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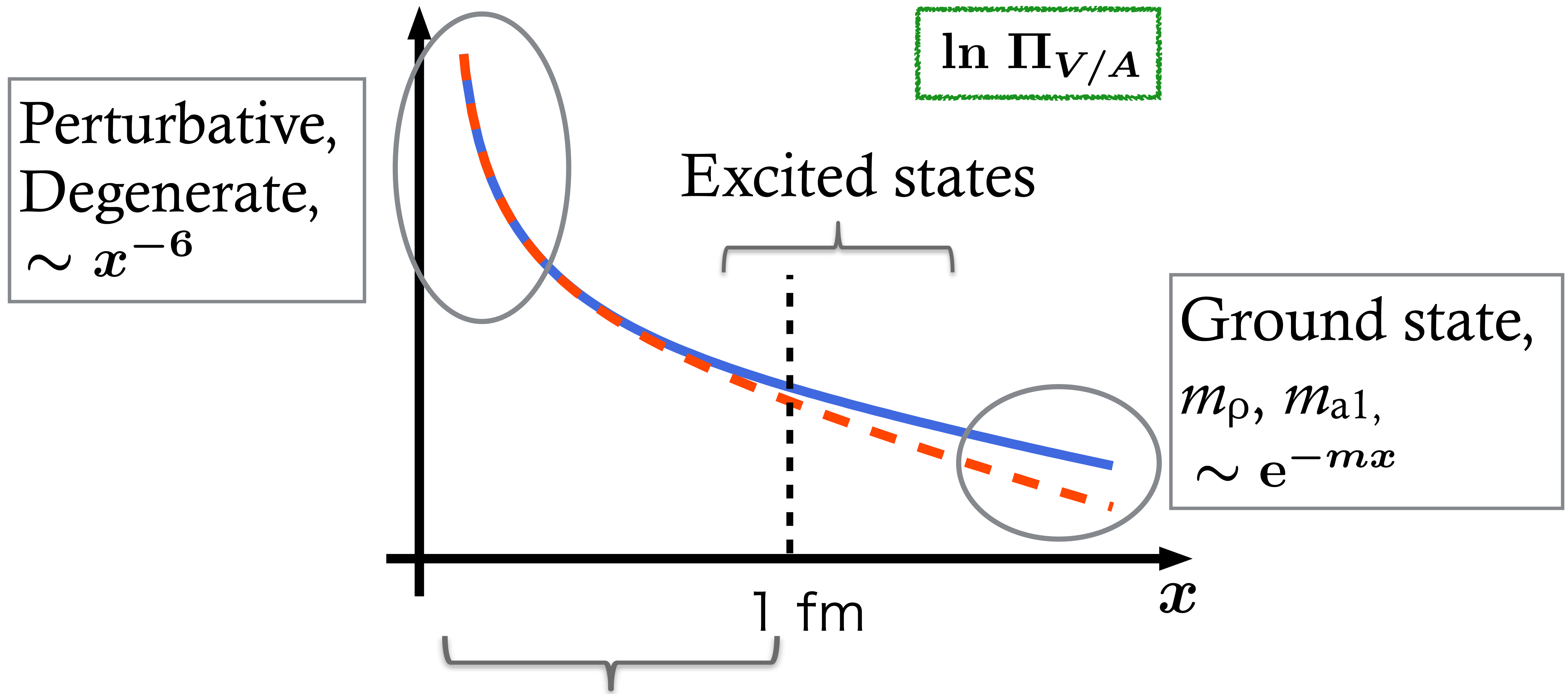
Current correlators in the coordinate space at short distances

Masaaki Tomii
&
Shoji Hashimoto

SOKENDAI
The Graduate University for Advanced Studies

Correlators in coordinate Sp.

$\Pi_{V/A}(x) = \sum_\mu \langle J_\mu^{V/A}(x) J_\mu^{V/A}(0)^\dagger \rangle$ defined in the Euclidean Sp.



α_s, <q̄q>, <GG>, …
Not studied by lattice a lot

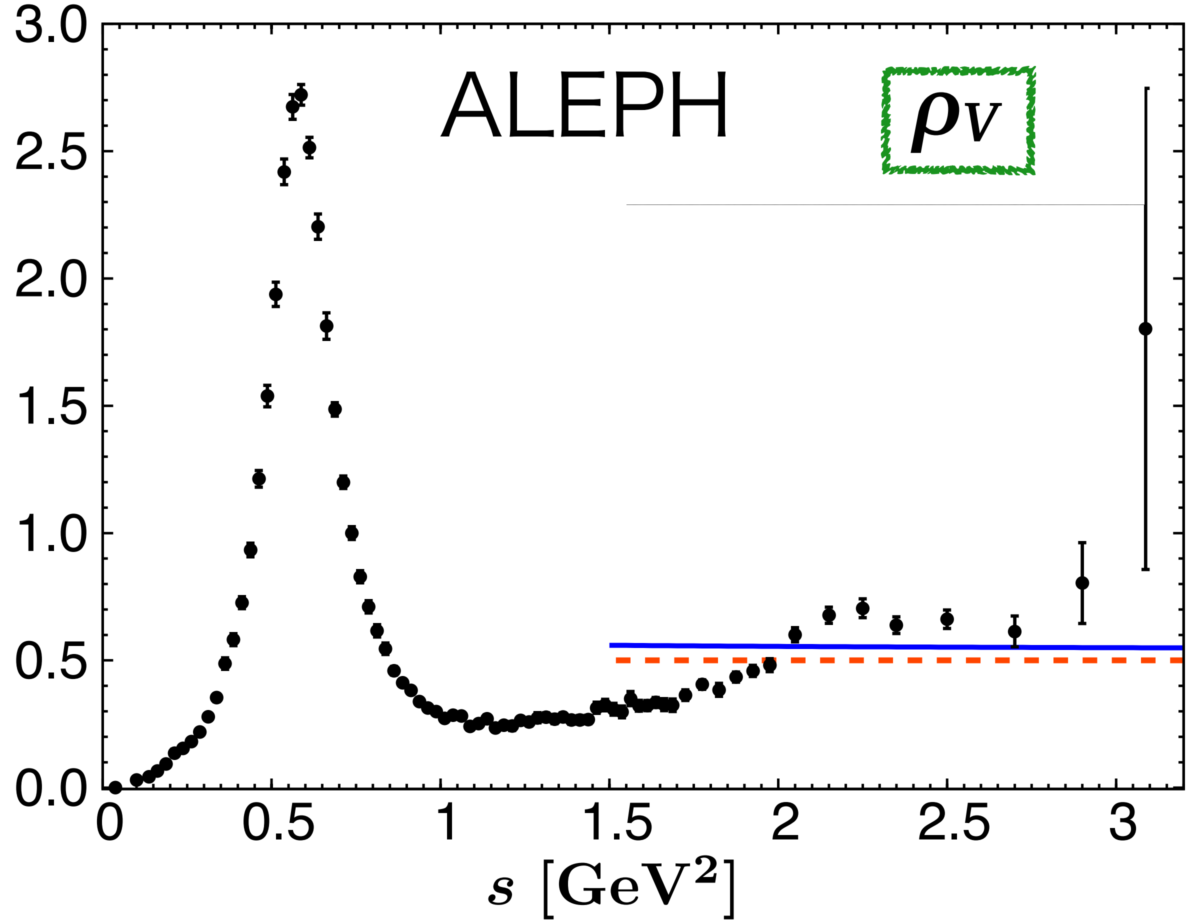
☆ Lattice can calculate them from 1st principle

