An a_0 resonance in strongly coupled $\pi\eta, K\bar{K}$ scattering from lattice QCD

PRD93 094506 (2016)

(with David Wilson and Robert Edwards)

Jozef Dudek

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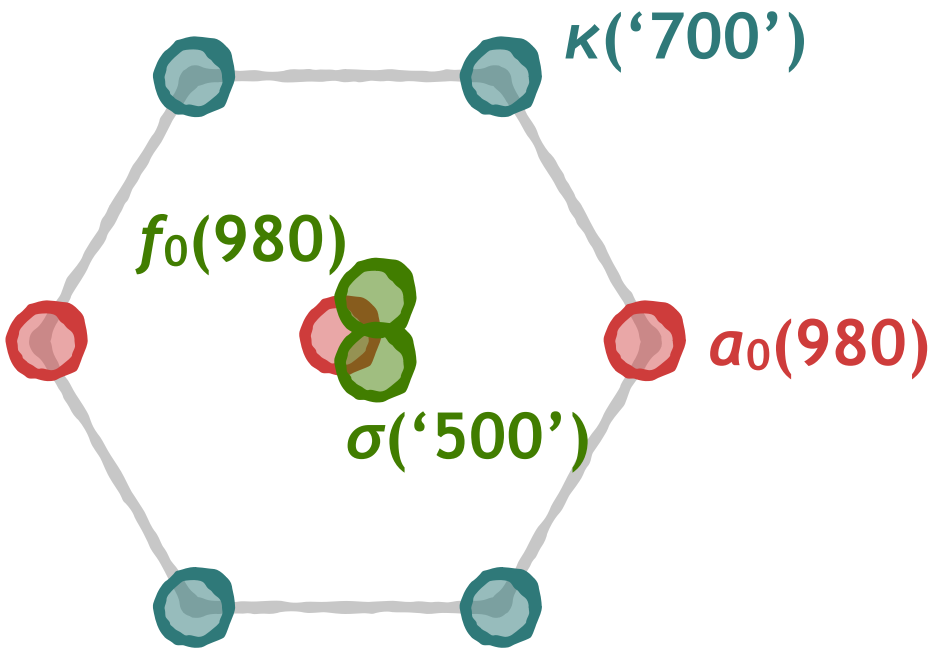
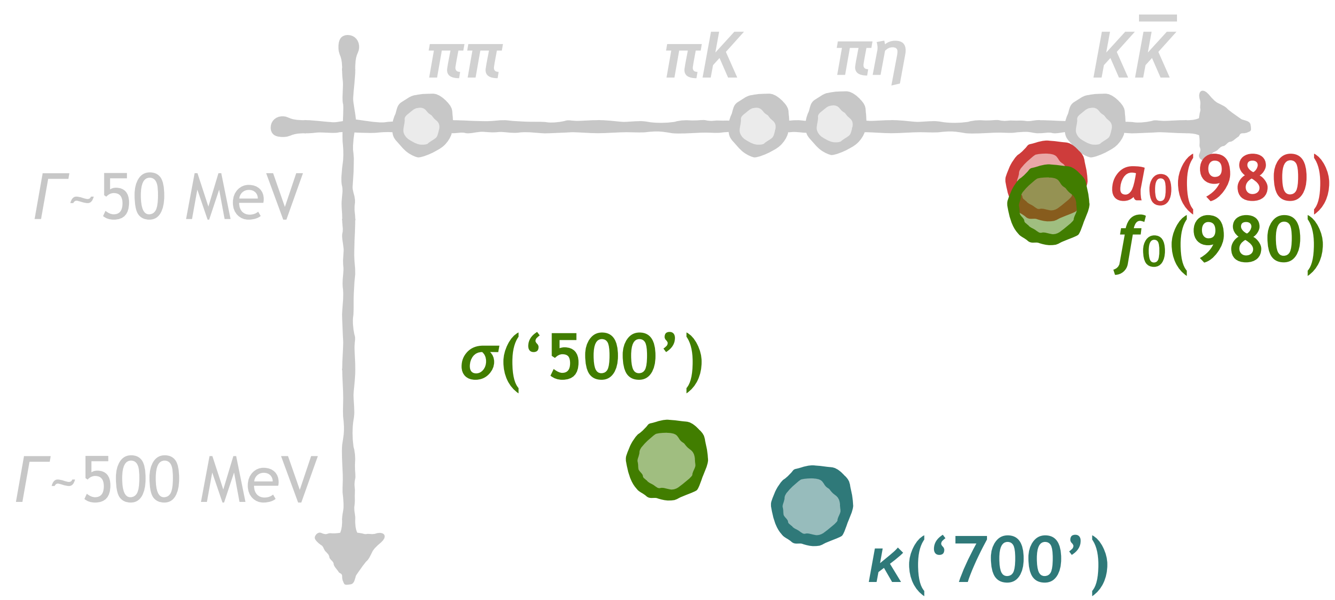
Jefferson Lab

the light scalar mesons - empirically

conventional to put them in an ‘inverted’ mass nonet



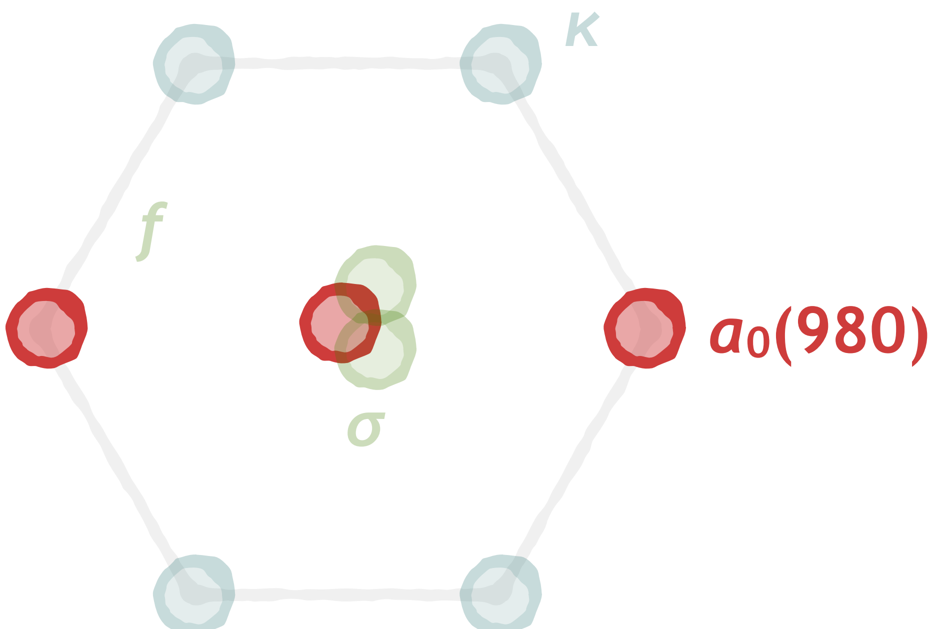
but how similar are they really ?



