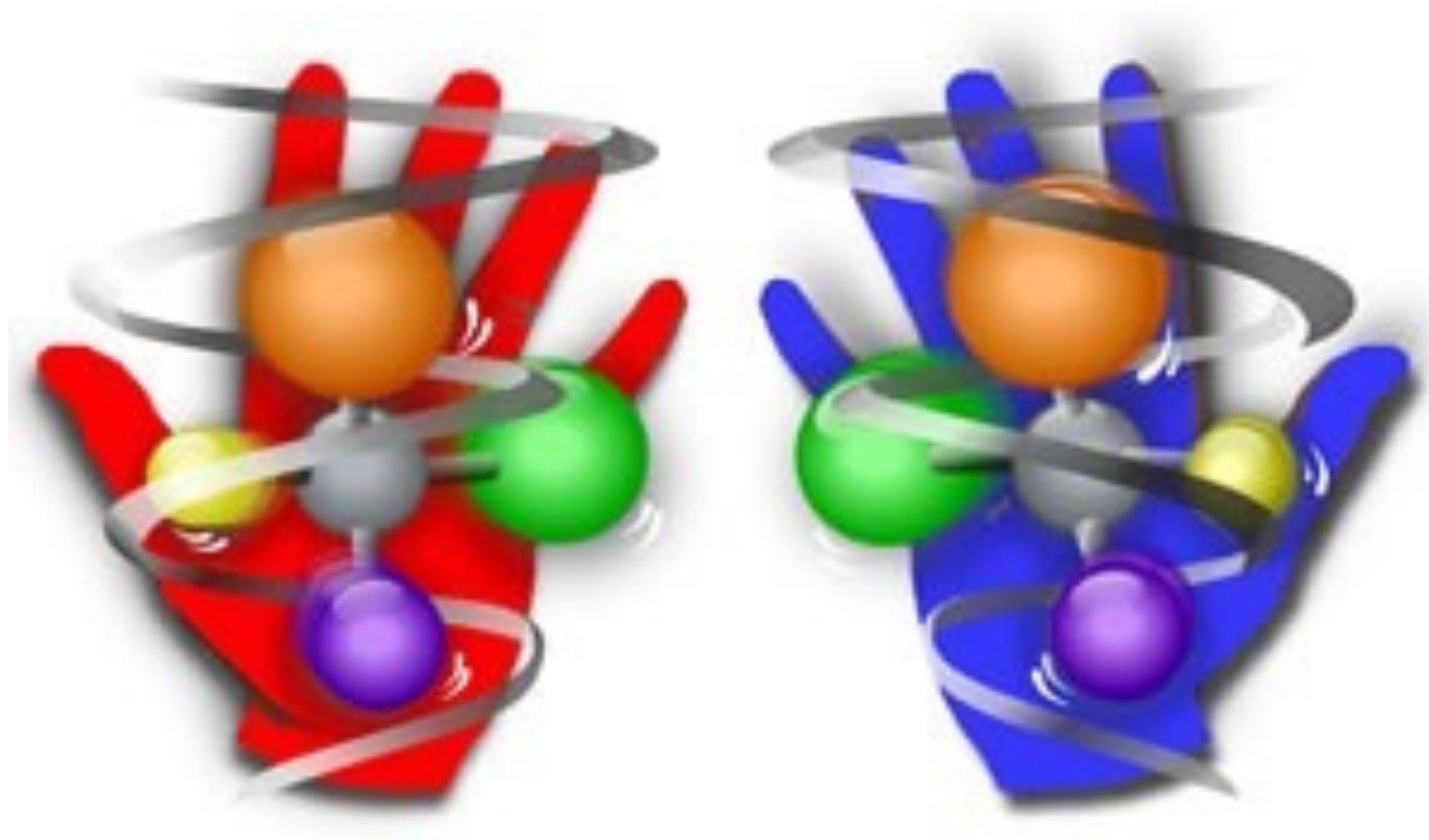
A New Perspective on Chiral Gauge Theories



Why an interest in chiral gauge theories?

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But of paramount importance:

The Standard Model is a χGT!

Nonperturbative definition ⇒

- **unexpected phenomenology?**
- **answers to outstanding puzzles?**

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= Weyl fermions in a real representation of the gauge group.

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We mean a product of eigenvalues...
...but there is no good eigenvalue problem for a chiral theory

Vector-like gauge theory with Dirac fermions:

$$\not{D}\psi = \begin{pmatrix} 0 & D_\mu\sigma_\mu \\ D_\mu\bar{\sigma}_\mu & 0 \end{pmatrix} \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} = \lambda \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

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So: $\Delta[A] = e^{i\delta[A]} \sqrt{|\det \not{D}|}$

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If δ=0, A & B would have same measure, same glue ball spectra...unlikely!

The fermion integral for a χGT: $\Delta[A] = e^{i\delta[A]}\sqrt{|\det \not{D}|}$

Alvarez-Gaume et al. proposal for perturbative definition (1984,1986):

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Well-defined eigenvalue problem with complex eigenvalues

Extra RH fermions decouple

Amenable to lattice regularization?

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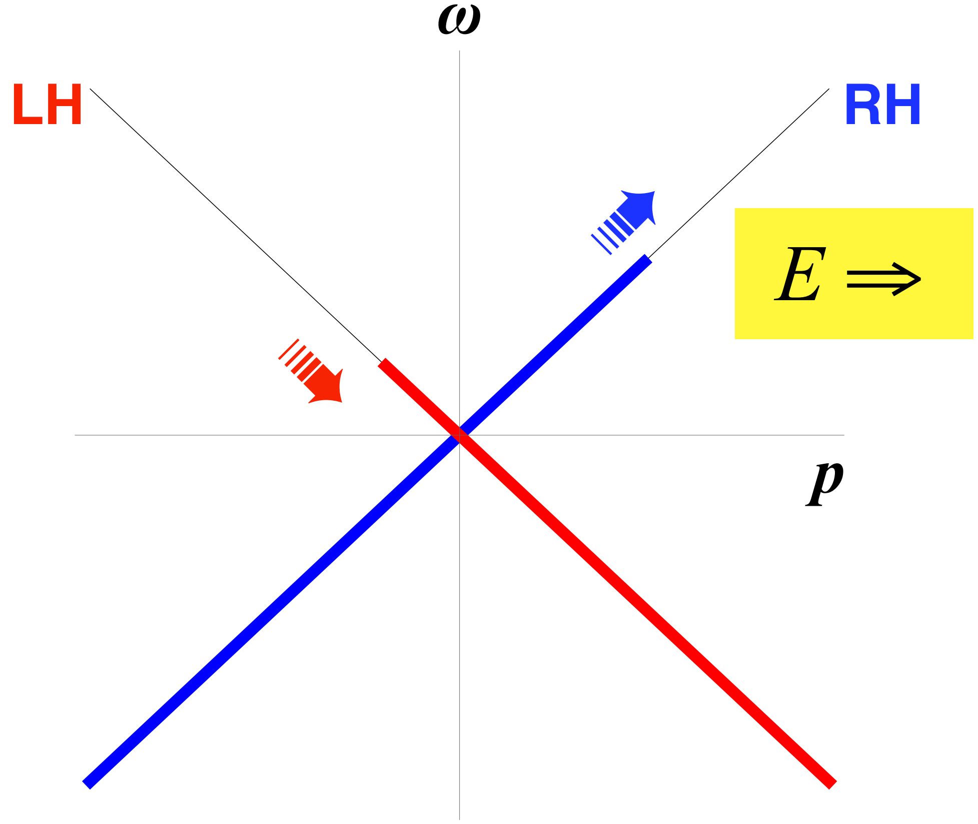
One way of looking at anomalies:

massless electrons in E field, 1+1 dim



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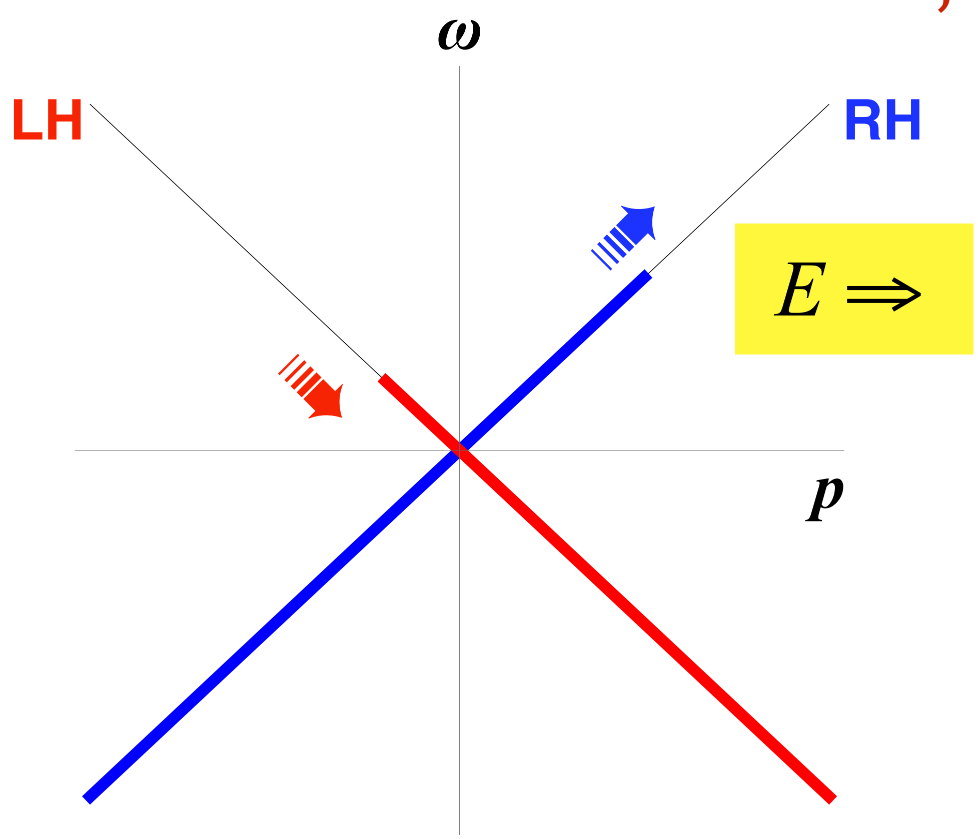

$$\text{d=(1+1):} \quad \partial_\mu j_5^\mu = \frac{qE}{\pi}$$

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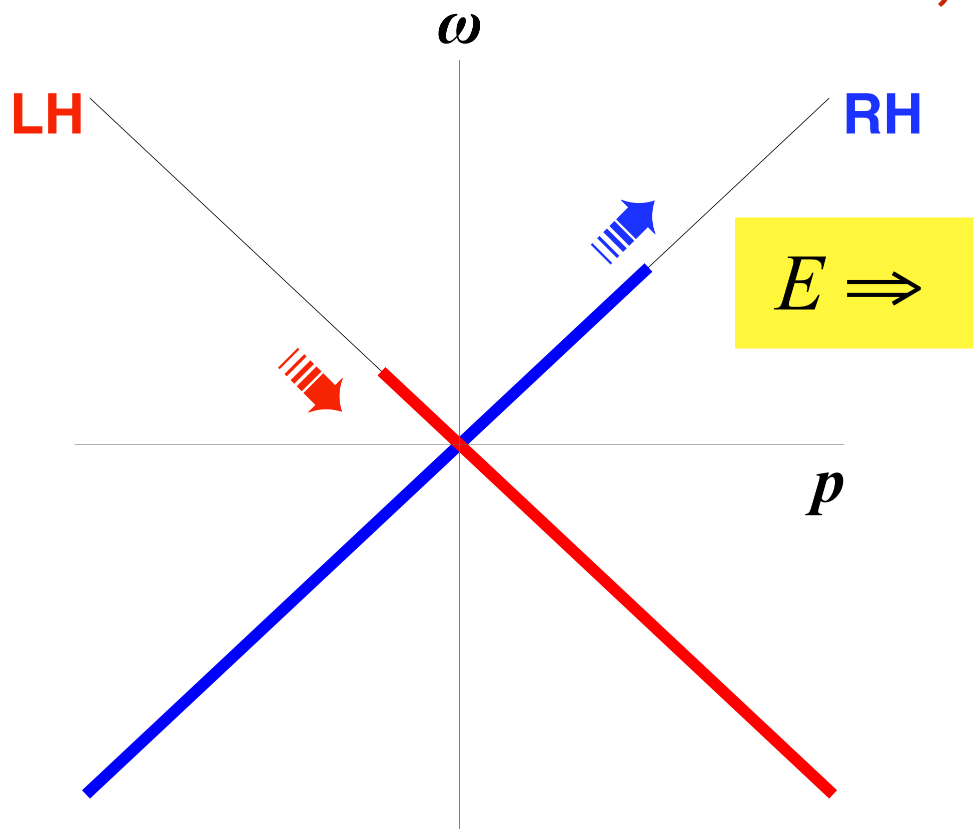

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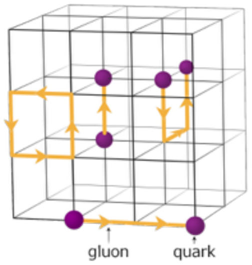
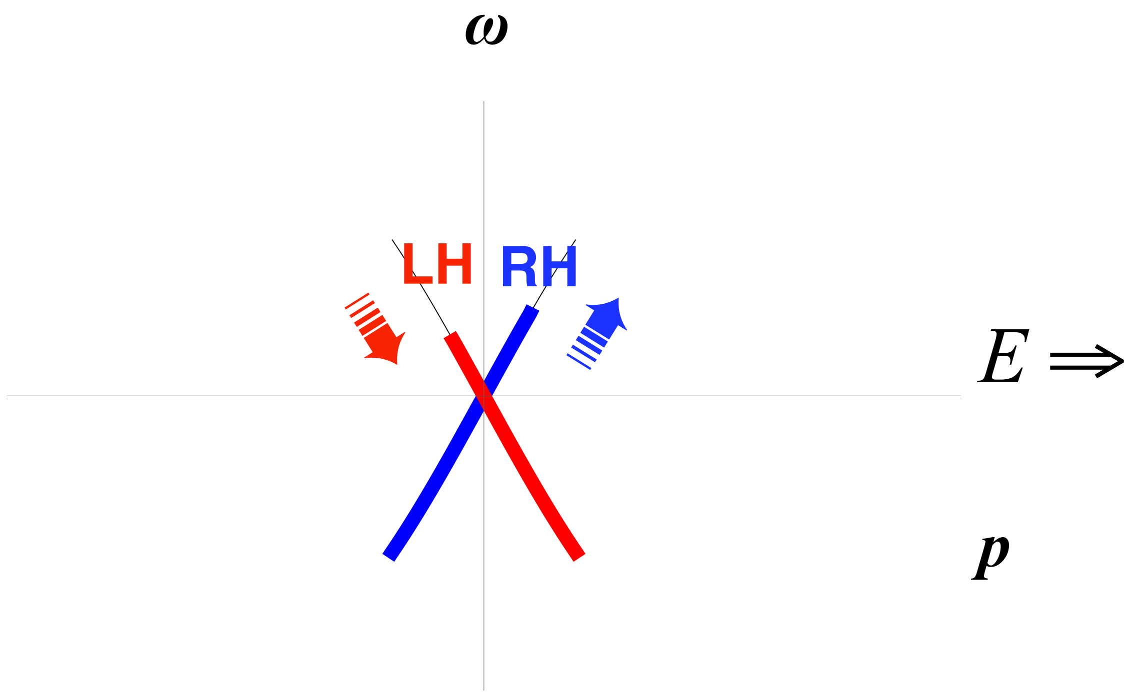
In the continuum, the Dirac sea is filled…but is a Hilbert Hotel which always has room for more

Not so on the lattice:

