}} \mathrm{e}^{k\,\log(m)}\ \mathrm{Det}(W(B_{Q=0}))$$

Chiral condensate get contribution from Q=1 sectors!

$$\langle\overline{\psi}\psi\rangle = \frac{1}{ZV}\ \sum_{B_{Q=1}} \mathrm{e}^{k\,\log(m)}\ \Big(\prod_{\lambda}\ \lambda(B_{Q=1})\Big)$$



fermion bag with Q = 1

Fermion Bag Analogy

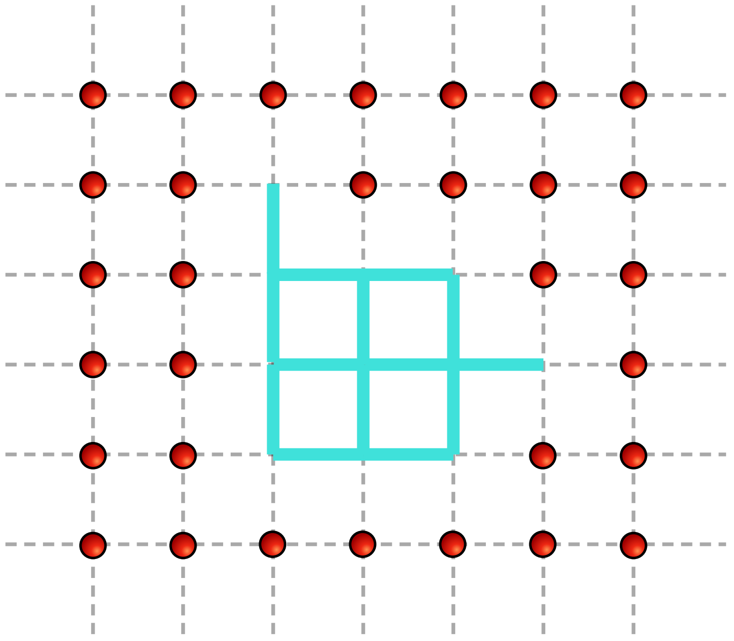
$$S = \frac{1}{2}\sum_{x,\alpha}\eta_{x,\alpha}\left(\overline{\psi}_x\psi_{x+\alpha} - \overline{\psi}_{x+\alpha}\psi_x\right) \; + \; m\sum_x \overline{\psi}_x\psi_x$$

Partition function gets contribution from Q = 0 sectors.

$$Z = \sum_{B_{Q=0}} \mathrm{e}^{k\,\log(m)}\ \mathrm{Det}(W(B_{Q=0}))$$

Chiral condensate get contribution from Q=1 sectors!

$$\langle\overline{\psi}\psi\rangle = \frac{1}{ZV}\ \sum_{B_{Q=1}} \mathrm{e}^{k\,\log(m)}\ \Big(\prod_{\lambda}\ \lambda(B_{Q=1})\Big)$$



fermion bag with Q = 1

Anomalous chiral symmetry breaking ~ Explicit chiral symmetry breaking

N_f = 2 QCD

N_f = 2 QCD

Chiral condensate vanishes due to additional chiral symmetries! $\langle \overline{\psi}\psi \rangle \; = \; 0$

$N_f = 2$ QCD

Chiral condensate vanishes due to additional chiral symmetries! $\langle\overline{\psi}\psi\rangle \;=\; 0$

Chiral susceptibility is non-zero and gets contribution from the sectors, Q = 0 and Q=1.

$$\chi \;=\; \frac{1}{ZV} \int [dA\; d\overline{\psi}\; d\psi]\; \mathrm{e}^{-S} \left\{ \int d^4x \overline{\psi}(x)\psi(x) \right\} \left\{ \int d^4y \overline{\psi}(y)\psi(y) \right\}$$

$$\chi \;=\; \frac{1}{ZV} \int [dA_{Q=0}]\; \mathrm{e}^{-S_G(A_{Q=0})}\; \mathrm{Det}(D(A_{Q=0})) \left\{ \sum_{\lambda} \frac{1}{\lambda^2} \right\} \;+\; \ldots$$

$N_f = 2$ QCD

Chiral condensate vanishes due to additional chiral symmetries! $\langle \overline{\psi}\psi \rangle \; = \; 0$

Chiral susceptibility is non-zero and gets contribution from the sectors, Q = 0 and Q=1.