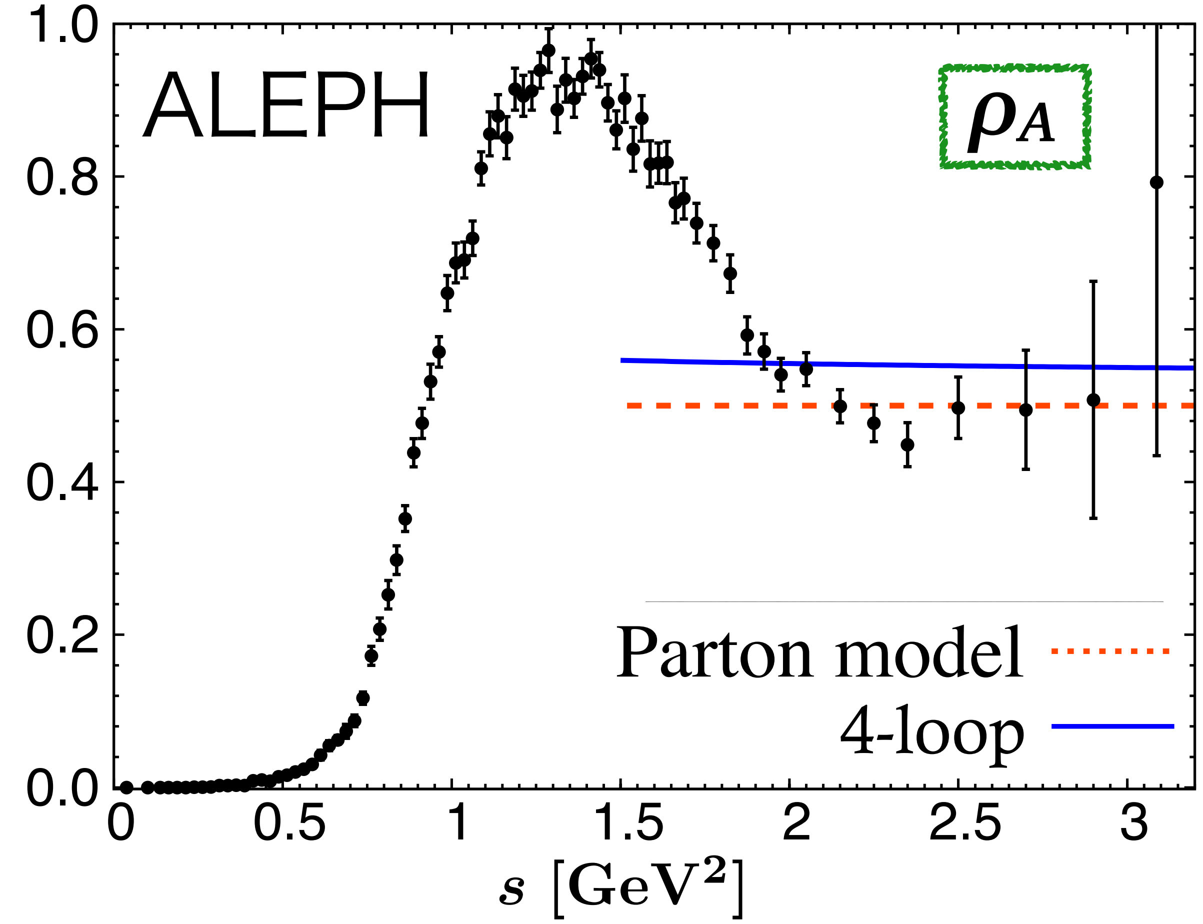
Spectral functions

- Correlators are related to spectral functions

$$\rho(s) \propto \mathrm{Im}\,\widetilde{\Pi}(s)$$

- Hadronic τ decays



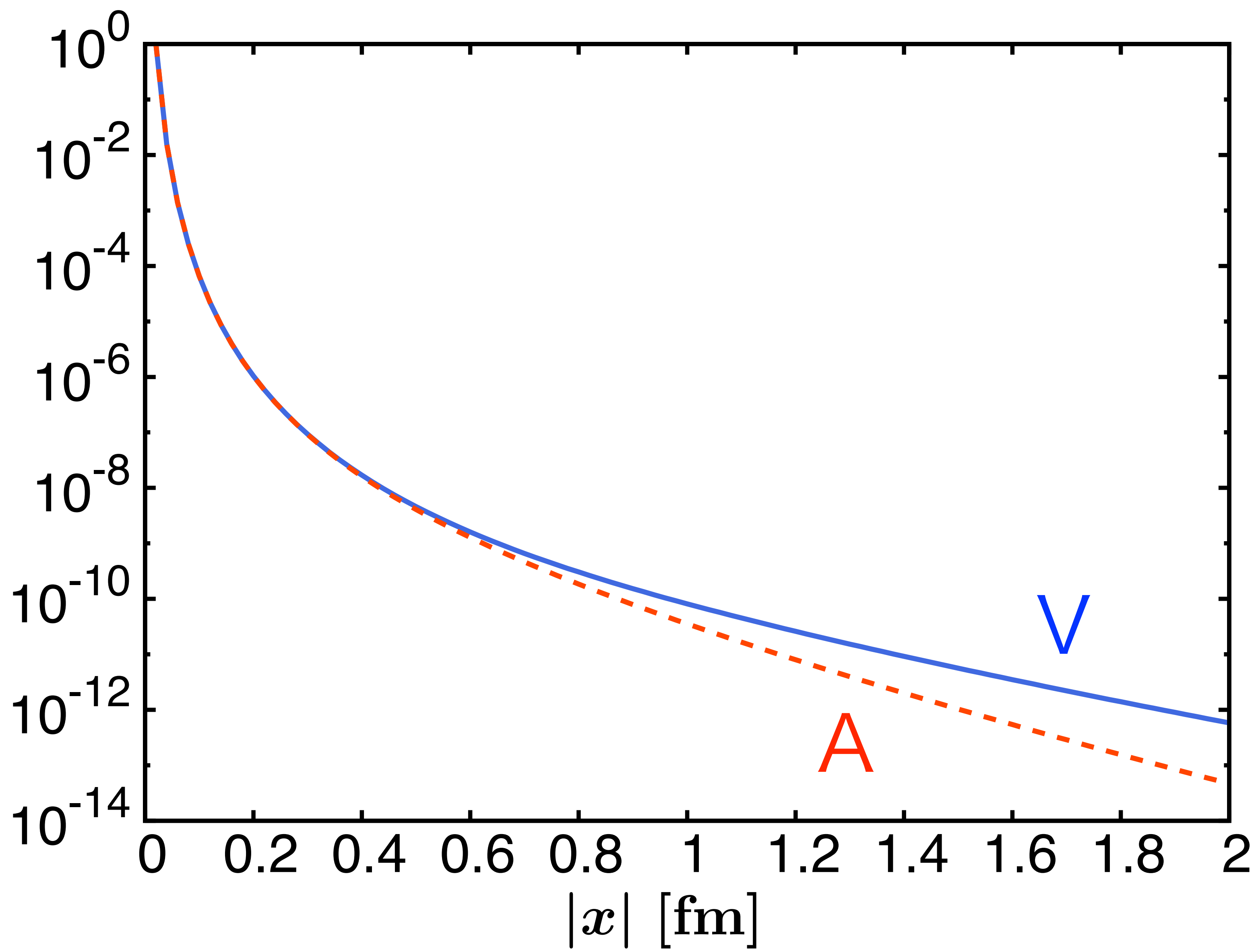


Experimental correlators

- Dispersion relation for Euclidean coordinate Sp.

$$\Pi_{V/A}(x) = \frac{3}{8\pi^4}\int_0^\infty ds\ s^{3/2}\rho_{V/A}(s)\frac{K_1(\sqrt{s}|x|)}{|x|}$$

Schafer, Shuryak, 2001



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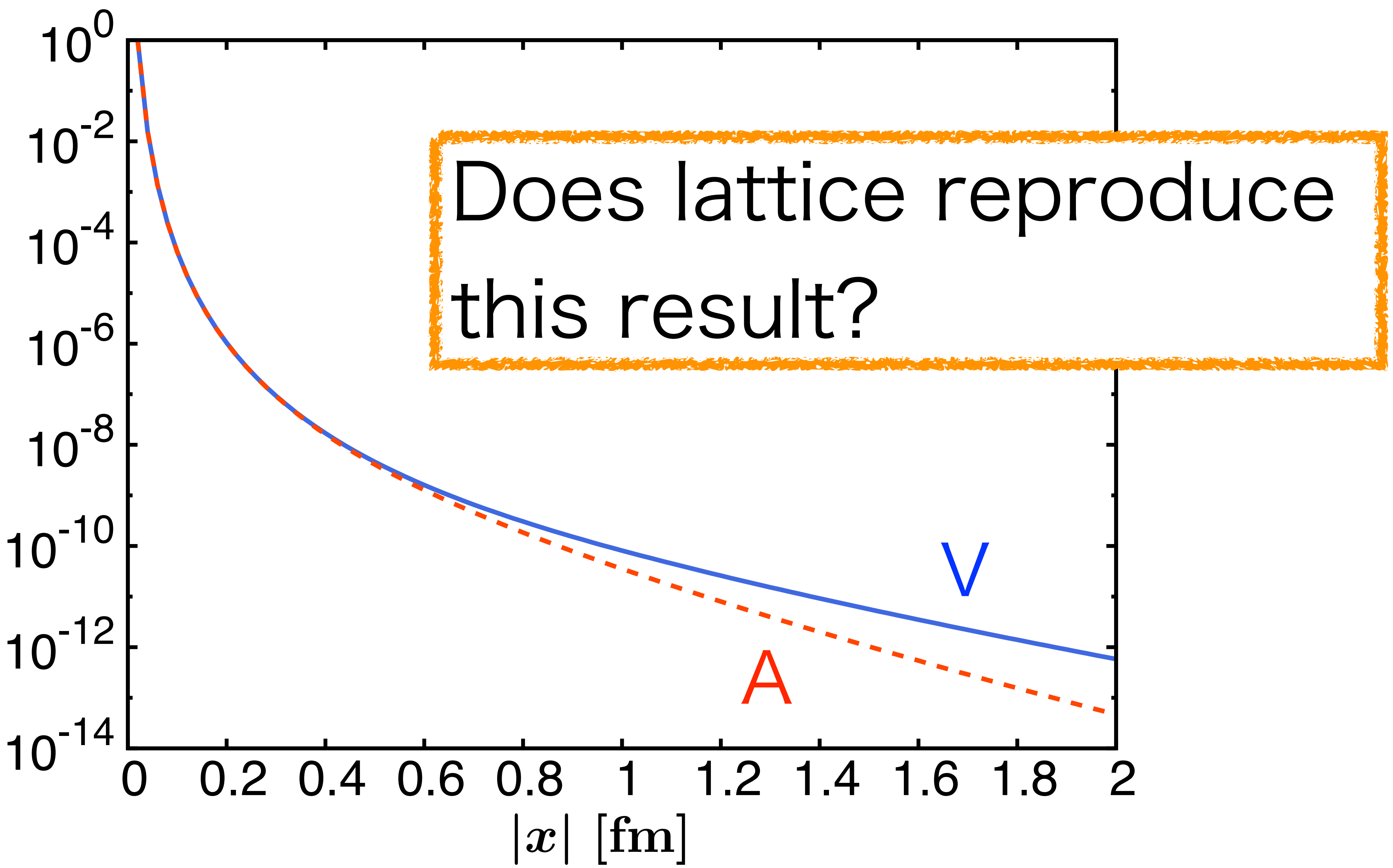


Outline

1. Lattice setup
   - 2+1 Möbius DW fermions
   - 3 cutoffs
   - 14 lattice ensembles

2. Consistency between lattice & experiment
   - V+A, V−A channels
   - Good agreement with experiment at short & middle distances

3. Test of OPE
   - Agreement at short distances < 0.3 fm

Lattice calculation

- We analyze

$$R_{V\pm A}(x) = \frac{\Pi_V(x) \pm \Pi_A(x)}{2\Pi_V^{\mathrm{free}}(x)|_{m_q=0}}$$

  - Clean separation of different contributions

- Lattice action
  - 2 + 1 Möbius DW fermions w/ 3-step stout link smearing
  - Symanzik improved gauge action
  - 3 cutoffs

- Renormalization factor is determined in MT et al, 1604.08702[hep-lat] from a matching to 4-loop PT

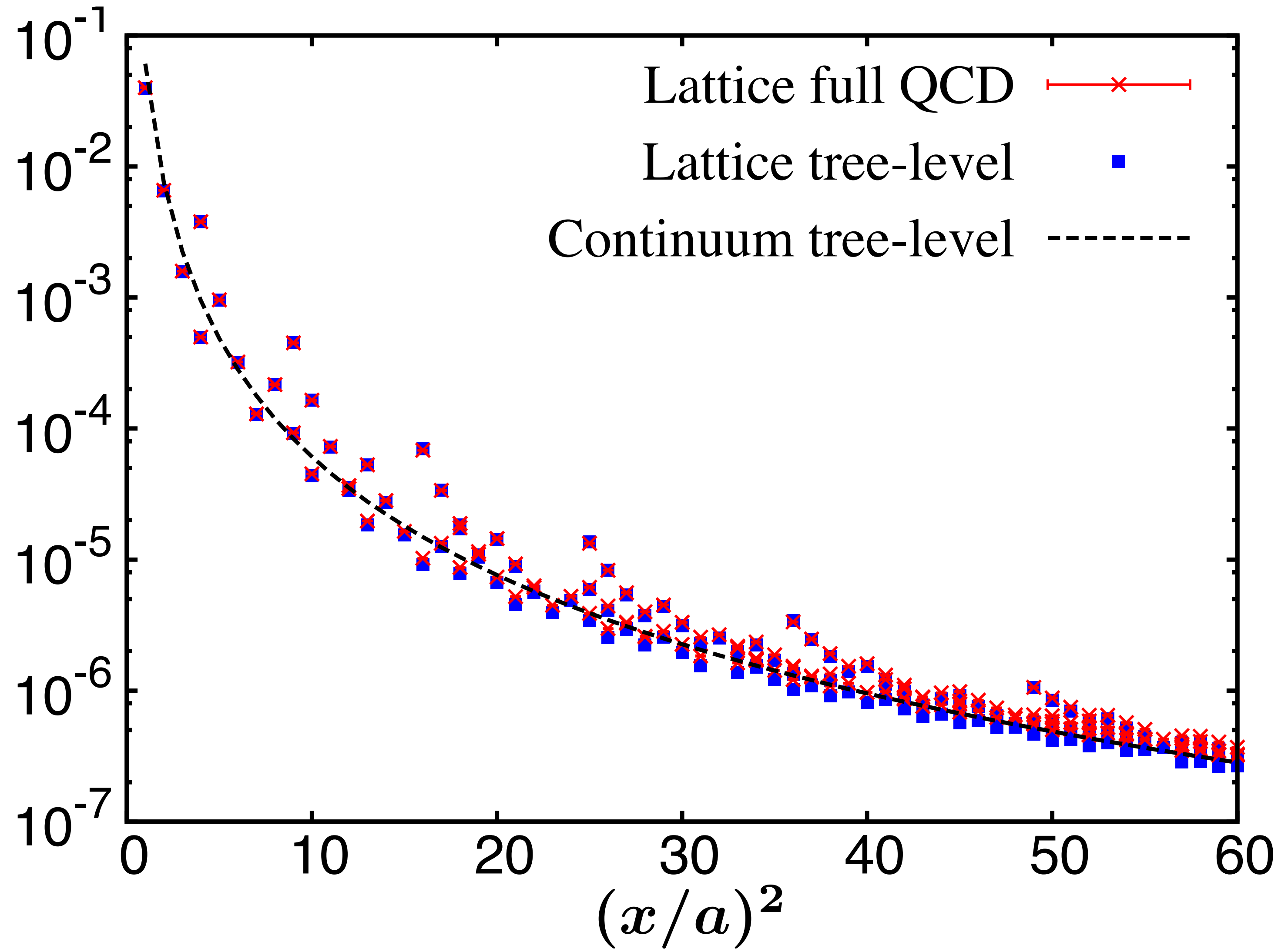
Ensembles

a [fm]	Volume	am_s	am_{ud} (M_π [MeV])
0.080	32^3x64x12	0.030	0.0070 (300), 0.0120 (400), 0.0190 (500)
		0.040	0.0070 (300), 0.0120 (400), 0.0190 (500)
	48^3x96x12	0.040	0.0035 (230)
0.055	48^3x96x8	0.018	0.0042 (300), 0.0080 (400), 0.0120(500)
		0.025	0.0042 (300), 0.0080 (400), 0.0120(500)
0.044	64^3x128x8	0.015	0.0030 (300)

