QCD with isospin density: pion condensation

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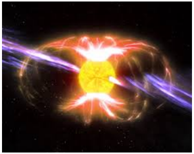
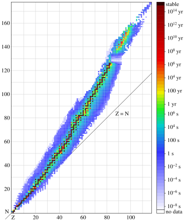
Outline

- introduction: QCD with isospin
- relevant phenomena
  - deconfinement/chiral symmetry breaking at low μ_I
    $\nwarrow$ next talk
  - pion condensation at high μ_I
    $\nwarrow$ this talk
- "λ-extrapolation"
  - naive method
  - new method
- outlook and summary

Introduction

- isospin density $n_I = n_u - n_d$
- $n_I < 0 \rightarrow$ excess of neutrons over protons
  $\rightarrow$ excess of π^- over π^+
- applications
  - neutron stars
  - heavy-ion collisions





- chemical potentials (3-flavor)

$$\mu_B = 3(\mu_u + \mu_d)/2 \qquad \mu_I = (\mu_u - \mu_d)/2 \qquad \mu_S = 0$$

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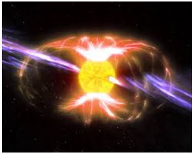

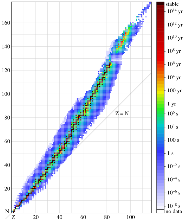

- chemical potentials (3-flavor)

$$\mu_B = 3(\mu_u + \mu_d)/2 \qquad \mu_I = (\mu_u - \mu_d)/2 \qquad \mu_S = 0$$

- here: zero baryon number but nonzero isospin

$$\mu_u = \mu_I \qquad \mu_d = -\mu_I$$

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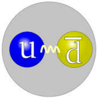

- QCD at low energies $\approx$ pions
- on the level of charged pions: $\mu_\pi = 2\mu_I$
  at zero temperature

$\mu_\pi < m_\pi$	vacuum state
$\mu_\pi = m_\pi$	Bose-Einstein condensation
$\mu_\pi > m_\pi$	undefined

- on the level of quarks: lattice simulations
  - no sign problem
  - conceptual analogies to baryon density
    (Silver Blaze, hadron creation, saturation)
  - technical similarities
    (proliferation of low eigenvalues)

Setup

Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1}$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V$$

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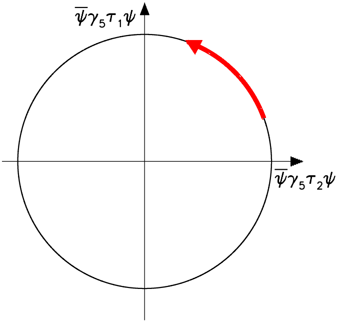

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$$\left\langle \bar\psi\gamma_5\tau_{1,2}\psi \right\rangle$$

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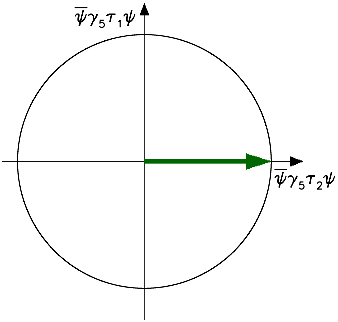

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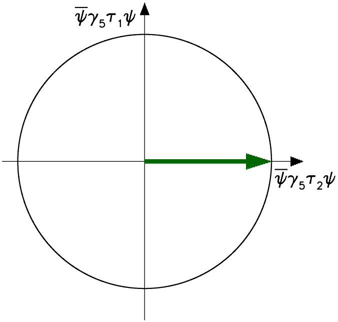

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- extrapolate results $\lambda \to 0$

Simulation details

- staggered light quark matrix with $\eta_5 = (-1)^{n_x+n_y+n_z+n_t}$

$$M = \begin{pmatrix} \not{D}_\mu + m_{ud} & \lambda\eta_5 \\ -\lambda\eta_5 & \not{D}_{-\mu} + m_{ud} \end{pmatrix}$$

