Static and non-static vector screening masses

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3. Transport properties of the QCD plasma
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   B.B.Brandt, A.Francis, M.Laine, H.B.Meyer, arXiv:1408.5917, PoS QM 2014

The Debye Screening Mass

in QED

$$k^2 + \Pi_{00}(0, k)|_{k^2=-m_E^2} = 0 \quad (1)$$

defines the static screening Debye mass as the pole of the longitudinal static photon self-energy

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in QCD the correct observable (has to be odd under Euclidean time-reversal) would be

$$\mathfrak{Im}\left(\mathrm{tr}\left[P\right]\right) \tag{2}$$

where P is Polyakov loop $\Rightarrow$ chromo-electric Debye mass

P.Arnold, L.G.Yaffe, hep-ph/9508280

Q: how well is a $U(1)$ electric field screened in QCD plasma?

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- there will be an effective vector screening mass $M_V \leftrightarrow$ screening length of a $U(1)$ electric field
- quark blob will contain chromo-electric Debye mass $m_E \leftarrow$ dealt with by EFT and LQCD

Effective Approach

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- for thermal gluons there are IR problems in the colour-magnetic dof
- therefore they cannot be treated perturbatively
- make use of scale hierarchy $g^2 T \ll gT \ll 2\pi T$ naturally arising in a thermal description

$$m_E^2 = g^2 T^2 \left(\frac{N_C}{3} + \frac{N_f}{6} \right)$$

Implementation of our problem (dimensional reduction)

the vector current screening correlator

$$G_{\mu\nu}^{(k_n)}(z) = \int_0^\beta \mathrm{d}\tau e^{ik_n\tau} \int_{\mathbf{x}} \left\langle \left(\overline{\psi}\gamma_\mu\psi\right)(\tau,\mathbf{x},z)\left(\overline{\psi}\gamma_\mu\psi\right)(0)\right\rangle \quad (3)$$

$$\to G_{\mu\nu}^{(k_n)}(z) = T\int_{\mathbf{x}} \left\langle V_\mu^{(k_n)}(\mathbf{x},z)V_\nu^{(-k_n)}(0)\right\rangle, \quad (4)$$

$$\mathbf{x} = (x_1, x_2)^T \to \text{transverse plane}$$

$$\overline{\psi}(\tau) = T\sum_{p_n} e^{-ip_n\tau}\overline{\psi}_{p_n}$$

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$$V_\mu^{(k_n)}(\mathbf{x},z) = T\sum_{p_n}\overline{\psi}_{p_n}(\mathbf{x},z)\gamma_\mu\psi_{p_n-k_n}(\mathbf{x},z)$$

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404

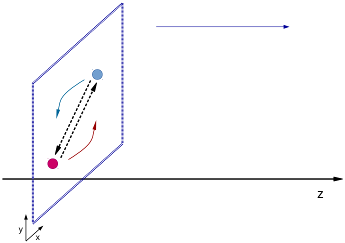
1-gluon exchange potential

non-relativistic auxiliary fields in the transverse plane

$$\psi = \frac{1}{\sqrt{T}} \begin{pmatrix} \chi \\ \phi \end{pmatrix} \tag{5}$$

remember: quarks carry momenta $\sim \pi T \gg gT \gg g^2 T$

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404



1-gluon exchange potential

dimensional reduction through Matsubara formalism

$$V^{+}_{\mathrm{LO}}(\mathbf{y}) = \frac{g_E^2 C_F}{2\pi}\left[\log\left(\frac{m_E y}{2}\right) + \gamma_E + K_0(m_E y)\right]$$

with $g_E^2 = g^2 T$ effective coupling of dimensionally reduced theory, m_E Debye mass and K_0 modified Bessel function

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with $g_E^2 = g^2 T$ effective coupling of dimensionally reduced theory, m_E Debye mass and K_0 modified Bessel function

the same effective potential enters the calculation of the dilepton production rate (cf. B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404; H.B.Meyer, arXiv:1512.06634, PoS Lattice 2015 and refs. therein)