let’s study their appearance within QCD …

the light scalar mesons - empirically

conventional to put them in an ‘inverted’ mass nonet



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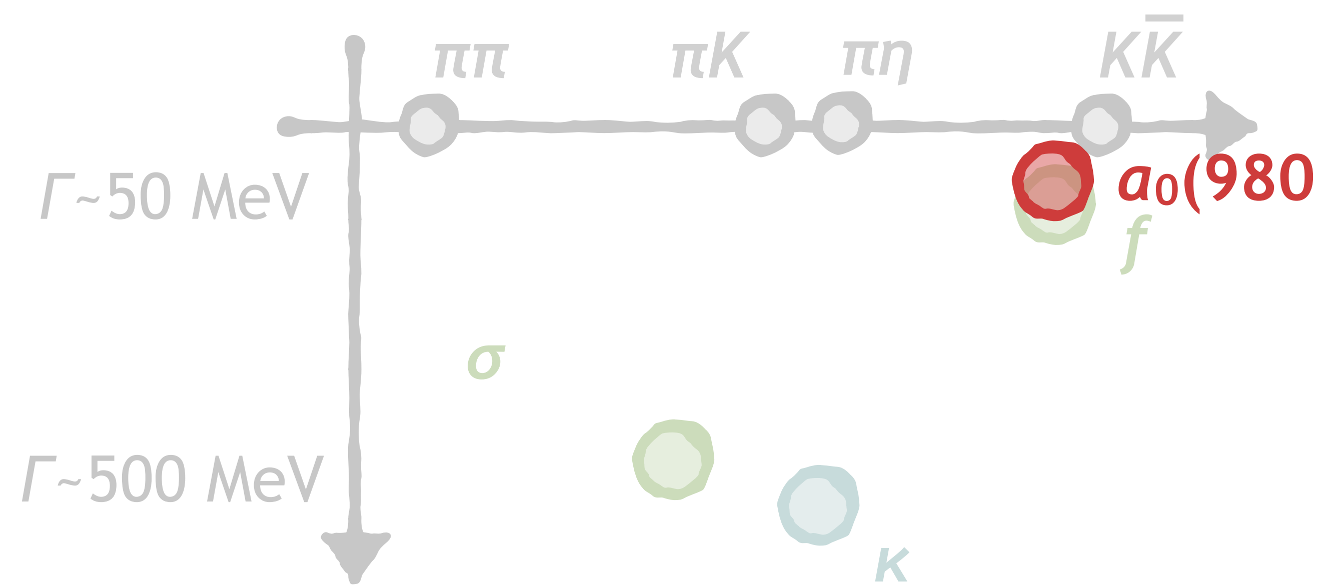
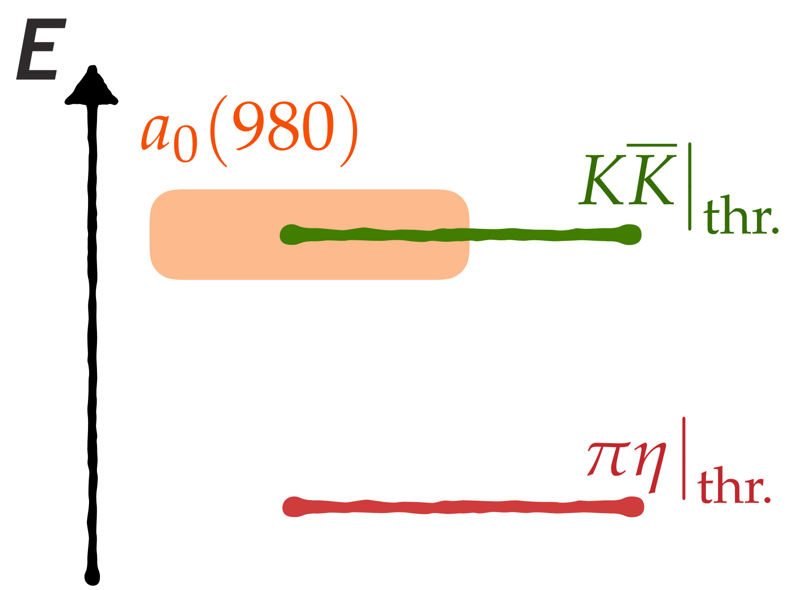


‘start’ with the a_0 resonance …

the a_0(980) as it really is - a *resonance*

- sharp experimental enhancement at $K\bar{K}$ threshold decaying to $\pi\eta$

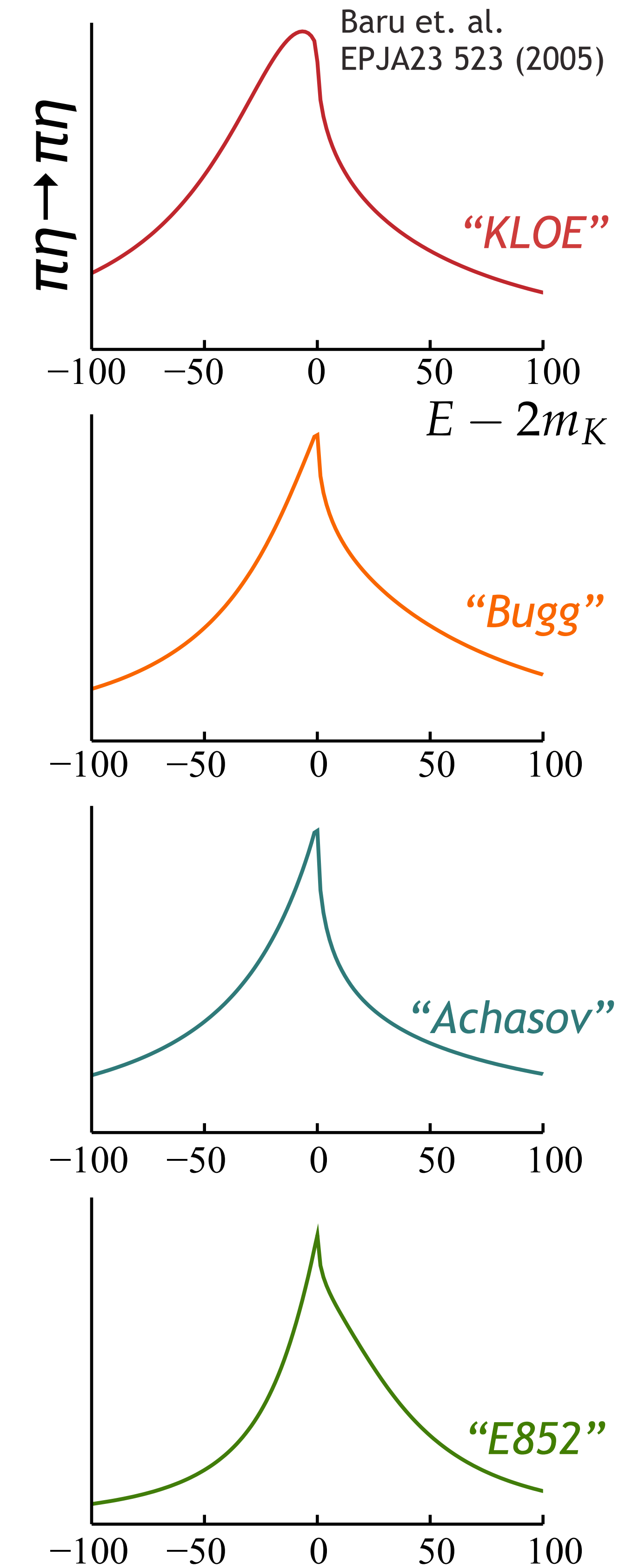


- usually observed in 'less-simple' production processes

e.g. $p\bar{p} \to \pi\pi\eta$

$\phi \to \gamma\pi\eta$

- amplitude models typically give $\dfrac{g^2(K\bar{K})}{g^2(\pi\eta)} \sim 1$



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the $a_0(980)$ as it really is - a *resonance*

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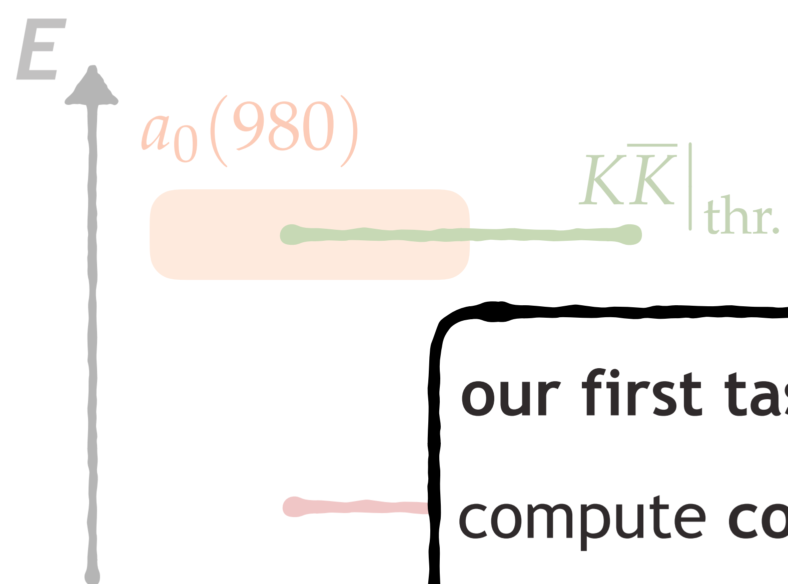