$$\chi \;=\; \frac{1}{ZV} \int [dA \; d\overline{\psi} \; d\psi] \; \mathrm{e}^{-S} \left\{ \int d^4 x \overline{\psi}(x)\psi(x) \right\} \left\{ \int d^4 y \overline{\psi}(y)\psi(y) \right\}$$

$$\chi \;=\; \frac{1}{ZV} \int [dA_{Q=0}] \; \mathrm{e}^{-S_G(A_{Q=0})} \; \mathrm{Det}(D(A_{Q=0})) \left\{ \sum_{\lambda} \frac{1}{\lambda^2} \right\} \; + \; \ldots$$

Spontaneous chiral symmetry breaking requires $\chi \;\sim \Sigma^2 V$

$N_f = 2$ QCD

Chiral condensate vanishes due to additional chiral symmetries! $\langle\overline{\psi}\psi\rangle \;=\; 0$

Chiral susceptibility is non-zero and gets contribution from the sectors, Q = 0 and Q=1.

$$\chi \;=\; \frac{1}{ZV} \int [dA \; d\overline{\psi} \; d\psi] \; \mathrm{e}^{-S} \left\{ \int d^4x \overline{\psi}(x)\psi(x) \right\} \left\{ \int d^4y \overline{\psi}(y)\psi(y) \right\}$$

$$\chi \;=\; \frac{1}{ZV} \int [dA_{Q=0}] \; \mathrm{e}^{-S_G(A_{Q=0})} \; \mathrm{Det}(D(A_{Q=0})) \left\{ \sum_\lambda \frac{1}{\lambda^2} \right\} \;+\; \ldots$$

Spontaneous chiral symmetry breaking requires $\chi \;\sim \Sigma^2 V$

This implies that the smallest non-zero eigenvalues in the Q=0 must scale as $\lambda \sim \dfrac{1}{\Sigma V}$

Nf = 2, Fermion Bag Analogy

Nf = 2, Fermion Bag Analogy

Action (Nf = 2 model)

$$S = \frac{1}{2}\sum_{x,\alpha,i=1,2} \eta_{x,\alpha}\left(\overline{\psi}_{x,i}\psi_{x+\alpha,i} - \overline{\psi}_{x+\alpha,i}\psi_{x,i}\right) \; - \; U\sum_{x}\overline{\psi}_{x,1}\psi_{x,1}\overline{\psi}_{x,2}\psi_{x,2}$$

Ayyar, SC, PRD91 (2015) 6, 065035, PRD93 (2016) 8, 081701, arXiv:1606.06312

Nf = 2, Fermion Bag Analogy

Action (Nf = 2 model)

$$S = \frac{1}{2}\sum_{x,\alpha,i=1,2} \eta_{x,\alpha}\Big(\overline{\psi}_{x,i}\psi_{x+\alpha,i} - \overline{\psi}_{x+\alpha,i}\psi_{x,i}\Big) \; - \; U\sum_{x}\overline{\psi}_{x,1}\psi_{x,1}\overline{\psi}_{x,2}\psi_{x,2}$$

Ayyar, SC, PRD91 (2015) 6, 065035, PRD93 (2016) 8, 081701, arXiv:1606.06312

Partition function

$$Z = \int [d\psi]\ \mathrm{e}^{-\frac{1}{2}\psi^T M\psi}\prod_x\big(1 \; + \; U\ \psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4}\big)$$

$$Z = \sum_B\ \mathrm{e}^{k\log(U)}\ \Big(\mathrm{Det}(W(B))\Big)^2$$

Nf = 2, Fermion Bag Analogy

Action (Nf = 2 model)