- Residual mass
  - O(1 MeV) for the coarsest lattice
  - Negligible for finer lattices

Reduction of discretization effect

- Tree-level correction



$$\Pi^{\rm lat}_{V/A}(x) \to \Pi^{\rm lat}_{V/A}(x) - \left(\Pi^{\rm lat,free}_{V/A}(x) - \Pi^{\rm cont,free}_{V/A}(x)\right)$$

Reduction of discretization effect

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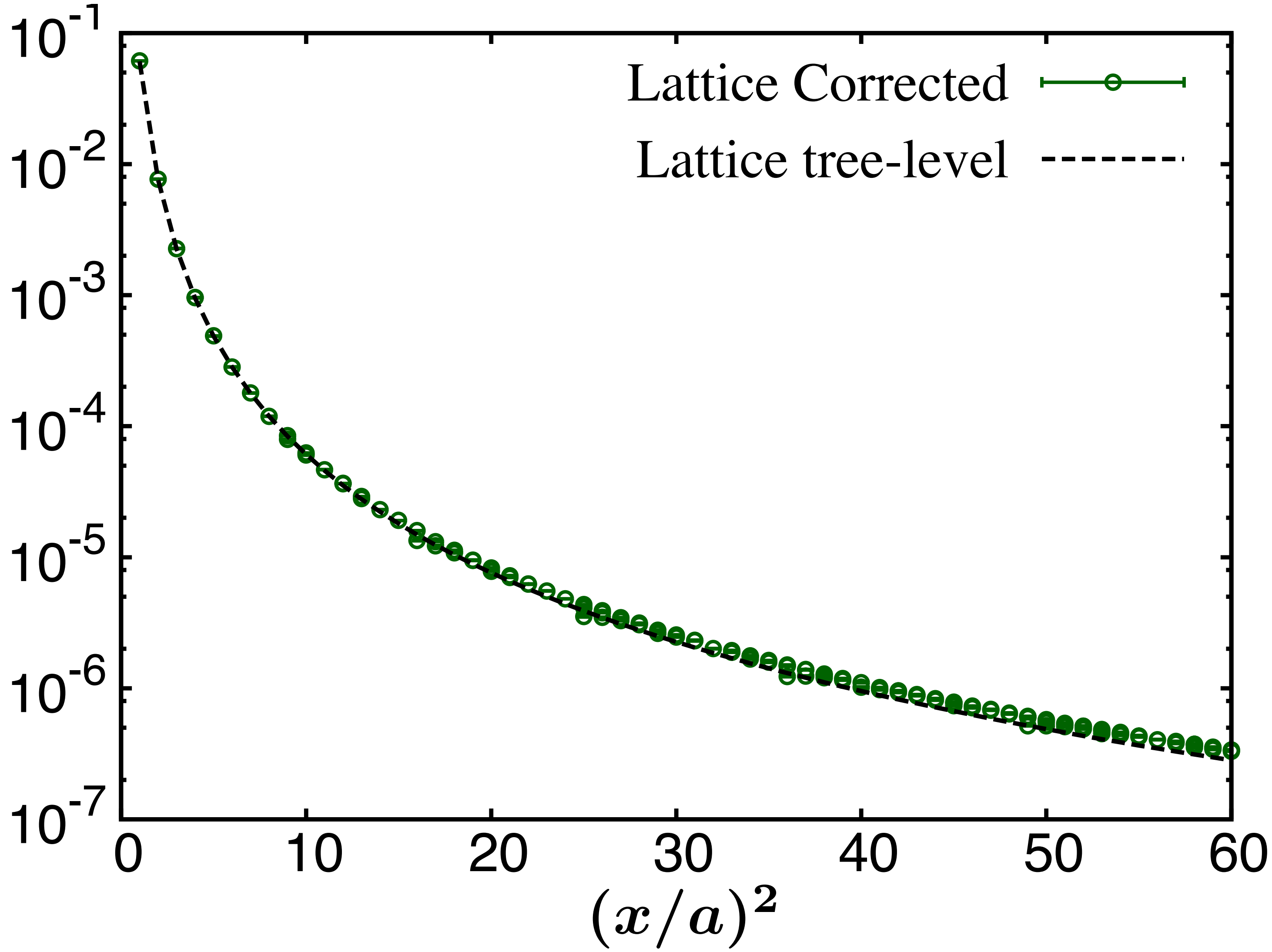
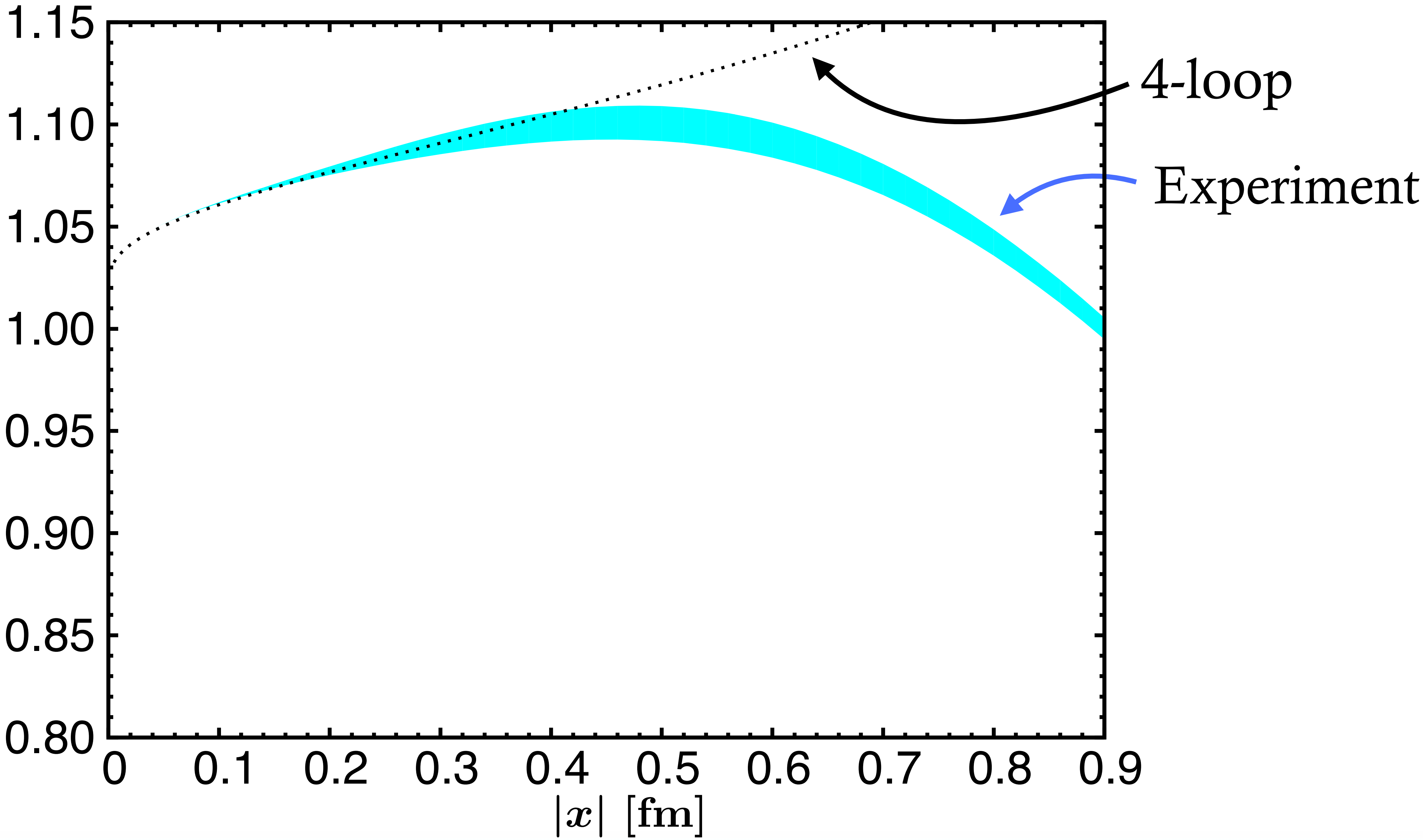


$$\Pi^{\mathrm{lat}}_{V/A}(x) \to \Pi^{\mathrm{lat}}_{V/A}(x) - \left(\Pi^{\mathrm{lat,free}}_{V/A}(x) - \Pi^{\mathrm{cont,free}}_{V/A}(x)\right)$$

