Approaching the conformal window in SU(2) field theory:
a systematic study of the spectrum for $N_f = 2, 4, 6$, and 8.

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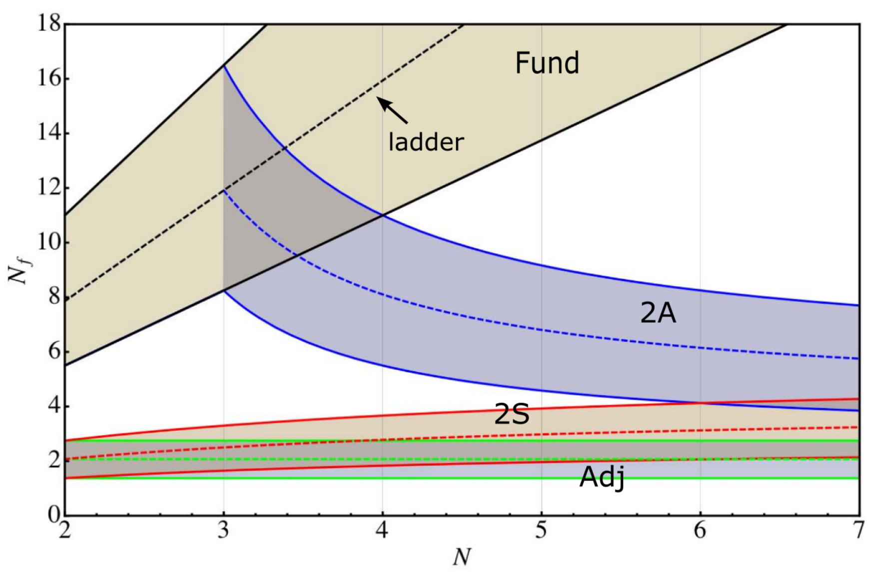
Introduction

Technicolor (TC) theories are SU(N) gauge field theories with N_f massless fermions. They solve i.e. the fine-tuning problem by substituting SM Higgs scalar with the chiral condensate. Standard Model fermion masses are typically assumed to arise from extended TC (ETC) interactions, since classical TC scenario does not explain them. However, ETC theory requires walking of the coupling constant i.e. $g \sim g_*$ over large scale separation.

Conformal window



Ref. [Appelquist, Lane, Mahanta, Cohen, Georgi, Yamawaki, Schrock, Sannino, Tuominen, Dietrich]

If the β-function of a theory $\beta = \mu\frac{dg}{d\mu}$ is zero at g_*, the theory has an infrared fixed point (IRFP). Conformal window is the range of N_f where the IRFP exists. The walking coupling can be found near the lower edge of the conformal window, whereas below it, chiral symmetry gets broken. A theory with IRFP has scale invariant long distance behaviour, and by observing the mass spectrum we can study if the theory is inside the conformal window. When $m_Q \to 0$, if

- pions gets massless as $m_\pi \propto m_Q^{1/2}$ → QCD-like chiral symmetry breaking.
- all states gets massless as $M \propto m_Q^{\frac{1}{1+\gamma(g_*^2)}}$ → IRFP
  - the factor $\gamma(g_*^2)$ is the mass anomalous exponent.

IRFP is found at strong coupling, where perturbative analysis is not valid and thus lattice simulations are required.

SU(2) theory with $N_f = 2, 4, 6$ and 8 fundamental representation fermions

On the lattice, there exists clear evidence for chiral symmetry breaking for $N_f = 2$ and 4, and the existence of an IRFP at $N_f = 8$ and 10 [1,2], but for $N_f = 6$ a clear picture has not yet emerged due to the conflicting results in the literature eg. [1,3–5].

Our goal is to study the hadron spectrum and scale-setting when approaching the conformal window.

- Method: we use
  - HEX smeared Wilson clover action for fermions
  - thin link Wilson + stout link Wilson for gauge fields.
- Lattice sizes: $24^3 \times 48$, and $32^3 \times 60$ for small am_Q.
- Number of configurations: 80-200.
- Scale setting with gradient flow.