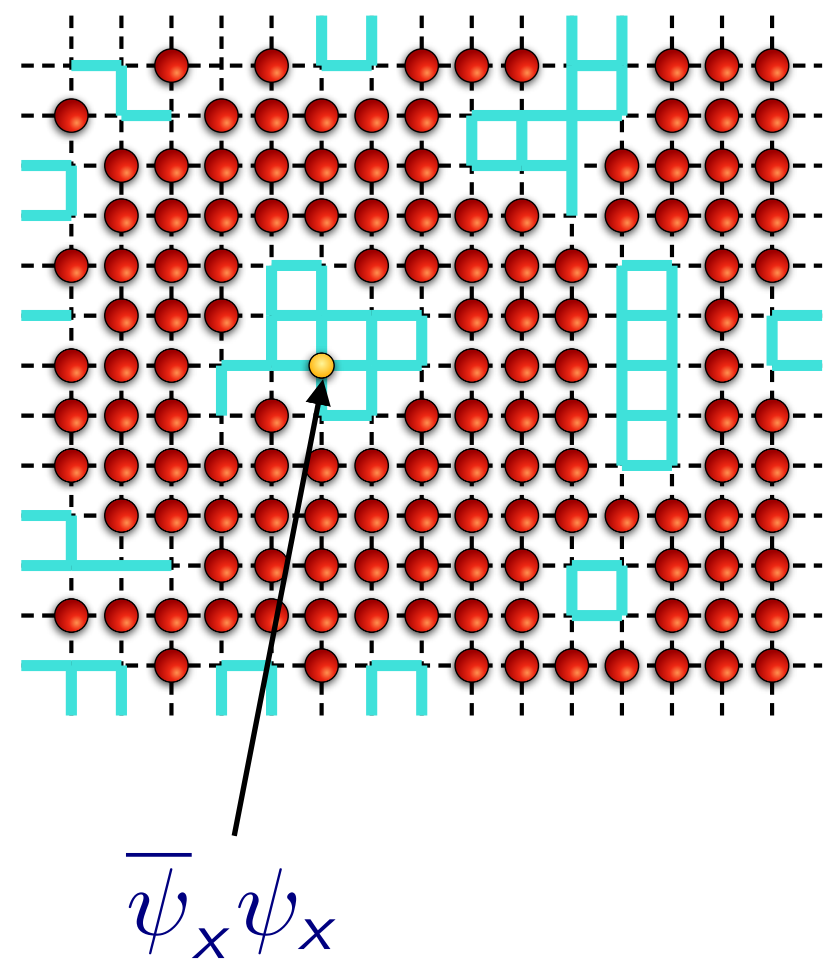
$$S = \frac{1}{2} \sum_{x,\alpha,i=1,2} \eta_{x,\alpha} \left(\overline{\psi}_{x,i} \psi_{x+\alpha,i} - \overline{\psi}_{x+\alpha,i} \psi_{x,i} \right) \; - \; U \sum_{x} \overline{\psi}_{x,1} \psi_{x,1} \overline{\psi}_{x,2} \psi_{x,2}$$

Ayyar, SC, PRD91 (2015) 6, 065035, PRD93 (2016) 8, 081701, arXiv:1606.06312

Partition function

$$Z = \int [d\psi] \; e^{-\frac{1}{2}\psi^T M \psi} \prod_{x} \left(1 \; + \; U \; \psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4}\right)$$

$$Z = \sum_{B} \; e^{k \log(U)} \; \left(\mathrm{Det}(W(B))\right)^2$$



Nf = 2, Fermion Bag Analogy

Action (Nf = 2 model)

$$S = \frac{1}{2}\sum_{x,\alpha,i=1,2} \eta_{x,\alpha}\left(\overline{\psi}_{x,i}\psi_{x+\alpha,i} - \overline{\psi}_{x+\alpha,i}\psi_{x,i}\right) \; - \; U\sum_{x}\overline{\psi}_{x,1}\psi_{x,1}\overline{\psi}_{x,2}\psi_{x,2}$$