V + A channel

$$R_{V+A}(x) = \frac{\Pi_V(x) + \Pi_A(x)}{2\Pi_V^{\text{free}}(x)|_{m_q=0}}$$

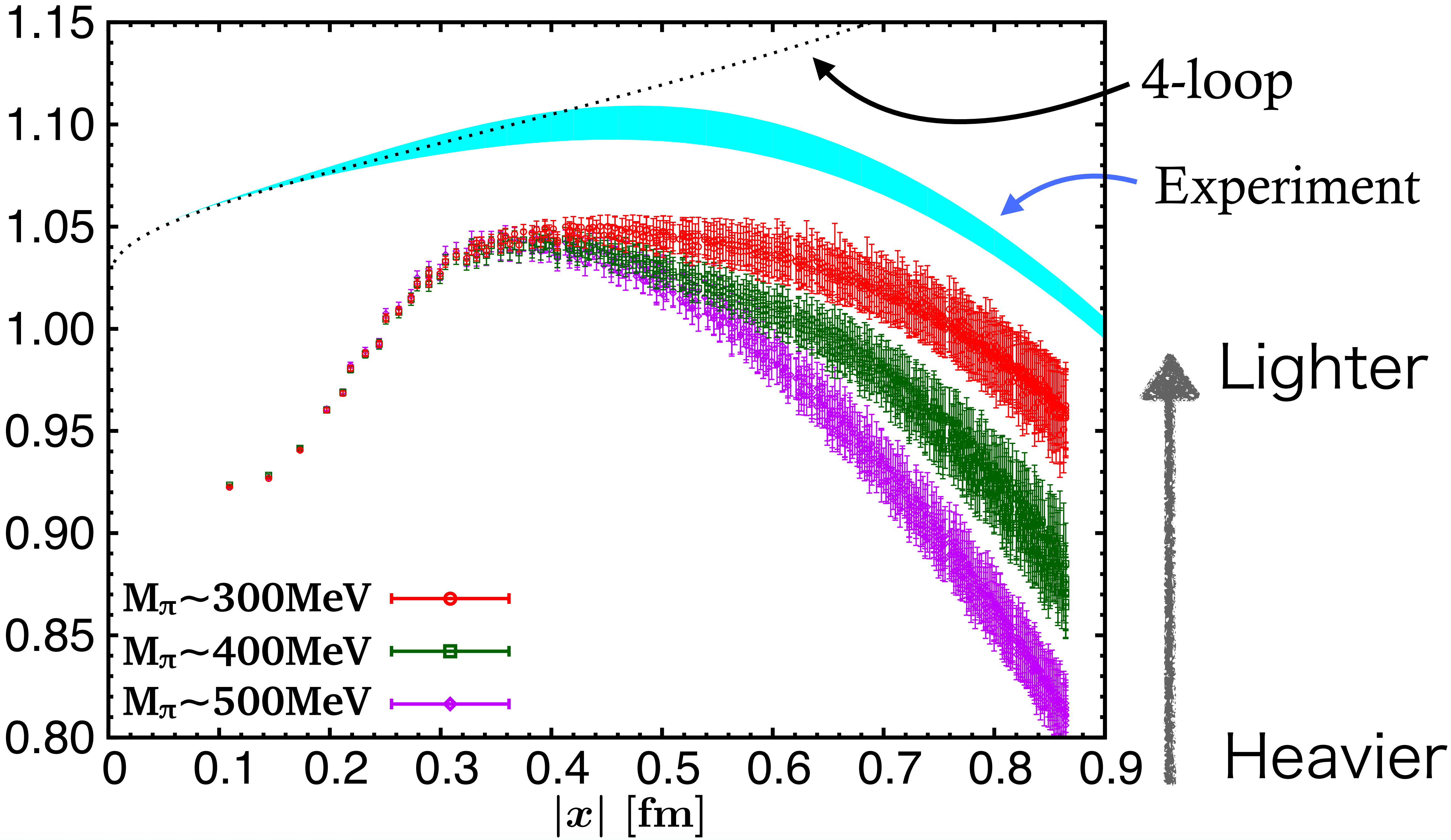


V + A channel

Dependence on light quark mass

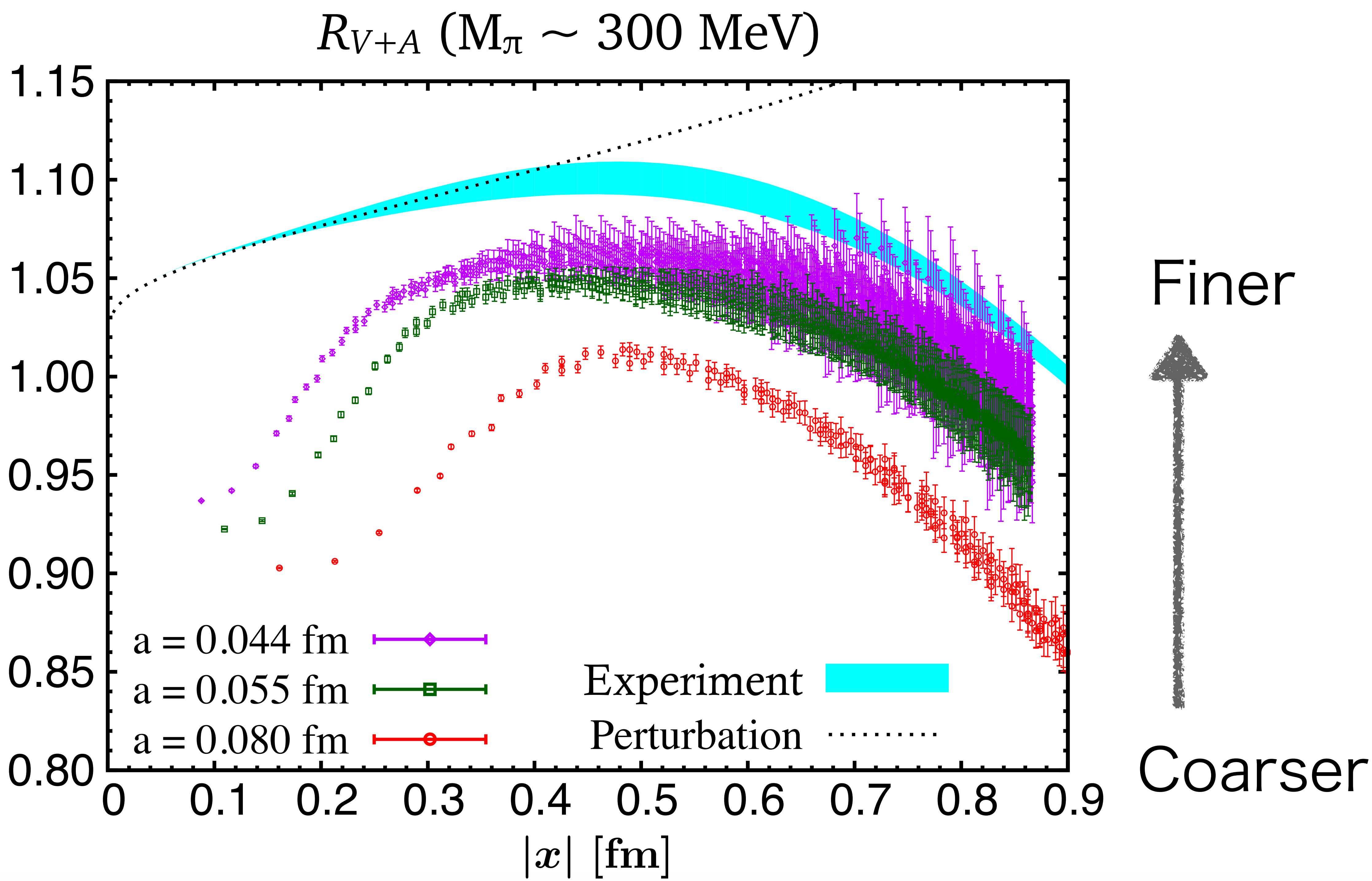
$$R_{V+A}(x) = \frac{\Pi_V(x) + \Pi_A(x)}{2\Pi_V^{\text{free}}(x)|_{m_q=0}}$$

R_{V+A} (a = 0.055 fm)