Extracting screening masses

The radial homogeneous part of the S-eqn. to be solved reads

$$\begin{aligned} \left\{-\frac{d^2}{d\overline{y}^2}-\frac{1}{\overline{y}}\frac{d}{d\overline{y}}+\frac{l^2}{\overline{y}^2}+\rho\left(\frac{2\pi V^{\pm}}{g_E^2 C_F}-\hat{E}^{(l)}\right)\right\}R_l &= 0 \\ \Rightarrow \left\{-\frac{d^2}{d\overline{y}^2}-\frac{1}{\overline{y}}\frac{d}{d\overline{y}}+\frac{l^2}{\overline{y}^2}+\rho\left(\hat{V}^{\pm}-\hat{E}^{(l)}\right)\right\}R_l &= 0 \end{aligned} \quad (6)$$

with dimensionless quantities

$$\overline{y}=m_E y, \quad \rho=\frac{g_E^2 C_F M_r}{\pi m_E^2}$$

and $g_E^2 = g^2 T,\ C_F = \frac{N_C^2-1}{2N_C}$

Extracting Screening Masses

$$\begin{aligned} E_{\text{full}} &= M_{cm} + \frac{g_E^2 C_F}{2\pi} \hat{E}^{(I)} \\ M_{cm} &= k_n + \frac{m_\infty^2}{2M_r}, \quad m_\infty^2 = \frac{g^2 T^2 C_F}{4}, \quad M_r = \left(\frac{1}{p_n} - \frac{1}{k_n - p_n} \right)^{-1} \end{aligned} \tag{7}$$

E_{full} can be understood as screening masses

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404

Screening amplitudes

the screening correlator exhibits as a long-distance behaviour; with a proper ansatz for the radial S-eqn. one finds

$$
\begin{aligned}
-\frac{G_{00}^{(k_n)}(z)}{T^3} &\approx \frac{N_c m_E^2 \mathcal{A}_0^+}{\pi T^2} e^{-|z| E_0^{(l=0)}} \\
-\frac{G_{T}^{(k_n)}(z)}{T^3} &\approx \frac{N_c m_E^4 \mathcal{A}_1^+}{\pi T^2} \left[\frac{1}{p_n^2} + \frac{1}{(k_n - p_n)^2} \right] e^{-|z| E_0^{(l=1)}}
\end{aligned}
$$

with

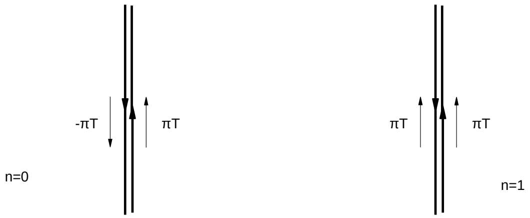
$$
\mathcal{A}_0^+ = \frac{|R_0(0)|^2}{\int_0^\infty \mathrm{d}\bar{y}\bar{y}|R_0(\bar{y})|^2}, \mathcal{A}_1^+ = \frac{|R_1'(0)|^2}{\int_0^\infty \mathrm{d}\bar{y}\bar{y}|R_1(\bar{y})|^2} \tag{8}
$$

for S-wave ($l = 0$) and P-wave ($l = 1$) channels, respectively

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404

Screening amplitudes

the situation is very similar for the static case



keep in mind: roles of transversal and longitudinal part of the correlator are reversed

Lattice setup

$N_\tau \cdot N_x^3 = 16 \cdot 64^3$ lattice with $a \approx 0.024\mathrm{fm}$ corresponding to

$$T = \frac{1}{N_\tau a} \cong 508\mathrm{MeV}$$

Lattice setup

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T &= \frac{1}{N_\tau a} \cong 508\mathrm{MeV} \\
\frac{6}{g_0^2} \approx 6.038, \quad \kappa &= 0.136238, \quad c_{\mathrm{sw}} = 1.51726
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with $N_f = 2$ $\mathcal{O}(a)$-improved Wilson-type fermions generated in Mainz on 'Clover' (using MP-HMC) exploiting $N_{\mathrm{cfg}} = 345$ and $N_{\mathrm{src}} = 64$ rnd src