Ayyar, SC, PRD91 (2015) 6, 065035, PRD93 (2016) 8, 081701, arXiv:1606.06312

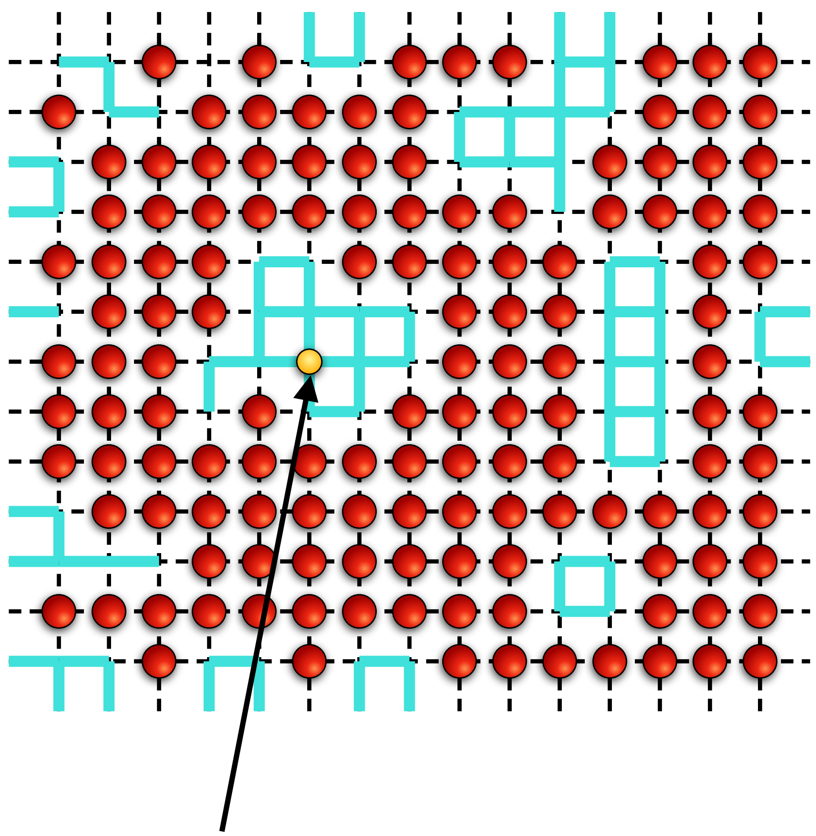
Partition function

$$Z = \int [d\psi]\; \mathrm{e}^{-\frac{1}{2}\psi^T M\psi} \prod_x \left(1 \; + \; U\; \psi_{x,1}\psi_{x,2}\psi_{x,3}\psi_{x,4}\right)$$

$$Z = \sum_B \mathrm{e}^{k\log(U)}\; \left(\mathrm{Det}(W(B))\right)^2$$

Chiral condensate vanishes

$$\langle\overline{\psi}\psi\rangle \; = \; 0$$



$$\overline{\psi}_x\psi_x$$

Fermion bilinear correlation function

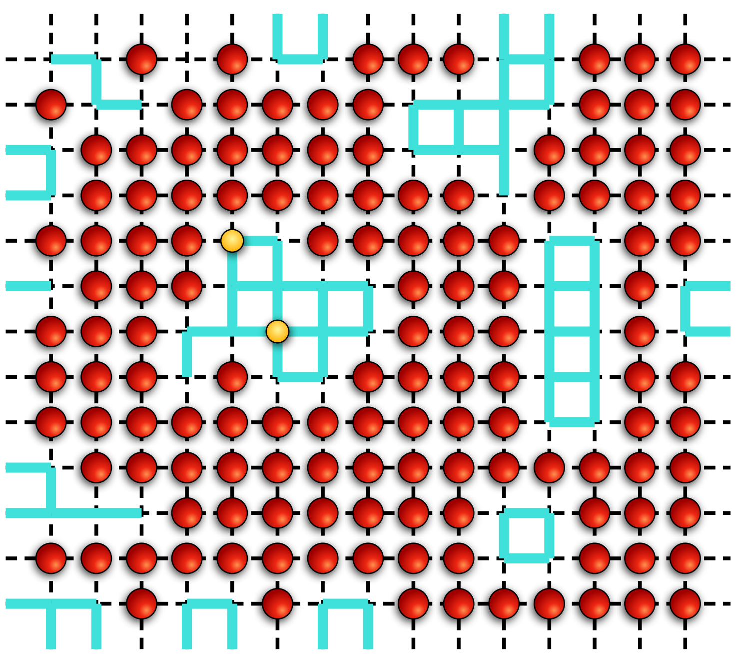
$$\langle \psi_{x,a}\psi_{x,b}\ \psi_{y,a}\psi_{y,b} \rangle \ = \int [d\overline{\psi} d\psi]\ \mathrm{e}^{-S}\ \psi_{x,a}\psi_{x,b}\ \psi_{y,a}\psi_{y,b}$$

If B = B1 + B2 + … then, it vanishes unless x and y are in the same bag!

Fermion bilinear correlation function

$$\langle \psi_{x,a}\psi_{x,b}\ \psi_{y,a}\psi_{y,b}\rangle \ = \ \int [d\overline{\psi} d\psi]\ \mathrm{e}^{-S}\ \psi_{x,a}\psi_{x,b}\ \psi_{y,a}\psi_{y,b}$$

If B = B1 + B2 + … then, it vanishes unless x and y are in the same bag!