Can reproduce continuum physics for long wavelength modes…





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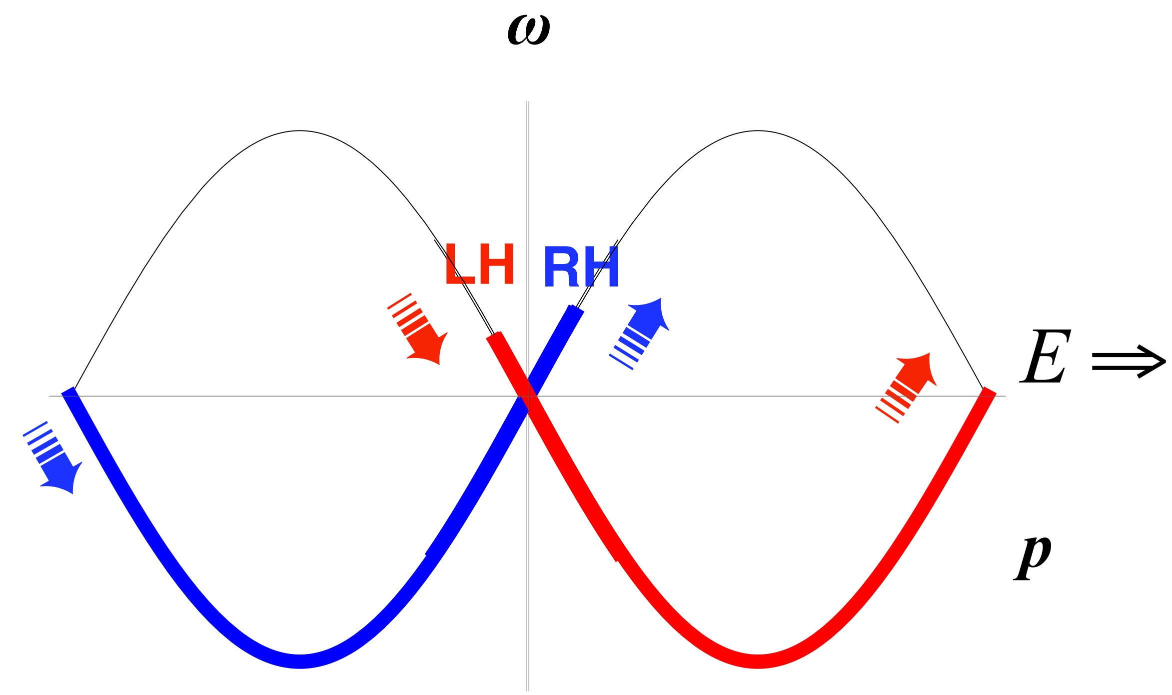

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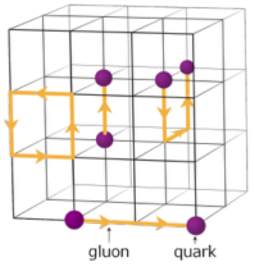

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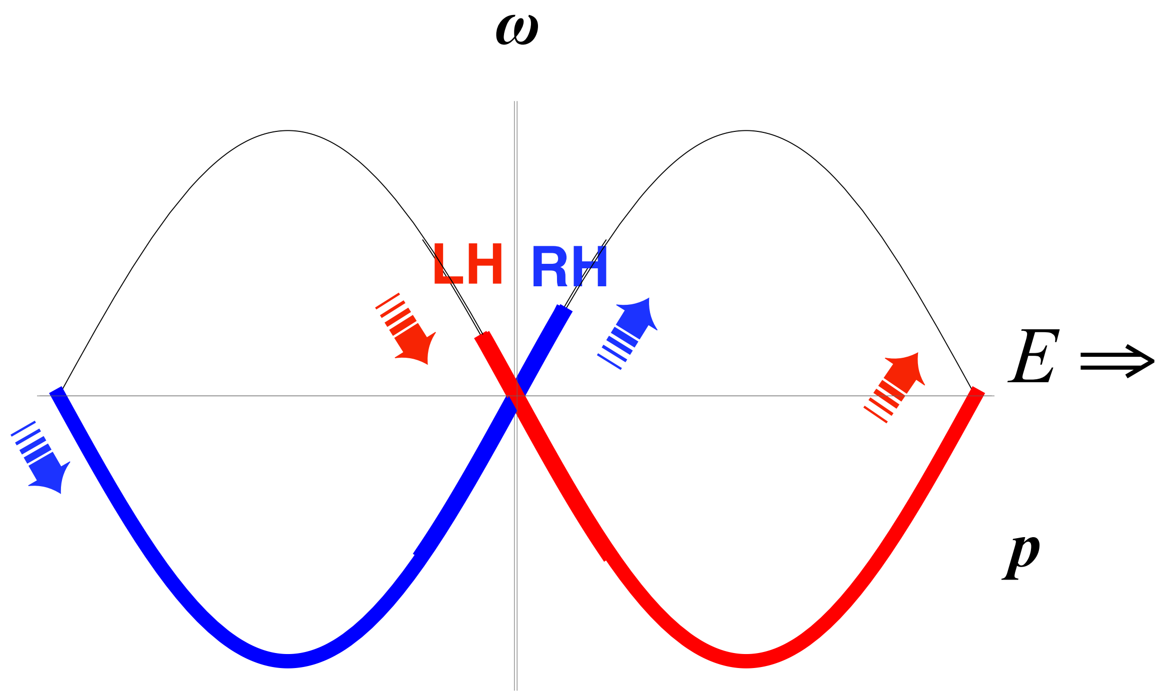

$$\partial_\mu j_5^\mu = 0$$

<u>anomalous symmetry</u> in the continuum

must be

<u>explicitly broken symmetry</u> on the lattice

How Wilson fermions reproduce the U(1)$_A$ anomaly in QCD:

Karsten, Smit 1980

Lattice covariant derivatives

$$\mathcal{L} = \bar{\psi}\left(\not{D} + m + aD^2\right)\psi$$

violates chiral symmetry

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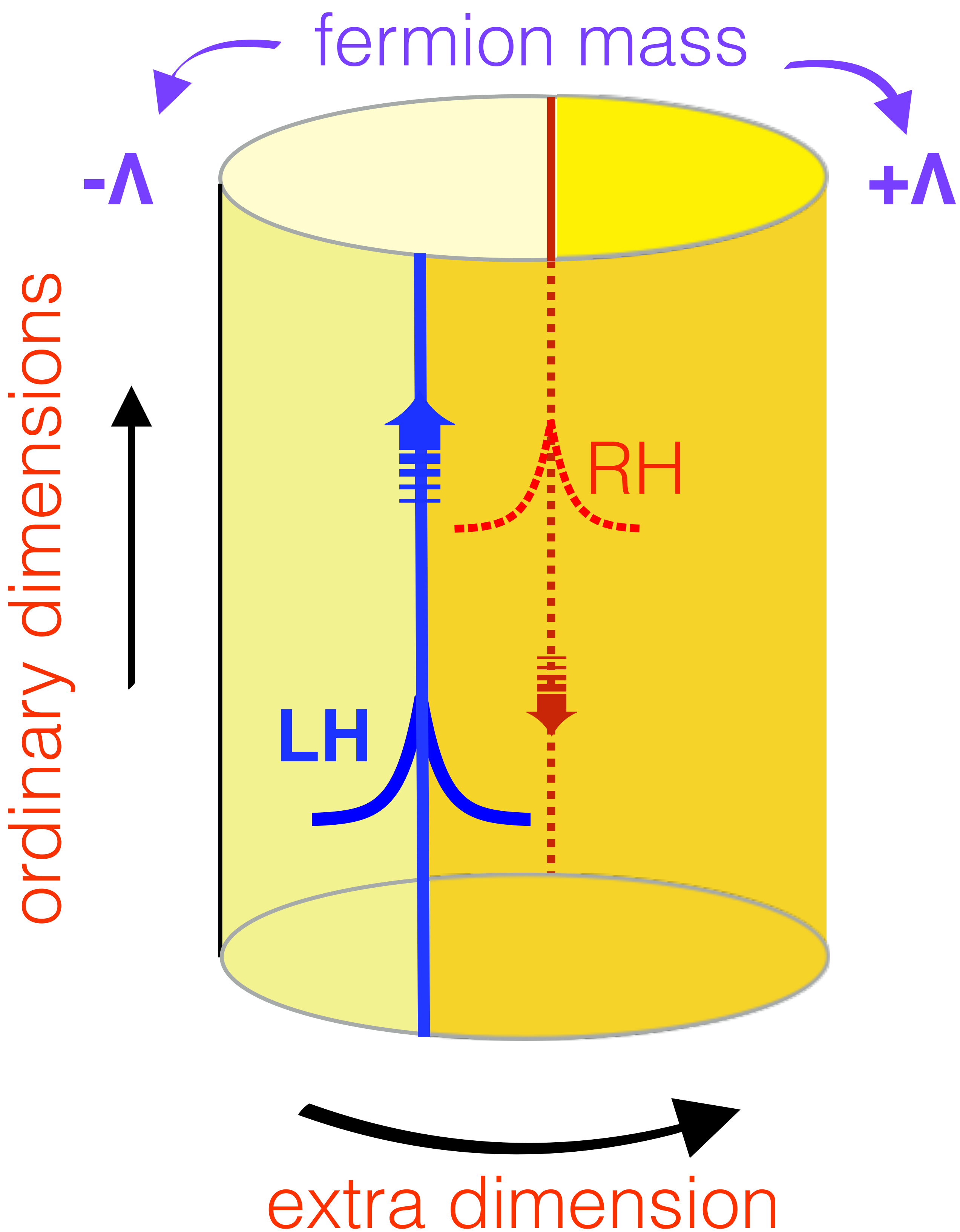
- fine tune m to continuum limit…find some chiral symmetry breaking does not decouple & correct anomalous Ward identities are found

How Domain Wall Fermions reproduce the U(1)$_A$ anomaly in QCD:

DBK, 1992



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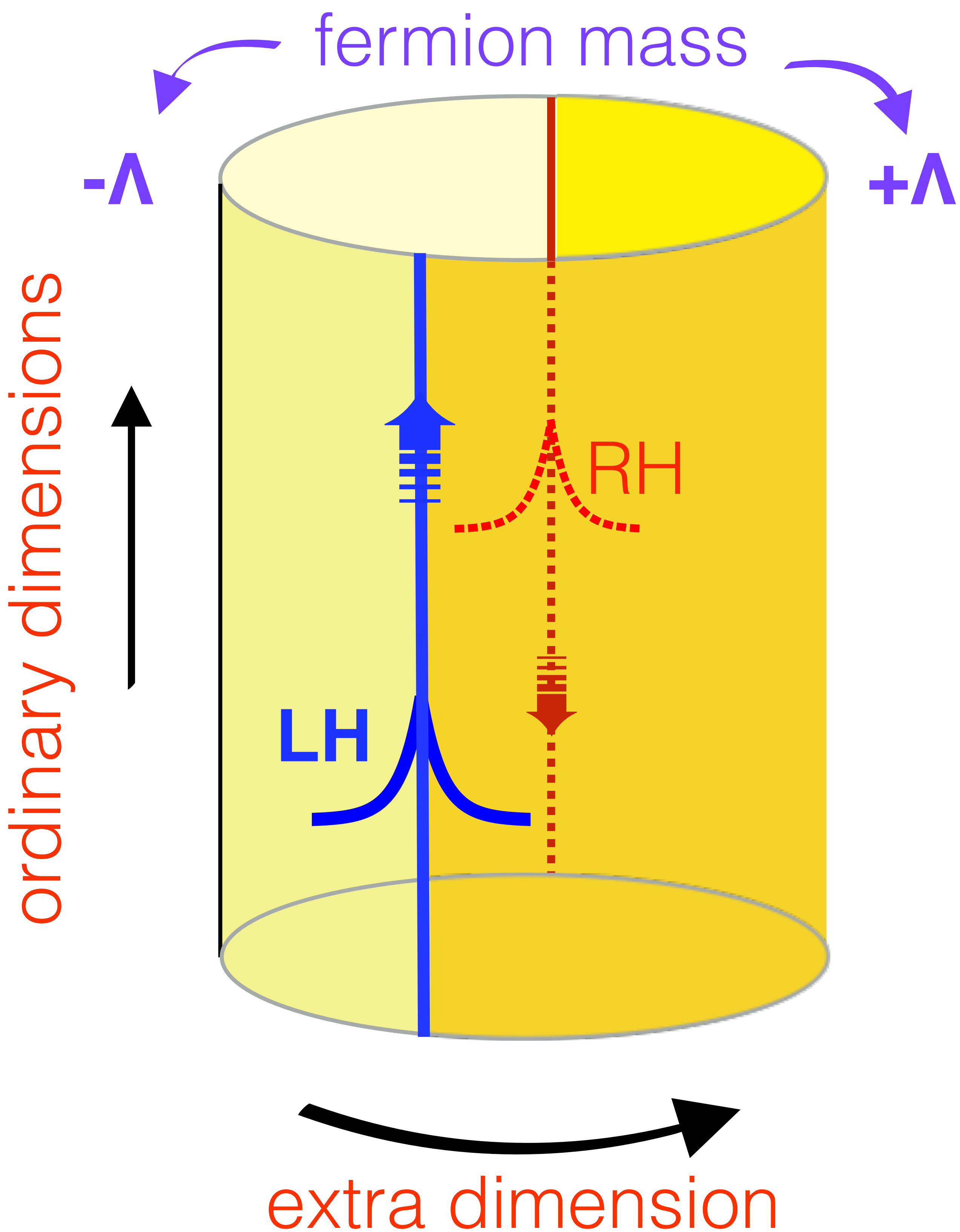

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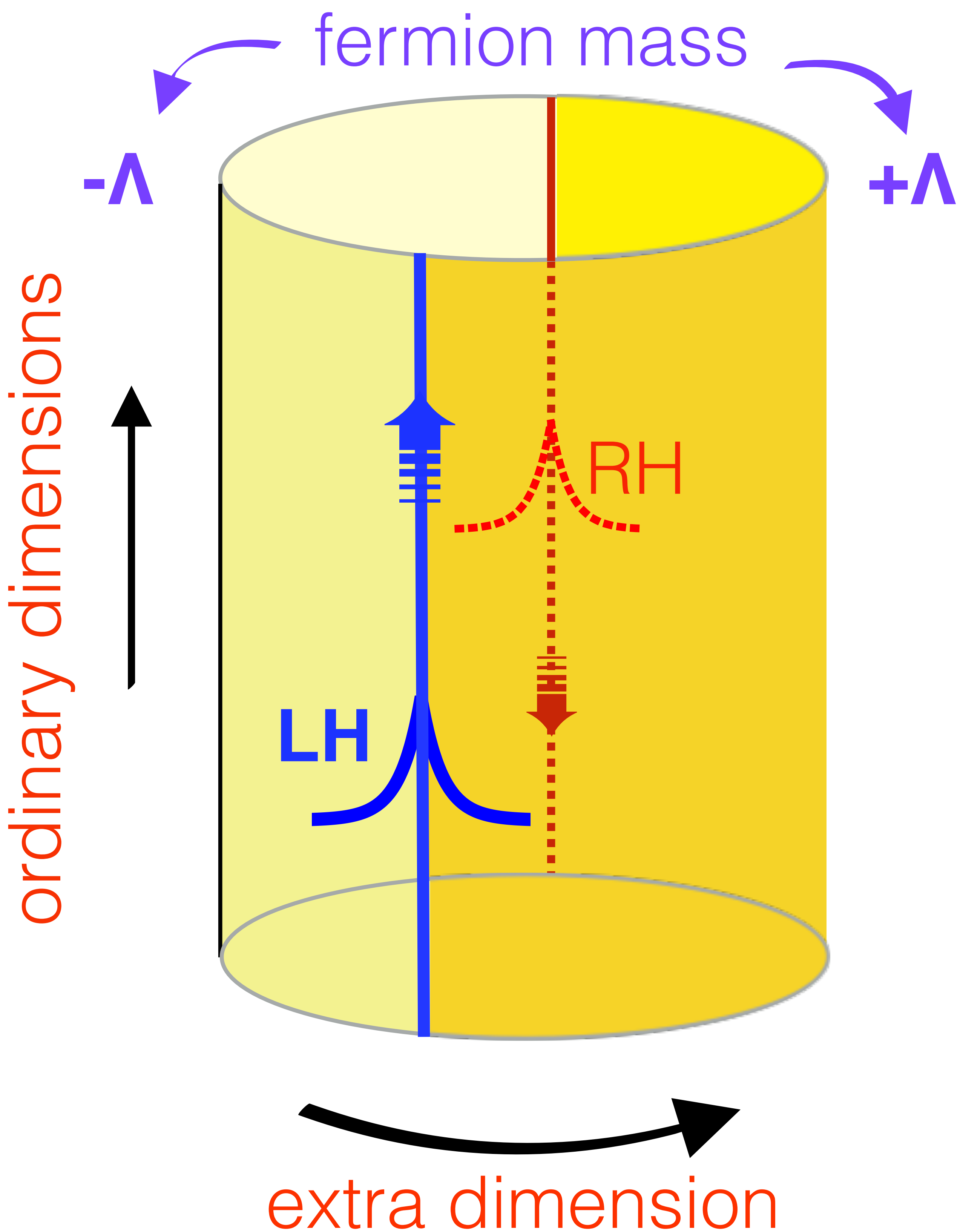