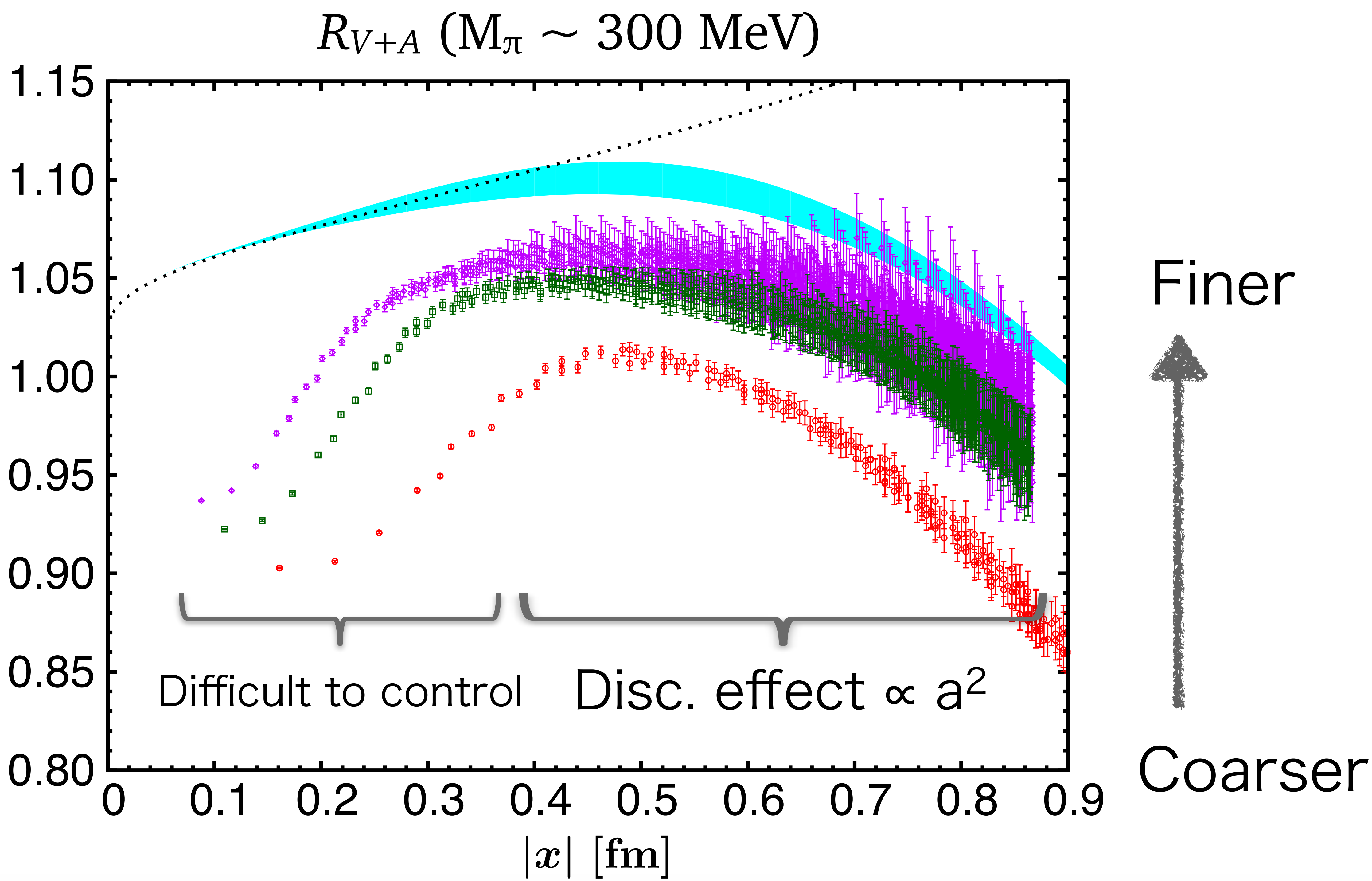
V + A channel

Dependence on *a*



V + A channel

Dependence on *a*



V + A channel

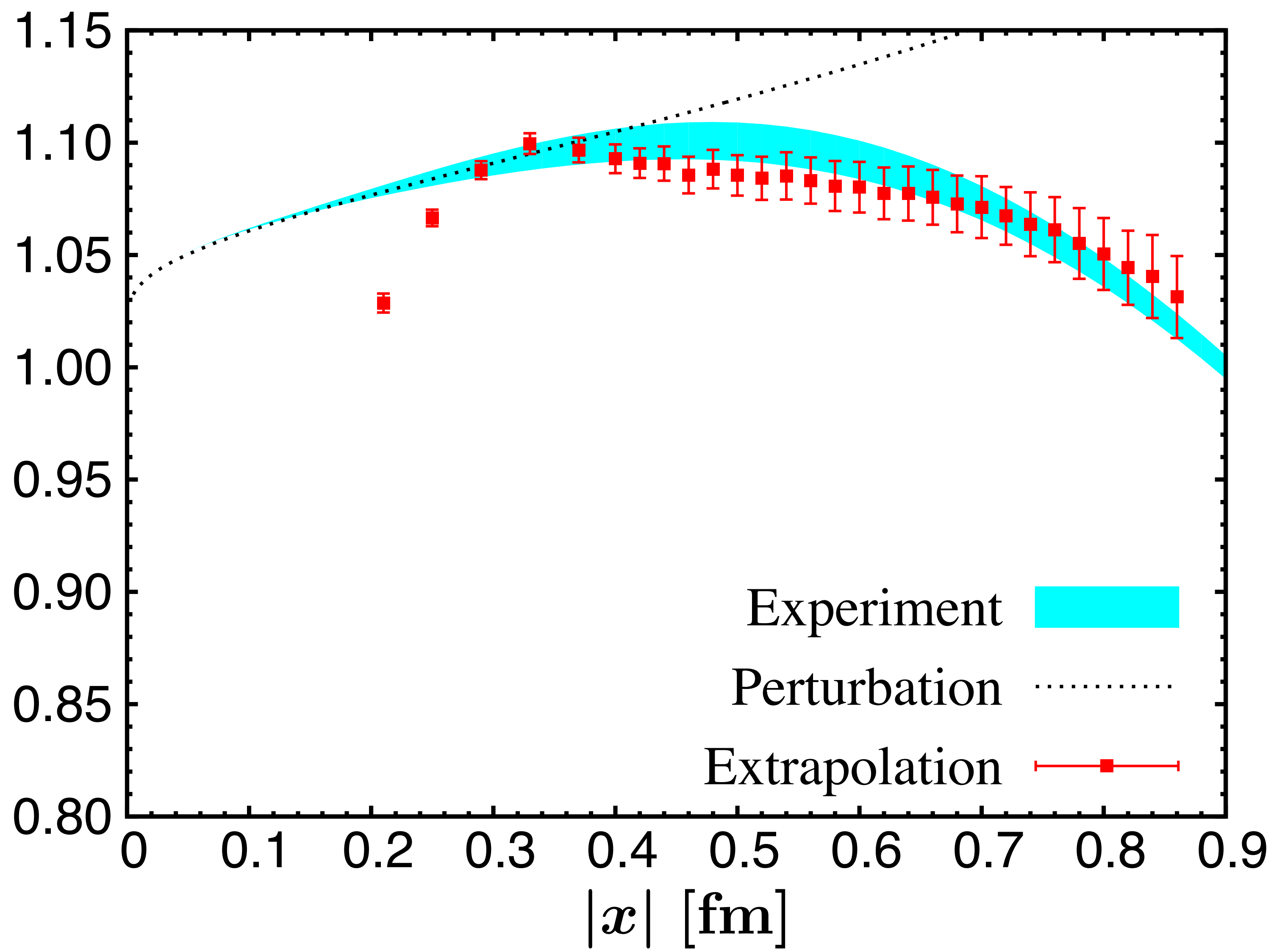
- Global fit

1. $R_{V+A}(a, M_\pi, x_c)$: average of $R_{V+A}(a, M_\pi, x)$ over $x \in [x_c - \delta x, x_c + \delta x]$

2. Chi2 fit by

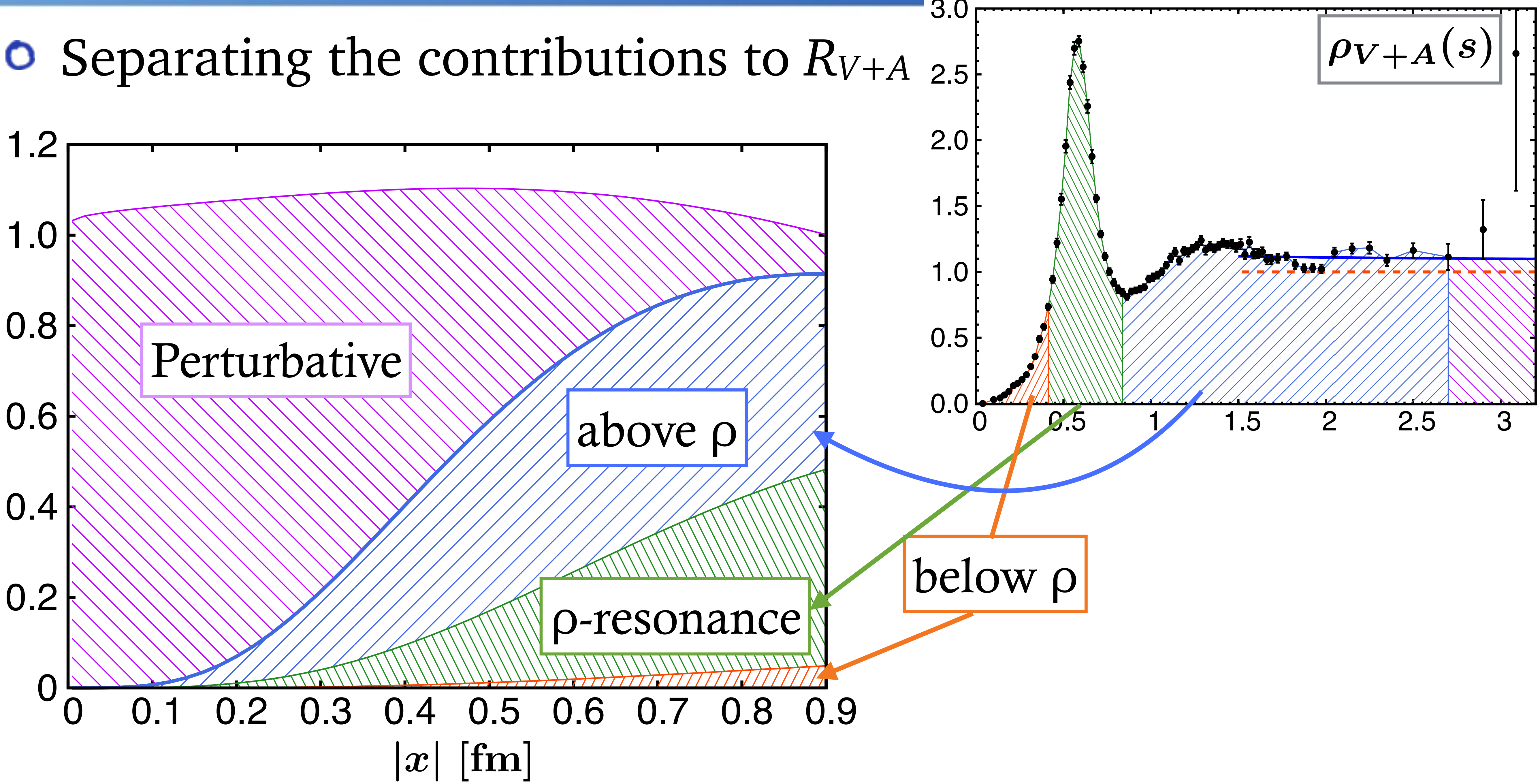
$$\begin{aligned} \boldsymbol{R_{V+A}(a, M_\pi, x_c)} & \\ &= \boldsymbol{R_{V+A}(0, m_\pi^{\text{phys}}, x_c)} \\ &\quad \boldsymbol{+C_1(M_\pi^2 - {m_\pi^{\text{phys}}}^2) + C_2 a^2} \end{aligned}$$

- Good agreement with Exp. for $x \gtrsim 0.3$ fm



Non-perturbative at 0.5 fm

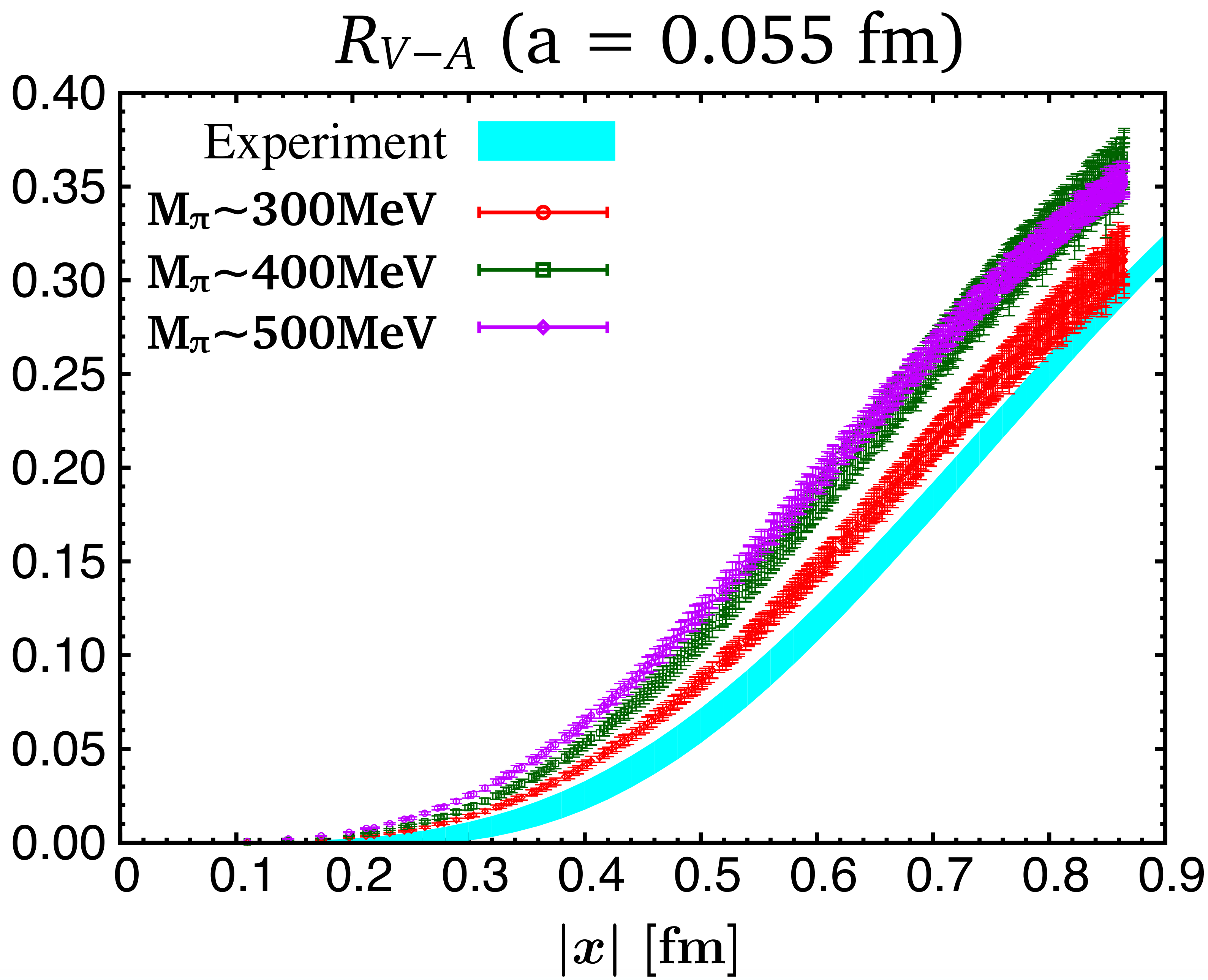
- Separating the contributions to R_{V+A}



* Non-perturbative effect is very significant already at 0.5 fm, where lattice can reproduce the Exp. data