- $\gamma_5\tau_1$-hermiticity

$$\eta_5\tau_1\not{D}_\mu\tau_1\eta_5 = \not{D}_\mu^\dagger$$

$\rightarrow$ determinant is real and positive

- first done by [Kogut, Sinclair '02]
- here: $N_f = 2+1$ rooted stout-smeared staggered quarks + tree-level Symanzik improved gluons

Pion condensate: definition and renormalization

- condensate

$$\langle\pi\rangle = \frac{T}{V}\frac{\partial \log \mathcal{Z}}{\partial\lambda}$$

- additive divergences cancel in

$$\lim_{\lambda\to 0}\langle\pi\rangle$$

- multiplicative renormalization

$$Z_\pi = Z_\lambda^{-1} = Z_{m_{ud}}^{-1}$$

- renormalization + convenient normalization

$$\Sigma_\pi \equiv m_{ud}\cdot\langle\pi\rangle\cdot\frac{1}{m_\pi^2 f_\pi^2}$$

- so that in leading-order chiral PT [Son, Stephanov '00]

$$\Sigma_{\bar\psi\psi}^2(\mu_I) + \Sigma_\pi^2(\mu_I) = 1$$

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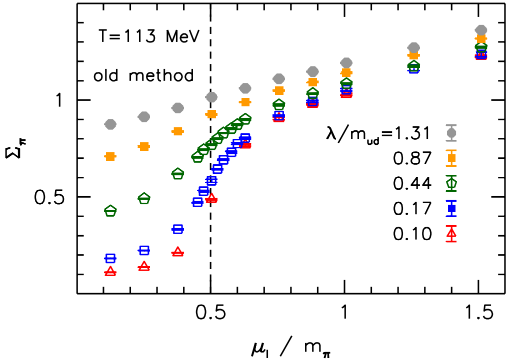
- traditional method [Kogut, Sinclair '02]
  measure full operator at nonzero λ (via noisy estimators)

$$\Sigma_\pi \propto \left\langle \mathrm{Tr} M^{-1} \eta_5 \tau_2 \right\rangle$$



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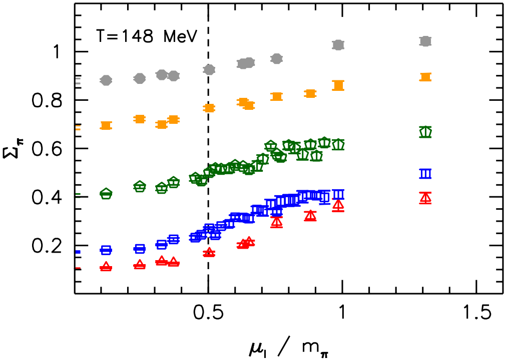

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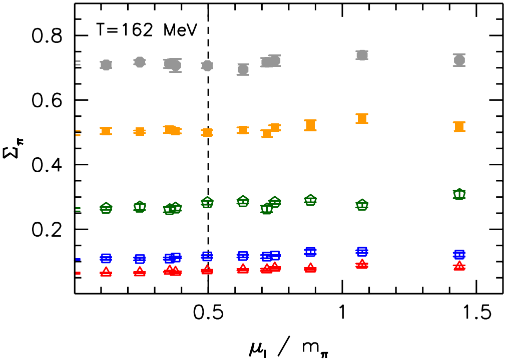

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Pion condensate: new method

Singular value representation

- pion condensate

$$\pi = i\mathrm{Tr}(M^{-1}\eta_5\tau_2) = \mathrm{Tr}\frac{2\lambda}{(\not{D}_\mu + m)^\dagger(\not{D}_\mu + m) + \lambda^2}$$

- singular values

$$(\not{D}_\mu + m)^\dagger(\not{D}_\mu + m)\,\psi_i = \xi_i^2\,\psi_i$$

- spectral representation

$$\pi = \frac{T}{V}\sum_i \frac{2\lambda}{\xi_i^2 + \lambda^2} = \int \mathrm{d}\xi\,\rho(\xi)\,\frac{2\lambda}{\xi^2 + \lambda^2} \xrightarrow{\lambda \to 0} \pi\rho(0)$$

first derived in [Kanazawa, Wettig, Yamamoto '11]