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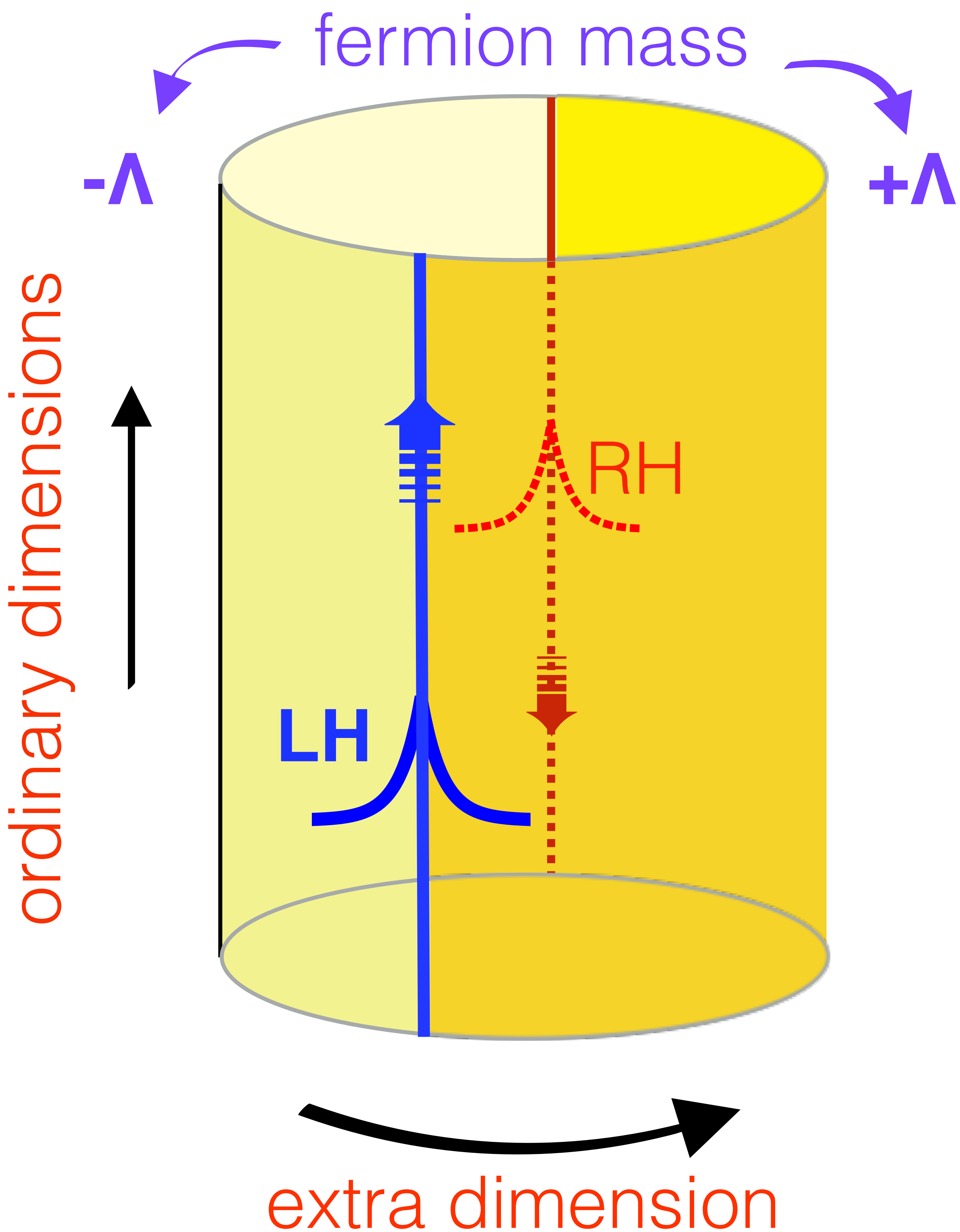

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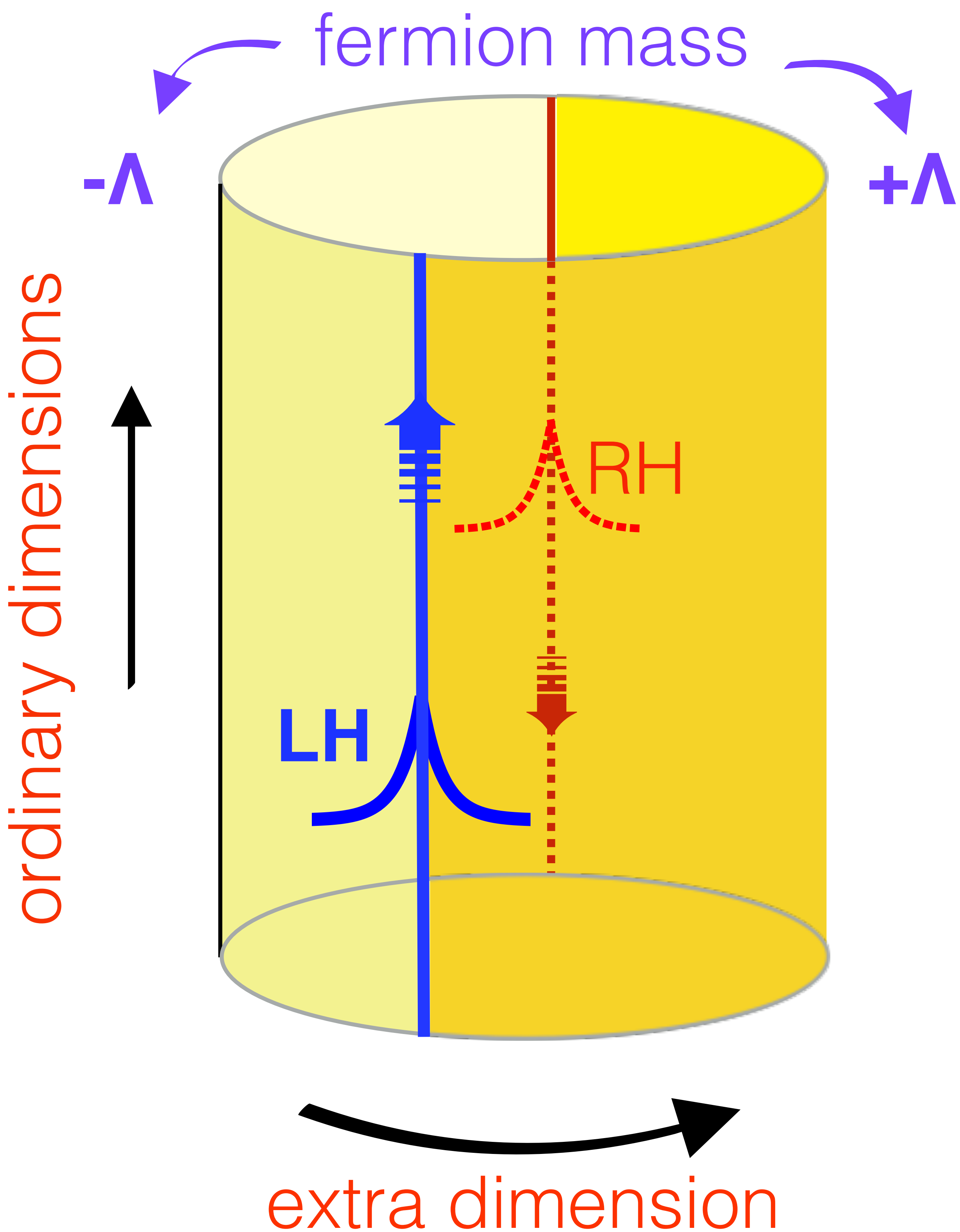

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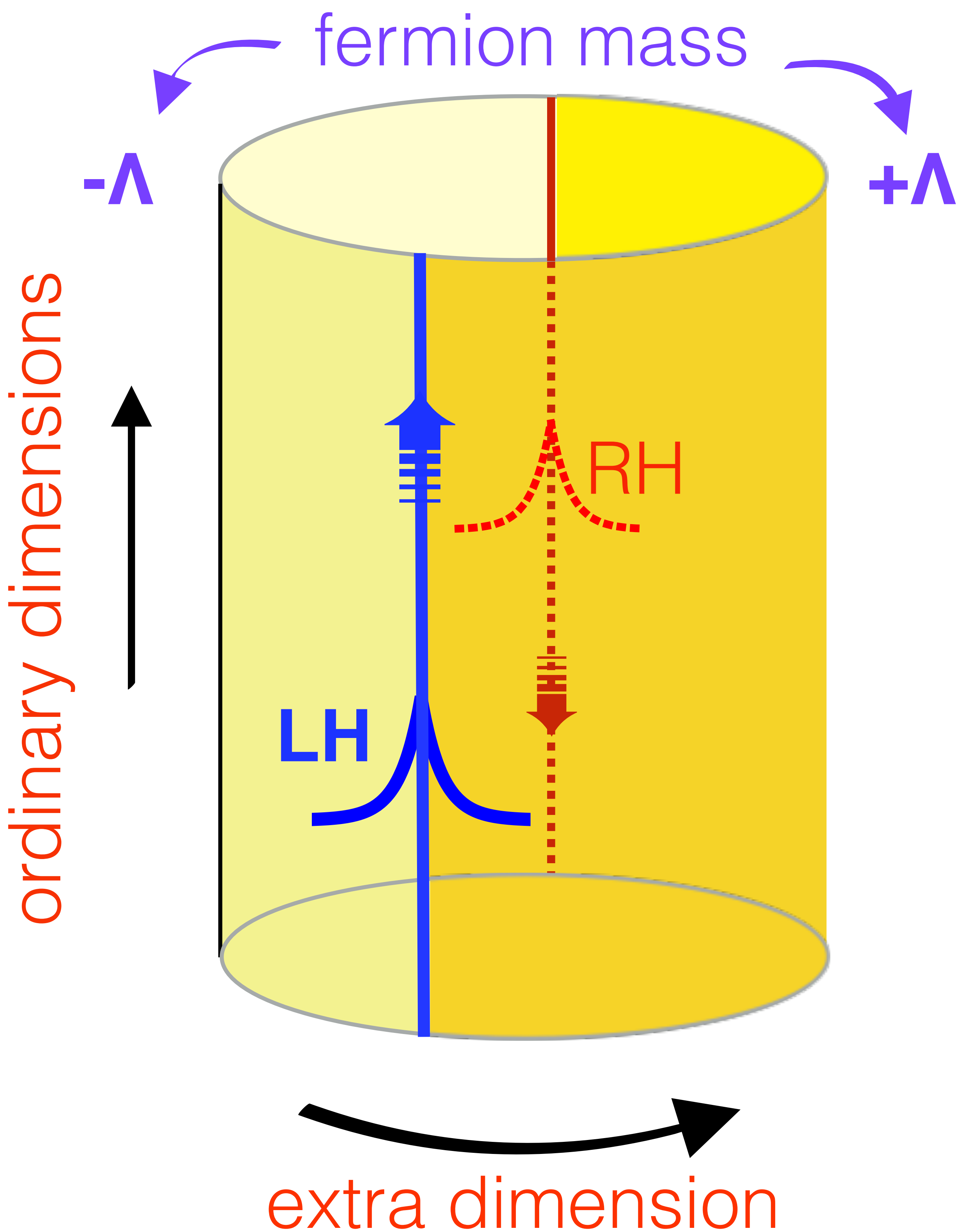

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"topological insulator"

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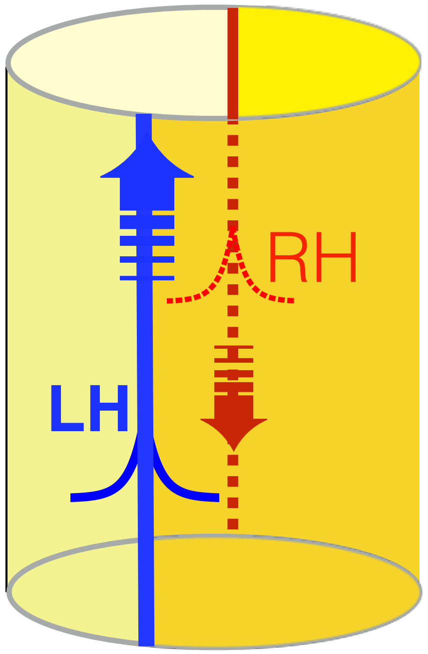

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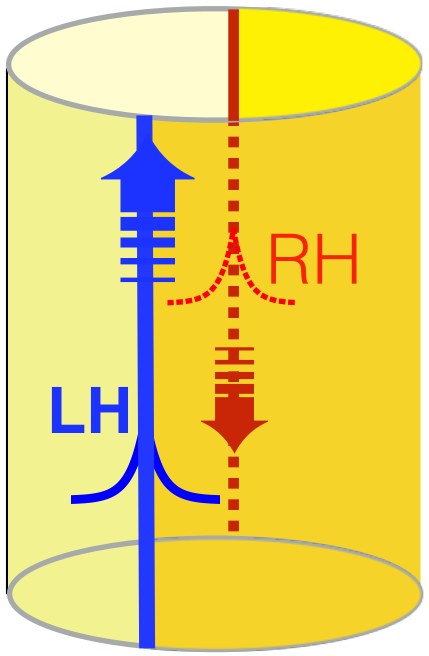

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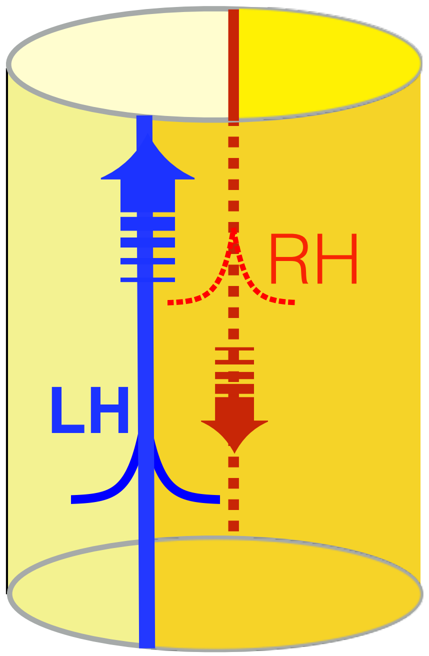

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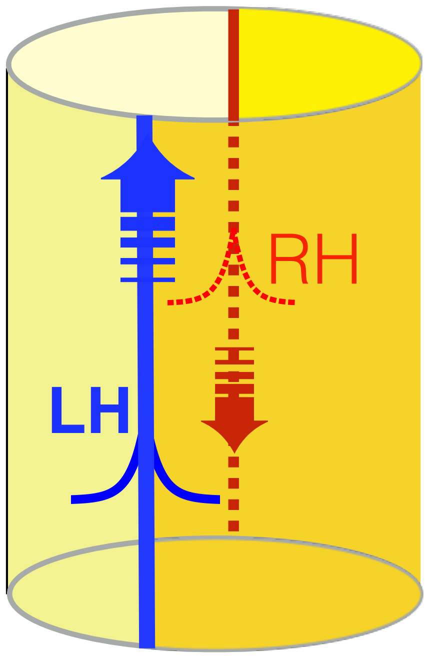

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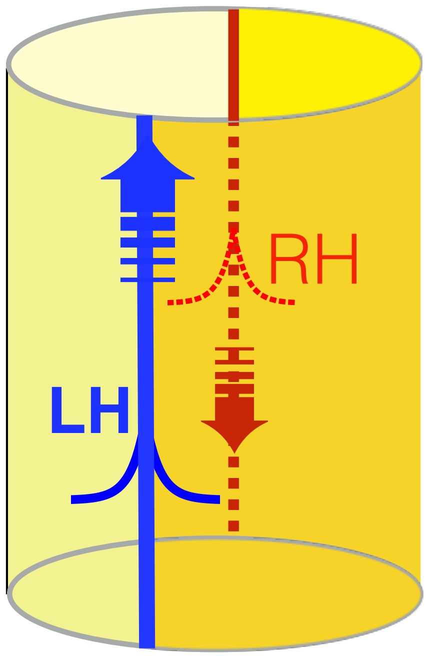

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Explicit chiral symmetry breaking

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…but if LH fermion is gauged and RH is neutral, X terms coupling them violate gauge symmetry!