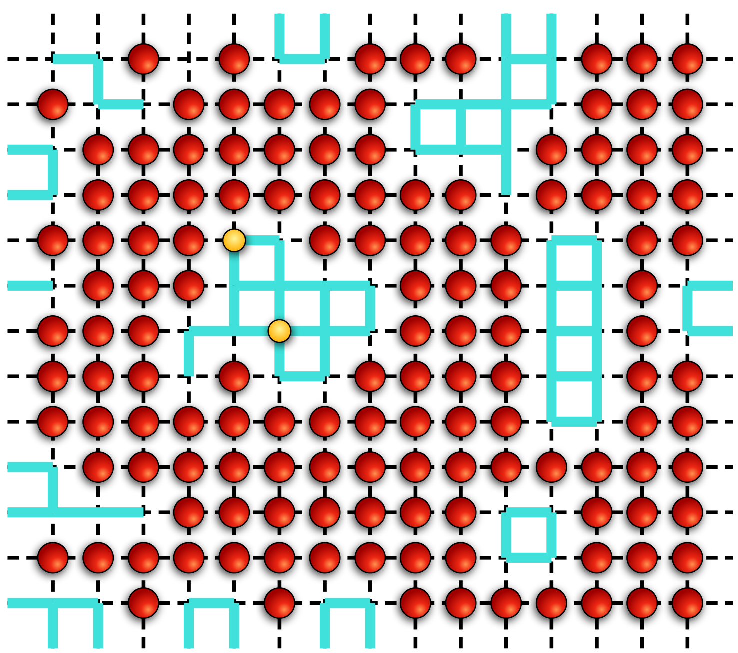


Non-zero!

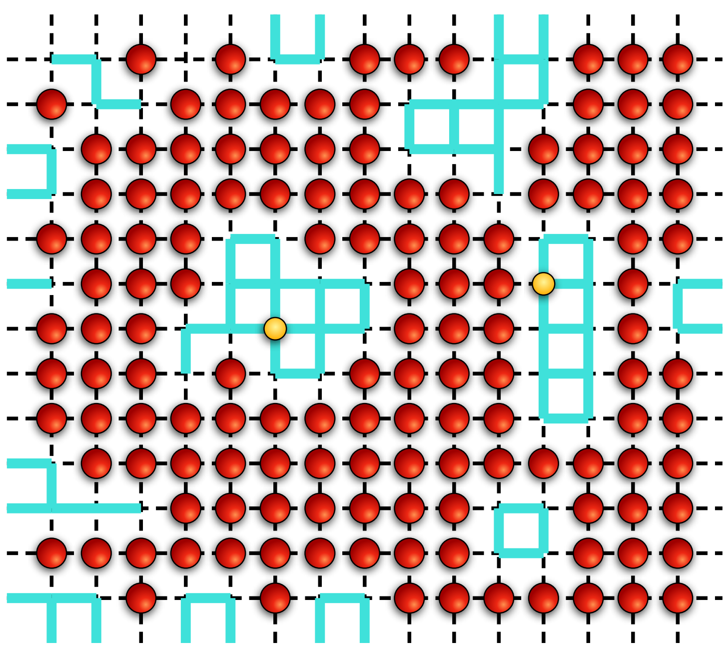
Fermion bilinear correlation function

$$\langle \psi_{x,a}\psi_{x,b}\ \psi_{y,a}\psi_{y,b}\rangle \;=\; \int [d\overline{\psi} d\psi]\ \mathrm{e}^{-S}\ \psi_{x,a}\psi_{x,b}\ \psi_{y,a}\psi_{y,b}$$

If B = B1 + B2 + … then, it vanishes unless x and y are in the same bag!



Non-zero!

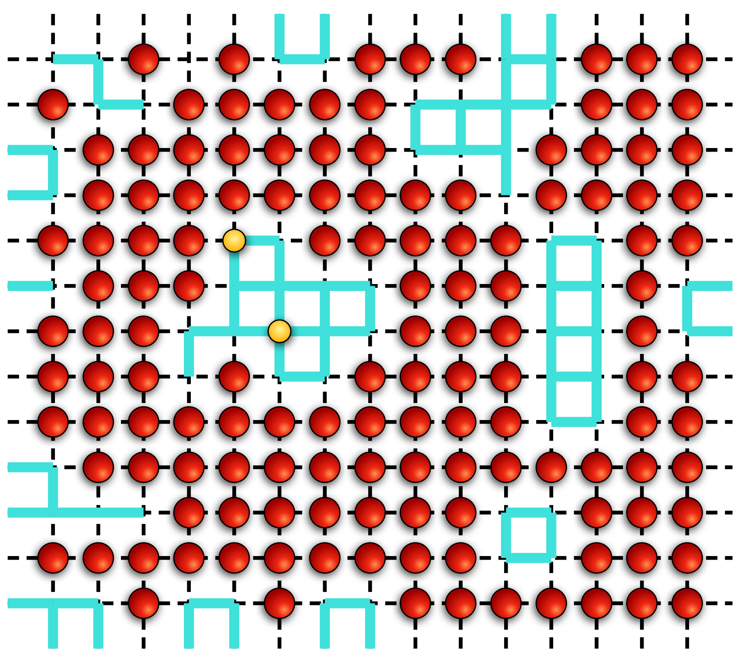


Zero!