Results

Hadron spectrum

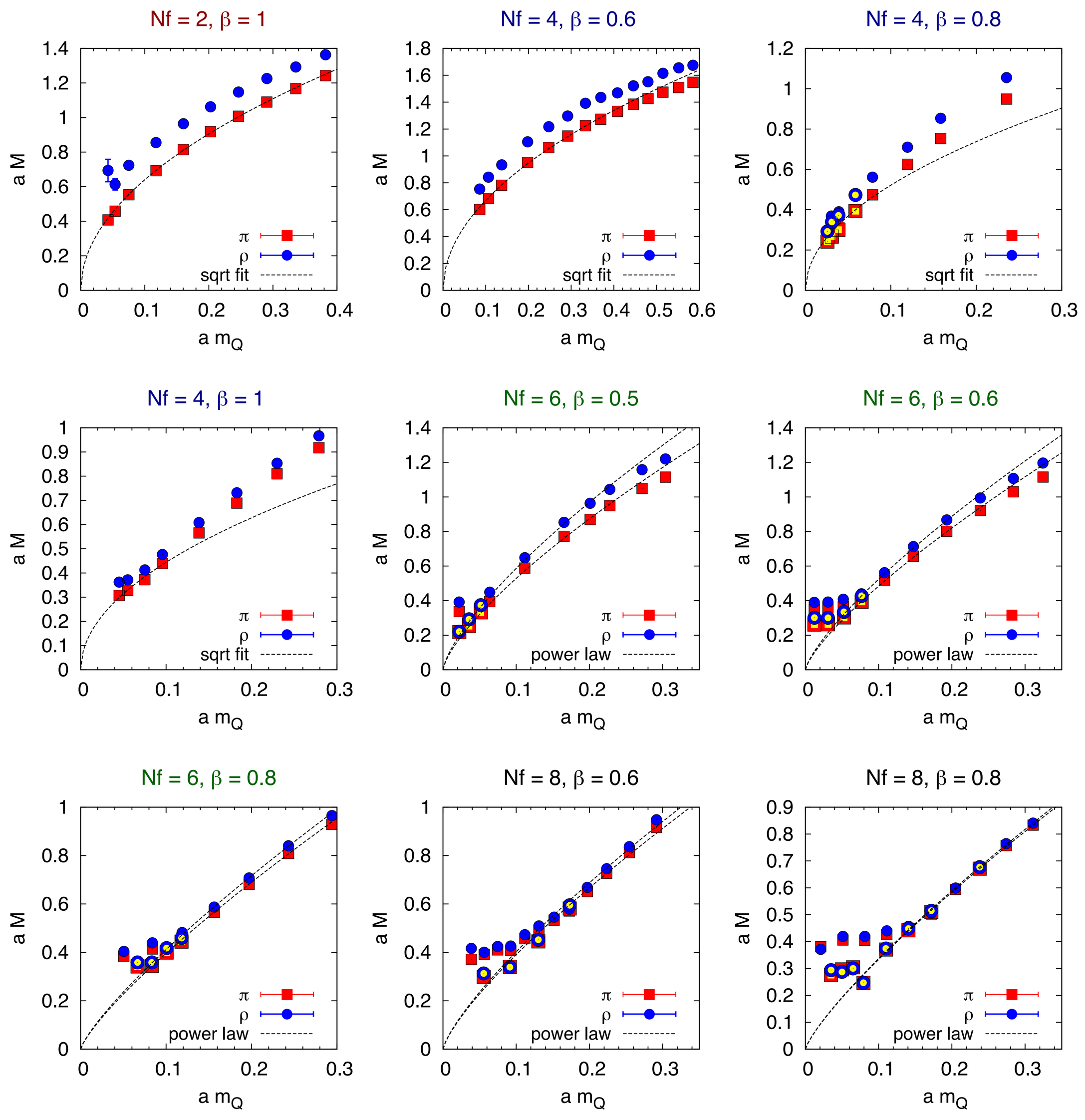


Figure 1: Pion and rho masses in $N_f = 2, 4, 6$ and 8 with different values of β. Square root dependence is fitted to pion mass in $N_f = 2$ and 4. In $N_f = 6$ and 8 $M \propto m_Q^{1/1+\gamma}$ with the same γ in pion and rho masses (see table 1). Points with yellow centers refer to $32^3 \times 60$ lattice, and others are with $24^3 \times 48$ lattice.