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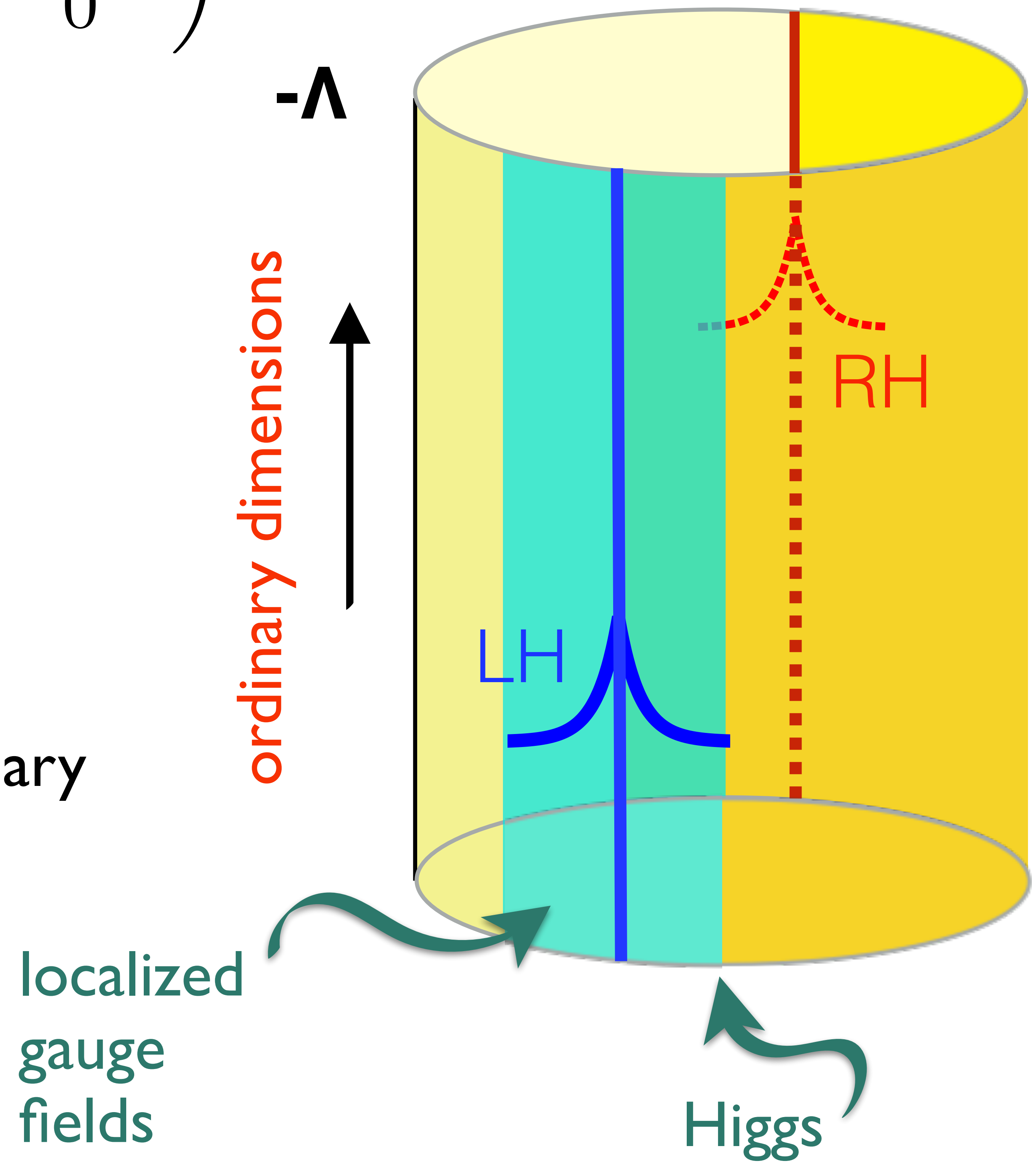
A natural way to examine the problem is with Domain Wall Fermions…

Motivation: $\mathcal{D}_\chi = \begin{pmatrix} 0 & D_\mu \sigma_\mu \\ \partial_\mu \bar{\sigma}_\mu & 0 \end{pmatrix}$

Suggests: localizing gauge fields near one domain wall

Requires "Higgs" field at boundary to maintain gauge invariance



Old attempts to use domain wall fermions for chiral gauge theory

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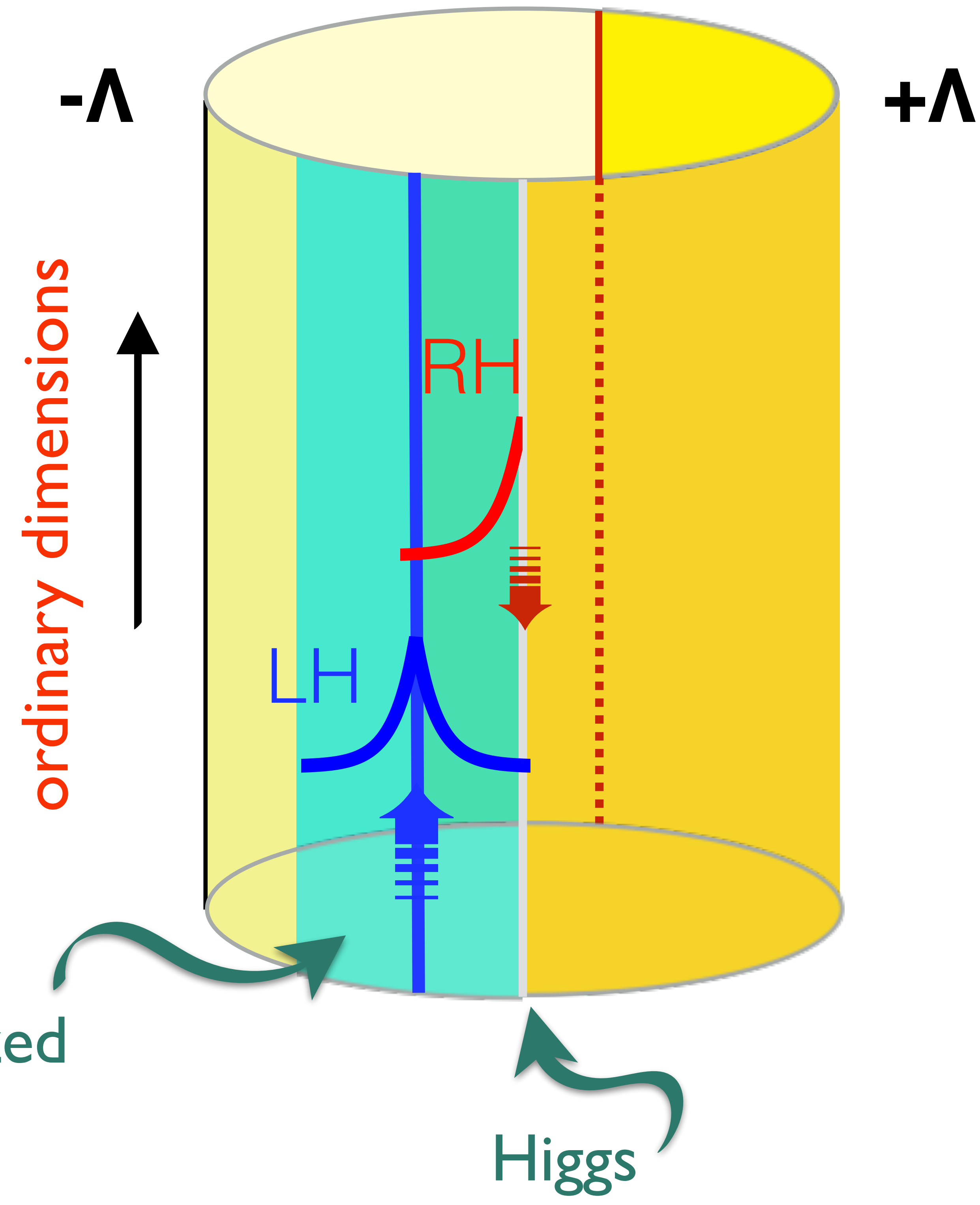

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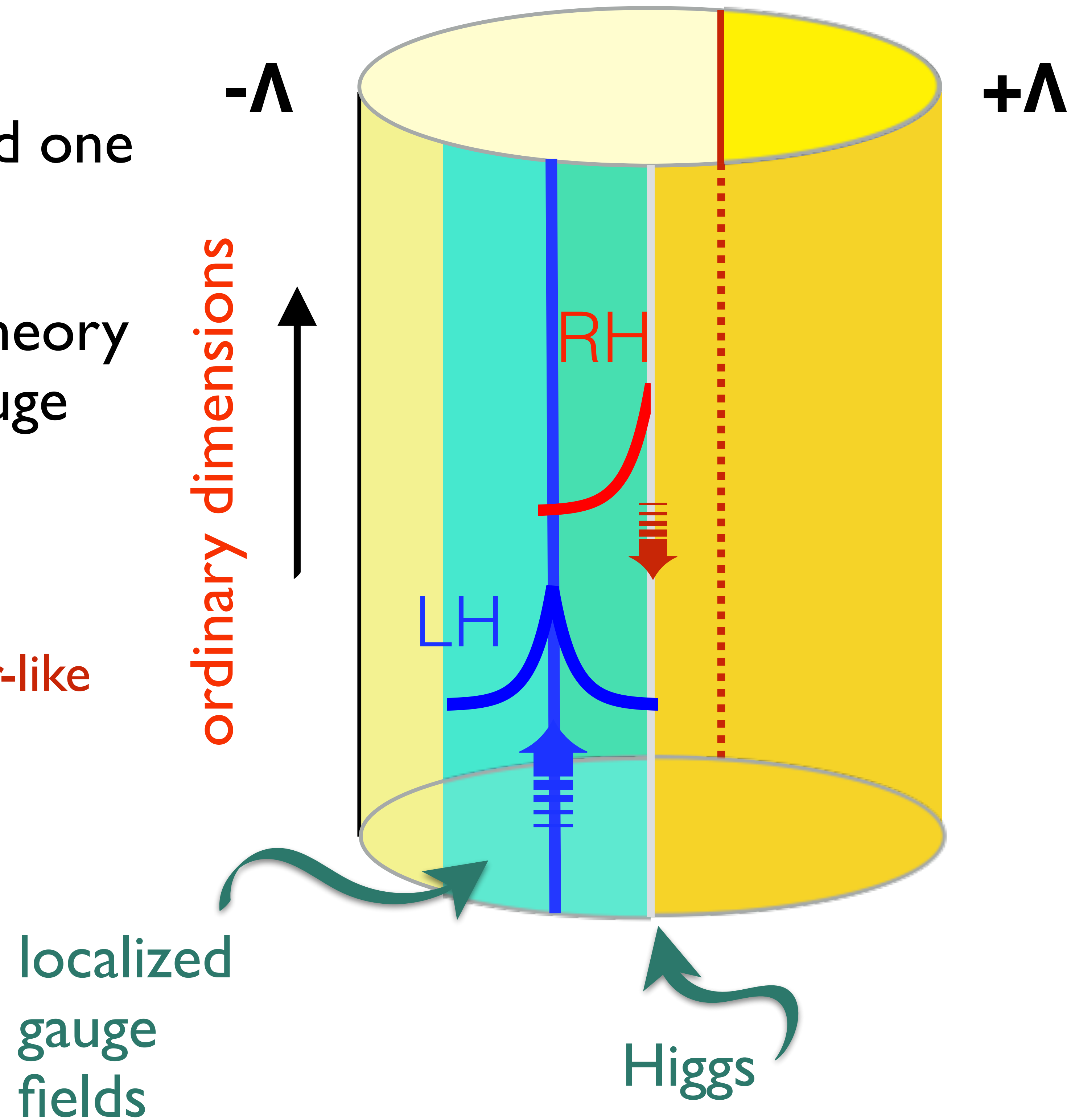
Never works.

One finds a Dirac fermion & vector-like gauge theory.

Golterman, Jansen, Vink 1993



New proposal: "localize" gauge fields using gradient flow

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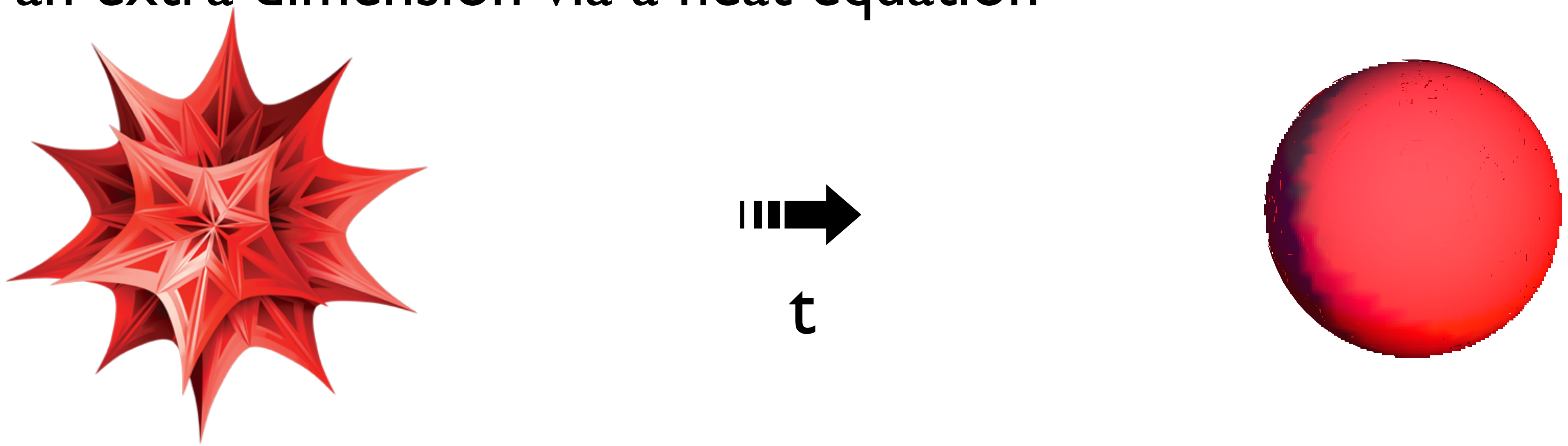
- work in progress

Gradient flow smooths out fields by evolving them classically in an extra dimension via a heat equation



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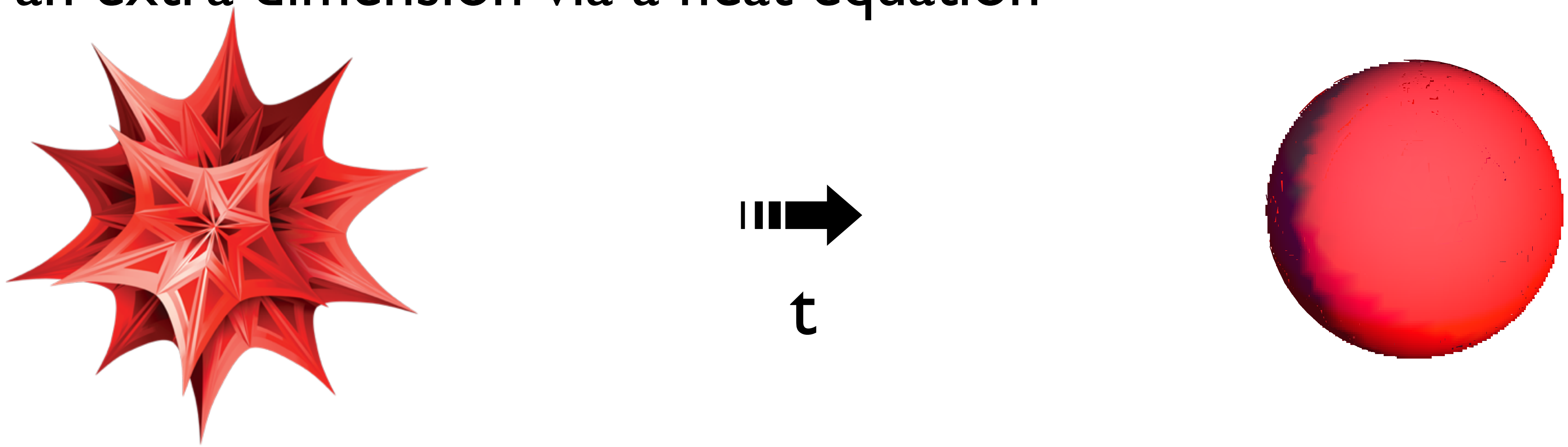

- Gradient flow uses an extra dimension…
- DWF uses an extra dimension…
- …maybe they fit together? What could go wrong?