Lattice formulation

$$G_{\mu\nu}^{(k_n)}(z) = \sum_{n=1}^{2} A_n \frac{\cosh[M_n(z - L_z/2)]}{\sinh[M_n L_z/2]}$$

Lattice formulation

$$\begin{aligned} G_{\mu\nu}^{(k_n)}(z) &= \sum_{n=1}^{2} A_n \frac{\cosh[M_n(z - L_z/2)]}{\sinh[M_n L_z/2]} \\ M_1 &= M_{\mathrm{eff}}\,, \quad M_2 = M_{\mathrm{exc}} \end{aligned} \tag{9}$$

A previous study

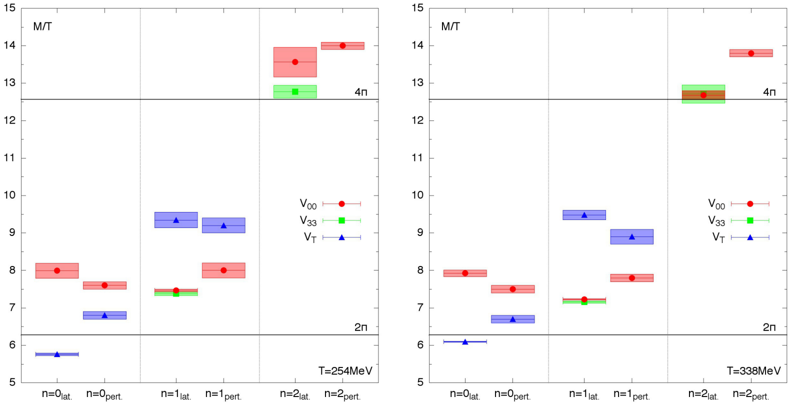

Figure: Screening masses at $T = 254\mathrm{MeV}$ (left panel) and $T = 338\mathrm{MeV}$ (right panel).

The fits, $n = 0$

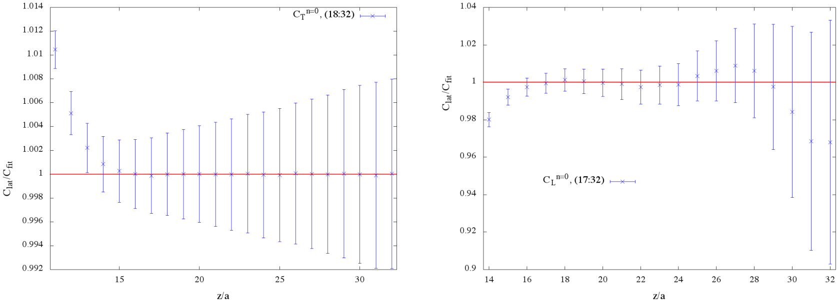

Figure: The correlator and two-state fit for the static case. Left: transversal channel. Right: longitudinal channel.

The fits, $n = 1$

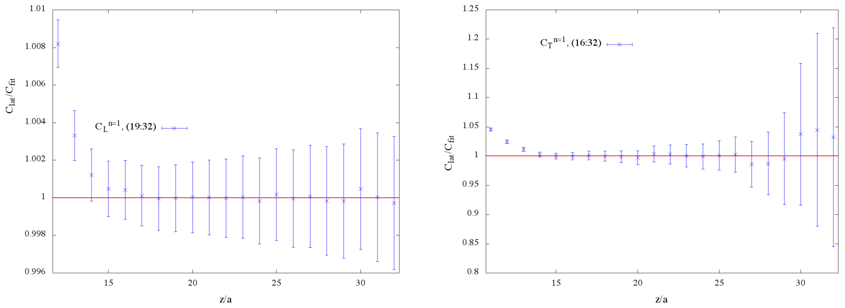

Figure: The correlator and two-state fit for the non-static case. Left: longitudinal channel. Right: transversal channel.

