An $a_0$ resonance in strongly coupled $\pi\eta, K\bar{K}$ scattering from lattice QCD

PRD93 094506 (2016)
(with David Wilson and Robert Edwards)
the light scalar mesons - empirically

conventional to put them in an ‘inverted’ mass nonet

but how similar are they really?

let’s study their appearance within QCD ...

an $a_0$ resonance ... | lattice 2016 | 7.26.16
the light scalar mesons - empirically

conventional to put them in an ‘inverted’ mass nonet

but how similar are they really?

‘start’ with the $a_0$ resonance ...
the $a_0(980)$ as it really is - a resonance

- sharp experimental enhancement at $K\bar{K}$ threshold decaying to $\pi\eta$

- usually observed in ‘less-simple’ production processes
  
  e.g. $p\bar{p}\to \pi\pi\eta$
  $\phi\to \gamma\pi\eta$

- amplitude models typically give $\frac{g^2(K\bar{K})}{g^2(\pi\eta)} \sim 1$
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- amplitude models typically give $\frac{g^2(K\bar{K})}{g^2(\pi\eta)} \sim 1$

our first task:
- compute coupled-channel $\pi\eta/K\bar{K}$ scattering amplitudes
- finite-volume spectra

$E$

\[ a_0(980) \]

$K\bar{K}_{\text{thr.}}$
correlation functions

- matrices of correlation functions with a large operator basis

“$q\bar{q}$”-like $\bar{\psi}\Gamma D\ldots D\psi$

$\pi\eta$-like $\sum_{\bar{p}_1,\bar{p}_2} C(\bar{p}_1, \bar{p}_2; \bar{P}) \pi(\bar{p}_1) \eta(\bar{p}_2)$

$K\bar{K}$-like $\sum_{\bar{p}_1,\bar{p}_2} C(\bar{p}_1, \bar{p}_2; \bar{P}) K(\bar{p}_1) \bar{K}(\bar{p}_2)$

$\pi\eta'$-like $\sum_{\bar{p}_1,\bar{p}_2} C(\bar{p}_1, \bar{p}_2; \bar{P}) \pi(\bar{p}_1) \eta'(\bar{p}_2)$

(with optimized pseudoscalar operators)
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(with optimized pseudoscalar operators)

many Wick contractions required, including annihilations ...

e.g.

... distillation
totally straightforward, massive reuse of propagators
\( I=1, G=- \) spectra

- spectra obtained from variational analysis

\[ m_\pi \sim 391 \text{ MeV} \]
\[ 16^3, 20^3, 24^3 \]
\[ a_s \sim 0.12 \text{ fm} \]
\[ a_t \sim a_s / 3.5 \]
coupled channel scattering

- finite-volume formalism established

\[ \det \left[ t^{-1}(E) + i\rho(E) - M(E, L) \right] = 0 \]

scattering matrix  \( t \)

phase space  \( t \)

known functions  \( t \)

in the two-channel case  \( t = \begin{bmatrix} t_{\pi\eta\rightarrow\pi\eta} & t_{\pi\eta\rightarrow K\bar{K}} \\ t_{K\bar{K}\rightarrow\pi\eta} & t_{K\bar{K}\rightarrow K\bar{K}} \end{bmatrix} \)
coupled channel scattering

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\det \left[ t^{-1}(E) + i\rho(E) - M(E, L) \right] = 0
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in the two-channel case

\[
t = \begin{bmatrix}
t_{\pi\eta\to\pi\eta} & t_{\pi\eta\to K\bar{K}} \\
t_{K\bar{K}\to\pi\eta} & t_{K\bar{K}\to K\bar{K}}
\end{bmatrix}
\]

- parameterizing the energy-dependence in a unitarity-preserving way

\[K\text{-matrix approach}\]

\[
t^{-1}(E) = K^{-1}(E) + I(E)
\]

\[\text{Im} \ (I(E))_{ij} = -\delta_{ij} \rho_i(E)\]

e.g. “Chew-Mandelstam” phase-space

\[\begin{aligned}
(K(E))_{ij} &= \sum_p \frac{g^{(p)}_i g^{(p)}_j}{m_p^2 - E^2} + \sum_n \gamma^{(n)}_{ij} (E^2)^n
\end{aligned}\]
$\pi\eta/K\bar{K}$ scattering describing the spectra

$K$-matrix parameterization: one pole plus constant matrix

( 6 free parameters )