New proposal: “localize” gauge fields using gradient flow

Dorota Grabowska, D.B.K.

- Phys.Rev.Lett. 116 211602 (2016) [arXiv:1511.03649]
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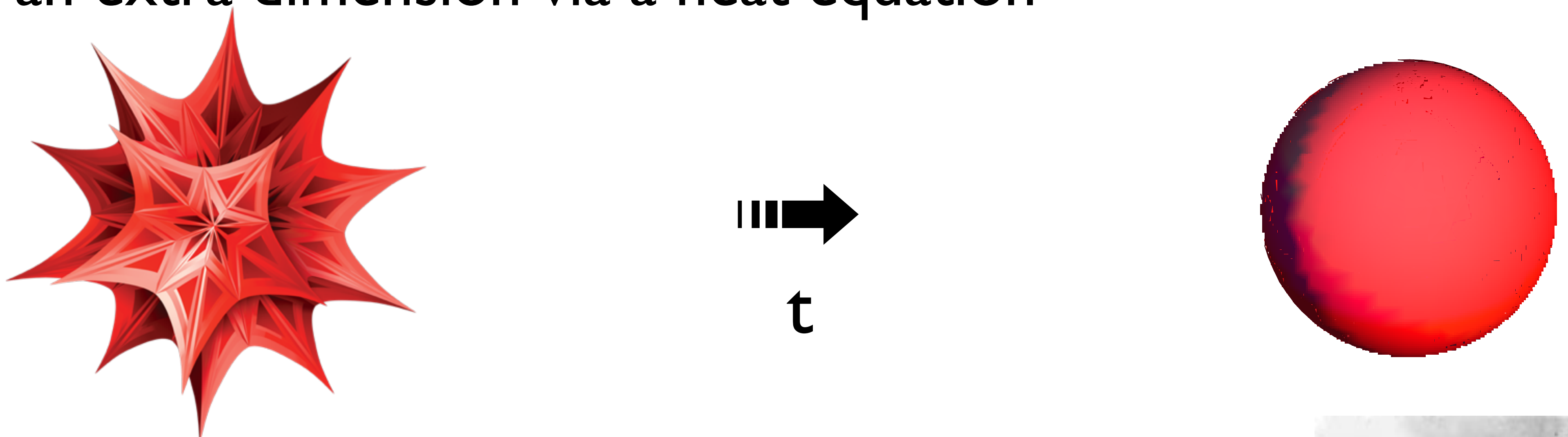

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Gradient flow (continuum version):

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4d world

$\bar{A}_\mu(x,t)$ lives in 5d bulk

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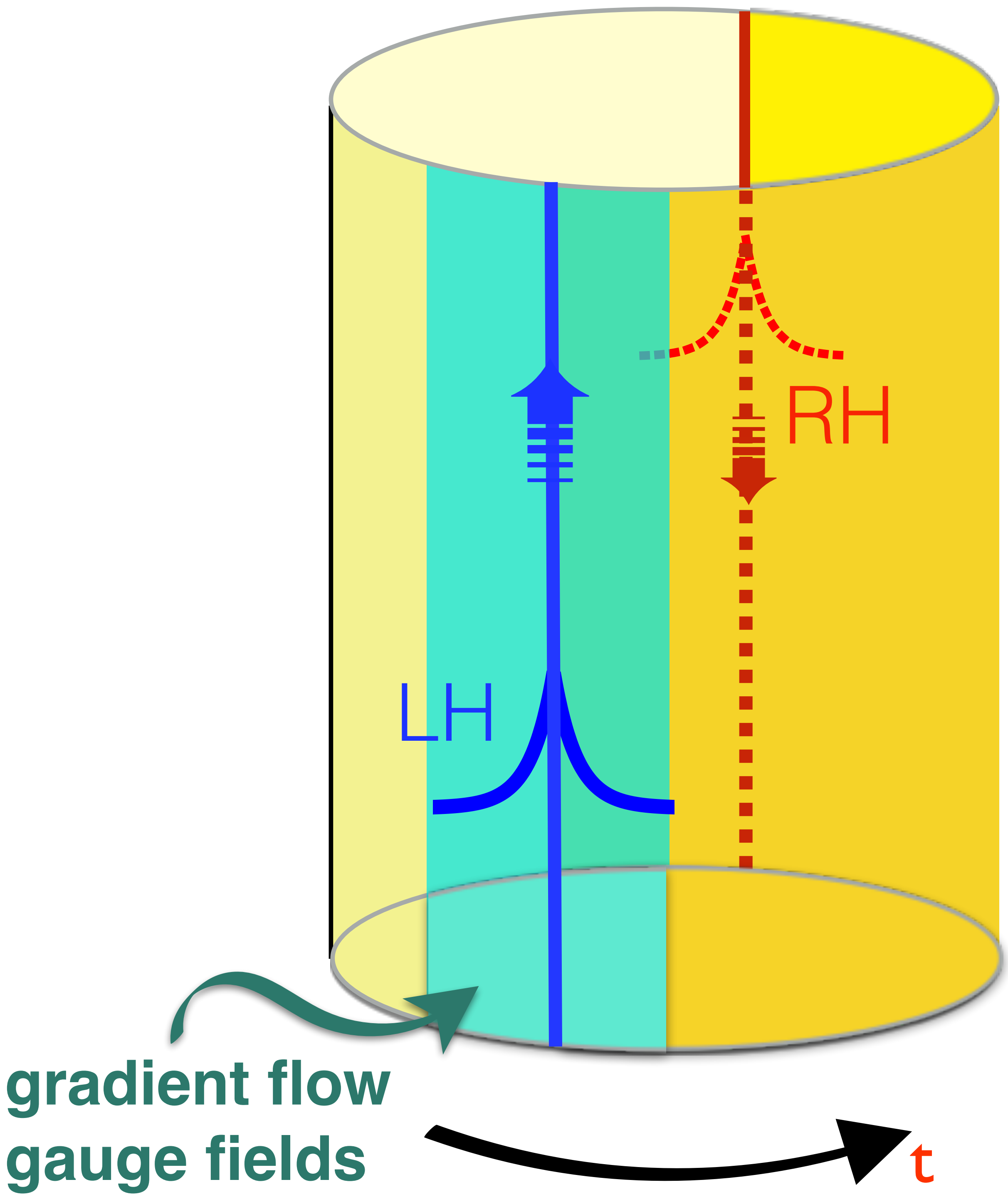
This will allow λ(p) to be localized near t=0 while maintaining gauge invariance

Combining gradient flow gauge fields with domain wall fermions:



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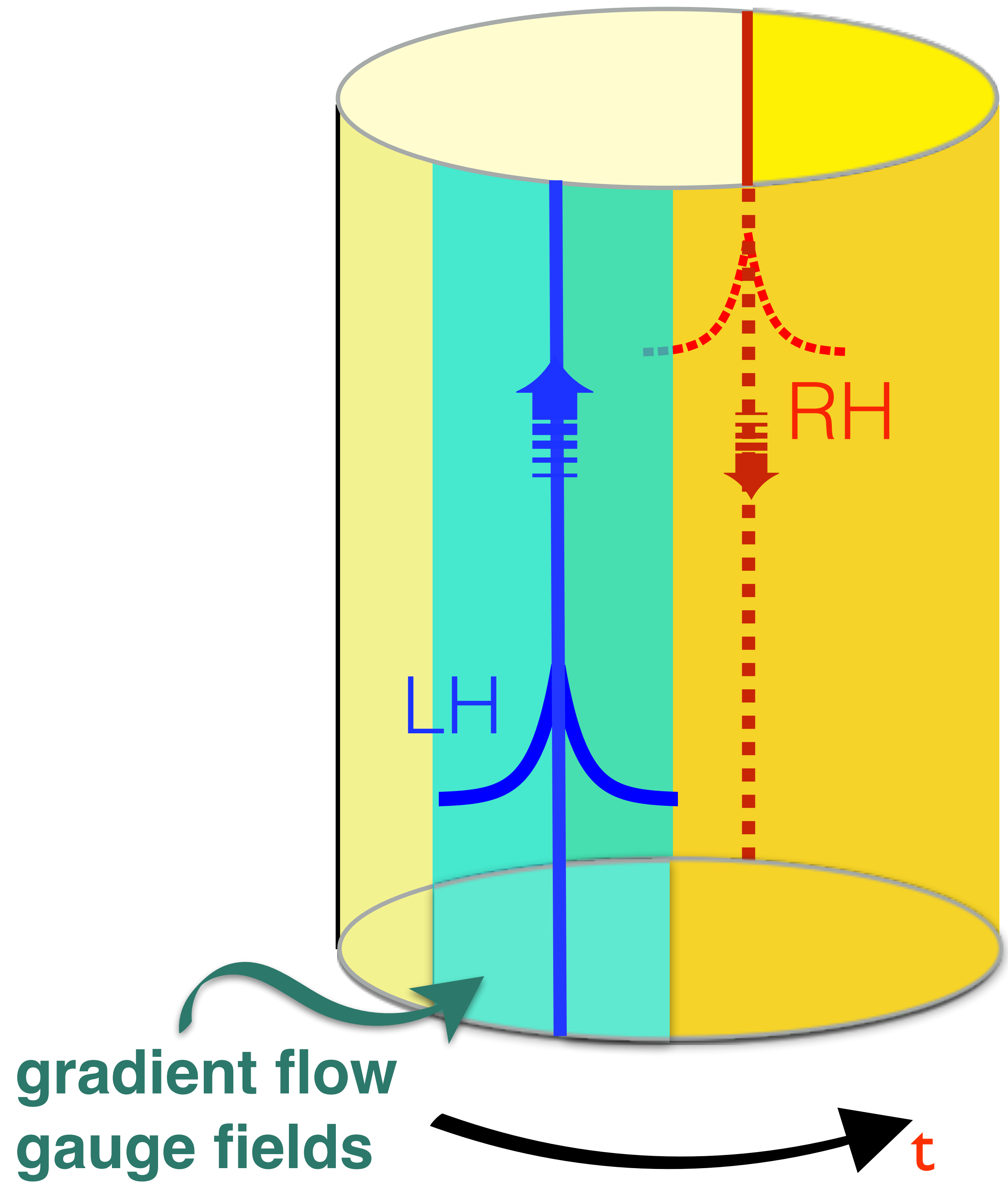

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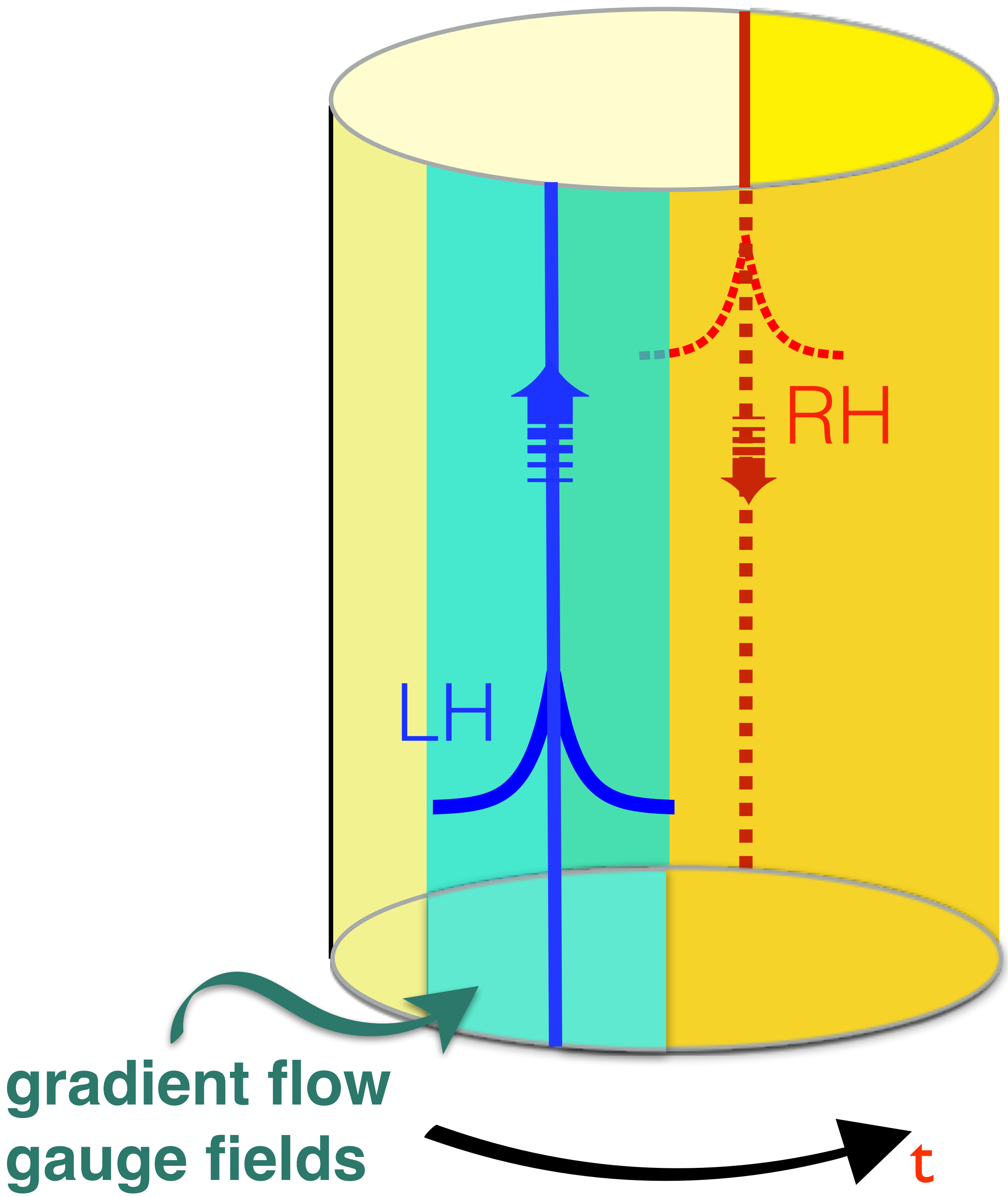