$T = 508\mathrm{MeV}$, screening mass

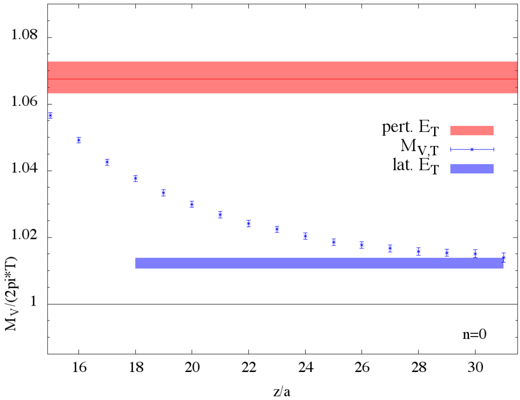

Figure: Static (n=0) transversal (S-wave) screening mass.

$T = 508\mathrm{MeV}$, screening mass

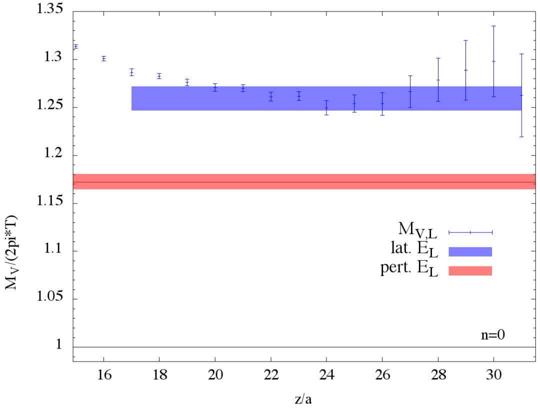

Figure: Static (n=0) longitudinal (P-wave) screening mass.

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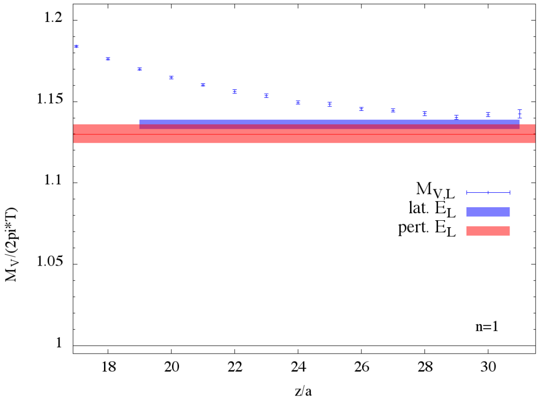

Figure: Non-static (n=1) longitudinal (S-wave) screening mass.

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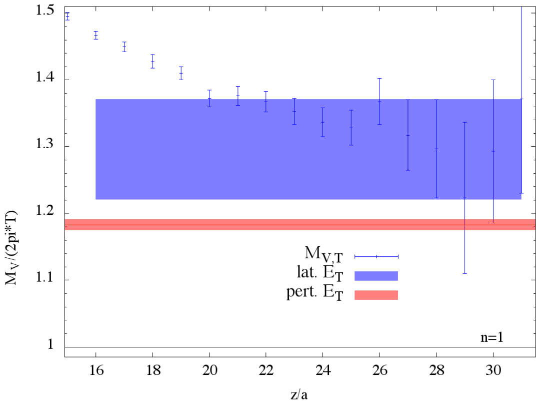

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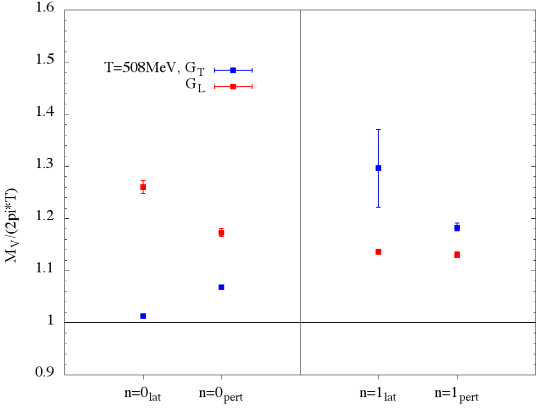
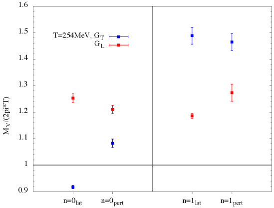
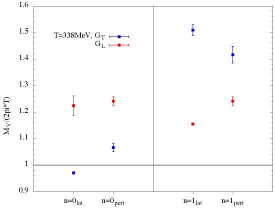


Figure: Comparison of the masses at different T.