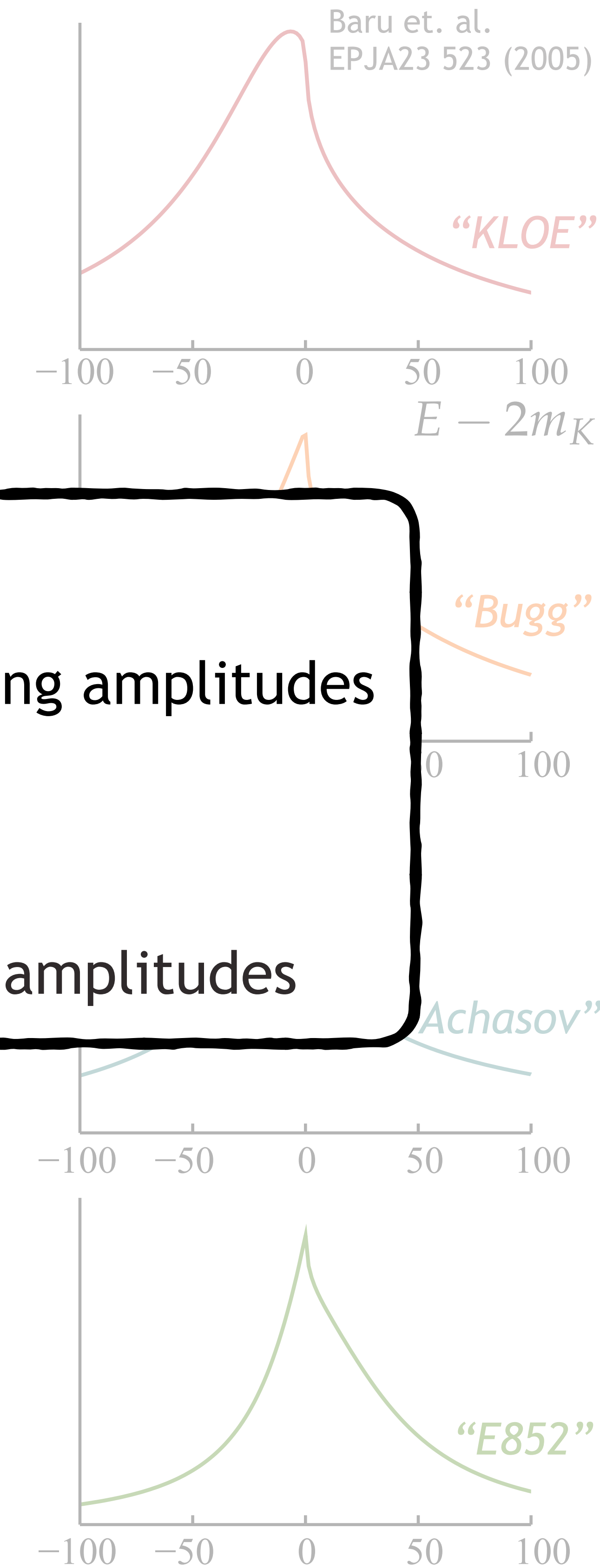

our first task:

compute **coupled-channel** $\pi\eta/K\bar{K}$ scattering amplitudes

finite-volume spectra → scattering amplitudes

- usually observed in "...

e.g. $p\bar{p} \to \pi\pi\eta$

$\phi \to \gamma\pi\eta$

- amplitude models typically give $\dfrac{g^2(K\bar{K})}{g^2(\pi\eta)} \sim 1$

correlation functions

- matrices of correlation functions with a large operator basis

"$q\bar{q}$"-like $\bar{\psi}\Gamma D \ldots D\psi$

$\pi\eta$-like $\sum_{\hat{p}_1,\hat{p}_2} C(\vec{p}_1,\vec{p}_2;\vec{P})\,\pi(\vec{p}_1)\eta(\vec{p}_2)$

$K\bar{K}$-like $\sum_{\hat{p}_1,\hat{p}_2} C(\vec{p}_1,\vec{p}_2;\vec{P})\,K(\vec{p}_1)\overline{K}(\vec{p}_2)$

$\pi\eta'$-like $\sum_{\hat{p}_1,\hat{p}_2} C(\vec{p}_1,\vec{p}_2;\vec{P})\,\pi(\vec{p}_1)\eta'(\vec{p}_2)$

(with optimized pseudoscalar operators)

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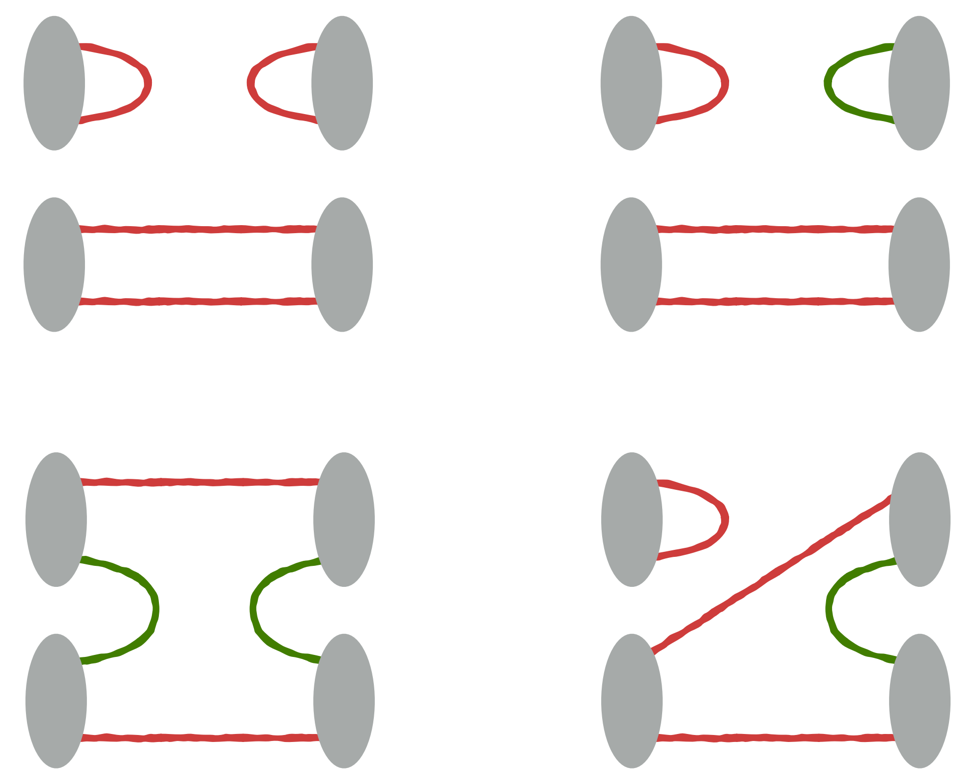
(with optimized pseudoscalar operators)

many Wick contractions required, including annihilations ...

e.g.



... **distillation**

totally straightforward, massive reuse of propagators

I=1, G=− spectra

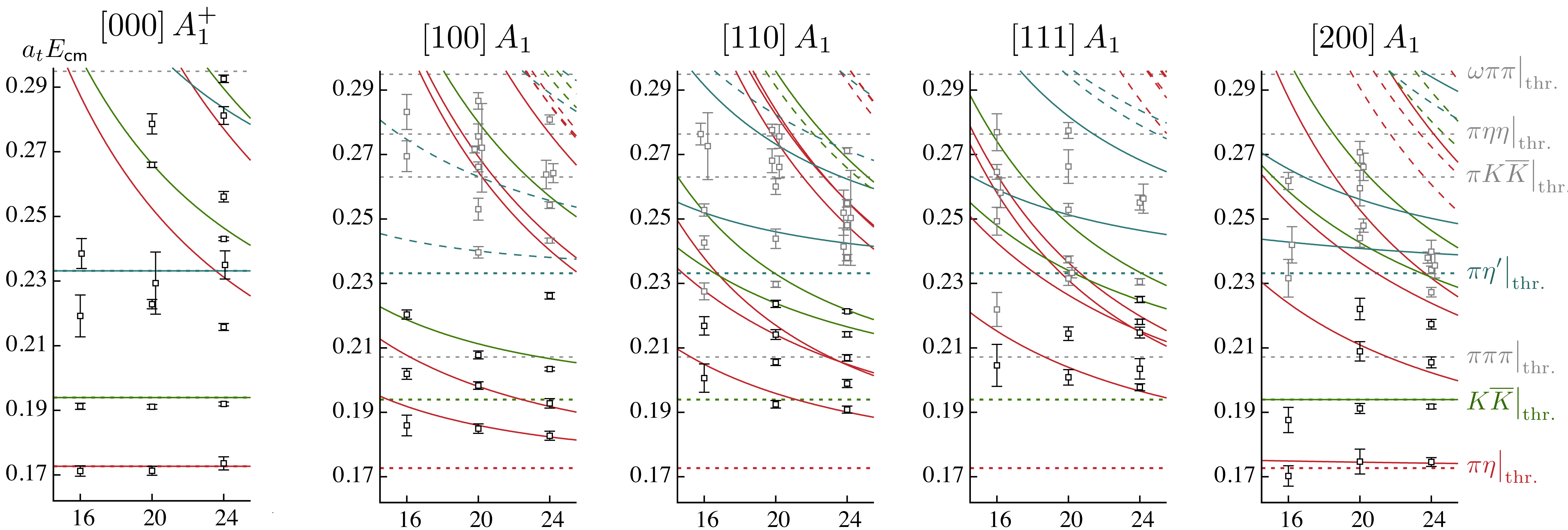
$m_\pi \sim 391\,\mathrm{MeV}$

16^3, 20^3, 24^3

a_s ~ 0.12 fm

a_t ~ $a_s/3.5$

- spectra obtained from variational analysis



(18) (11) (21) (12) (16) (19) (12) (26) (17) (11) (12) (13) (13) (14) (20)