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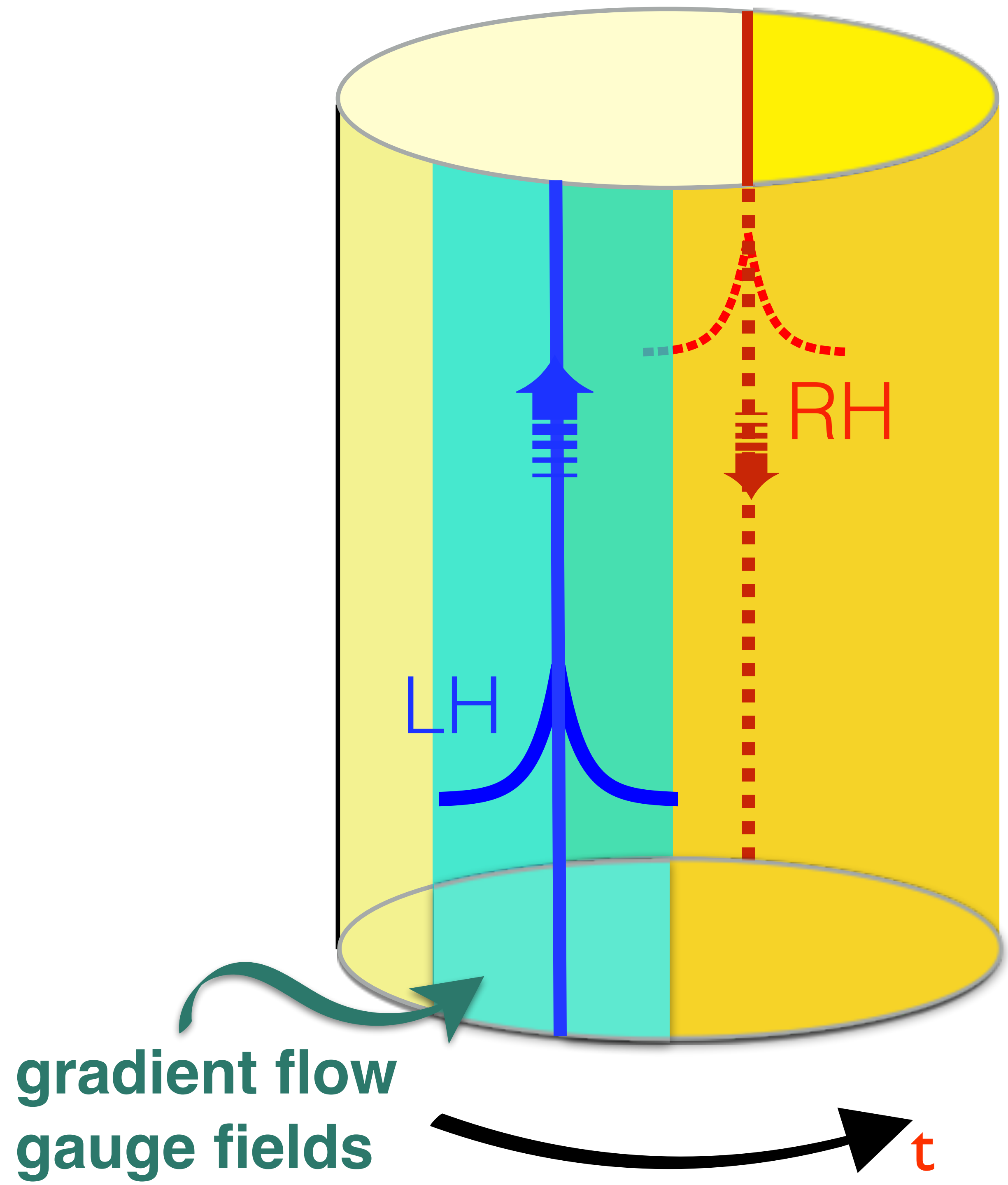

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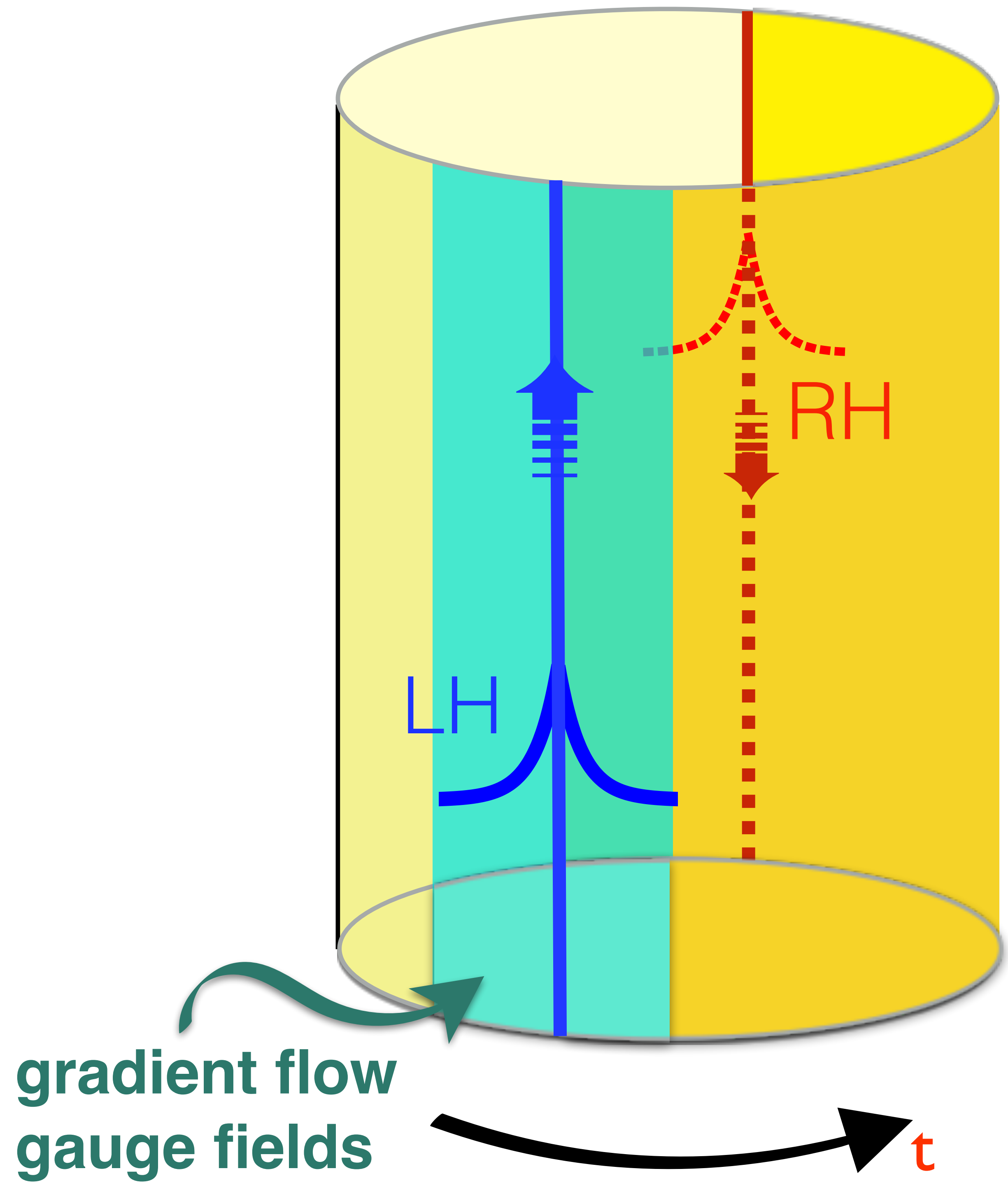

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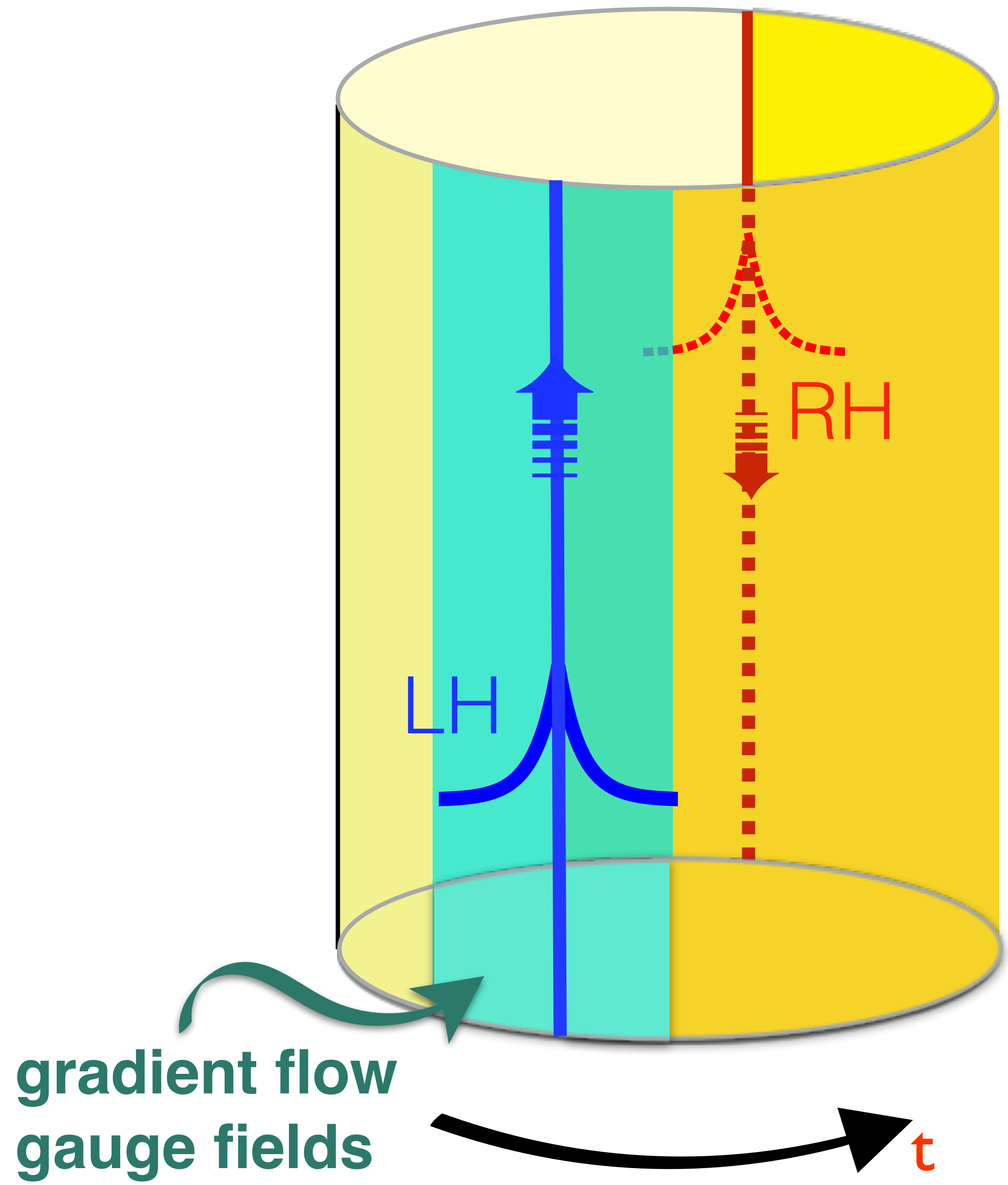

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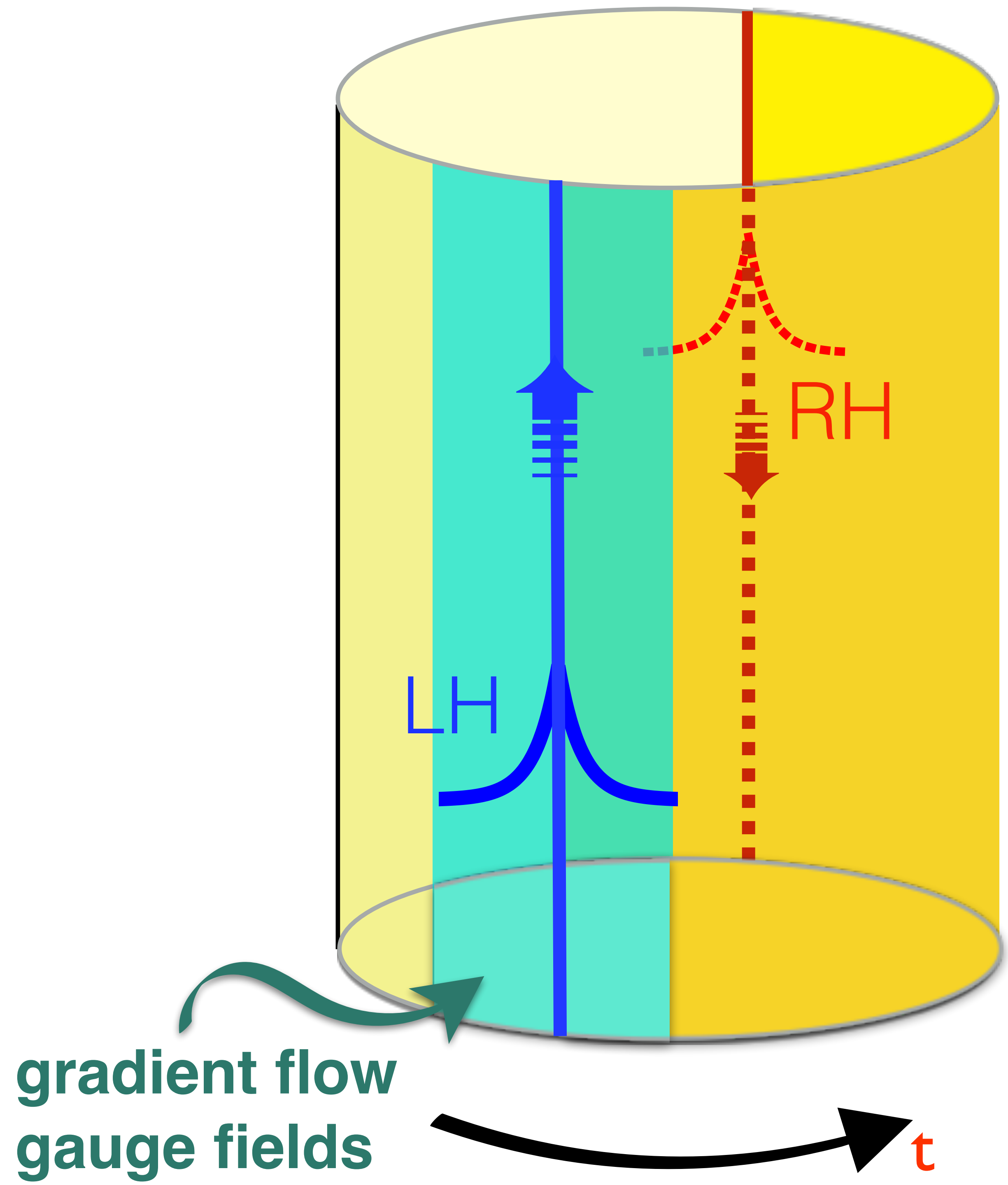