$m_\pi \sim 391$ MeV

$\chi^2/N_{dof} = \frac{58.0}{47 - 6} = 1.4$
πη/KK scattering in J^P = 0^+

\[ m_\pi \sim 391 \text{ MeV} \]

\[ \rho_i \rho_j |t_{ij}|^2 \]

\[ \pi\eta \rightarrow \pi\eta \]

\[ \pi\eta \rightarrow KK \]

\[ \pi\eta' \rightarrow KK \]

\[ a_t E_{cm} \]

\[ \pi\eta \] thr.

\[ 0.18 \]

\[ 0.19 \]

\[ 0.20 \]

\[ 0.21 \]

\[ 0.22 \]

\[ 0.23 \]

\[ 0.1 \]

\[ 0.2 \]

\[ 0.3 \]

\[ 0.4 \]

\[ 0.5 \]

\[ 0.6 \]

\[ 0.7 \]
parameterization variation

\[ \rho_i \rho_j |t_{ij}|^2 \]

\( K\bar{K} \rightarrow K\bar{K} \)

\( \pi\eta \rightarrow \pi\eta \)

\( \pi\eta \rightarrow K\bar{K} \)

\( m_\pi \sim 391 \text{ MeV} \)
πη/KK scattering in \( J^P = 0^+ \)

Strong cusp in \( πη \) at KK threshold

Rapid turn-on of KK amplitudes

Indicative of a nearby resonance?
**πη/KK** scattering in $J^P = 0^+$

**Strong cusp** in $\pi\eta$ at $KK$ threshold

**Rapid turn-on** of $KK$ amplitudes indicative of a nearby resonance?

**How do we determine rigorously if an amplitude is resonant?**

**Look for a pole singularity at complex energy**

$$t_{ij}(s) \sim \frac{g_i g_j}{s_0 - s}$$

$$\text{Re}[\sqrt{s_0}] \sim \text{‘mass’}$$

$$2 \cdot \text{Im}[\sqrt{s_0}] \sim \text{‘width’}$$

$m_\pi \sim 391 \text{ MeV}$

$\rho_i \rho_j |t_{ij}|^2$
\[ \pi \eta / K \bar{K} \text{ scattering in } J^P = 0^+ \]

- we find a single dominant (nearby) pole

\[ m_\pi \sim 391 \text{ MeV} \]

**Complex Energy Plane**

**Complex Momentum Plane**

<table>
<thead>
<tr>
<th>Sheet</th>
<th>( \text{Im} k_{\eta \eta} )</th>
<th>( \text{Im} k_{K \bar{K}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>II</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>IV</td>
<td>+</td>
<td>-</td>
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\( \pi \eta / K \bar{K} \) scattering in \( J^P = 0^+ \)

- we find a single dominant (nearby) pole

\[ m_\pi \sim 391 \text{MeV} \]
πη/KK scattering in $J^P = 0^+$

scale setting using the Ω baryon mass

$m_\pi \sim 391$ MeV

- resonance found near $K\bar{K}$ threshold

pole position

$$\sqrt{s_0} = \left((1177 \pm 27) + \frac{i}{2}(49 \pm 33)\right)\text{MeV}$$

$$\text{Re}\sqrt{s_0} - 2m_K = 79 \pm 27$$

pole coupling ratio

$$\left|\frac{g_{K\bar{K}}}{g_{\pi\eta}}\right| = 1.3(4)$$
Raul Briceño (previous session) presented $\pi\pi$ elastic scattering in $I=0$ with the $\sigma$
- bound state at $m_\pi=391$ MeV
- broad resonance at $m_\pi=236$ MeV

extension to coupled channel ($\pi\pi$, $KK$, ...) and likely $f_0('980')$ coming up

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$\pi K$, $\eta K$ scattering already done at $m_\pi=391$ MeV
... at $m_\pi=236$ MeV to come soon ...

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$\pi\eta$, $K\bar{K}$ at $m_\pi=236$ MeV to come soon ...

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κ as a virtual bound-state?

- [PRL113 182001 (2014)]
- [PRD91 054008 (2015)]
what are the scalar mesons?

• what tools do we have at our disposal?
  • quark mass dependence of the pole positions and channel couplings
  • distribution of poles across Riemann sheets
    may be relatable to loose meson-meson molecule versus tightly bound object
  • coupling to external currents
    form-factors from residue at the resonance pole
  • and things we haven’t thought of yet ...

finite-volume formalism demonstrated with $\gamma\pi \to \pi\pi$

$\gamma^* \pi \to \pi\pi$

$Q^2 = 0$

$Q^2 = 0.803 \text{GeV}^2$

$E_{\pi\pi}/m_\pi$

$A(E_{\pi\pi},Q^2)$

PRL115 242001 (2015)
PRD93 114508 (2016)

$\pi\pi \to \pi\pi$

$m_\pi \sim 391 \text{MeV}$
an $a_0$ resonance ...

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