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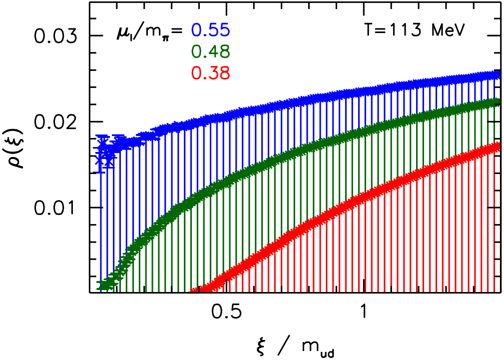
- compare to Banks-Casher-relation at $\mu_I = 0$

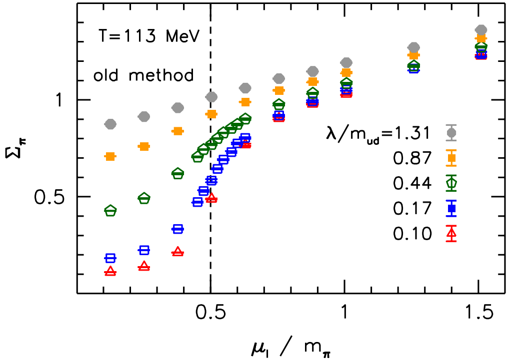
Singular value density

- spectral densities at $\lambda/m_{ud} = 0.17$



Density at zero

- scaling with λ is improved drastically



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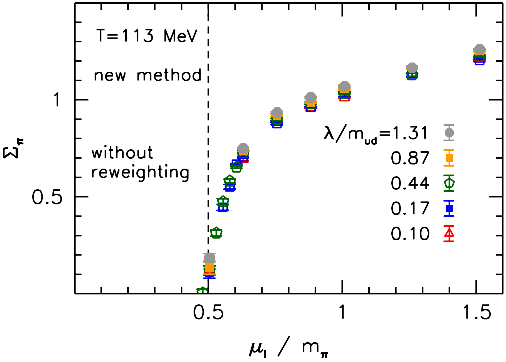

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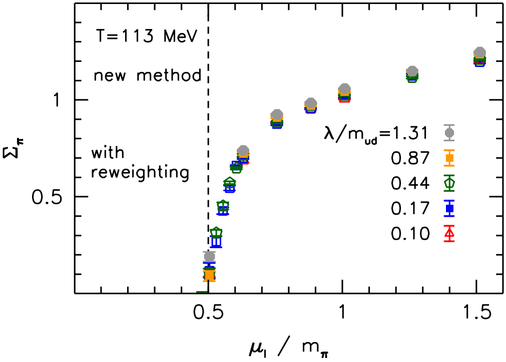
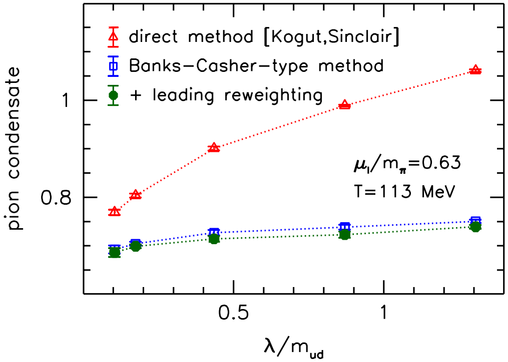


- leading-order reweighting

$$\langle \pi \rangle_{\rm rew} = \langle \pi W_\lambda \rangle \,/\, \langle W_\lambda \rangle \qquad\qquad W_\lambda = \exp[-\lambda V_4 \pi + \mathcal{O}(\lambda^2)]$$