$T = 508\mathrm{MeV}$, masses

$T = 254\mathrm{MeV}$ $T = 338\mathrm{MeV}$ $T = 508\mathrm{MeV}$





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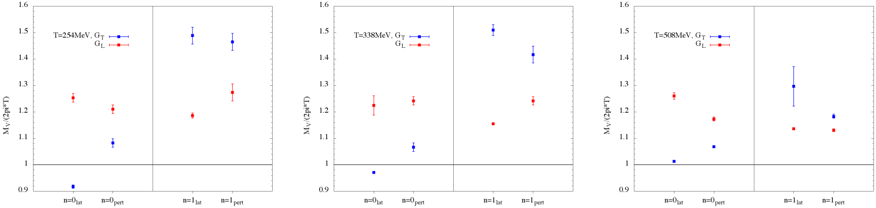

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$T = 508\mathrm{MeV}$, amplitudes

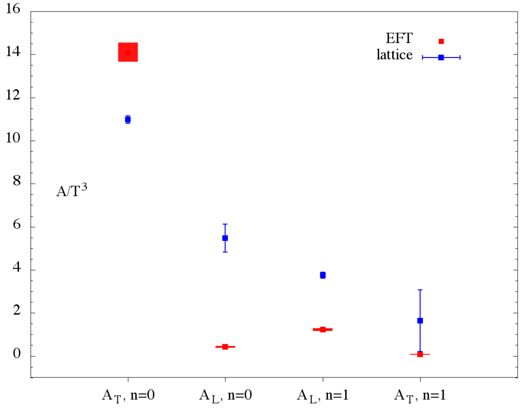

Figure: Comparison of the amplitudes.

Summary and Outlook

- vector screening masses as probes of PT
- EFT for 1-gluon exchange potential between quarks coupling to a photon in the medium
- lattice ansatz
- comparison
- FUTURE: Diffusion coefficient from analytic continuation

The Debye Screening Mass

thermal gluon propagator in Fourier space

$$\left\langle \widetilde{A}^a_\mu(K)\widetilde{A}^b_\nu(Q)\right\rangle \overset{K\approx 0}{\propto} \frac{\delta^{ab}\delta_{\mu\nu}}{K^2+\delta_{\mu 0}\delta_{\nu 0}m_E^2}$$

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$$m_E^2 = g^2T^2\left(\frac{N_C}{3}+\frac{N_f}{6}\right)$$

at leading order:
color-electric fields get screened in a thermal plasma
color-magnetic fields are not screened

Basics of Thermal Field Theory, M.Laine, A.Vuorinen

Charge density correlator

$$G_{00}^{(k_n)}(z) = \lim_{\mathbf{y},\mathbf{y}'\to\mathbf{0}} T \int_{\mathbf{x}} \left\langle V_0^{(k_n)}(\mathbf{x}, z; \mathbf{y}) V_0^{(-k_n)}(0; -\mathbf{y}') \right\rangle$$

with the point-splitting

$$V_0^{(k_n)}(\mathbf{x}, z; \mathbf{y}) \equiv \sum_{0<p_n<k_n} \phi^\dagger(\mathbf{x} + \frac{\mathbf{y}}{2}, z) W_{\mathbf{y},z} \phi_{p_n-k_n}(\mathbf{x} - \frac{\mathbf{y}}{2}, z)$$