(*conservatively*) **47** levels
in the relevant energy region

coupled channel scattering

- finite-volume formalism established

$$\det\left[\mathbf{t}^{-1}(E) + i\boldsymbol{\rho}(E) - \mathbf{M}(E,L)\right] = 0$$

scattering matrix *phase space* *known functions*

He, JHEP 0507 011
Hansen, PRD86 016007
Briceno, PRD88 094507
Guo, PRD88 014051

in the two-channel case $\mathbf{t} = \begin{bmatrix} t_{\pi\eta\to\pi\eta} & t_{\pi\eta\to K\overline{K}} \\ t_{K\overline{K}\to\pi\eta} & t_{K\overline{K}\to K\overline{K}} \end{bmatrix}$

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scattering matrix — phase space — known functions

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- parameterizing the energy-dependence in a unitarity-preserving way

_K_-matrix approach

$$\mathbf{t}^{-1}(E) = \mathbf{K}^{-1}(E) + \mathbf{I}(E)$$

$$\mathrm{Im}\,\big(\mathbf{I}(E)\big)_{ij} = -\delta_{ij}\,\rho_i(E)$$

e.g. "Chew-Mandelstam" phase-space

e.g. poles plus polynomial form

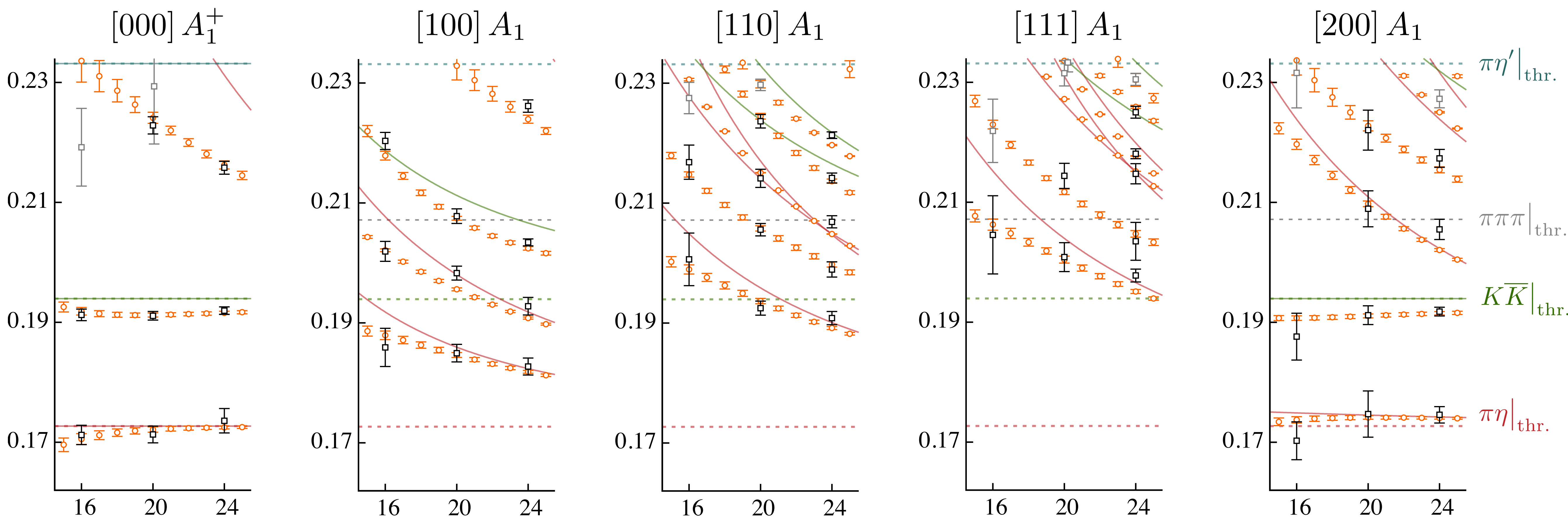
$$\big(\mathbf{K}(E)\big)_{ij} = \sum_p \frac{g_i^{(p)} g_j^{(p)}}{m_p^2 - E^2} + \sum_n \gamma_{ij}^{(n)} (E^2)^n$$

$\pi\eta/K\bar{K}$ scattering describing the spectra

$m_\pi \sim 391\,\mathrm{MeV}$

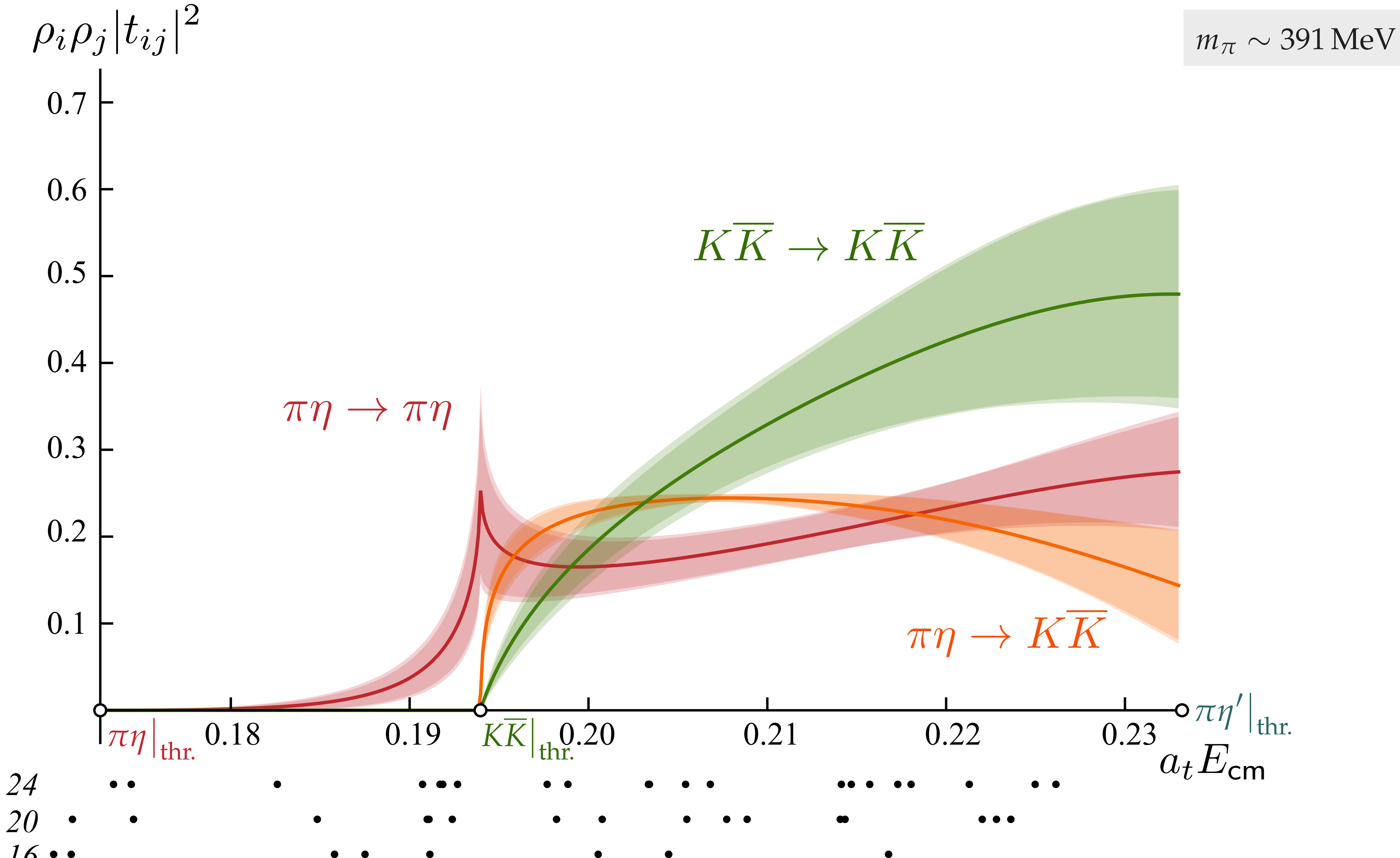
K-matrix parameterization: one pole plus constant matrix

(6 free parameters)