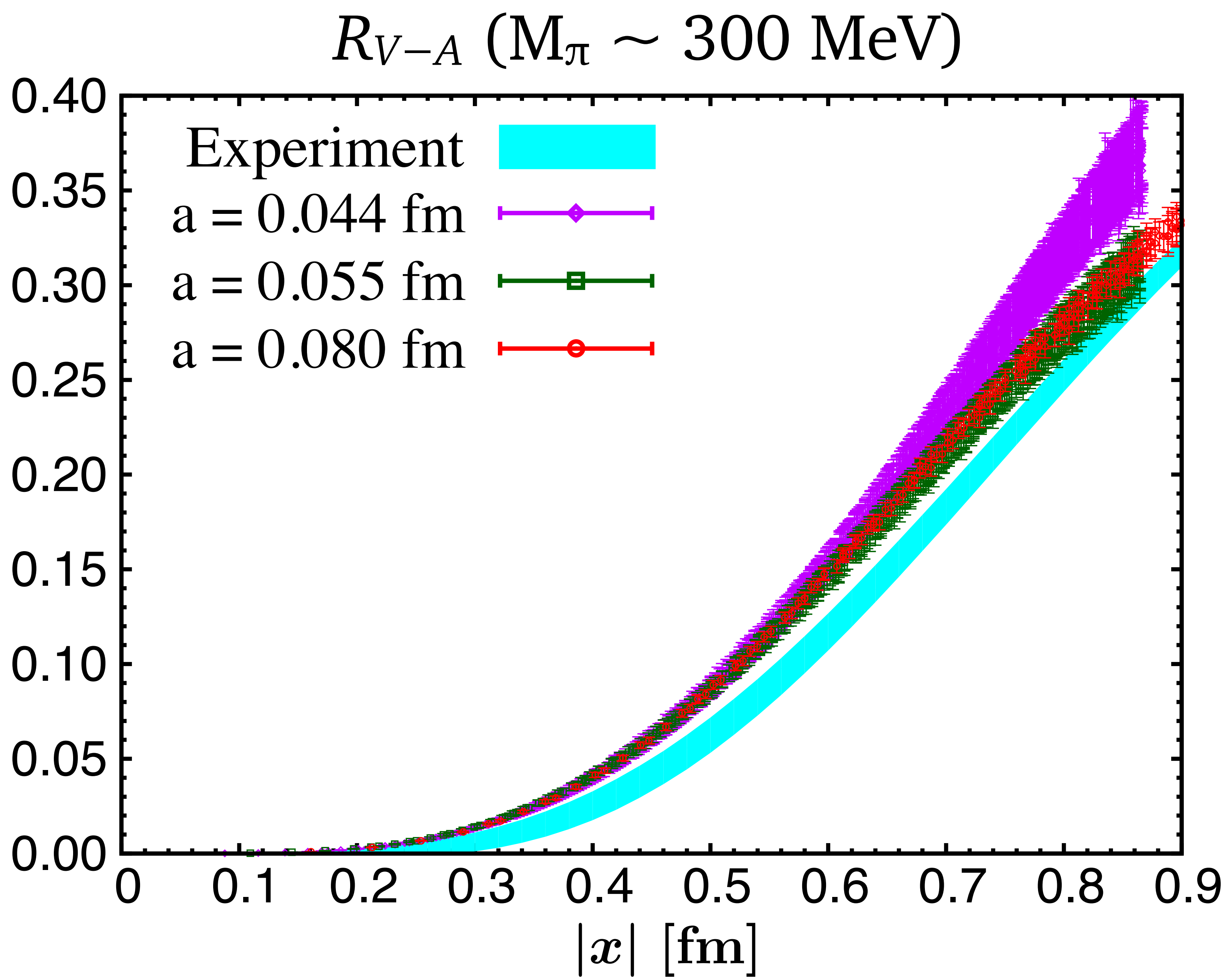
V − A channel

- Perturbatively vanish in the chiral limit
- Dependence on input mass
  - Good chiral symmetry
    → vanish near $x = 0$
  - Non-perturbative effect significant for $x > 0.2$ fm
  - Mass dependence clear at short distances



V − A channel

- Dependence on a



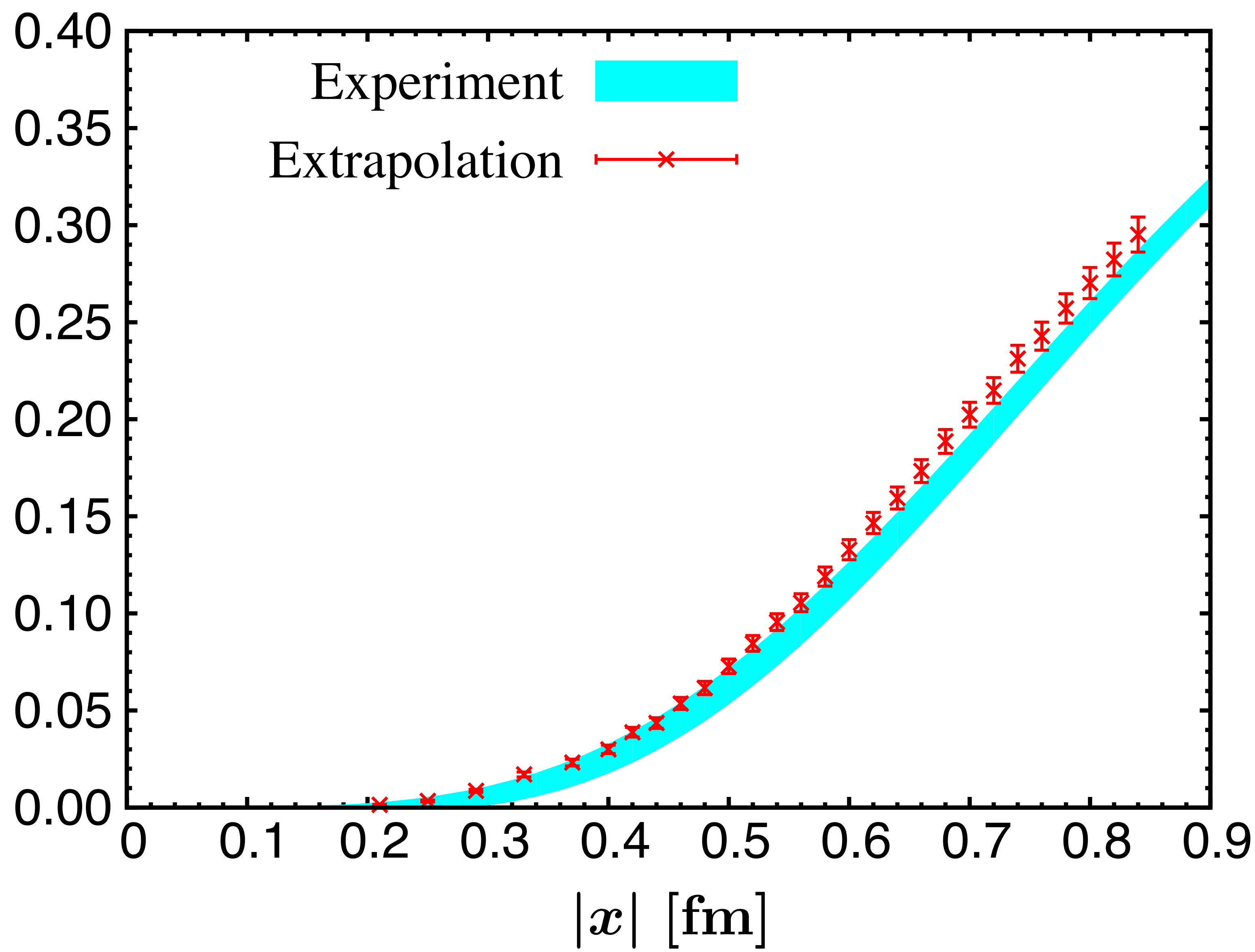
- No significant a-dependence below 0.5 fm

V − A channel

- Extrapolation

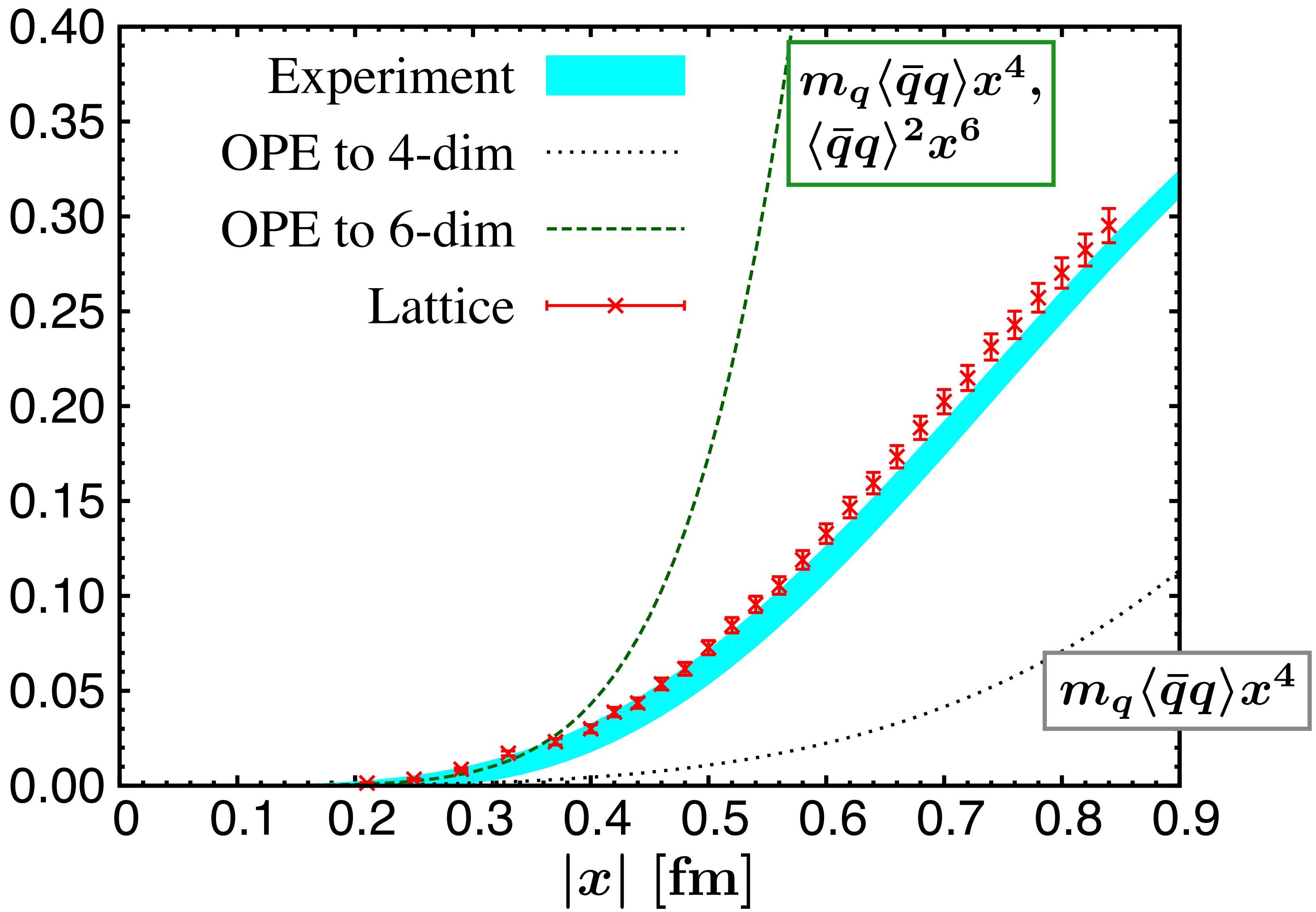
$$R_{V-A}(a, M_\pi, x_c) = R_{V-A}(0, m_\pi^{\rm phys}, x_c) + C_1(M_\pi^2 - m_\pi^{\rm phys,2}) + C_2 a^2$$

- Agreement at short distances (∼ 0.2 fm) as well as long distances



Does OPE describe Exp & Lat?