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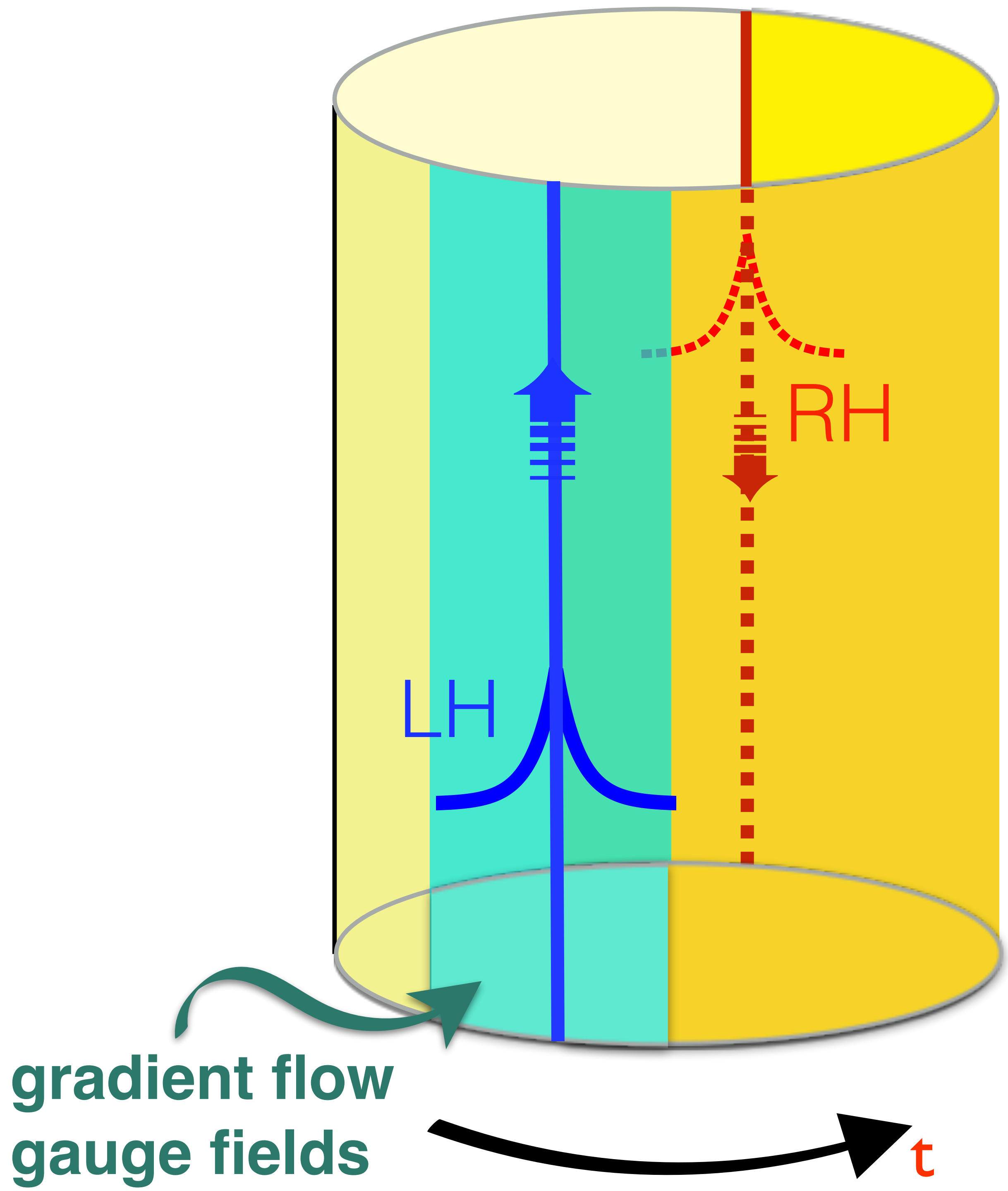

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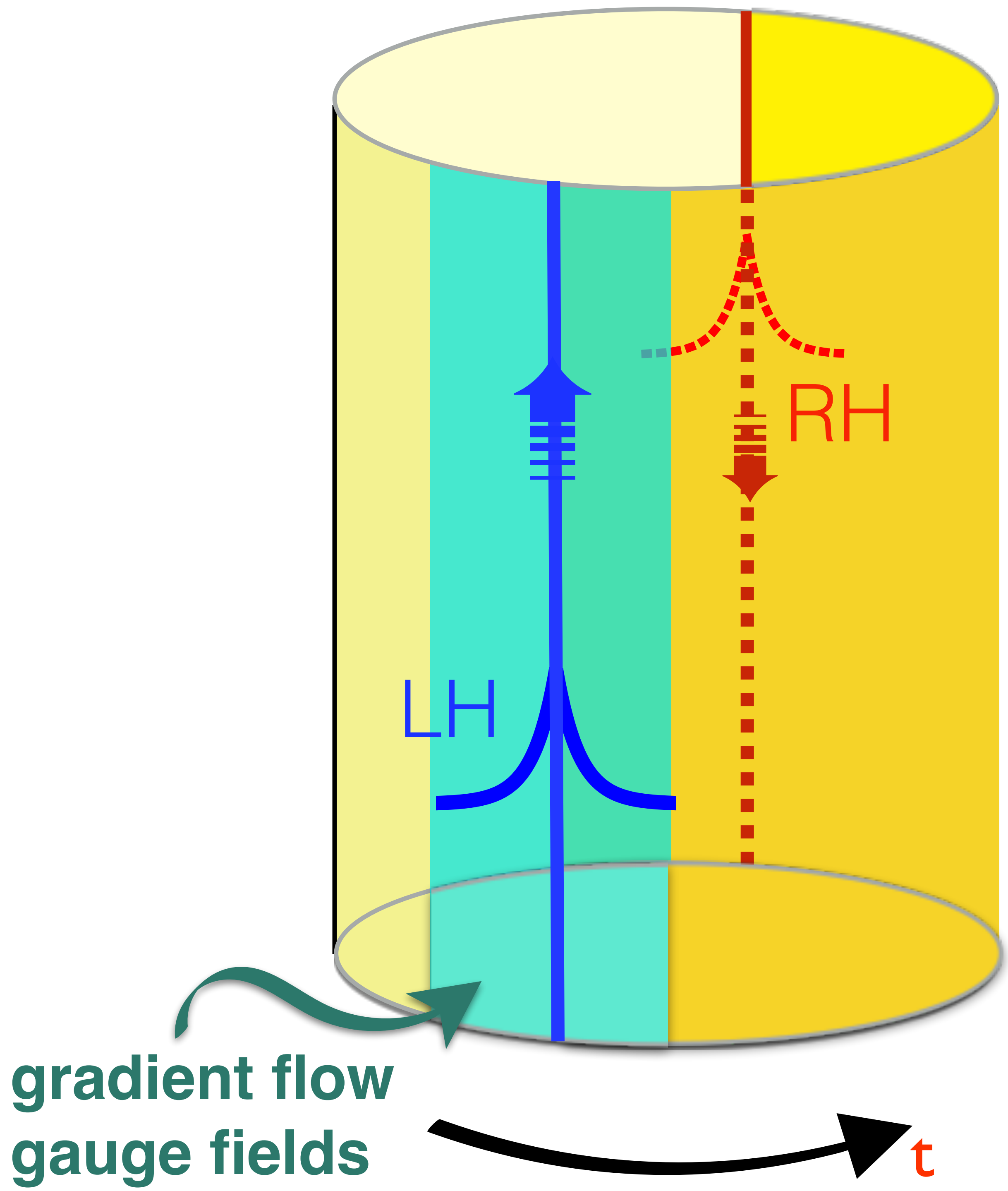

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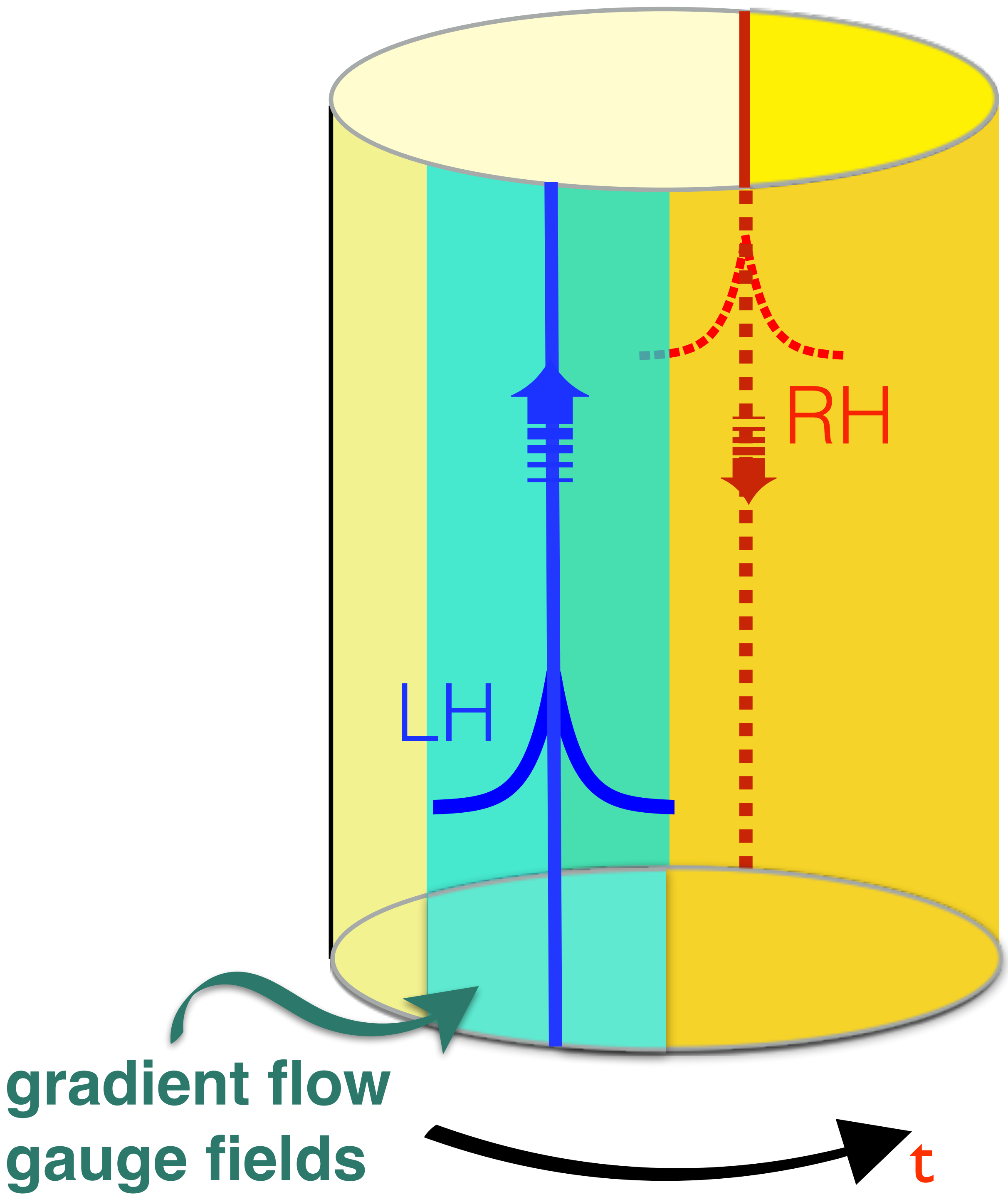

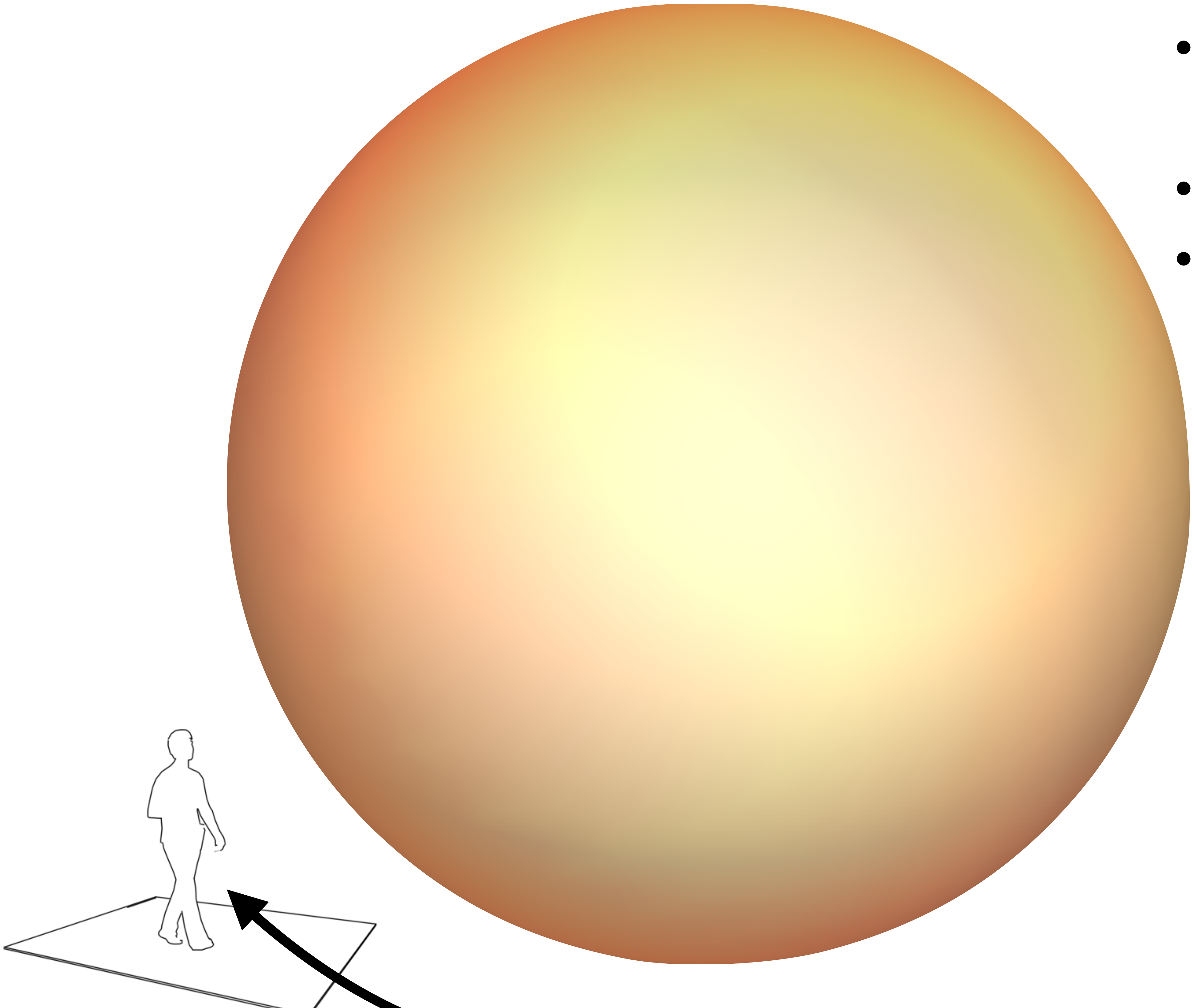

- Mirror top quark (fluff)
- mass = 170 GeV
- couples only to radio waves?

- lattice gauge theorist

Decoupling mirror fermions as soft fluff in a gauge invariant way:

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- 🙂 Can show that this could only lead to a local 4d quantum field theory if the fermion representation has no gauge anomalies
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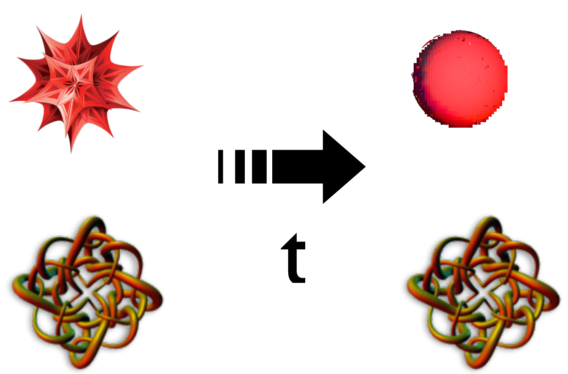

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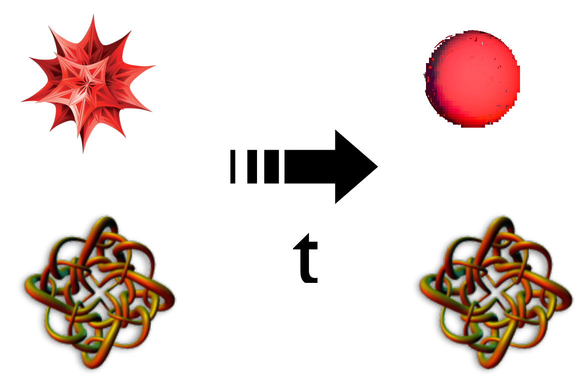

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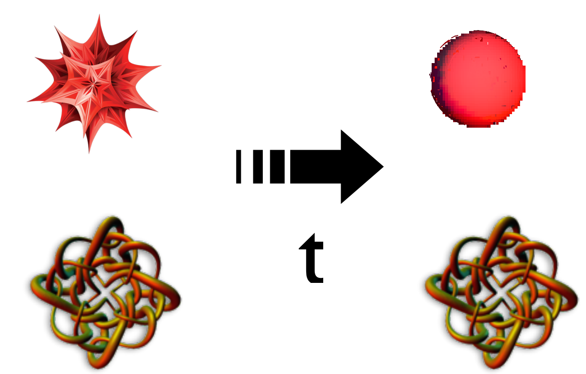

Suggests taking $t \to \infty$ limit first…gradient flow like a projection operator $A \to A_\star$

$t \to \infty$ limit suggests finding an overlap operator for this system

The overlap operator for vector theories:

Neuberger, Narayanan 1993-1998

$$\boxed{\mathcal{D}_V = 1 + \gamma_5 \epsilon}$$

$$\epsilon \equiv \epsilon(H_w) = \frac{H_w}{\sqrt{H_w^2}}$$

$$\gamma_5 H_w = \left[\tfrac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) - \tfrac{1}{2}\nabla_\mu \nabla_\mu^* - m\right]$$