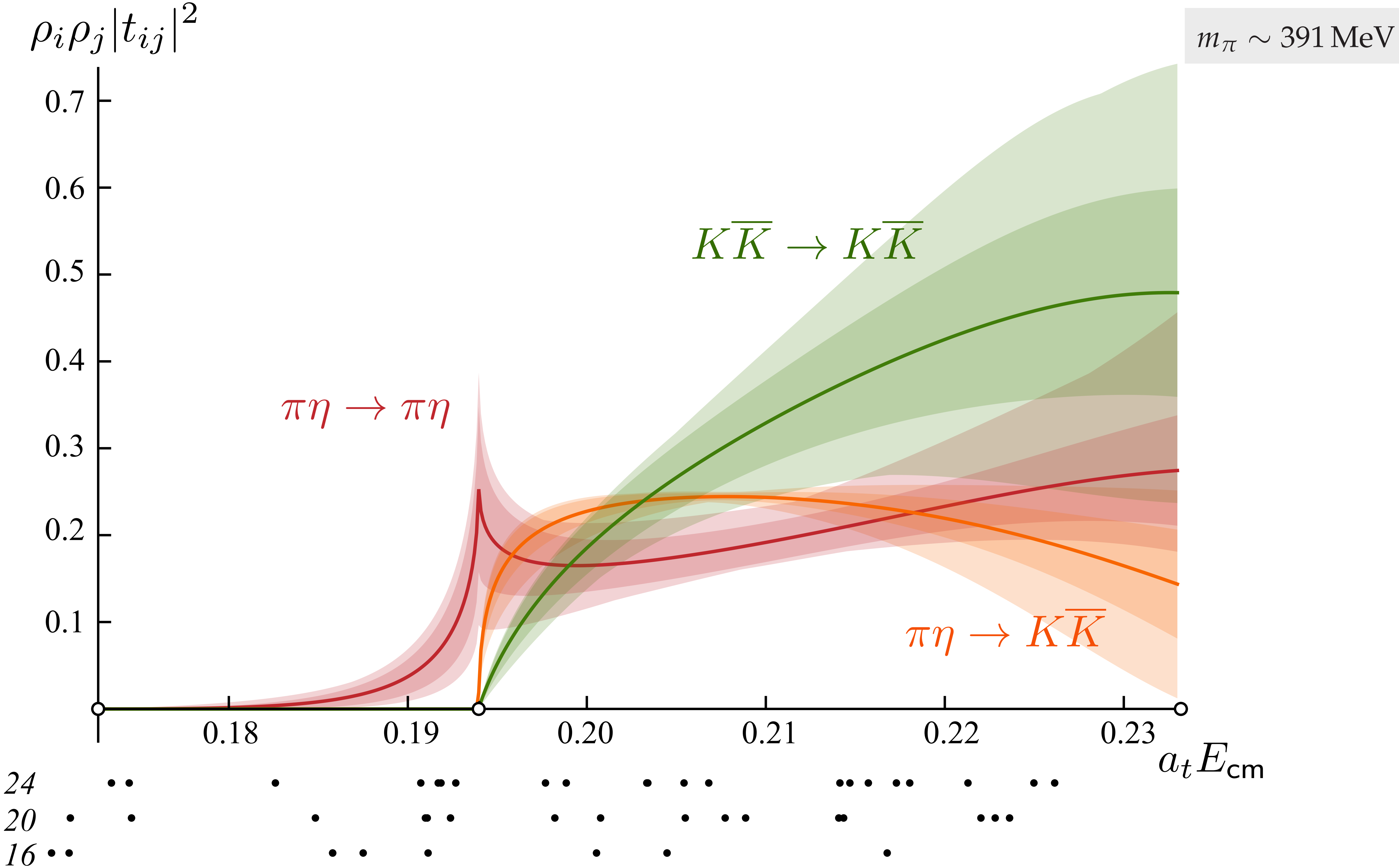


$$\chi^2/N_{\mathrm{dof}} = \frac{58.0}{47-6} = 1.4$$

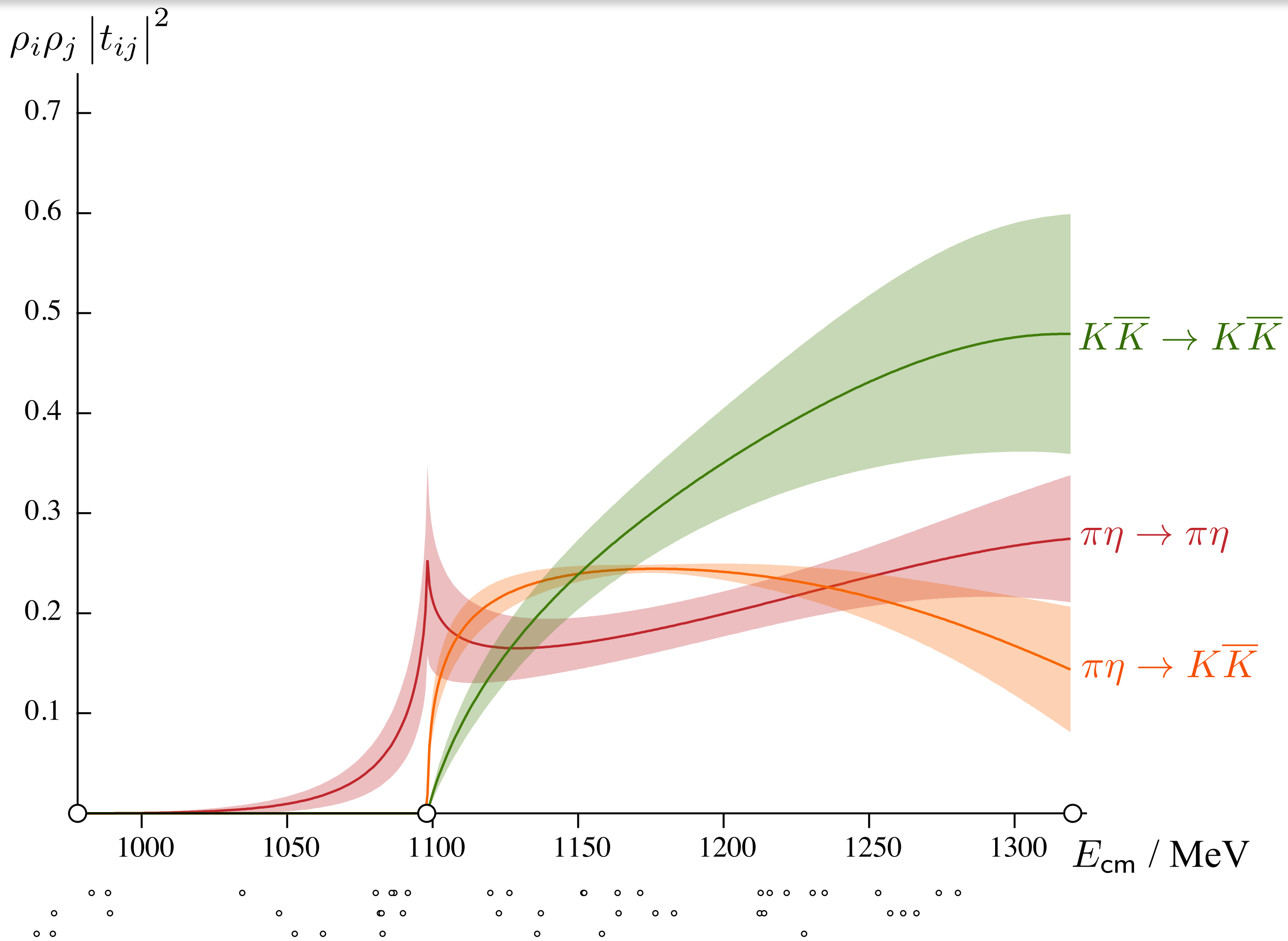
$\pi\eta/K\bar{K}$ scattering in $J^P = 0^+$