In fig. 1 the pion and the rho masses are presented as a function of the quark mass with different N_f and β. We found:

- $N_f = 2$: pions behave exactly as expected, i.e. they become massless $\propto \sqrt{m_Q}$. Due to the familiarity of this scenario we do not attempt to reach very small quark masses.
- $N_f = 4$: finite volume effects are stronger than what we expected, and the square root behaviour can be found only at small am_Q.
- $N_f = 6$: pion and rho masses $\propto m_Q^{1/1+\gamma}$, the values of γ can be found from table 1. The finite volume effects can be clearly seen.
- $N_f = 8$: spectrum shows very strong finite size effects already at quite big am_Q → mass measurements unreliable?

If we want to reach small quark masses, we should use bigger volumes (which quickly becomes expensive) or use really strong bare coupling (small β).

N_f	β	γ	γ_{SD}
6	0.5	0.382(12)	0.280(2)
	0.6	0.314(7)	0.231(2)
	0.8	0.248(3)	~ 0.16
8	0.6	0.293(30)	~ 0.13
	0.8	0.238(31)	0.111(1)

Table 1: We have also measured the factor γ of these theories by using the spectral density method (details: poster by Joni Suorsa [6]). Mass spectrum suggests bigger γ than what the spectral density theorem γ_{SD} gives.

Decay constants

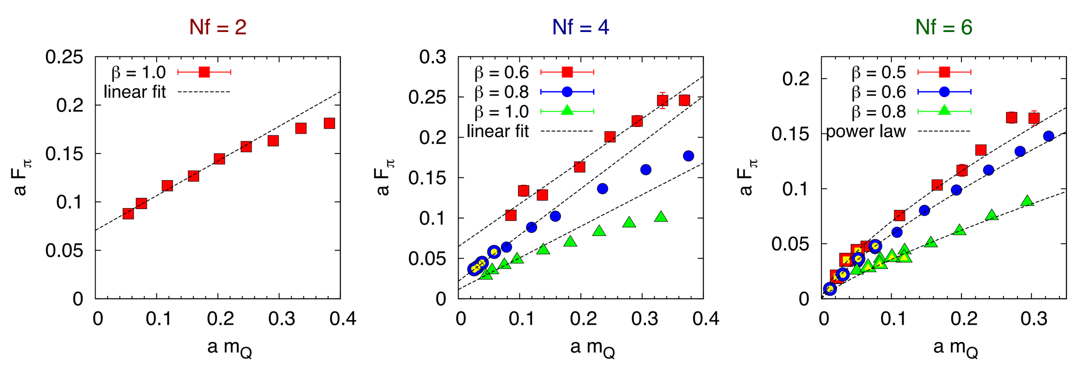


Figure 2: Pion decay constant. Yellow centers refer to the bigger lattice.

Pion decay constants as a function of quark mass are presented in fig. 2. We fitted a linear function to the decay constant in $N_f = 2$ and 4, since a theory with chiral symmetry breaking should have a finite intercept as $m_Q \to 0$.

- $N_f = 2$ is clear: F_π has a finite value, when $m_Q \to 0$.
- The case $N_f = 4$ is more difficult:
  - $\beta = 0.6$: behaves as expected and has a positive F_π at $m_Q = 0$.
  - $\beta = 0.8$ and 1: F_π gets too small compared to LF_π, L is the lattice size, that we cannot say for sure what happens, when $m_Q \to 0$.
- $N_f = 6$: pion decay constant $\propto m_Q^{1/1+\gamma}$ as we assume a theory with IRFP to behave.