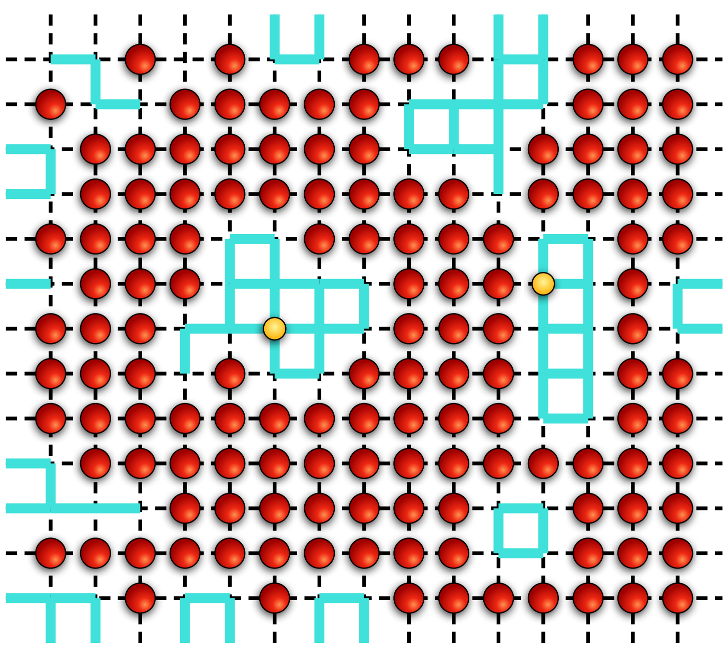
Fermion bilinear correlation function

$$\langle \psi_{x,a}\psi_{x,b}\ \psi_{y,a}\psi_{y,b}\rangle \;=\; \int [d\overline{\psi}d\psi]\ \mathrm{e}^{-S}\ \psi_{x,a}\psi_{x,b}\ \psi_{y,a}\psi_{y,b}$$

If B = B1 + B2 + … then, it vanishes unless x and y are in the same bag!



Non-zero!



Zero!

At sufficiently large coupling U
all two point correlation functions will exponentially decay!

Analogy with QCD?

Analogy with QCD?

As in QCD Chiral Susceptibility can get contributions from Q=0,1 sectors.

Analogy with QCD?

As in QCD Chiral Susceptibility can get contributions from Q=0,1 sectors.

As in QCD spontaneous chiral symmetry breaking requires $\chi \sim \Sigma^2 V$

Analogy with QCD?

As in QCD Chiral Susceptibility can get contributions from Q=0,1 sectors.

As in QCD spontaneous chiral symmetry breaking requires $\chi \sim \Sigma^2 V$

This implies that the smallest non-zero eigenvalues in the Q=0 must sector scale as $\lambda \sim \frac{1}{\Sigma V}$

Analogy with QCD?

As in QCD Chiral Susceptibility can get contributions from Q=0,1 sectors.

As in QCD spontaneous chiral symmetry breaking requires $\chi \sim \Sigma^2 V$

This implies that the smallest non-zero eigenvalues in the Q=0 must sector scale as $\lambda \sim \dfrac{1}{\Sigma V}$

A necessary condition is that fermion bags span the entire volume for SSB!

Analogy with QCD?

As in QCD Chiral Susceptibility can get contributions from Q=0,1 sectors.

As in QCD spontaneous chiral symmetry breaking requires $\chi \sim \Sigma^2 V$

This implies that the smallest non-zero eigenvalues in the Q=0 must sector scale as $\lambda \sim \frac{1}{\Sigma V}$

A necessary condition is that fermion bags span the entire volume for SSB!

But at large U fermion bags are small. Smallest eigenvalues $\lambda \not\sim \frac{1}{\Sigma V}$

Analogy with QCD?

As in QCD Chiral Susceptibility can get contributions from Q=0,1 sectors.