parameterization variation



$\pi\eta/K\overline{K}$ scattering in $J^P = 0^+$



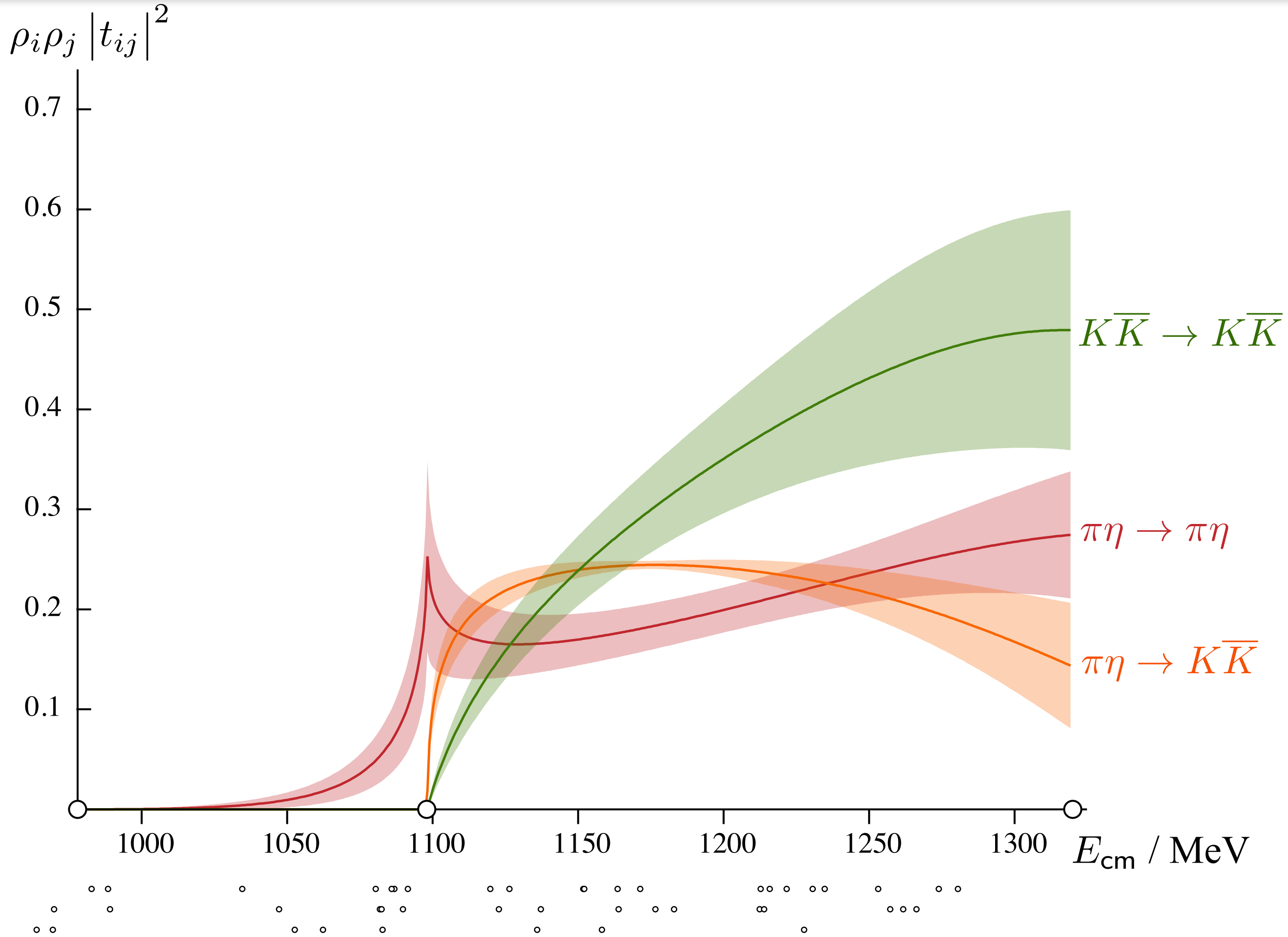
$m_\pi \sim 391\,\mathrm{MeV}$

strong cusp in $\pi\eta$ at $K\overline{K}$ threshold

rapid turn-on of $K\overline{K}$ amplitudes

indicative of a nearby **resonance** ?

$\pi\eta/K\bar{K}$ scattering in $J^P = 0^+$



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strong cusp in $\pi\eta$ at $K\bar{K}$ threshold

rapid turn-on of $K\bar{K}$ amplitudes

indicative of a nearby **resonance** ?

how do we determine rigorously if an amplitude is resonant ?

look for a **pole singularity** at **complex energy**

$$t_{ij}(s) \sim \frac{g_i \, g_j}{s_0 - s}$$

Re[$\sqrt{s_0}$] ~ ‘mass’

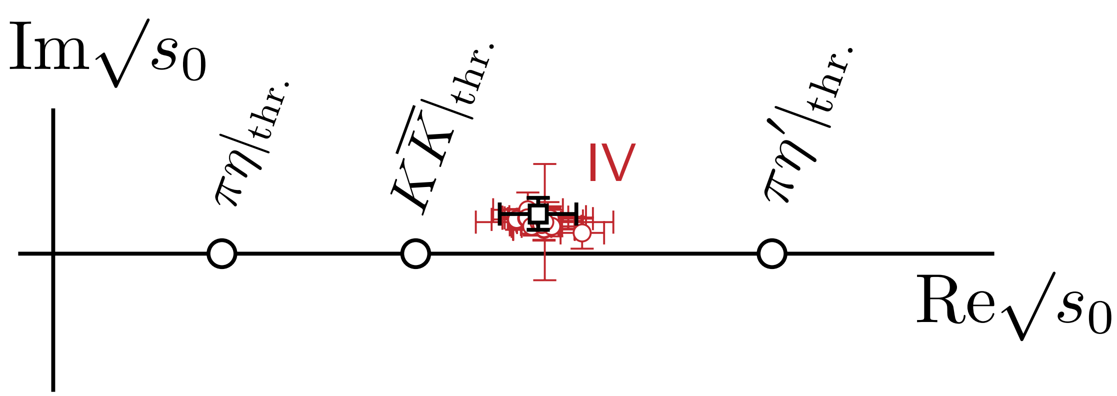
2·Im[$\sqrt{s_0}$] ~ ‘width’