- OPE : $R^{\mathrm{OPE}}_{V-A}(x) = W_4 m_q \langle \bar{q}q \rangle x^4 + W_6 \langle \bar{q}q \rangle^2 x^6 + \cdots$ Shifman et al, 1979

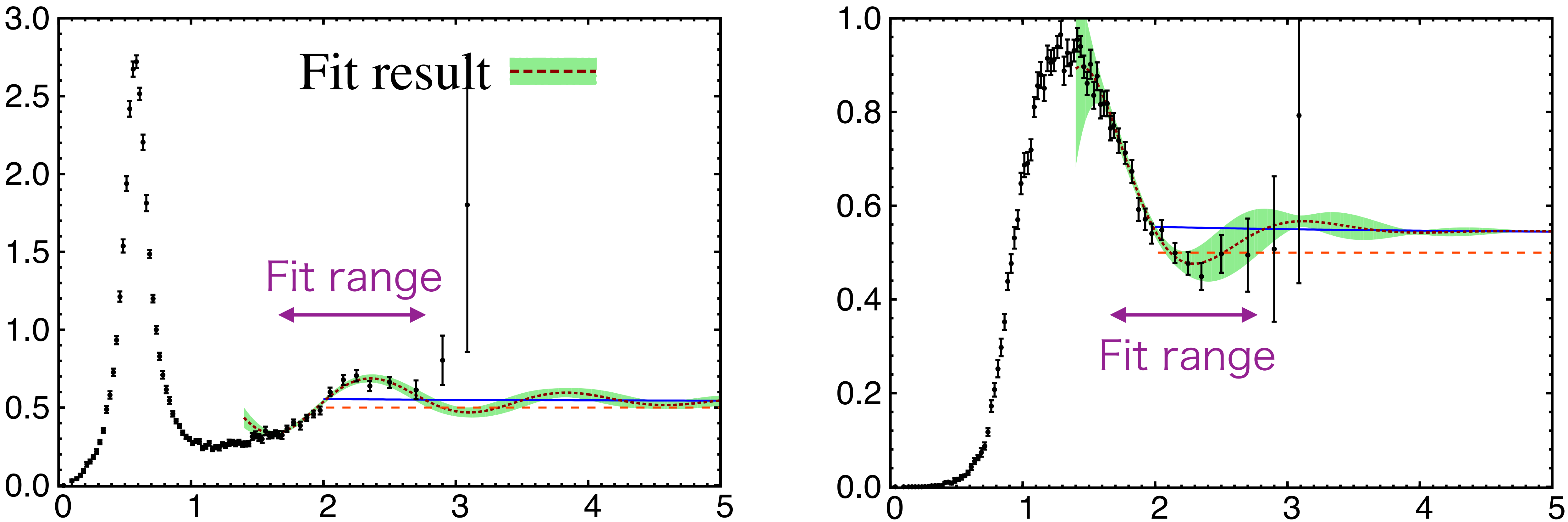


- Useful only at short distances $\lesssim 0.3$ fm
- 6-dim is significant at 0.3 fm

Spectral functions at $s > m_\tau^2$

- Experimental data present only in $s < m_\tau^2$
- Quark-Hadron duality violation (DV)
  - $\rho_{V/A}(s)$ at $s > 0$ disagrees with perturbation & OPE
  - Separation: $\rho_{V/A}(s) = \rho^{\text{pert}}_{V/A}(s) + \rho^{\text{DV}}_{V/A}(s)$
  - Resonance model (large N_C & Regge theory)

$$\rho^{\text{DV}}_{V/A}(s) = \kappa_{V/A} e^{-\gamma_{V/A} s} \sin(\alpha_{V/A} + \beta_{V/A} s)$$

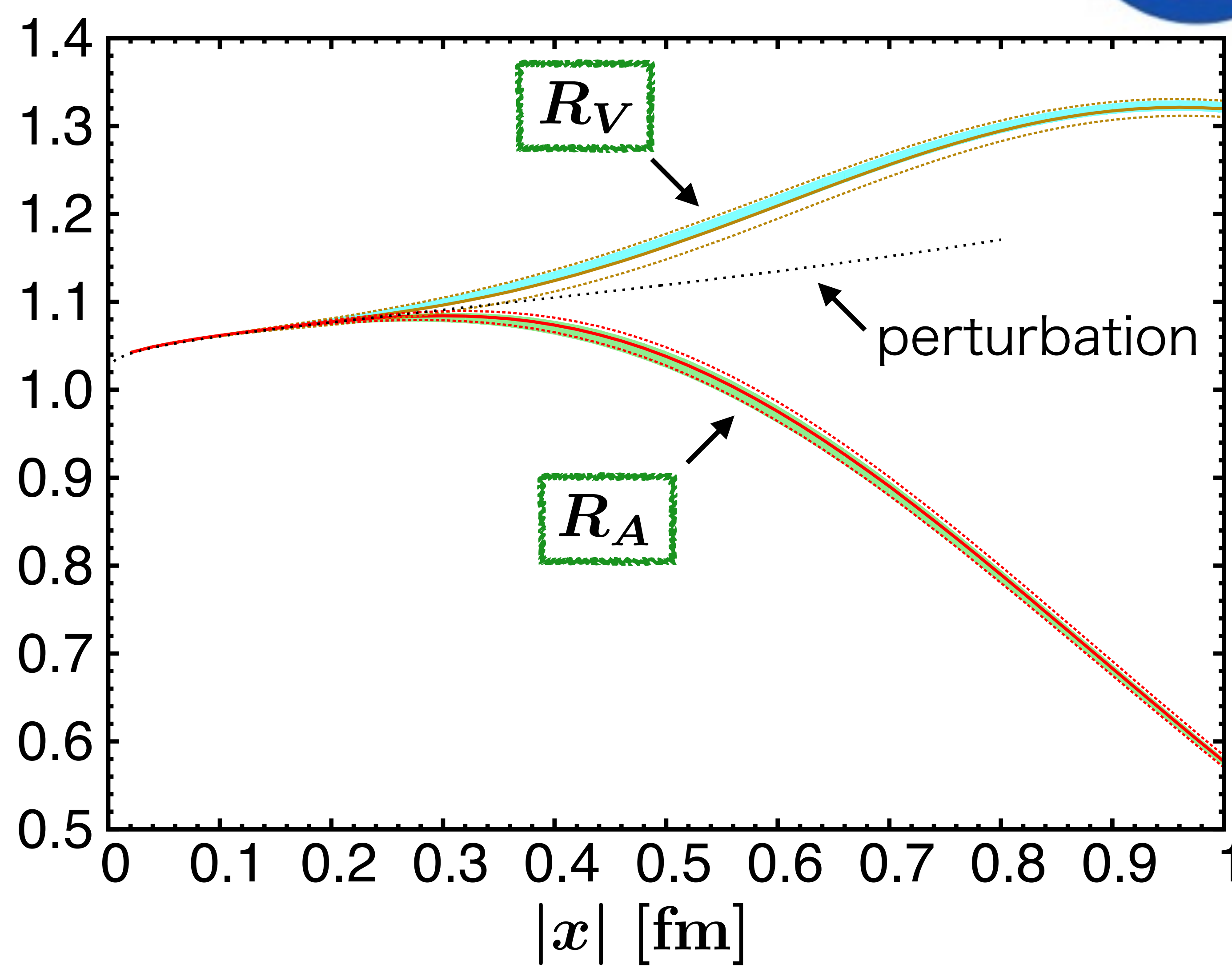


Correlators from experiments

- $R_{V/A}(x) = \dfrac{\Pi^{(1)}_{V/A}(x)}{\Pi^{\text{free}}_{V/A}(x)|_{m_q=0}}$

- key

	ALEPH+PT	ALEPH+model
R_V		
R_A		



- In Euclidean, DV is absent and 'PT' is close to 'model'
- In $|x| < 0.2$ fm, mostly perturbative
  - If lattice calculation at $|x| \sim 0.2$ fm is possible