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properties:

$$\{\mathcal{D}_V^{-1}, \gamma_5\} = a\gamma_5$$

$$\lim_{a\to 0} \mathcal{D}_V = \frac{1}{am}\begin{pmatrix} 0 & D_\mu\sigma_\mu \\ D_\mu\bar{\sigma}_\mu & 0 \end{pmatrix}$$

$\epsilon(H_w)$ arises as $\displaystyle\lim_{n\to\infty}\frac{1-T^n}{1+T^n}$

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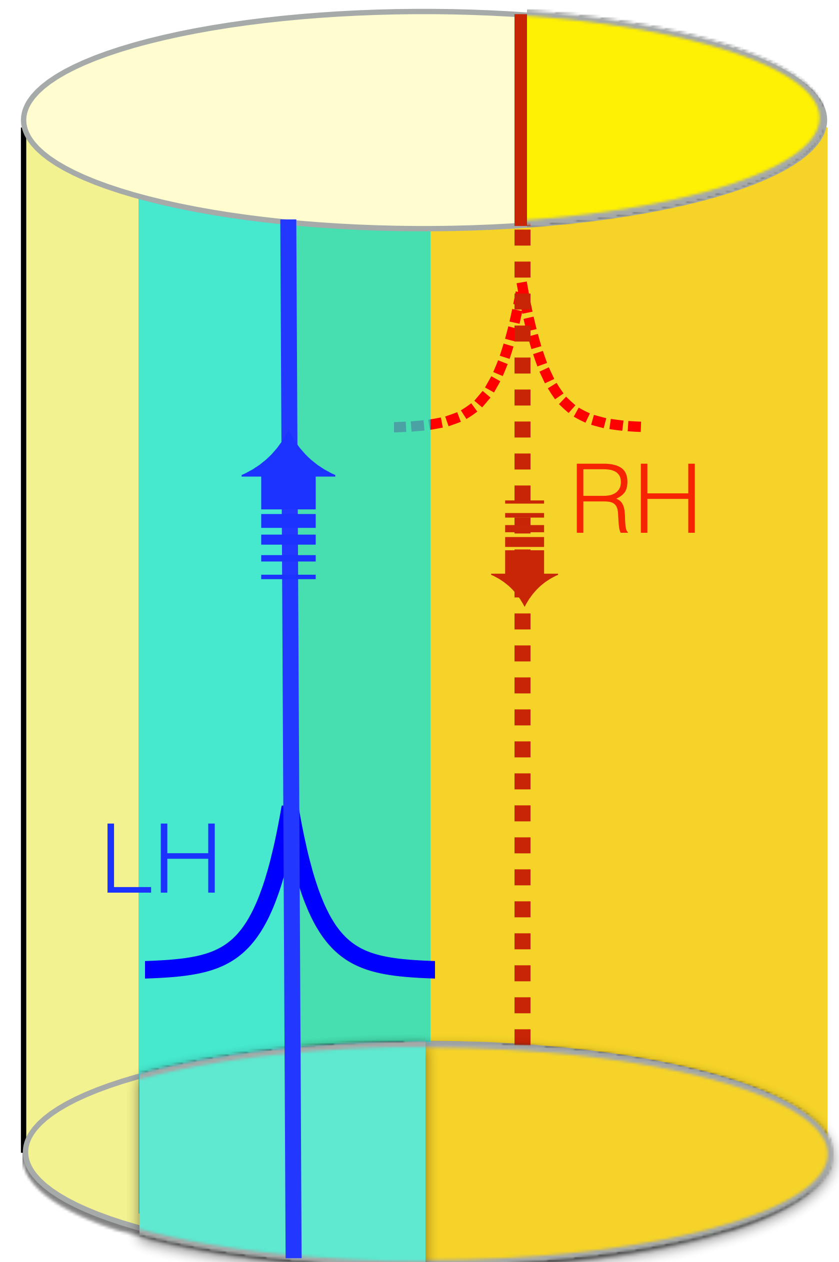
To compute overlap operator for DWF with flowed gauge field, need only replace



$$T^n \to T_\star^{n/2} T^{n/2}$$

gauge field A

gauge field A⋆

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Can construct a gauge invariant overlap operator

(DG, DBK, to appear):

$$\mathcal{D}_\chi = 1 + \gamma_5 \left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon\,\epsilon_\star + 1} (1 - \epsilon) \right]$$

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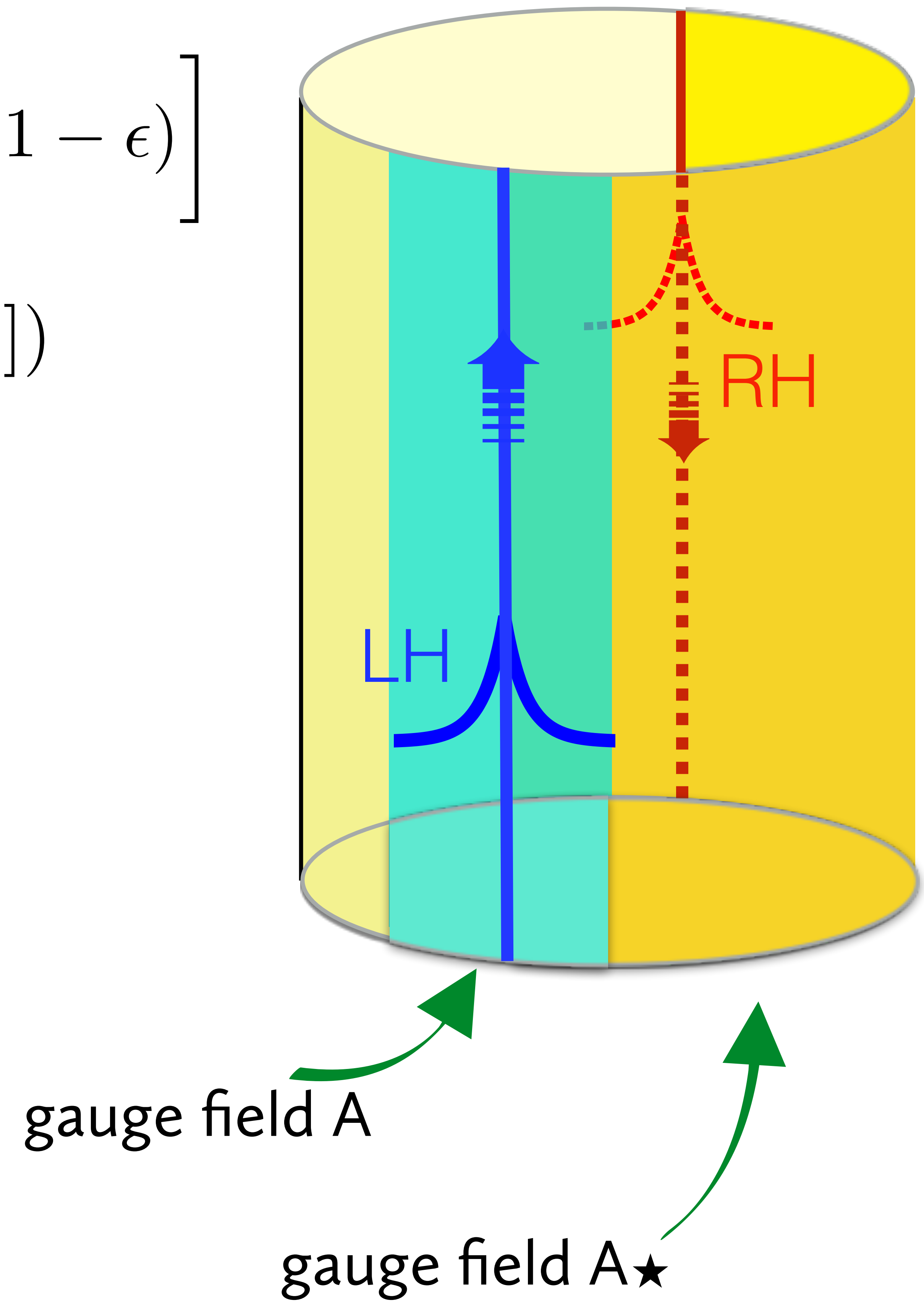

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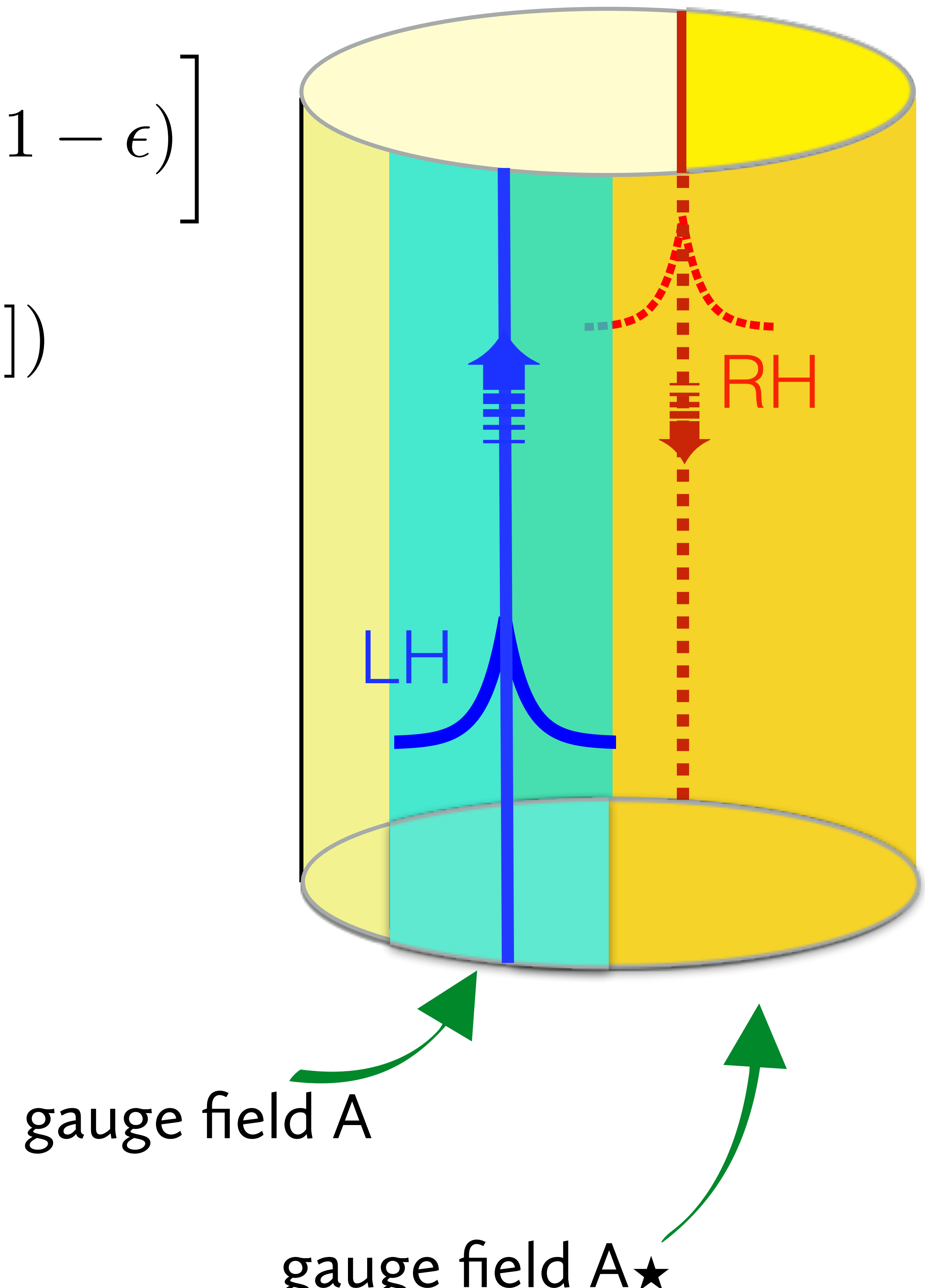

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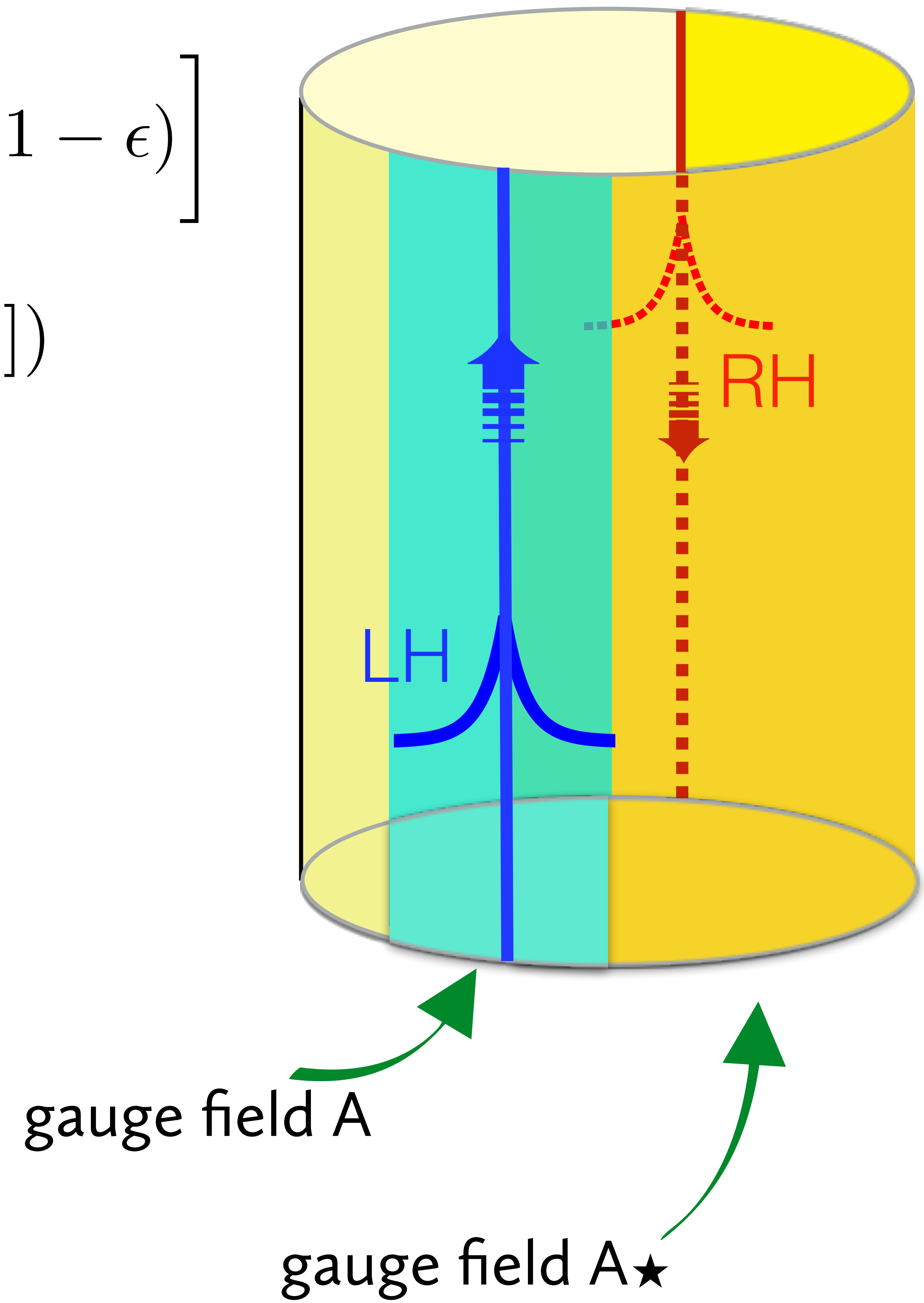

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- How the gauge invariant and gauge variant forms differ;
- how it can fail if fermion representation has a gauge anomaly;
- whether it can reproduce known results for a vector-like theory
- whether the U(1) chiral gauge theory constructed this way has a connection to Lüscher's implicit GW construction...

…and if these ideas don't work, try something else!