$\pi\eta/K\bar{K}$ scattering in $J^P = 0^+$

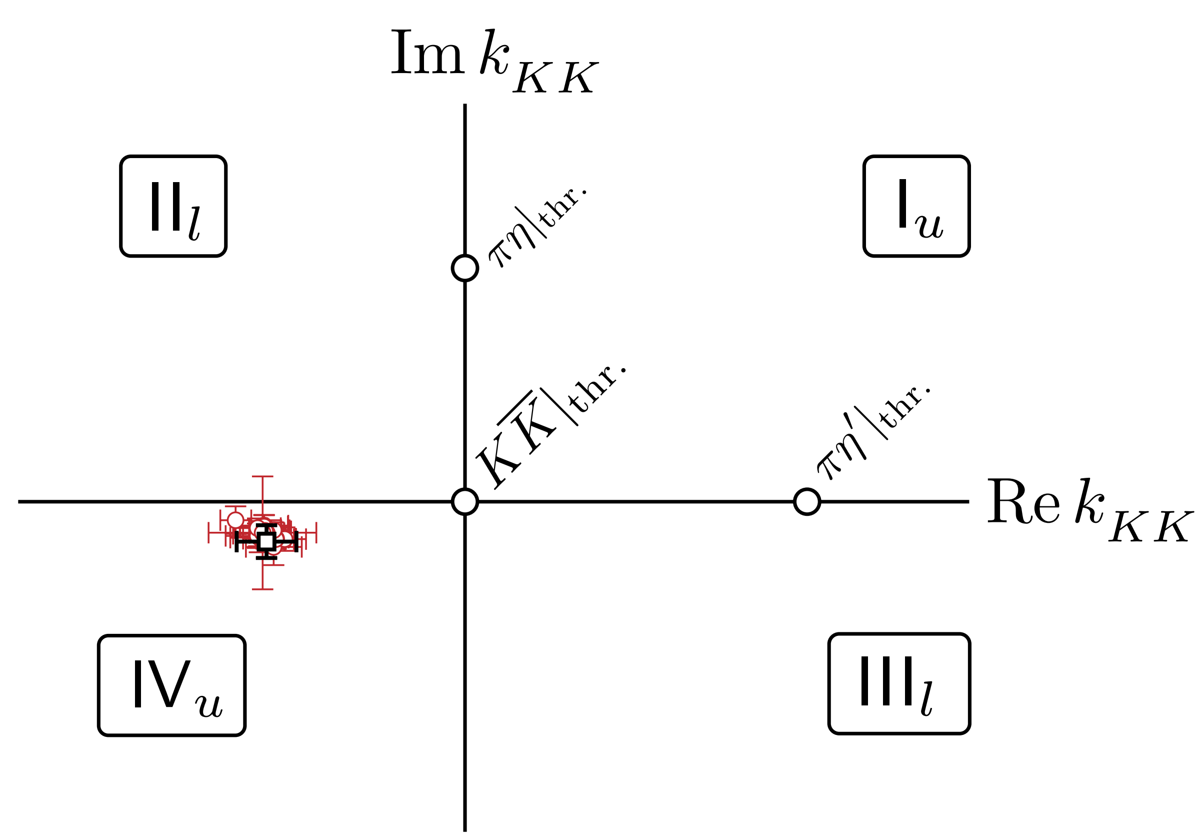
$m_\pi \sim 391\,\text{MeV}$

- we find a single dominant (nearby) pole

COMPLEX ENERGY PLANE



COMPLEX MOMENTUM PLANE



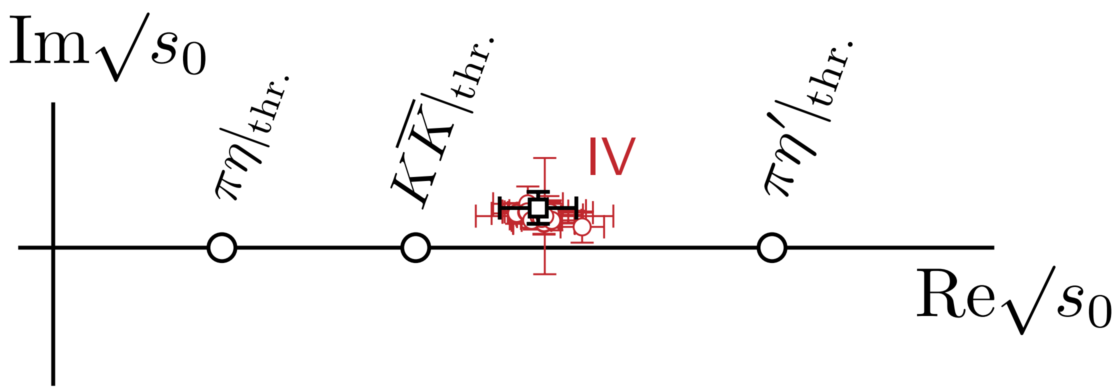
Sheet	$\text{Im}k_{\pi\eta}$	$\text{Im}k_{K\bar{K}}$
I	+	+
II	−	+
III	−	−
IV	+	−

$\pi\eta/K\overline{K}$ scattering in $J^P = 0^+$

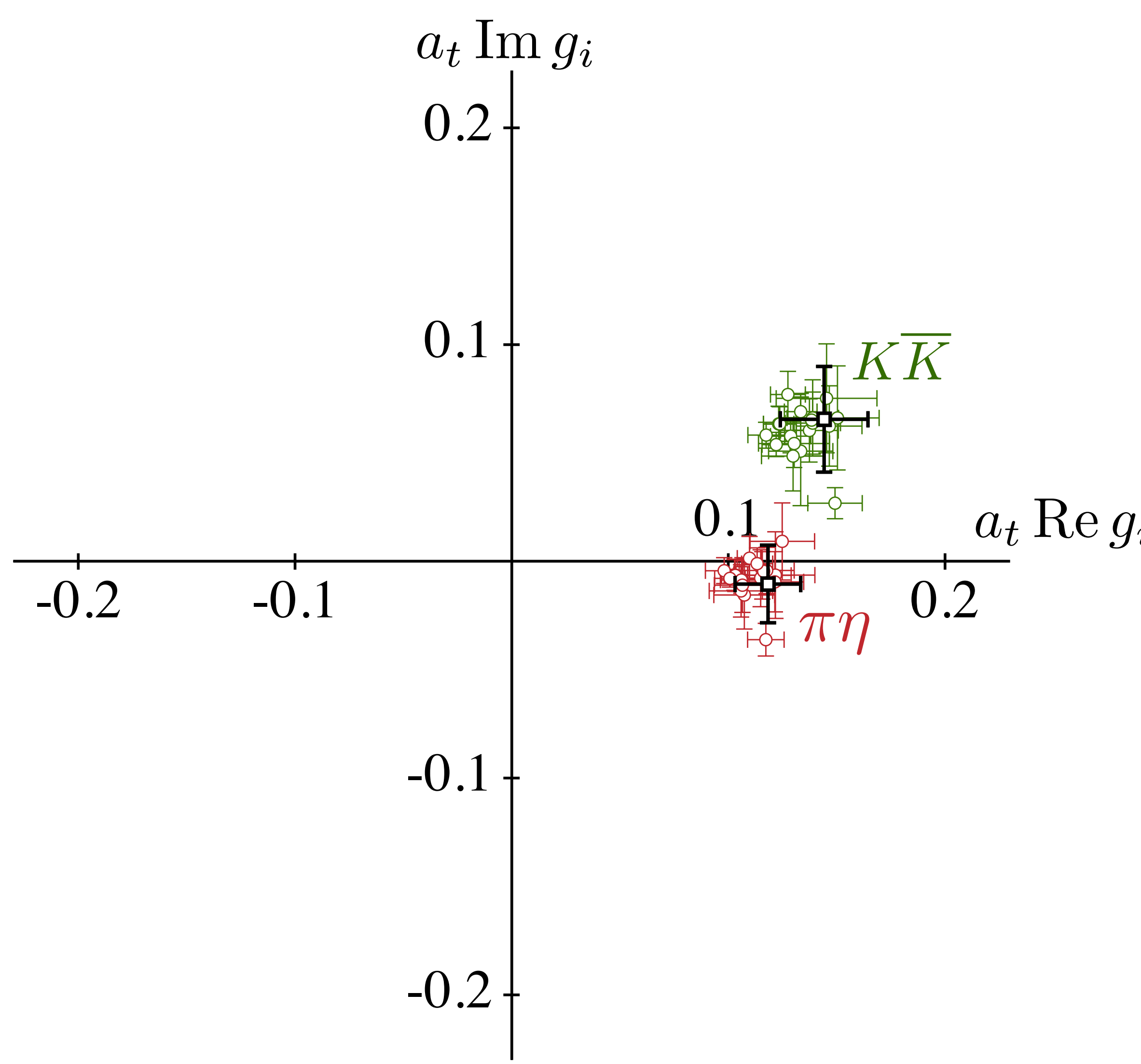
- we find a single dominant (nearby) pole