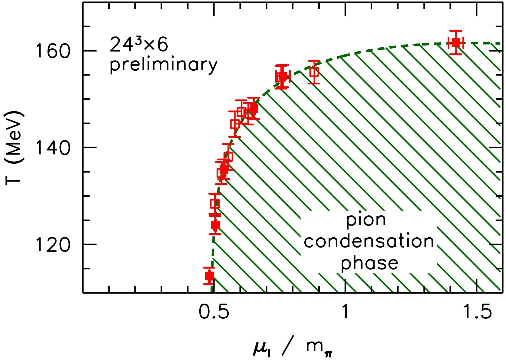
Comparison between old and new methods

- extrapolation in λ gets almost completely flat



Phase boundary

- interpolate $\rho(0)$ as function of μ_I to find phase boundary



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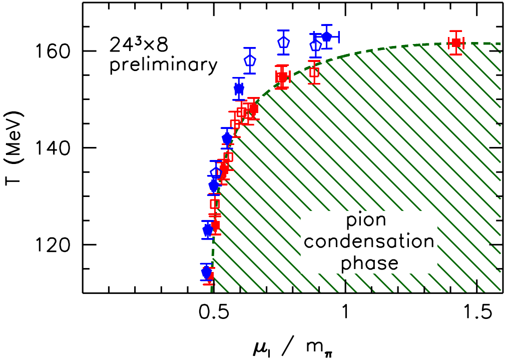

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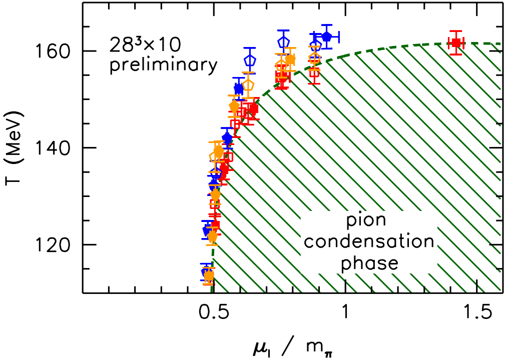

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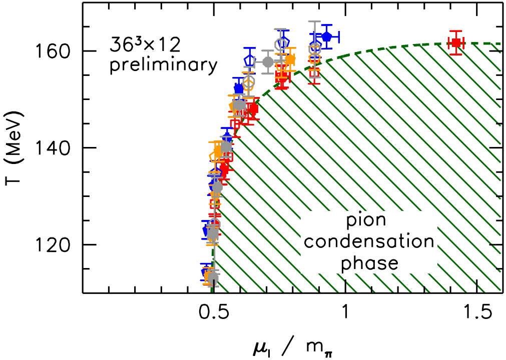

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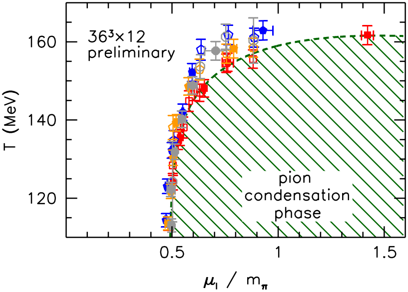

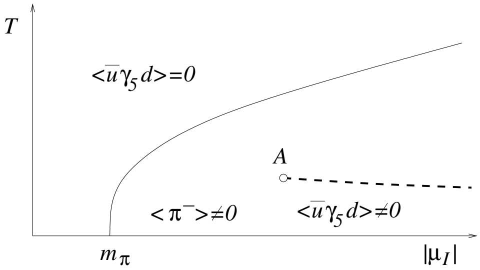

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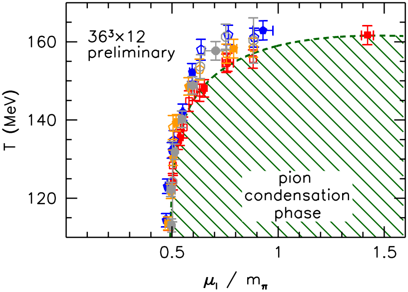