As in QCD spontaneous chiral symmetry breaking requires $\chi \sim \Sigma^2 V$

This implies that the smallest non-zero eigenvalues in the Q=0 must sector scale as $\lambda \sim \frac{1}{\Sigma V}$

A necessary condition is that fermion bags span the entire volume for SSB!

But at large U fermion bags are small. Smallest eigenvalues $\lambda \not\sim \frac{1}{\Sigma V}$

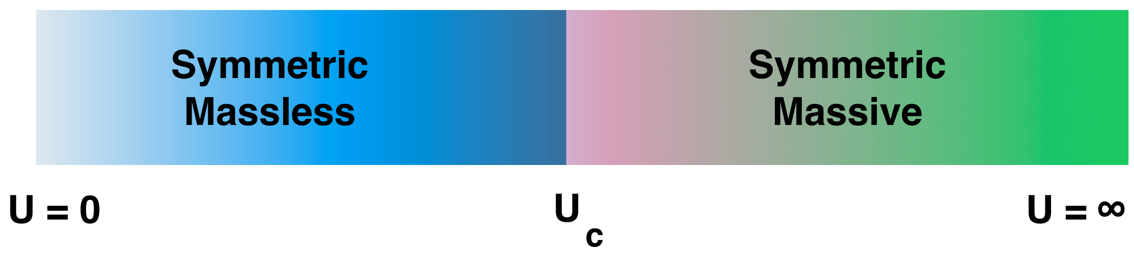
At large U, unlike QCD no SSB, but fermions are still massive!

Results in the Nf=2 model

Results in the Nf=2 model

In 3D there is evidence for a single exotic transition

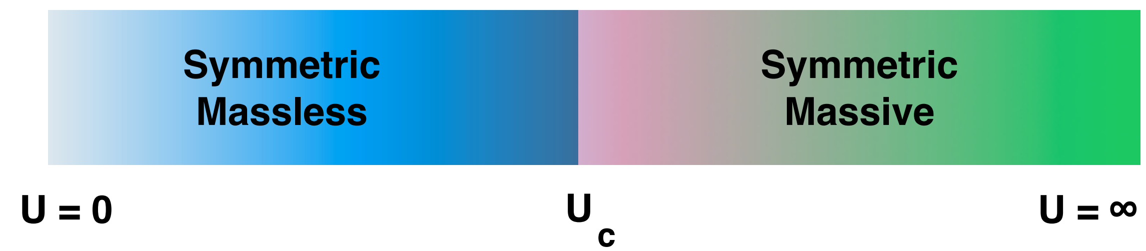
PRD91 (2015) 6, 065035,
PRD 93 (2016), 081701



Results in the Nf=2 model

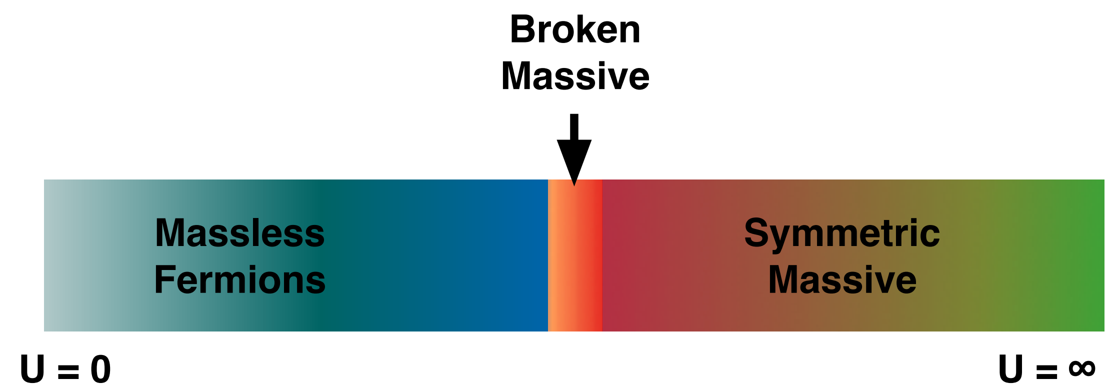
In 3D there is evidence for a single exotic transition