$m_\pi \sim 391\,\mathrm{MeV}$

COMPLEX ENERGY PLANE

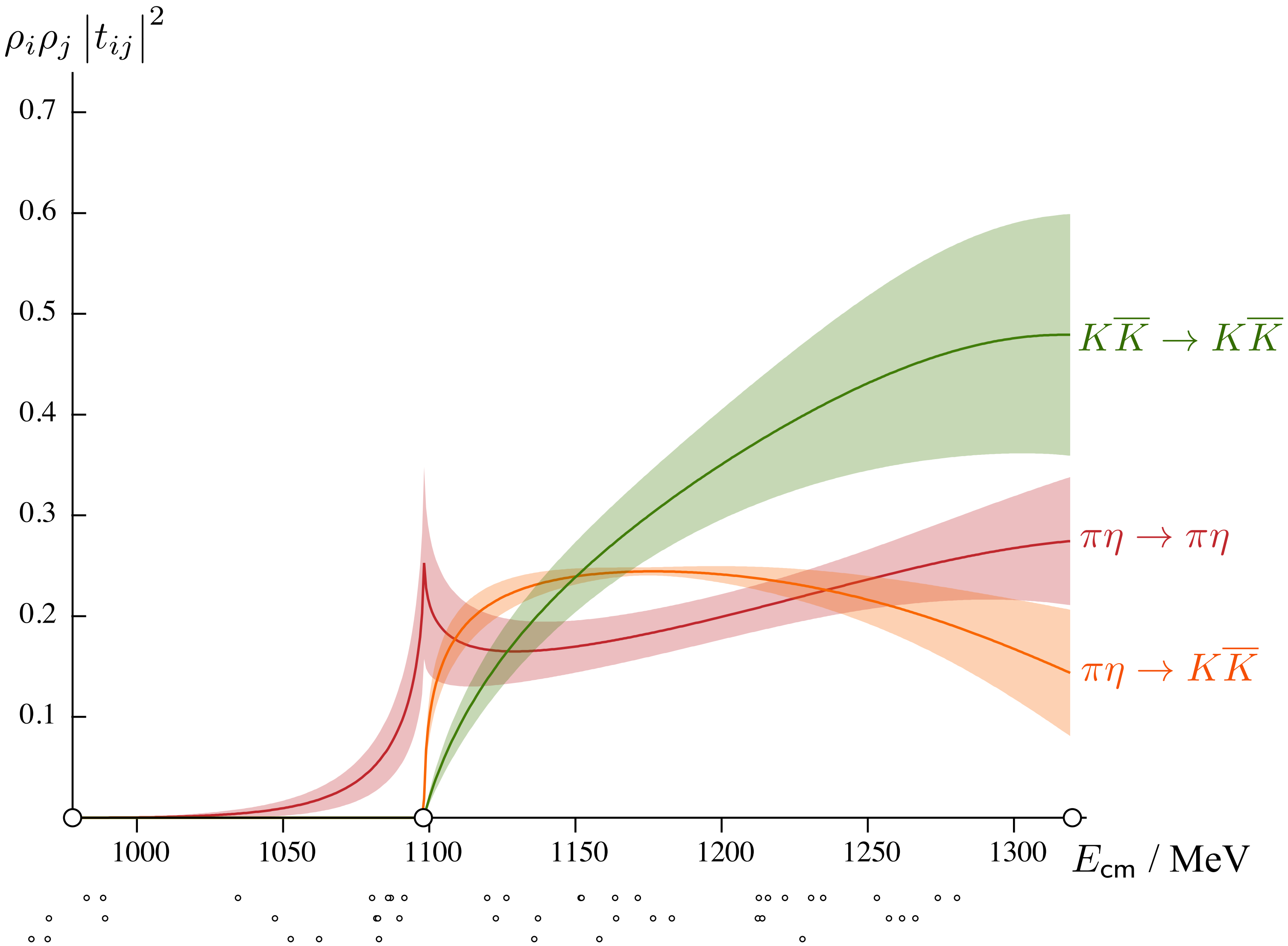


RESONANCE POLE COUPLINGS



$\pi\eta/K\overline{K}$ scattering in $J^P = 0^+$

scale setting using the Ω baryon mass



$m_\pi \sim 391$ MeV

- resonance found near $K\overline{K}$ threshold

pole position

$$\sqrt{s_0} = \Big((1177 \pm 27) + \tfrac{i}{2}(49 \pm 33) \Big) \text{ MeV}$$

$$\text{Re}\sqrt{s_0} - 2m_K = 79 \pm 27$$

pole coupling ratio

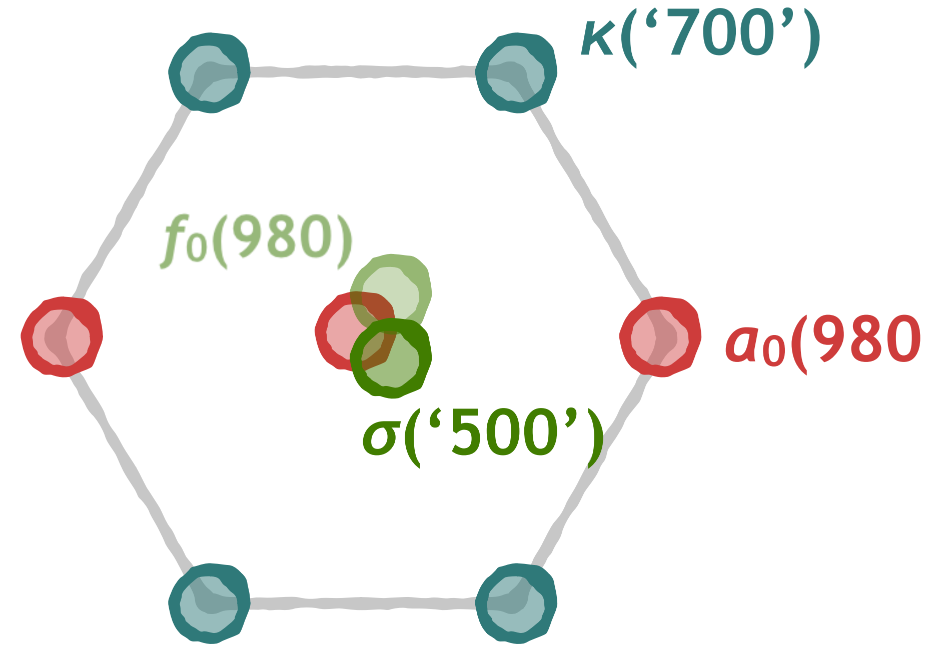
$$\left| \frac{g_{K\bar{K}}}{g_{\pi\eta}} \right| = 1.3(4)$$

complete study of scalar mesons ?

at unphysical quark mass (for now)

- Raul Briceño (previous session) presented $\pi\pi$ elastic scattering in I=0 with the σ
  - bound state at m_π=391 MeV
  - broad resonance at m_π=236 MeV ARXIV:1607.05900

  extension to coupled channel ($\pi\pi$, $K\bar{K}$, …) and likely f_0('980') coming up



- πK, ηK scattering already done at m_π=391 MeV
  … at m_π=236 MeV to come soon …

  κ as a virtual bound-state ?

  PRL113 182001 (2014)
  PRD91 054008 (2015)

- $\pi\eta$, $K\bar{K}$ at m_π=236 MeV to come soon …

what are the scalar mesons ?

- what tools do we have at our disposal ?

  - quark mass dependence of the pole positions and channel couplings

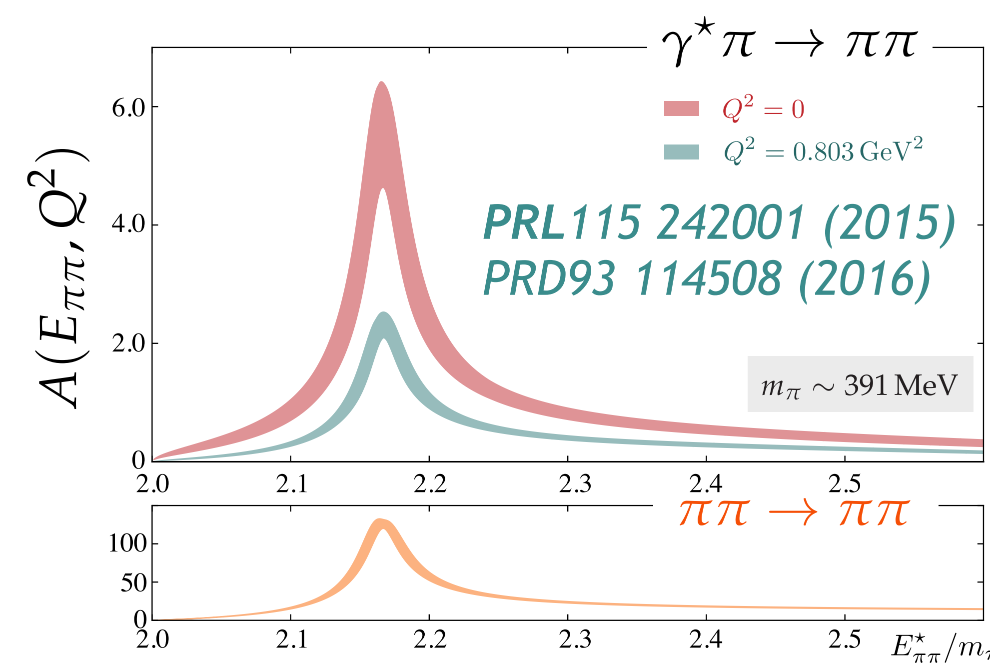
  - distribution of poles across Riemann sheets

    may be relatable to loose meson-meson molecule versus tightly bound object

  - coupling to external currents

    form-factors from residue at the resonance pole

    finite-volume formalism demonstrated with $\gamma\pi \rightarrow \pi\pi$



  - and things we haven't thought of yet …

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MESON SPECTRUM

PRL103 262001 (2009)	$I=1$
PRD82 034508 (2010)	$I=1, K^\star$
PRD83 111502 (2011)	$I=0$
JHEP07 126 (2011)	$c\bar{c}$
PRD88 094505 (2013)	$I=0$
JHEP05 021 (2013)	D, D_s

BARYON SPECTRUM

PRD84 074508 (2011)	$(N,\Delta)^\star$
PRD85 054016 (2012)	$(N,\Delta)_{\rm hyb}$
PRD87 054506 (2013)	$(N\ldots\Xi)^\star$
PRD90 074504 (2014)	$\Omega^\star_{ccc}$
PRD91 094502 (2015)	$\Xi^\star_{cc}$

HADRON SCATTERING

PRD83 071504 (2011)	$\pi\pi\ I=2$
PRD86 034031 (2012)	$\pi\pi\ I=2$
PRD87 034505 (2013)	$\pi\pi\ I=1, \rho$
PRL113 182001 (2014)	$\pi K, \eta K : K^\star$
PRD91 054008 (2015)	$\pi K, \eta K : K^\star$
PRD92 094502 (2015)	$\pi\pi, K\bar{K} : \rho$
PRD93 094506 (2016)	$\pi\eta, K\bar{K} : a_0$
ARXIV:1607.05900	$\pi\pi\ I=0, \sigma$
ARXIV:1607.07093	$D\pi, D\eta, D_s\bar{K}$

MATRIX ELEMENTS

PRD90 014511 (2014)	$f_{\pi^\star}$
PRD91 114501 (2015)	$M' \to \gamma M$
PRL115 242001 (2015)	$\gamma^\star\pi \to \pi\pi$
PRD93 114508 (2016)	$\gamma^\star\pi \to \pi\pi$

LATTICE TECH.

PRD79 034502 (2009)	lattices
PRD80 054506 (2009)	distillation
PRD85 014507 (2012)	$\vec{p} > 0$