Scale setting: gradient flow

The scale setting procedure in lattice QCD is needed in order to relate the lattice scale to some physically known quantity. Presently, the gradient flow approach [7] has become the preferred choice for that. It is an artificial way to set the scale, but it is known to be very precise, cheap and straightforward to implement.

- Wilson flow
  - solve t_0 from: $t_0^2\langle E(t_0)\rangle = 0.3$
    - $E(t)$ = continuum-like action density at flow time t
  - related observable w_0 [8]: solve $t\frac{d}{dt}(t^2\langle E(t)\rangle)|_{t=w_0^2} = 0.3$

These two scales, t_0 and w_0, do not differ much, so here we use t_0.

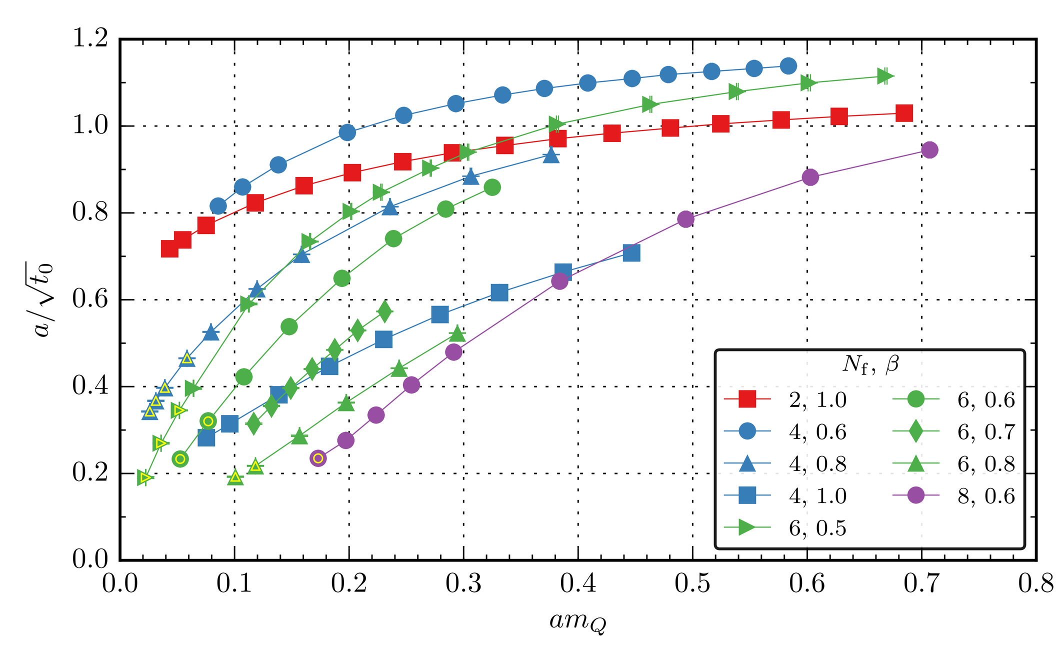


Figure 3: Results of the $a/\sqrt{t_0}$ scale. Yellow symbols refer to the $32^3 \times 60$ lattice.

The behaviour of $a/\sqrt{t_0}$ is shown in fig. 3. If we use $\sqrt{t_0}$ to fix the scale:

- smaller $a/\sqrt{t_0}$ means a smaller lattice spacing a → for fixed am_Q the quarks are heavier.
- harder to reach the small physical quark mass region, when N_f or β are increased.

If the theory has IRFP, the scale should be invariant in the infrared, and we expect $a/\sqrt{t_0} \to 0$ as $m_q \to 0$. $N_f = 2$ and 4 seem to a finite intercept at $m_Q = 0$, whereas $N_f = 6$ and 8 are not obvious. Especially at $N_f = 8$ we expect strong finite volume effects at small m_Q.