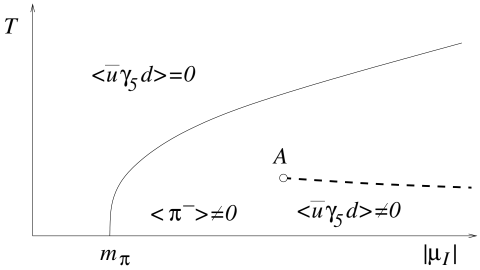

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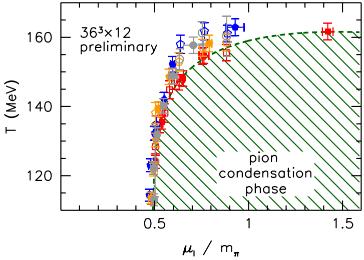
- no pion condensate above $T \approx 160$ MeV

Outlook



- order of transition?
- deconfinement/chiral symmetry breaking transition?
- asymptotic-μ_I limit?
- BCS phase at large μ_I?
- analogies to two-color QCD [Holicki, Thu]
- test Taylor-expansion in μ_I

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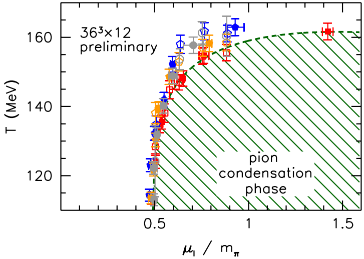

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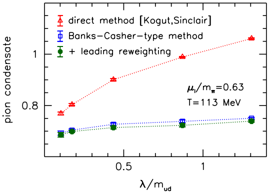
- stay for next talk [Brandt, Thu]

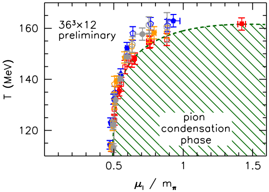
Summary

- QCD with isospin chemical potentials via lattice simulations at the physical point

- determine pion condensate via Banks-Casher-type relation
  $\rightsquigarrow$ flat extrapolation in pion source

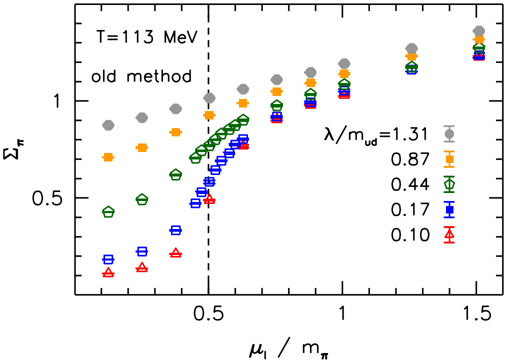


- phase boundary surprisingly flat for intermediate μ_I
- chance to test effective theories and low-energy models



Backup

Order of the transition – fits



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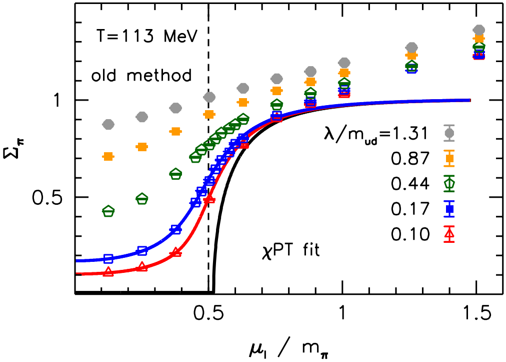

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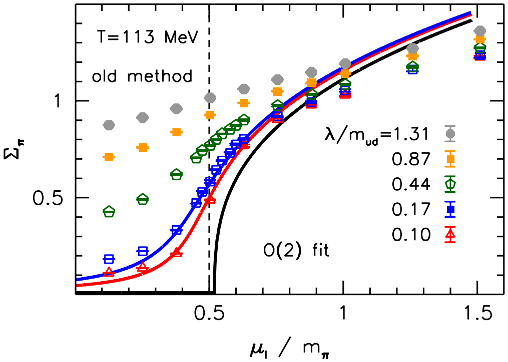

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  - $\mathrm{O}(2)$ scaling [Ejiri et al '09]