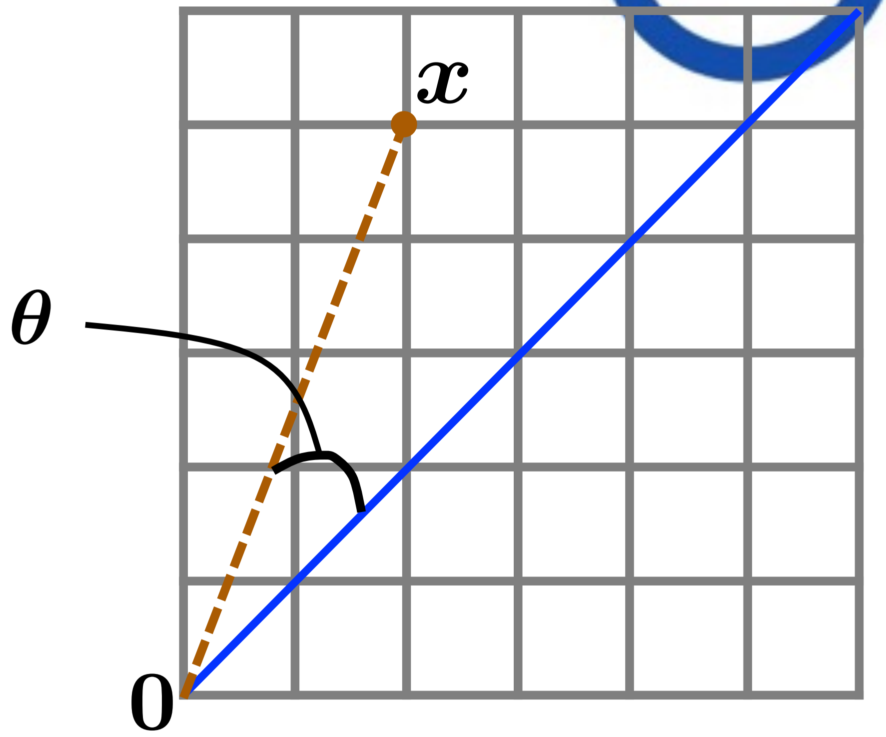
    ➡ α_s can be determined

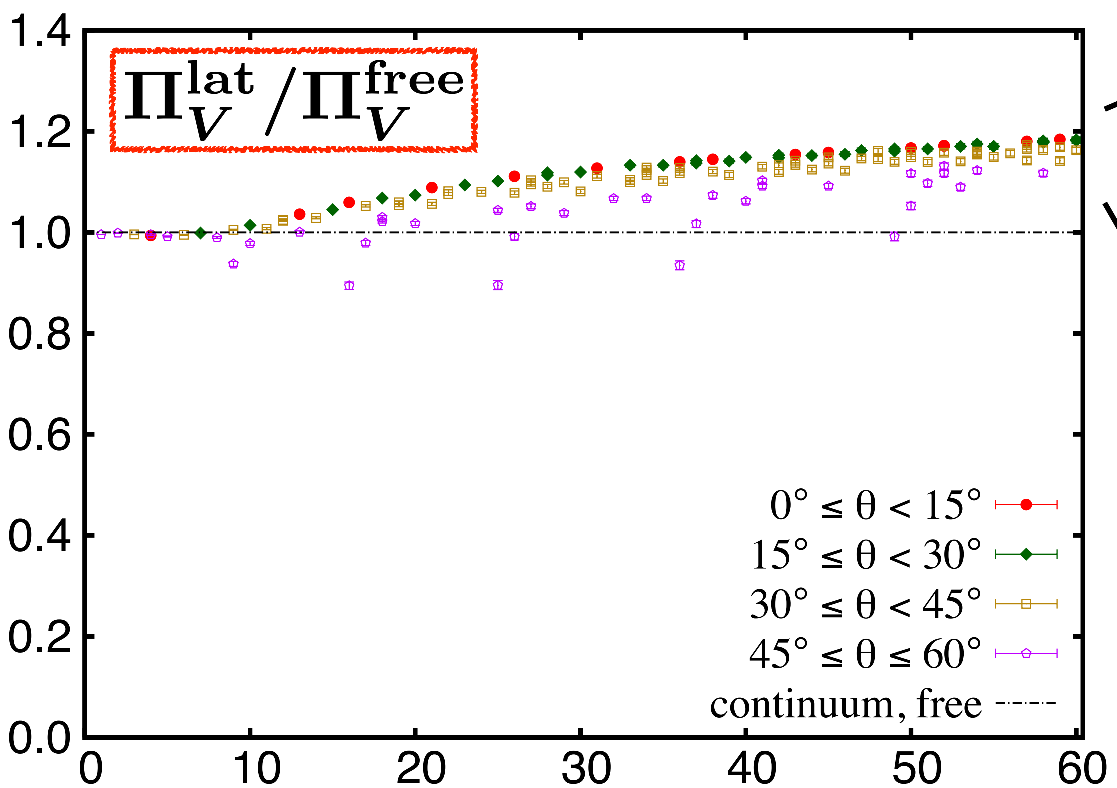
Summary

- Current correlators at short & middle distances
- Good agreement between lattice and experiment even at short distances
  - $\gtrsim$ 0.3 fm (V+A)
  - $\gtrsim$ 0.2 fm (V−A)
- Prospects
  - Determination of α_s, $<\bar{q}q>$, …
  - Continuous link between perturbative and non-perturbative dynamics

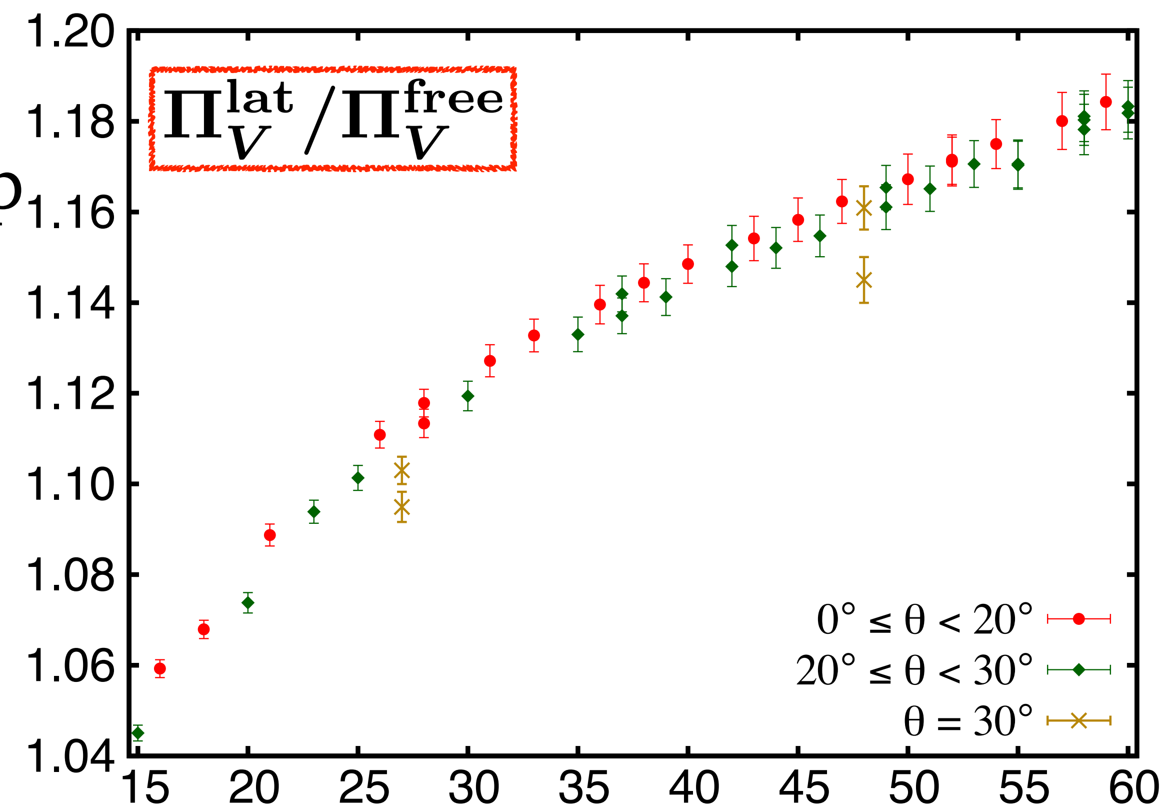
Diagonal cut

- θ : angle between *x* & (1,1,1,1)
- large θ ⇔ large discretization effect
  Chu et al, 1993; Cichy et al 2012
- Disc. eff. at 30° is already complicated
- Discard θ ≥ 30°





zoom up



Definitions & Dispersion relation

- Vector & Axial-vector correlators

$$\Pi_V(x) = \sum_\mu \langle V_\mu(x) V_\mu(0)^\dagger \rangle, \qquad \Pi_A(x) = \sum_\mu \langle A_\mu(x) A_\mu(0)^\dagger \rangle$$

- Non-singlet local operators are used

$$V_\mu(x) = \bar{\mathrm{u}}\gamma_\mu \mathrm{d}(x), \qquad A_\mu(x) = \bar{\mathrm{u}}\gamma_\mu\gamma_5 \mathrm{d}(x)$$

- Experimental correlators through dispersion relation
  - $$\Pi_{V/A}(x) \equiv \frac{3}{8\pi^4}\int_0^\infty \mathrm{d}s\; s^{3/2}\rho_{V/A}(s)\frac{K_1(\sqrt{s}|x|)}{|x|}$$
  - $\rho_{V/A}(s)$: Spectral function

    Experimentally measured

Correlators from experiments

- Momentum space

$$\widetilde{\Pi}_{V/A,\mu\nu}(q) = \int d^4x \; e^{-iqx}\Pi_{V/A,\mu\nu}(x)$$
$$= (\delta_{\mu\nu}q^2 - q_\mu q_\nu)\widetilde{\Pi}^{(1)}_{V/A}(-q^2) - q_\mu q_\nu \widetilde{\Pi}^{(0)}_{V/A}(-q^2)$$

- Dispersion relation

$$\widetilde{\Pi}^{(J)}_{V/A}(-q^2) = \pi^{-1}\int_0^\infty ds \frac{\mathrm{Im}\,\widetilde{\Pi}^{(J)}_{V/A}(s)}{(s+q^2)} - \text{subtractions}$$

$$\Rightarrow \Pi_{V/A}(x) = \frac{1}{4\pi^3}\int_0^\infty ds\; s^{3/2}(3\,\underline{\mathrm{Im}\,\widetilde{\Pi}^{(1)}_{V/A}(s)} - \mathrm{Im}\,\widetilde{\Pi}^{(0)}_{V/A}(s))\frac{K_1(\sqrt{s}|x|)}{|x|}$$

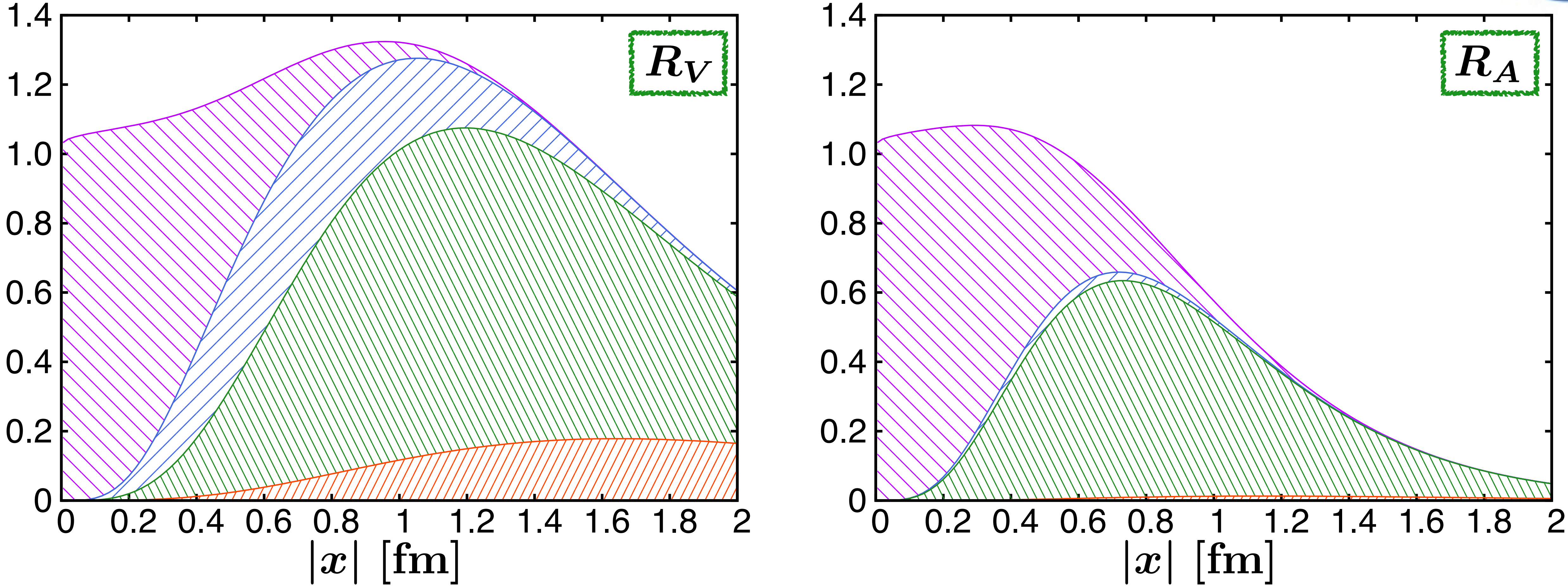
experimentally measured

K_1: modified Bessel function

- We analyze the projection to $J = 1$
  - $$\Pi^{(1)}_{V/A}(x) \equiv \frac{3}{8\pi^4}\int_0^\infty ds\; s^{3/2}\rho_{V/A}(s)\frac{K_1(\sqrt{s}|x|)}{|x|}$$
  - $\rho_{V/A}(s) \equiv 2\pi\;\mathrm{Im}\,\Pi^{(1)}_{V/A}(s)$: spectral function

Separating the region of s

- Separation of contributions of $\rho_{V/A}$ in the four region of s



: $s > 2.7$ GeV^2
: $(m_{\rho/a1} + \delta m_{\rho/a1})^2 < s < 2.7$ GeV^2
: $(m_{\rho/a1} - \delta m_{\rho/a1})^2 < s < (m_{\rho/a1} + \delta m_{\rho/a1})^2$
: $0 < s < (m_{\rho/a1} - \delta m_{\rho/a1})^2$

} perturbation (or model)

} experiments