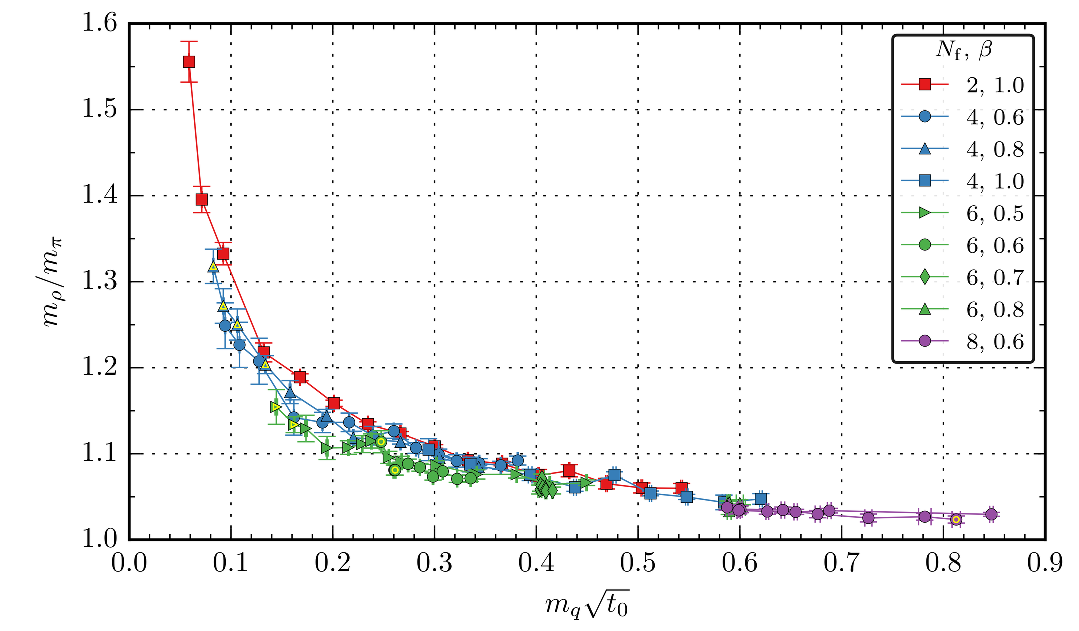


Figure 4: Results in physical units. All of our measurements fall on a "universal" curve. The reasons for this scaling is currently unclear.

In fig. 4 we plot m_ρ/m_π against the "physical" quark mass $m_Q\sqrt{t_0}$. Remarkably, all of our measurements fall on an "universal" curve, independent of N_f or the lattice spacing. We found:

- $N_f = 2$: the ratio diverges as $m_Q \to 0$.
- $N_f = 4$: follows $N_f = 2$, but much more slowly. Small β is required to reach large m_ρ/m_π.
- $N_f = 6$: two types of behaviour
  - $\beta = 0.5$ and 0.6: the points slowly grow along the curve.
  - $\beta = 0.7$ and 0.8: the points remain fixed at $m_Q\sqrt{t_0} = 0.4$ and 0.6, respectively.
- $N_f = 8$: the points turn around at $m_Q\sqrt{t_0} = 0.6$ → finite size effects?

Conclusions

In our study of SU(2) theory with N_f =2, 4, 6 and 8 fundamental representation fermions we have focused on the hadron spectrum, decay constants and the gradient flow scale setting. For the mass spectrum, our central finding is that the finite volume effects are stronger than expected, making it difficult to reach the chiral small quark mass regime even at $N_f = 4$. Therefore we are forced to use small β (strong bare coupling) in order to reach the small quark masses.

Considering the scale setting, our main observation is that smaller physical quark masses can be reached by decreasing N_f or β. By plotting the mass ratio m_ρ/m_π against $m_Q\sqrt{t_0}$, we observe that all our measurements fall on a universal curve, but the reason for that is currently unknown.

Forthcoming Research

- Define the glueball masses and the string tension.
- Do the scalar measurement.
- In the future: SU(3) theory with fundamental fermions.

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