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and non-relativistic auxiliary fields

$$\psi = \frac{1}{\sqrt{T}} \begin{pmatrix} \chi \\ \phi \end{pmatrix} \qquad (10)$$

remember: quarks carry momenta$\sim \pi T \gg gT \gg g^2 T$

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404

Effective Potential

after weak-coupling expansion and taking the $\mathbf{y}'$-limit

$$G_{00}^{(k_n)}(z) = -\sum_{0<p_n<k_n} 2N_c T \lim_{\mathbf{y}\to\mathbf{0}} w_{\mathrm{LO}}(z,\mathbf{y}) + \mathcal{O}(\alpha_s)$$

$$w_{\mathrm{LO}}(z,\mathbf{y}) = \int_{\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{y}-(M_{cm}+\frac{q^2}{2M_r})|z|}$$

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$$\begin{aligned}\left(\partial_z + M_{cm} - \frac{\nabla^2}{2M_r}\right) w_{\rm LO}(z,\mathbf{y}) &= 0, \\ w_{\rm LO}(0,\mathbf{y}) &= \delta^{(2)}(\mathbf{y})\end{aligned}$$

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404

Effective Potential and Schrödinger Equation

after NLO corrections and suppressing $\mathbf{y}'$

$$
\begin{aligned}
(\partial_z + M_{cm})\, w_{\mathrm{NLO}}(z,\mathbf{y}) &\overset{z\to\infty}{=} -V^{+}_{\mathrm{LO}}(\mathbf{y}) w_{\mathrm{LO}}(z,\mathbf{y}) \\
V^{+}_{\mathrm{LO}}(\mathbf{y}) &= \frac{g_E^2 C_F}{2\pi}\left[\log\left(\frac{m_E y}{2}\right) + \gamma_E + K_0(m_E y)\right]
\end{aligned}
$$

with $g_E^2 = g^2 T$ effective coupling of dimensionally reduced theory, m_E Debye mass and K_0 modified Bessel function

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with $g_E^2 = g^2 T$ effective coupling of dimensionally reduced theory, m_E Debye mass and K_0 modified Bessel function
with initial condition and a Fourier transform

$\rightarrow$ z-independent inhomogeneous Schrödinger eqn.

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404

Static Sector $k_n = 0$

S-wave contribution from the transverse part of $G_{\mu\nu}(z)$

$$G_T^{(0)} = \lim_{\mathbf{y},\mathbf{y}' \to \mathbf{0}} T \sum_{i=1}^{2} \int_{\mathbf{x}} \left\langle V_i^{(0)}(\mathbf{x}, z; \mathbf{y}) V_i^{(0)}(0; -\mathbf{y}') \right\rangle$$

inducing now

$$V_{\mathrm{LO}}^{-}(\mathbf{y}) = \frac{g_E^2 C_F}{2\pi} \left[\log\left(\frac{m_E y}{2}\right) + \gamma_E - K_0(m_E y) \right]$$

$$M_{cm} = 2p_n + \frac{m_\infty^2}{2M_r}, \quad M_r = \frac{p_n}{2}$$

B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404

$$-\frac{G_T^{(0)}(z)}{T^3} \approx \frac{4N_c m_E^2 \mathcal{A}_0^-}{\pi T^2} e^{-|z|E_0^{(l=0)}}$$
$$-\frac{G_{00}^{(0)}(z)}{T^3} \approx \frac{4N_c m_E^4 \mathcal{A}_1^-}{\pi T^2 p_n^2} e^{-|z|E_0^{(l=1)}}$$

with

$$\mathcal{A}_0^- = \frac{|R_0(0)|^2}{\int_0^\infty \mathrm{d}\overline{y}\overline{y}|R_0(\overline{y})|^2}, \mathcal{A}_1^- = \frac{|R_1'(0)|^2}{\int_0^\infty \mathrm{d}\overline{y}\overline{y}|R_1(\overline{y})|^2} \tag{11}$$

for S-wave ($l = 0$) and P-wave ($l = 1$) channels, respectively (reverse order!) B.B.Brandt,A.Francis,M.Laine,H.B.Meyer, arXiv:1404.2404