PRD91 (2015) 6, 065035,
PRD 93 (2016), 081701



In 4D we find evidence for a narrow intermediate spontaneously broken phase

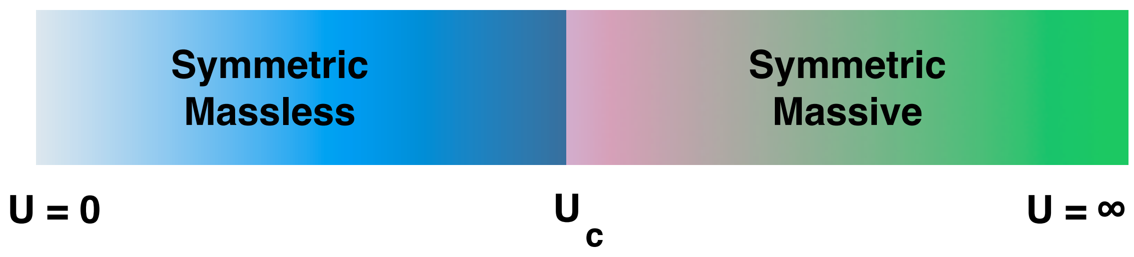
arXiv:1606.06312



Results in the Nf=2 model

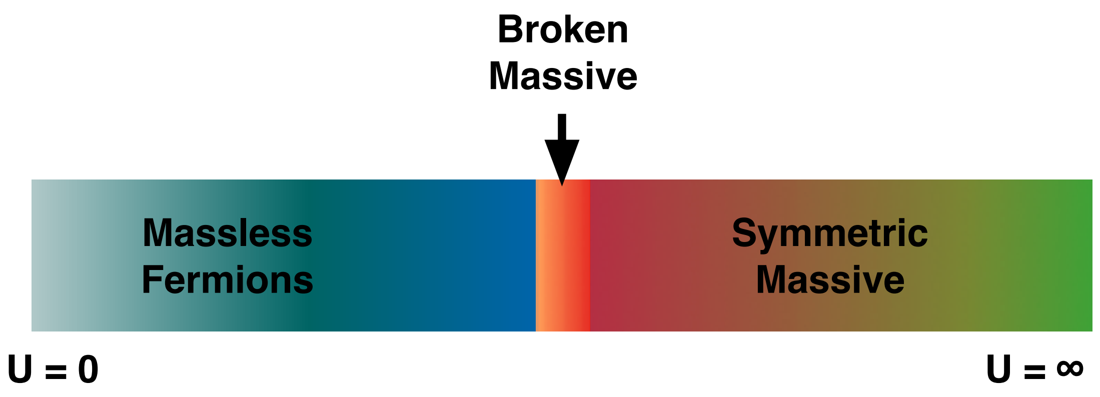
In 3D there is evidence for a single exotic transition

PRD91 (2015) 6, 065035,
PRD 93 (2016), 081701



In 4D we find evidence for a narrow intermediate spontaneously broken phase

arXiv:1606.06312



In 2D we find evidence for a single asymptotically free (massive) phase

work in progress

Origin of Fermion Masses

Origin of Fermion Masses

Bilinear Condensate Mechanism: A

Fermion bilinear condensates form due to anomalous or explicit chiral symmetry breaking through topological space-time defects.
Examples: Nf=1 QCD, Nf =1 massive free staggered fermions

Origin of Fermion Masses

Bilinear Condensate Mechanism: A

Fermion bilinear condensates form due to anomalous or explicit chiral symmetry breaking through topological space-time defects.
Examples: Nf=1 QCD, Nf =1 massive free staggered fermions

Bilinear Condensate Mechanism: B

Fermion bilinear condensates form due to spontaneous breaking of symmetries that prevent their formation.
Nf = 2 QCD is an example of this phenomena.

Origin of Fermion Masses

Bilinear Condensate Mechanism: A

Fermion bilinear condensates form due to anomalous or explicit chiral symmetry breaking through topological space-time defects.
Examples: Nf=1 QCD, Nf =1 massive free staggered fermions

Bilinear Condensate Mechanism: B

Fermion bilinear condensates form due to spontaneous breaking of symmetries that prevent their formation.
Nf = 2 QCD is an example of this phenomena.

Four-fermion Condensate Mechanism: C

Four-fermion condensate can form while preserving symmetries that ensure fermion bilinear condensates vanish. But, fermions can still become massive. Nf = 2 interacting fermion model is one example.

Summary

The concepts of topology and index theorems
that arise in the context of QCD,
have analogies in simple fermion lattice field theories
with staggered fermions,
when formulated in the fermion bag approach

This connection gives a more complete perspective
on fermion mass generation mechanisms,
including a mechanism where fermions acquire a mass
through four-fermion condensates instead of
fermion bilinear condensates.

Can non-Abelian gauge theories also demonstrate
this alternate mechanism of fermion mass generation?